
Application of Genetic-Based Wavelet Packet Watermarking for Copyright Protection

Hsiang-Cheh Huang¹ and Yueh-Hong Chen²

¹ National University of Kaohsiung,
Kaohsiung 811, Taiwan, R.O.C.
huang.hc@gmail.com
<https://sites.google.com/site/hch888dr/>

² Far East University,
Tainan 744, Taiwan, R.O.C.
yuehhong@gmail.com

Summary. Copyright protection is mostly required for lots of applications, and watermarking is a scheme for making this possible. In this chapter, we propose a genetic watermarking scheme based on the wavelet packet transform (WPT) for the applications of copyright protection and trusted communication. Wavelet packet transform can be regarded as a generalization of the discrete wavelet transform (DWT). For the WPT, a best wavelet basis in the sense of evaluation metric can be found within a large library of permissible bases. In addition, genetic algorithm (GA) is employed to select an appropriate basis from permissible bases of wavelet packet transform to meet the requirements of our watermarking algorithm. After the training with GA, the secret information corresponding to copyright owners can be embedded into original images, and the watermarked image is obtained. During transmission of watermarked images, intentional signal processing is expected, and our watermarking algorithm is considered to keep the integrity for the trusted communication. Experimental results demonstrate that the proposed method can increase the capability to resist intentional signal processing for the use of GA. Moreover, the watermarked image quality can be guaranteed based on the inherent characteristics of the watermarking algorithm. Better performances can also be observed over existing methods in literature, and our algorithm is suitable for the application of copyright protection and trusted communication.

7.1 Introduction

Multimedia contents can be easily accessed in our daily lives. For instance, people can capture images or video with digital cameras or mobile phone cameras at any time. Besides, with the development of computer industry and the widespread use of the Internet and wireless networks, a lot of digital media, including image, audio, and video, have been copied, stolen, and altered by anyone easily and unlimitedly. Because these digital media could be acquired quickly and easily over the Internet, how to provide the necessity for copyright protection and trusted communication has

become a more and more important task for both academic researches and industrial applications [1]. Under this scenario, digital watermarking has been regarded as an effective solution [2, 3].

Digital watermarking is a scheme to put the secret information into the multimedia contents. With the aim of copyright protection for watermarking, the secret information, relating to the copyright owners, would be hidden into the digital contents. After embedding the secret information with algorithms designed by researchers, the watermarked contents would be delivered to the receiver. During the delivery, some signal processing, called attacks, might apply to the watermarked contents, and this would make copyright protection unavailable. After reception, people are expected to extract the secret information from the marked contents after attacks. Owner's copyright can be retained once the extracted information can be recognizable, or the correlation between the embedded and extracted information lie above some confidence level.

From practical point of view, taking the characteristics of image contents for instance, by use of altering the pixel values in the spatial domain [4, 5], the codewords in the vector quantization (VQ) domain [6], and the coefficients in the frequency domain, including the discrete cosine transform (DCT) [7, 8] and the discrete wavelet transform (DWT) [9], we observe that these commonly employed techniques could lead watermarking a possible means for copyright protection and trusted communication in literature.

Because digital images are the most commonly acquired media on the web, in this chapter, we focus the multimedia contents on the digital images. Image watermarking is to embed the watermark (or the secret information) into images for copyright protection and trusted communication. The watermarked content needs to be as resemble as its original counterpart as possible. Also, secret information should be extracted at the decoder, which should lie above the threshold of the confidence level to retain the capability for copyright protection. As discussed in literature, a good image watermarking scheme would be expected to have the following characteristics [2, 10]:

1. *High fidelity*: A watermarked image should be perceptually identical or very similar to its original counterpart. The Peak Signal-to-Noise Ratio (PSNR) is generally considered as the representation of fidelity between the original image X , and watermarked one, X' . For image size of $M \times N$, the PSNR can be calculated by

$$\text{PSNR} = 10 \times \log_{10} \left(\frac{255^2}{\frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N (X(i, j) - X'(i, j))^2} \right). \quad (7.1)$$

Larger PSNR values imply less distortions induced, hence the better outcomes can be reached.

2. *Good robustness*: The watermark can be successfully detected or extracted based on the application of algorithm, even though some intentional attacks, or image processing operations, had been applied on the watermarked image. People often use the bit correct rate (BCR), with the definition in Eq. (7.2), to assess

the robustness of the watermarking algorithm. The normalized cross-correlation (NC) between the embedded watermark and the extracted one may also be considered to assess the survivability for the extracted watermark. Let the embedded and extracted watermarks be W and W' , respectively, with length of W_N . With its ease of calculation, we choose the BCR values to measure the robustness of extracted watermarks. The BCR is

$$\text{BCR} = \left(1 - \frac{1}{W_N} \sum_{i=1}^{W_N} [W(i) \oplus W'(i)] \right) \times 100\%, \quad (7.2)$$

where \oplus denotes the exclusive-or (XOR) operation. The larger the BCR value, the better the result for copyright protection.

3. *Large data capacity*: The larger the data capacity is, the more secret information can be embedded into original images. On the other hand, if error control codes (ECC) is employed, enhanced performance of extracted information can be increased with the expense of the added redundancy. Larger capacity can reside the produced redundancy, and provide the flexibility for the selection of error control codes.

We can easily see that for obtaining high fidelity, as less alteration to the original image should be performed as possible. Intuitively, in the spatial domain, the secret information can be embedded to the least significant bits, while in the frequency domain, it should be hidden into higher frequency bands due to the energy distribution of frequency coefficients. On the contrary, the commonly employed attacks, JPEG for instance, tend to remove the data at higher frequency bands, causing the secret information vanished. From the first two requirements, there are conflicts between each other. Furthermore, comparing between the first and the third requirements, if more capacity is embedded, more alteration to the original content should be performed, leading to the degradation of output image quality. Following this induction, because conflicts between each other can be easily observed, some compromises among the requirements need to be reached. Therefore, designing an optimal watermarking scheme to meet all the requirements has been a difficult and interesting problem.

Embedding watermarks into images can be referred to as a constrained optimization problem by taking the three requirements mentioned above into consideration. Hence, genetic algorithm (GA) can possibly be used to solving this kind of problem [10]. A detail explanation of GA can be found in [11]. Different from typical watermarking schemes, conventional genetic watermarking looks for the optimal coefficients in frequency domain in the sense of a certain evaluation function to embed watermarks. Under the constraint of keeping these requirements acceptable, and to simplify the optimization problem to some extent, GA can be employed to optimize the robustness of watermarks, the fidelity of watermarked images, or both, under the assumption of a fixed number of capacity for embedding [7]. The maximized PSNR and BCR values corresponding to the optimized combinations of embedding bands can be obtained with GA.

A number of methods have been proposed to embed robust and invisible watermarks. Some operate directly in the spatial domain [5], others in the transform domain, such as Fourier [12], DCT [13], or wavelet domains [14]. Among all methods, wavelet packet transform (WPT) has attracted much attention [15, 16, 17, 18]. WPT can be regarded as a generalization of the discrete wavelet transform. In the usual dyadic wavelet decomposition, as depicted in Fig. 7.1(a), only the low-pass-filtered subband is recursively decomposed and it thus can be represented by a logarithmic tree structure. However, a wavelet packet decomposition, shown in Fig. 7.1(b), allows the process to be represented by any pruned subtree of the full tree topology. Therefore, this representation of the decomposition topology is isomorphic to all permissible subband topologies. The leaf nodes of each pruned subtree represent one permissible orthonormal basis [17]. Thus, a best wavelet basis in the sense of evaluation metric can be found within a large library of permissible bases in WPT [18, 19].

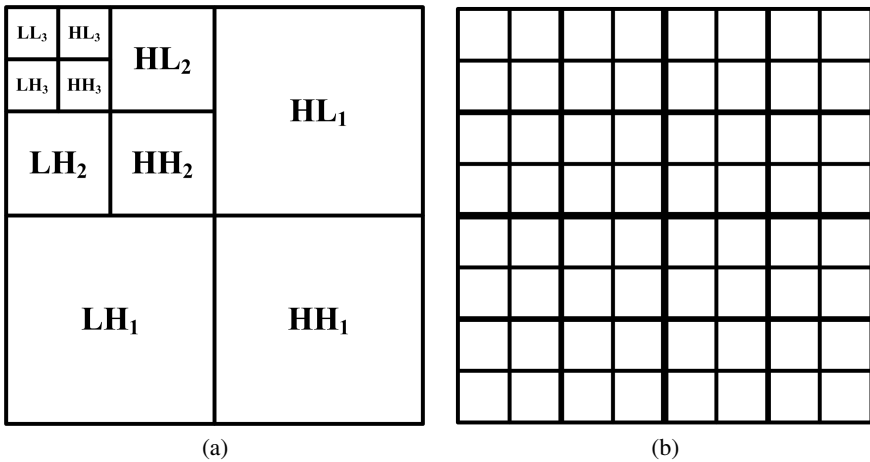


Fig. 7.1. Comparisons of frequency domain representations between (a) the three-level decomposition of DWT, and (b) the fully decomposed three-level WPT.

In [20], authors indicated that the watermarking scheme employing the zerotree of WPT provided the best performance in terms of Peak Signal-to-Noise Ratio in comparison with schemes employing DWT and DCT. On the contrary, the robustness against various types of attacks of the scheme employing WPT was somewhat decreased. Therefore, we propose a genetic watermarking scheme based on the wavelet packet transform in this chapter in order to obtain both the better image quality and the more resistance to intentional attacks. Better image quality can be expected with the characteristics of the proposed embedding algorithm in Sec. 7.2, and the more robustness for extracted watermark can be obtained with the training in GA. In the proposed method, GA is employed to select an appropriate basis from permissible bases of WPT to increase the robustness of the embedded watermarks.

Experimental results demonstrate that proposed method can increase the capability to resist image processing methods if an appropriate fitness function of GA is adopted.

7.2 Genetic Watermarking in Wavelet Packet Transform

As people know, the goals for DWT and WPT are to get the minimal representation of data relative to particular fitness function. Wavelet transform is applied to low-pass results recursively. In Fig. 7.1(a), it represents the three-level decomposition of DWT. Only the low-pass bands are supposed to be further decomposed. On the contrary, for WPT, both the low-pass and high-pass bands can be selected for further decomposition. Figure 7.1(b) denotes the three-level decomposition of WPT when all the bands are fully decomposed. With our algorithm, we are going to hide our secret message into the selected bands in WPT.

Assuming that the watermark for embedding consists of 0's and 1's, and all bits in the watermark are embedded into an image with the same manner accordingly. To embed the watermark, the genetic algorithm is first used to select an appropriate basis of WPT, and then a number of coefficients are randomly chosen and modified. The random seed and the WPT decomposition tree for watermark embedding play the roles as secret keys. Genetic algorithm for basis selection is described in Sec. 7.2.1 and the method for embedding watermarks into images is introduced in Sec. 7.2.2, respectively. Application aspects and limitations with our algorithm are briefly addressed in Sec. 7.2.3.

7.2.1 Best Basis Selection with GA

Conventional search techniques are often incapable of optimizing non-linear functions with multiple variables. One scheme called the “genetic algorithm”, based on the concept of natural genetics, is a directed random search technique. The exceptional contribution of this method was developed by Holland [21] over the course of 1960s and 1970s, and finally popularized by Goldberg [11].

In genetic algorithms, the parameters are represented by an encoded binary string, called the “chromosome.” And the elements in the binary strings, or the “genes,” are systematically adjusted to minimize or maximize the fitness value. The fitness function generates its fitness value, which is composed of multiple variables to be optimized by GA. For every training iteration, a pre-determined number of individuals would correspondingly produce fitness values associated with the chromosomes. Figure 7.2 demonstrates the flow chart for a typical binary GA. It begins by defining the optimization parameters, the fitness function, and the fitness value, and it ends by testing for convergence. According to the applications for optimization, designers need to carefully define the necessary elements for training with GA. Next, we are able to evaluate the fitness function in addition to the terminating criteria with the natural selection, crossover, and mutation operations in a reasonable way. With the aid of Fig. 7.2, the core components are depicted as follows.

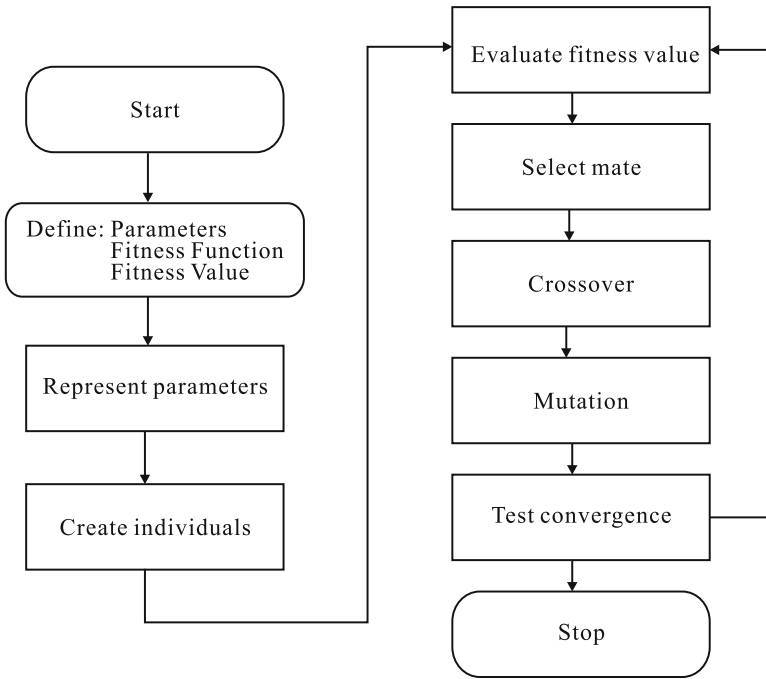


Fig. 7.2. The flow chat of genetic algorithms

1. *Mate selection.* A large portion of the low fitness individuals is discarded through this natural selection step. Of the N individuals in one iteration, only the top N_{good} individuals survive for mating, and the bottom $N_{\text{bad}} = N - N_{\text{good}}$ ones are discarded to make room for the new offspring in the next iteration. Therefore, the selection rate is $p_s = \frac{N_{\text{good}}}{N}$.
2. *Crossover.* Crossover is the first way that GA explores a fitness surface. Two individuals are chosen from N_{good} individuals to produce two new offsprings. A crossover point is selected between the first and last chromosomes of the parents' individuals. Then, the fractions of each individual after the crossover point are exchanged, and two new offspring are produced.
3. *Mutation.* Mutation is the second way that GA explores a fitness surface. It introduces traits not in the original individuals, and keeps GA from converging too fast. The pre-determined mutation rate, p_m , should be low. It is accomplished by intentionally flipping the value of selected bits based on the mutation rate. Most mutations deteriorate the fitness of an individual, however, the occasional improvement of the fitness adds diversity and strengthens the individual.

Chromosomes of the genetic algorithm used in the proposed scheme consist of a binary string. Meanwhile, referring to Fig. 7.1(b), the length of the binary string, L , is equal to

$$\begin{aligned}
L &= 1 + 4 + 4^2 + \dots + 4^{D-1} \\
&= \frac{(1 - 4^D)}{(1 - 4)} \\
&= \frac{(4^D - 1)}{3},
\end{aligned} \tag{7.3}$$

where D is decomposition level of WPT. Bit ‘1’ in the string indicates that the corresponding subband should be decomposed further, and bit ‘0’ denotes the termination of subband decomposition. If the value at index i is equal to ‘1’, the indices of the resulting four subbands, i_m , can be derived by Eq. (7.4):

$$i_m = 4 \times i + m, \quad m \in \{1, 2, 3, 4\}. \tag{7.4}$$

An example of the chromosome and its corresponding WPT decomposition tree is shown in Fig. 7.3. Suppose that at most two levels are decomposed, shown in Fig. 7.3(a). $D = 2$, meaning that a chromosome is composed of $\frac{4^2-1}{3} = 5$ bits. At the first level, if bit 0 in Fig. 7.3(b) is ‘1’, meaning that further decomposition is needed, and the values of remaining four bits are determined accordingly.

The fitness function in the proposed scheme is described as follows:

$$\text{fitness value} = \prod_{i=1}^K P_i, \tag{7.5}$$

where P_i denotes the percentage of the watermark bits that still survive after applying certain attacks, or the BCR in Eq. (7.2), and K means the number of attack method adopted for robustness evaluation. From Eq. (7.5), we can see that since $P_i \in [0, 1]$, the maximum of the possible fitness value would be 1.0. By doing so, all the expected attacks can be conquered because we choose the product of all extracted BCR values for optimization, and each value of P_i needs to be as large as possible. If the BCR values of extracted watermarks after experiencing some of the attacks get too low, such a kind of watermark embedding fails to exist with the fitness function in our training iterations. Thus, the goal for GA optimization is to search for the maximum of the fitness in Eq. (7.5). Because the watermarked image quality would lead to the optimized PSNR value by using the proposed algorithm, which will be discussed in Sec. 7.2.2, only the robustness of the watermarking algorithm is taken into consideration in Eq. (7.5).

7.2.2 Adaptive Watermarking in Wavelet Packet Transform

The method for embedding one binary value into an image is the extension to the method in [22]. When one bit of the watermark is embedded, a pre-specified number of coefficients are chosen randomly. These coefficients are then modified such that the first one, in the order of being chosen, is the largest if a ‘1’ is embedded. If a ‘0’ is embedded, the coefficients should be modified such that the first one is the smallest.

Due to its simplicity in implementation, authors are suggested to refer to [22] in watermark embedding and extraction for more details.

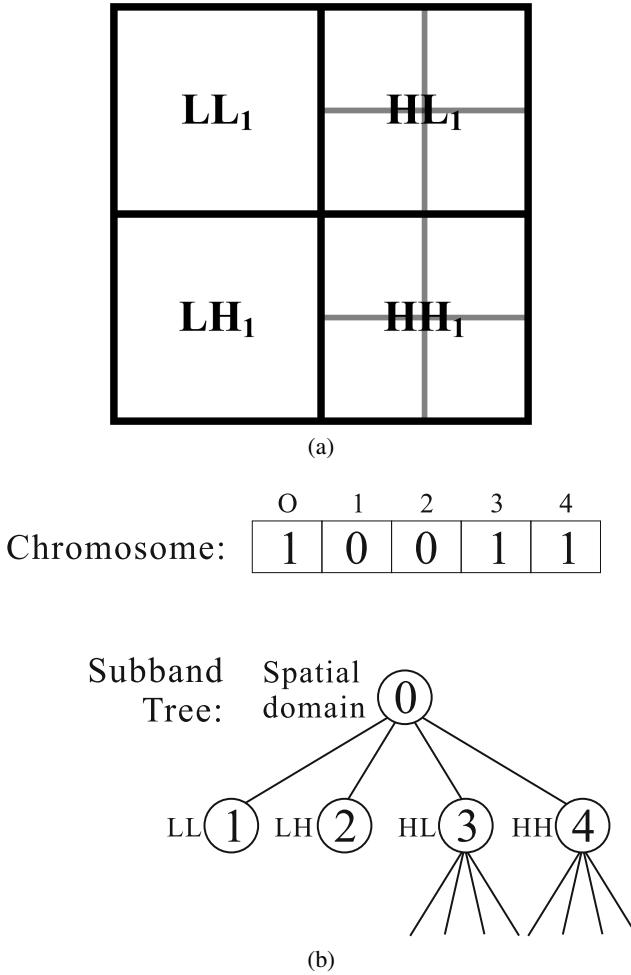


Fig. 7.3. An example for our algorithm. (a) The representation of decomposed subbands in WPT. (b) The binary representation of chromosome and its corresponding subband tree.

7.2.3 Limitations and Flexibilities with the Proposed Method

Our method aims at obtaining both the better image quality of watermarked image, and the enhanced result in watermarking robustness. For achieving the goals, we employ GA for selecting suitable bands with WPT. For training with GA, it takes some time to calculate the results in each training iteration, and hence our method may have limitations for real-time processing. This issue is addressed in Sec. 7.3 shortly. For the offline applications such as archival of multimedia with trusted communication, our method may work well.

The proposed method also offers flexibilities for users to obtain the optimized result. For instance, the levels of decomposition, the compositions of elements in

the fitness function, or the crossover and mutation rates in GA, are flexible to meet the users' needs.

7.3 Experimental Results

In this section, we present some experimental results to demonstrate the performance of the proposed method from three aspects:

1. the output image quality of the watermarked image,
2. the robustness of watermarking algorithm after selected attacks, and
3. the time consumption for completing the training procedures.

Two 512×512 , 8 bit/pixel gray scale images, Lena and F16, are employed as the test images. A watermark, 1000-bit in length, is the randomly generated bitstream. The number of chromosomes is 200, and the maximal decomposition level is set to $D = 8$, meaning that the length of the chromosome is $\frac{4^8-1}{3} = 21845$ bits. Additionally, selection and mutation rates are set to $p_s = 0.5$ and $p_m = 0.04$, respectively, and the number of training iteration is set to 450. Three image processing operations, including JPEG compression with quality factor = 30, Gaussian filtering, and sharpening, are applied on the watermarked images to evaluate fitness value of the chromosomes. The mask for 3×3 Gaussian filtering can be addressed with Eq. (7.6):

$$\begin{bmatrix} \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \\ \frac{2}{16} & \frac{4}{16} & \frac{2}{16} \\ \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \end{bmatrix}. \quad (7.6)$$

In comparison with [22], the watermark could only be embedded into subbands decomposed from *LH2*, *HL2* and *HH2* in Fig. 7.3. Note that parameters employed in the watermarking scheme here are adopted from those in [22].

With the best basis evolved by GA, the watermarked images are demonstrated in Fig. 7.4 for subjective evaluation. Objective quality, represented by PSNR, are 44.80 and 42.27 dB, respectively. The image quality of both images should be acceptable both subjectively and objectively. Next, robustness in this paper is represented by bit correct rate (BCR). These results of applying three image processing operations in Fig. 7.4, including JPEG with quality factor (QF) of 30, the 3×3 Gaussian filtering, and image sharpening, are represented in Table 7.1 for Lena, and Table 7.2 for F16, respectively.

We can see that the BCR values are high enough to make copyright protection possible with WPT. Moreover, results with the proposed algorithm are better than those in [22], meaning that watermarking with WPT and GA has the potential to overcome conventional schemes. It is because a proper basis is selected by GA aiming at the three image processing methods. Thus, for specific image processing methods, the proposed method can generate a best WPT basis to increase the robustness of the watermark.

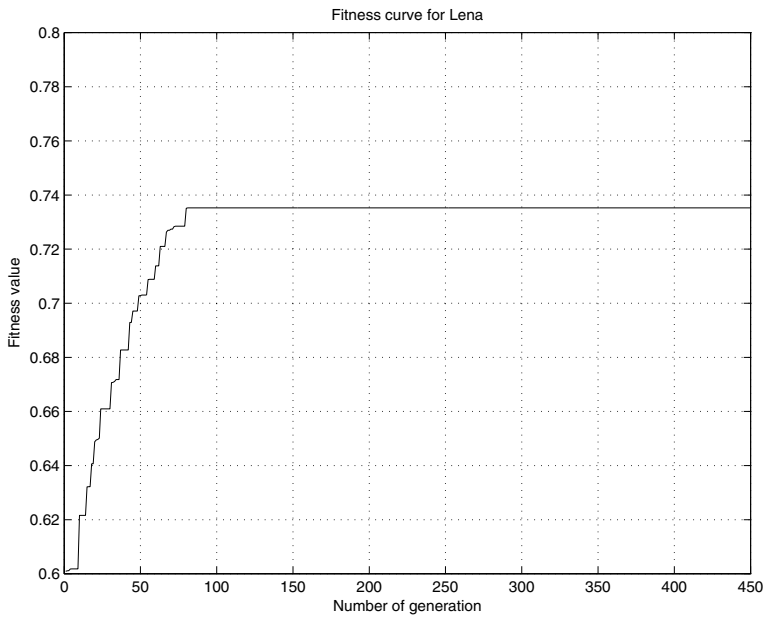


(a)

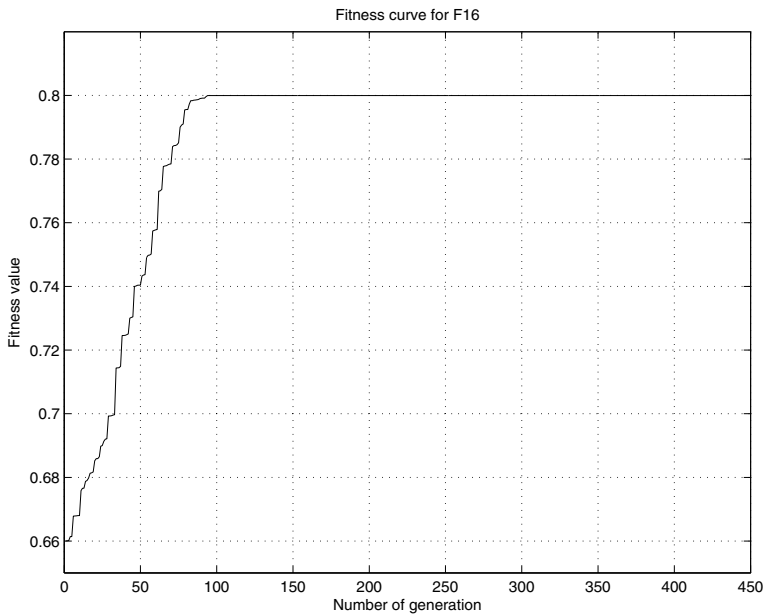


(b)

Fig. 7.4. The watermarked images with the proposed GA watermarking scheme. (a) Watermarked *Lena*, with the objective quality of 44.80 dB. (b) Watermarked *F16*, with the objective quality of 42.27 dB.



(a)



(b)

Fig. 7.5. The curve of fitness value vs. the number of generation. (a) With watermarked Lena. (b) With watermarked F16.

Table 7.1. The bit correct rates for extracted watermark after applying three different attacks on the watermarked Lena image.

Attack method	Results with proposed method	Results in [22]
JPEG (Q=30)	83.4%	76.6%
Gaussian Filtering	88.0%	83.1%
Sharpening	99.8%	99.7%

Table 7.2. The bit correct rates for extracted watermark after applying three different attacks on the watermarked F16 image.

Attack method	Results with proposed method	Results in [22]
JPEG (Q=30)	87.1%	81.4%
Gaussian Filtering	90.0%	84.8%
Sharpening	99.8%	99.8%

The fitness values in the GA training process can be evaluated in Fig. 7.5(a) for Lena, and Fig. 7.5(b) for F16, respectively. We observe that both curves are non-decreasing functions, with the maximum values of 0.73526 in Fig. 7.5(a), and 0.79997 in Fig. 7.5(b). Because the fitness value is the product of survival probabilities in Eq. (7.5), results in Fig. 7.5(a) and (b) seem reasonable. From the simulation results, we can observe the advantages with the proposed algorithm.

Finally, as described in Sec. 7.2, GA is mainly used to optimize non-linear functions with multiple variables. These optimization problems are usually difficult to solve directly. Therefore, comparing to other algorithms, GA often outputs promising solutions with more time consumption. From the viewpoints in theory or in practice, an analysis on time complexity of a complicated algorithm could help users to select proper algorithms for their applications. Since the fitness function used by our method are, however, composed of several image processing operations or attacks, these attacks adopted would have a significant impact on the performance of the watermark embedding process. Besides, the proposed method is performed in

Table 7.3. The maximum, average, and minimum duration among all the 450 iterations for GA training with our method for Lena and F16 images.

Duration (in sec.)	Lena	F16
Maximum	257.48	274.53
Average	233.81	246.36
Minimum	209.27	221.18

the wavelet packet domain; therefore, the average decomposition levels of wavelet packet trees generated with GA chromosomes would also have an effect on execution time. Thus, it is difficult to evaluate time complexity of the proposed method theoretically.

In this chapter, the execution time of all the 450 iterations of GA in our experiments has been measured and summarized to illustrate the performance of our method. Table 7.3 represents the maximum, the minimum, as well as the average durations of all iterations in our experiments. Though GA is time-consuming, the experimental results depicted in Table 7.3 are still effective because the main memory is utilized as a cache to store the original image and some intermediate results. With the statistics in Table 7.3, we can observe that for each training iteration in our watermarking algorithm, the time consumption for Lena and F16 lies between 209.27 to 274.53 seconds. These values would vary according to the different test images, the decomposition levels in WPT, and the compositions of the fitness function. As we pointed out in Sec. 7.2.3, time consumption with this order limits the application of our method for real-time processing. For the archival of multimedia with trusted communication, our method may have the potential for real applications.

7.4 Conclusion

Embedding watermarks into images can be referred to as a constrained optimization problem. Hence, genetic algorithm can be used to solving this problem. In this paper, we proposed a genetic watermarking scheme based on the wavelet packet transform. Genetic algorithm is used to select an appropriate basis from permissible bases of wavelet packet transform to increase the robustness of the embedded watermarks. Robustness can be optimized by GA, and fidelity can be reached with the inherent characteristics of the proposed algorithm. Experimental results demonstrate that the proposed method can increase the capability for resisting some image processing

attacks if an appropriate fitness function of GA was adopted. Therefore, proposed method in this paper is suitable for the application of trusted communication. In the future, efficient approaches to finding the best basis for watermark embedding would be another topic for further studies.

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