A Proposal for Combination of Category Theory and λ–Calculus in Formalization of Autopoiesis

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Abstract. There have recently been some computational or mathematical formalization studies on closedness of living systems such as autopoiesis and (M,R) systems. In particular, some have mentioned relationships between cartesian closed categories and λ–calculus. Following this line, the paper proposes a framework to formalize autopoiesis by combining category theory and λ–calculus more strictly, by introducing an equivalence between the category of cartesian closed categories and that of λ–calculi while providing a formalization of the distinction between organization and structure in autopoietic systems.

Keywords: Autopoiesis, category theory, λ-calculus, operational closure, Cartesian closed category, organization, structure.

1 Introduction

Autopoiesis gives a framework in which a system exists as an organism through physical and chemical processes, based on the assumption that organisms are machinery [8,9]. This system is organized as a network of processes of production of components, where these components continuously regenerate and realize the network that produces them, and constitute the system as a distinguishable unity in the domain in which they exist. However, the system description of autopoiesis includes circular closedness of relationships between components, and it is hard to interpret the definition from the perspective of the existing computational and dynamical systems. For solving this difficulty, some formal models have been proposed to represent its characteristics. McMullin [10] has studied a computational model of autopoiesis as 2-D biological cells. Bourgine and Stewart [1] proposed a mathematical formalization of autopoiesis as random dynamical systems, and explored the relationships between autopoiesis and cognitive systems. Egbert and Di Paolo [4] proposed an artificial chemistry model to represent autopoiesis.

Moreover, some research works have mentioned the similarity of autopoiesis with metabolism-repair ((M,R)) systems, which are an abstract mathematical model of biological cells proposed by Rosen [14], from the perspective of closedness of the systems. Letelier et al., [7] reviewed (M,R) systems and provided them with an algebraic example which suggested the relationship with autopoiesis. Chemero and Turvey [3] proposed a system formalization based on hyperset theory and found a similarity between (M,R) systems and autopoiesis on closedness.

The author also proposed some mathematical models of autopoiesis while connecting between closedness of autopoiesis and (M,R) systems, based on category theory [12,13]. On the other hand, recently, Mossio, Longo, and Stewart [11] showed that closedness of (M,R) systems can be formalized within λ–calculus by using category theory, that is, some properties of Cartesian closed categories corresponding to λ–calculus. Moreover, Cárdenas et al., [2] critically discussed their work. In the sense that a Cartesian closed category is used in the model of autopoiesis by the author, these studies lead to a common framework for discussing relationships between closedness of autopoiesis and its implementation within computational formal systems.

For encouraging the discussion about closedness of autopoiesis and its computational formalization, in particular, about closedness in organizations and dynamics in structures, this paper proposes a framework of a research program by a combination of category theory and λ–calculus, based on the models previously proposed.

2 Completely Closed Systems: Revisited

The author proposed "completely closed systems" under entailment between components in a category with specific properties and a distinction between organization of components and structure among elements by introducing functors between the categories [12,13]. As mentioned in the previous section, the systems are defined within a cartesian closed category [6].

We assume that an abstract category $\mathscr{C}$ has a final object 1 and product object $A \times B$ for any pair of objects A and B. The category of all sets is an example of this category. Moreover, we describe the set of morphisms from A to B as $H_{\mathscr{C}}(A,B)$ for any pair of objects A and B. A element of $H_{\mathscr{C}}(1,X)$ is called a morphic point on X. For a morphism $f \in H_{\mathscr{C}}(X,X)$ and a morphic point x on X, x is called a fixed point of f iff $f \circ x = x$ ($\circ$ means concatenation of morphisms) [15]. Morphic points and fixed points are respectively abstraction of elements of a set and fixed points of maps in the category of sets.

The fact that the components reproduce themselves in a system implies that the components are not only operands but also operators. The easiest method for realizing this implication is the assumption of the existence of an isomorphism from the space of operands to the space of operators [5].

When there exists the power object Y^X for objects X and Y (that is, the functor $\cdot \times X$ on $\mathscr{C}$ has the right adjoint functor $\cdot^X$ for X), note that there is a natural one–to–one correspondence between $H_{\mathscr{C}}(Z \times X, Y)$ and $H_{\mathscr{C}}(Z, Y^X)$ for any objects X, Y, Z satisfying the diagram in the upper figure of figure 1. Thus, there is a natural one–to–one correspondence between morphic points on Y^X and morphisms from X to Y satisfying the diagram in the lower figure of figure 1. This property is the condition for which $\mathscr{C}$ is a cartesian closed category.

Now, we assume an object X with powers and an isomorphism $f : X \simeq X^X$ in $\mathscr{C}$. Then, there uniquely exists a morphic point p on $(X^X)^X$ corresponding to f in the above sense, that is , $p' = f$. Since the morphism from X^X to $(X^X)^X$ entailed by the functor $\cdot^X$, f^X, is also isomorphic, there uniquely exists a morphic point q on X^X such that $f^X \circ q = p$. We can consider that p and q entail each other by f^X. Furthermore, there

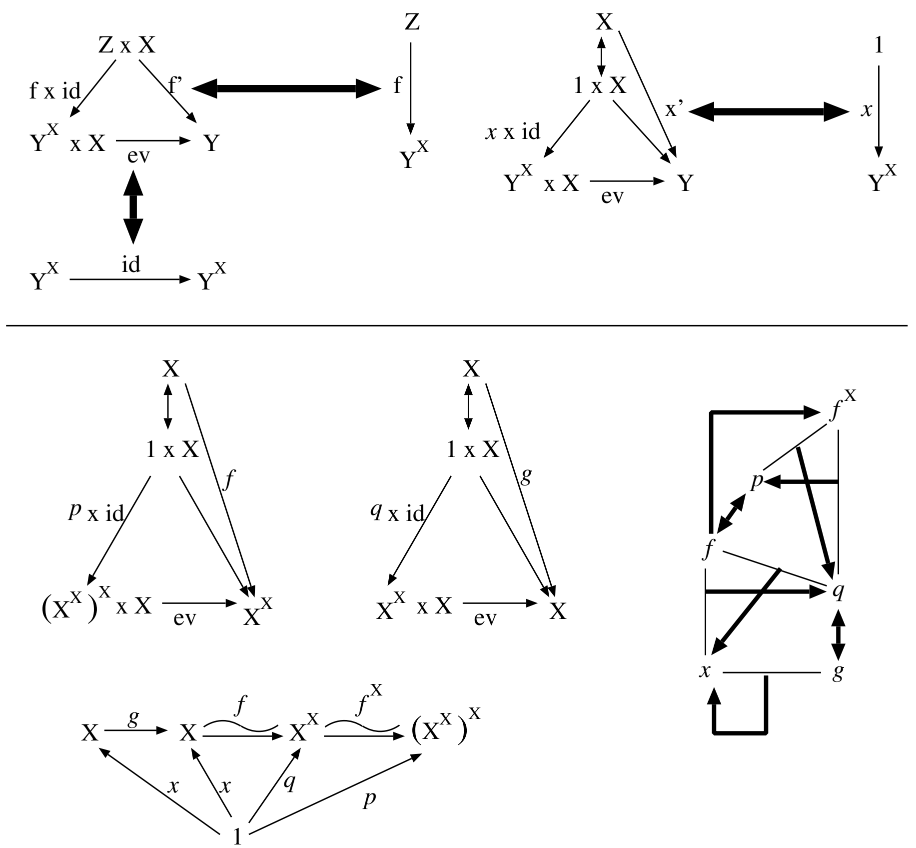

Fig. 1. The Diagrams of a Completely Closed System and the Entailment Relations based on Natural One–To–One Correspondence

uniquely exists a morphic point x on X such that $f \circ x = q$ because f is isomorphic. Since we can consider that x and q entail each other by f, and f and p entail each other by the natural correspondence, the system consisting of x, q, p, f, and f^X is completely closed under entailment. Moreover, if x is a fixed point of $g : X \to X$ naturally corresponding to q, that is, $g \circ x = x$, we can consider that x entails itself by g. The lower figure of Figure 1 shows the diagrams of this completely closed system and the entailment relations.

3 Distinction between Organization and Structure: A Combination with λ–Calculus

In [12,13], the author proposed a model of distinction of structures and organizations in autopoiesis. If circular relations between components and their production process network are closed under entailment, this closedness may be hard to formalize in a general category such as state spaces. On the other hand, the structure of an autopoietic system must be realized in a state space as a physical one (as shown in the upper figure

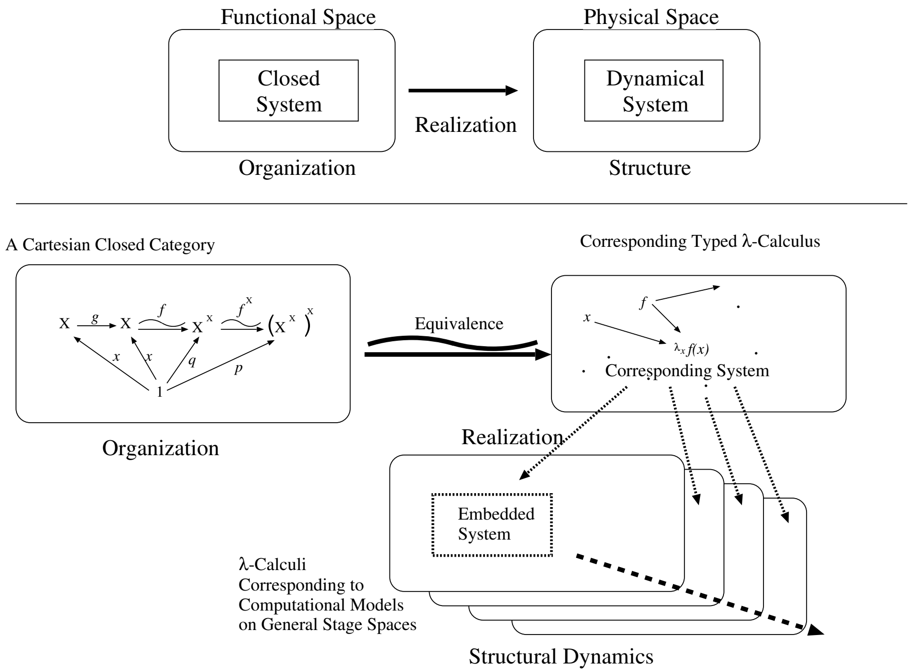

Fig. 2. Aspect of Autopoiesis based on Distinction between Organization and Structure, and Its Category Theoretical Formalization

of Figure 2)[1]. In the model, the organization is formalized in a specific category, that is, as a completely closed system in a Cartesian closed category. Then, the structure is formalized in the category of general state spaces, and realization from the organization to the structure is represented by a functor between the categories.

However, this framework does not argue for any concrete definition of the category of structure or functors. Moreover, the model consists of a family of Cartesian closed categories which include completely closed systems representing the same organization, and one general state space. The model can represent a structural dynamics on a state space based on the organization. However, it cannot include higher dynamics in which the state space itself changes, as, for example, occurs in metamorphoses of life systems. To overcome these problems, the paper proposes the introduction of categorical equivalence between cartesian closed categories and typed λ–calculi into the distinction between organization and structure in the model of autopoiesis.

According to Lambek and Scott [6], a cartesian closed category generates a category of typed λ–calculus, a category of typed λ–calculus generates a cartesian closed category, and the functors by these generations induce the equivalent relation between the category of cartesian closed categories and that of typed λ–calculi. The framework to be proposed in the paper consists of the following items (shown in the lower figure of Figure 2):

[1] This distinction is mentioned in Maturana and Varela's original literature [9].

1. A completely closed system as an organization is formalized in a cartesian closed category.
2. There exists a system in the typed λ–calculus corresponding to the completely closed system.
3. Realization is formalized by embedding the system in the typed λ–calculus into a more general λ–calculus corresponding to a computational model in a state space.
4. In order that components of the embedded system are repeatedly entailed within the organization, another general λ–calculus is found and the original system is repeatedly embedded into it.

4 Discussion

The framework proposed in the paper differs from the study of Mossio et al., [11], which showed a possible formalization of closedness of (M,R) systems on λ–calculus. Although autopoiesis requires distinction between organization and structure, the form of (M,R) systems does not include the explicit distinction between closed organizations and structures realized in state spaces, and these concepts are confused [13]. Although Mossio et al., [11] used some properties of cartesian closed category, closedness of (M,R) systems is discussed only on the category of structure. In the proposed framework, closedness of a system is dealt with on cartesian closed categories, and then the corresponding structure is discussed.

The framework proposed in the paper has an advantage. Cartesian closed categories on which completely closed systems are defined are a specific subcategory in which an isomorphism exists between operands and operators. By considering the relationship between this specific category and the corresponding subcategory in the category of typed λ–calculi, what type of computational model is needed to realize systems with operational closure, (that is, what class of computation is required for formalization of operational closure) can be investigated. More strictly, we can investigate whether the form of λ–calculus corresponding to a completely closed system can be embedded into general λ–calculi corresponding to computational models on general state spaces, and whether operationally closed systems can be formalized as computational models, by this investigation.

The framework in the paper is currently at the stage of a proposal. It is most important to clarify the form of closed organization in typed λ–calculus based on mathematically strict relationships between Cartesian closed categories and typed λ–calculi, and the form of embedding from the specific typed λ–calculus to general λ–calculi. Moreover, it should be extended to more general systems with operational closure.

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