Parameterized Complexity and Fixed-Parameter Tractability of Description Logic Reasoning

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An important goal of research in description logics (DLs) and related logic-based KR formalisms is to identify the worst-case complexity of reasoning. Such results, however, measure the complexity of a logic *as a whole*. For example, reasoning in the basic DL $\mathcal{ALCI}$ is ExpTime-complete, which means that $\mathcal{ALCI}$ constructors can be used in a way so that exponential time is strictly required for solving a reasoning problem. It is, however, well known that, given two $\mathcal{ALCI}$ knowledge bases of roughly the same size, reasoning with one knowledge base may be much more difficult than with the other, depending on the interaction of the axioms in the KBs. Thus, existing worst-case complexity results provide only a very coarse measure of reasoning complexity, and they do not tell us much about the "hardness" of each individual knowledge base.

Parameterized complexity [2] provides us with a framework for a more fine-grained analysis of the difficulty of reasoning. The general idea is to measure the "hardness" of a problem instance of size n using a nonnegative integer *parameter* k, and the goal is to solve the problem in time that becomes polynomial in n whenever k is fixed. A particular goal is to identify *fixed parameter tractable* (FPT) problems, which can be solved in time $f(k) \cdot n^c$, where c is a constant and f is an arbitrary computable function that depends *only* on k.

Each problem is clearly in FPT if the parameter is the problem's size, so a useful parameterization should allow increasing the size arbitrarily while keeping the parameter bounded. Various problems in AI were successfully parameterized using the graph-theoretic notions of *tree decompositions* and *treewidth* [3–5], and many FPT results have been obtained using the Courcelle's Theorem [1]. Applying these ideas to formalisms such as description and modal logics seems difficult: due to existential and universal quantifiers, solving a reasoning problem may require exploring very large structures.

In my talk I will present an overview of parameterized complexity, fixed-parameter tractability, and treewidth, and I will briefly discuss how these notions can be used to obtain FPT results for formalisms such as propositional logic and answer set programming. Furthermore, I will discuss the difficulties in applying these ideas to logics with existential quantifiers, such as DLs. I will then present a particular parameterization for DL knowledge bases. This result is based on a novel notion of a *decomposition*—a structure inspired by tree decompositions, but extended in a way that captures the effects of quantifiers. I will also discuss

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a fundamental tradeoff between decomposition *width* and *length*—the two parameters that characterize the difficulty of DL reasoning. Finally, I will present what I believe to be the first result FPT result for DL reasoning.

References

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