Heteroglossia in Multilingual Mathematics Classrooms

Richard Barwell

Abstract What does linguistic or cultural diversity look like in a mathematics classroom? How does such diversity influence the teaching or learning of mathematics? In this chapter, I address these and related questions. Specifically, I draw on Bakhtin's notion of heteroglossia to analyse the literature on teaching and learning mathematics in linguistically diverse classrooms. Based on this analysis, I describe and discuss four tensions that arise in linguistically diverse mathematics classrooms: tensions between school and home languages; between formal and informal language in mathematics; between language policy and mathematics classroom practice; and between a language for learning mathematics and a language for getting on in the world. These tensions can all be traced to an underlying tension between what Bakhtin calls centripetal and centrifugal forces in language. I conclude by considering some of the implications of my analysis for equity in mathematics teaching.

What does linguistic or cultural diversity look like in a mathematics classroom? How does such diversity influence the teaching or learning of mathematics? And what does it mean for equity in mathematics classrooms? My own experience of teaching and researching in mathematics classrooms around the world leads to an awareness that linguistic and cultural diversity is itself diverse. Attempts to categorise different 'types' of multilingual mathematics classroom (e.g. Barwell 2005a; Clarkson 2009), for example, while not entirely unhelpful, tend to vastly oversimplify the full richness and complexity of language use that exists in mathematics classrooms around the world. To at least partially answer my opening question, let me give some examples.

Example 1 The children of immigrants to Québec must generally attend French medium schools. If they do not speak French, they spend their first year in a classe d'accueil, the main purpose of which is for them to learn French. I have recently been visiting a classe d'accueil made up of eighteen 9–11 year-olds from South America, West Africa, the middle East and South Asia. The languages they speak

R. Barwell (✉)
University of Ottawa, Ottawa, Canada
e-mail: richard.barwell@uottawa.ca

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at home include Spanish, Swahili, Hindi and Arabic. During their mathematics lessons, the requirement that they use French presents challenges. For example, during some work on the properties of different geometric forms, students struggle to explain their thinking e.g. why they think a shape is non-convex. They also make extensive use of deixis (words like 'this' or, in French, 'ça') often combined with gestures e.g. one student explains that a shape is curved by saying comme ça (like this) and sketching a curve in the air with his pencil. In this class, the acquisition of French is the primary objective, even in mathematics. After the year is over, the students will join mainstream classes. Their various other languages are acknowledged and referred to from time to time but they are expected to use French. And they are expected to learn mathematics even as they learn French.

Example 2 In a classroom in northern Pakistan, mathematics is taught in a mixture of English, Urdu and Burushaski. The school in which I taught was established by the local community to provide an English-medium education. As such, parents pay relatively high fees and the children all live close to the school. English is seen as the language of the elite and ruling classes in Pakistan and widely used in higher education, the civil service and the army. Urdu is the national language and Burushaski is the local language. Mathematics textbooks are in English. Mathematics teachers in the school do not necessarily have teaching qualifications and are not necessarily highly proficient in English. Furthermore, if complex topics like the formal arithmetic properties of associativity, distributivity, etc., are taught entirely in English, many students will not understand. Hence teachers use a mixture of three languages. English is at least used for key terms like associativity and sentences from the textbook, Urdu for more surrounding explanation and discussion and Burushaski for more informal discussion (see Halai 2009, for a more detailed discussion of mathematics classrooms in Pakistan).

Example 3 A few years ago, I spent a year visiting a class of 9–10 year-olds in an inner city school in England. The class included middle class and working class White children, middle class and working class Black children, recently arrived children from Hong Kong and a girl from Somalia whose family had reached England via the Netherlands and Wales (and who consequently knew some Dutch and Welsh). There were also several children from Pakistani backgrounds who had spent various periods in school in Pakistan as well as in England and who spoke Urdu and Punjabi and were learning English. My research focused particularly on word problems (see, for example, Barwell 2005b). The students who were learners of English struggled to make sense of word problems, although my research suggested that they were able to relate mathematics to 'real life' contexts if they created these contexts themselves.

It is apparent from these briefly sketched classrooms that mathematics is taught and learned in a wide range of diverse settings, each diverse in their own particular way. In Québec, all the students in the class are learners of French and all are recent arrivals to the province, but they come from widely differing linguistic and cultural backgrounds. In Pakistan, the children were all from close-by and, in broad terms, shared a common cultural background. And yet they studied mathematics in the context of great linguistic diversity, through a language that they were not likely to encounter much outside of school. In the third class I described, the teacher had to take into account a wide range of proficiency in English and a wide range of familiarity with life in the UK (these two not necessarily coinciding). In terms of mathematics teaching and learning, do these situations and others like or unlike them have anything in common? Are there common issues or challenges that arise? How are language, diversity and mathematics related? How can teachers deal equitably with such diversity?

In this chapter, I will address these questions with a specific focus on language-related issues, although the discussion applies by extension to questions of cultural diversity. I begin with a summary of the main substantive findings of research on teaching and learning mathematics in linguistically diverse classrooms. I then set out a theoretical framework for language diversity based on the work of Bakhtin. These ideas are used to discuss four tensions that arise across the existing research and in a range of different contexts. A key point that emerges from this analysis is that these tensions are an inevitable part of linguistic diversity. In the final part of the paper, I return to the above questions and consider some possible ways in which mathematics teachers could respond to linguistic diversity in the light of my analysis.

1 Research on Teaching and Learning Mathematics in Linguistically Diverse Classrooms

There is a growing body of research on teaching and learning mathematics in multilingual, bilingual or second language classrooms i.e. any setting in which participants could draw on a repertoire of different languages. This work has produced a number of useful substantive findings.

First, research has shown that there is a relationship between students' proficiency in the language used for teaching and assessing mathematics and their attainment, although this relationship is not straightforward. In particular, students who have a high level of proficiency in their first language and who develop a similar level of proficiency in the classroom language tend to outperform monolingual students in mathematics. By contrast, students who do not develop a suitable level of proficiency in any language tend to under-perform in mathematics. Students who develop such proficiency in one language, whether at home or at school, tend to perform at a similar level of monolingual students. These findings have been found in a variety of contexts, including ESL settings in Australia (Clarkson 1992, 2007; Clarkson and Galbraith 1992) and in immersion settings in Ireland (Ní Ríordáin and O'Donoghue 2009) and Canada (Bournot-Trites and Reeder 2001; Lapkin et al. 2003; Swain and Lapkin 2005; Turnbull et al. 2001).

Second, research has established that teaching mathematics in multilingual settings presents challenges for teachers. Such challenges relate to choice of language, to enabling students' expression of mathematics and to the socio-political context. Such issues have been identified in a variety of settings including in South Africa

(Adler 2001; Setati 1998, 2005), in the context of immigration in Catalonia (Gorgorió and Planas 2001) and in Spanish-English bilingual education settings in the United States (Khisty 1995; Moschkovich 2002).

Finally, research has examined some of the strategies that bilingual students use to participate in and make sense of school mathematics. Such strategies include the use of two or more languages, sometimes known as code-switching; and connecting mathematics problems with their experience outside of school. Code-switching in mathematics classrooms has been investigated in, among other places, Australia (Parvanehnezhad and Clarkson 2008), South Africa (Setati 1998), Malta (Farrugia 2009a) and Pakistan (Halai 2009). Bilingual students' ways of engaging with mathematical problems have been examined in ESL settings in the UK (Barwell 2003, 2005b, 2005d), in bilingual education classrooms in the United States (Moschkovich 2008) and in indigenous language contexts in Brazil (Mendes 2007).

This growing body of work underlines the tremendously diverse nature of language diversity in mathematics classrooms around the world and the range of challenges this diversity can sometimes present. These challenges are related to the need to develop proficiency in the language of teaching and learning mathematics at the same time as learning mathematics, as well as the use of different languages by teachers and learners in the mathematics classroom. In this chapter, however, I want to go beyond these basic substantive findings to examine some of the underlying issues and tensions that emerge from all this research. These issues and tensions arise from language diversity but have repercussions for teaching and learning mathematics. To take forward this collected work, I propose a deeper theorisation of language diversity in mathematics classrooms based on Bakhtin's notion of heteroglossia. These ideas are set out in the next section.

2 The Diversity of Language

Much of the history of linguistics is based on the idea that languages are unified and describable—that there is, for example, a standard kind of English or French, for which the rules can be determined. This idea has deep roots, drawing on an ideology of language as a unifying force heavily implicated in the politics of national cohesion. This ideology becomes apparent, for example, in calls for immigrants to be compelled to learn the national language (see, e.g., Blackledge 2002). The European origins of this view of language have been highlighted in African (e.g. Makoni and Meinhof 2004) and South Asian (e.g. Canagarajah 2009) critiques of linguistics: a model of unified, discrete languages does not fit well with the kind of continuous variation in languages found in these parts of the world. Indeed, one of the linguistic legacies of colonialism is the struggle to apply a model of unified national languages in the midst of a language diversity for which it is ill suited.

In mathematics education, meanwhile, there is an assumption that mathematical language, often referred to as 'the mathematical register' (Halliday 1978) is as unified and describable as French or Japanese. This kind of assumption is apparent both in research, such as O'Halloran's (2005) extensive analysis, and in mathematics curricula (cf. Barwell 2005c) and often works in concert with a view of mathematics as equally unified and as language and culture free. From this perspective, the different students in the classe d'accueil referred to above do not bring different mathematics, only different languages. Similarly, the students in the mathematics class in Pakistan may use any language to learn mathematics, since the mathematics is always the same.

Bakhtin's work offers a somewhat different perspective from those I have summarised. Working in Russia during the Soviet era, Bakhtin was primarily a literary theorist—he was interested, for example, in what makes a novel a novel and not, say, poetry. His examination of questions relating to literature led him to develop a highly distinctive theory of language. In particular, his thinking about language is relational rather structural (cf. Holquist 1981). For example, he was more interested in the relationship between actual utterances and the contexts that affect how they are interpreted, than in a structural account of meaning or context. His work is also notable for its attention to the great diversity of language as it is used. Bakhtin's ideas have not been widely influential in mathematics education, although traces are apparent in Sfard's (2008) thinking. Van Oers (2001) draws particularly on some of the ideas that I refer to in this chapter, although, like many interpreters of Bakhtin in the field of education (e.g. Wells 1999), his reading seems to me to be more oriented to psychology than to linguistics. In this chapter, then, I take up Bakhtin's distinction between unitary language and heteroglossia.

Bakhtin (1981) used the term 'unitary language' to refer to the sense of language as coherent and unified. Unitary language, he writes, "gives expression to forces working toward concrete verbal and ideological unification and centralisation, which develop in vital connection with the processes of socio-political and cultural centralization" (p. 271). Unitary language is, however, a theoretical construct; language is not inherently unitary, although the idea of unitary language reflects the common patterns that make meaning possible. Language does not exist in this theoretical form; language is always instantiated in use. Language in use, however, is inherently diverse, multiple and fluid:

> At any given moment of its evolution, language is stratified not only into linguistic dialects in the strict sense of the word [...] but also [...] into languages that are socio-ideological: languages of social groups, "professional" and "generic" languages, languages of generations and so forth. (Bakhtin 1981, pp. 271–272)

A key point about this stratification and about language use is that specific utterances can reflect multiple layers simultaneously. Speech necessarily entails 'the social diversity of speech types' (Bakhtin 1981, p. 263), for which Bakhtin's translators introduce the Latinate term heteroglossia. This idea captures well the nature of interaction in the three mathematics classrooms described above. Each classroom swirled with bits of different languages, some more prominent than others, some filtered through others. Each classroom swirled with the 'social languages' of each student's background: the languages of their social class, race, gender and so on. And each classroom swirled with the social languages of school: the languages of mathematics, of curriculum, of textbooks, of students being students and of teachers being teachers.

For Bakhtin, these two ideas of language—unitary language and heteroglossia—are in constant struggle, a struggle characterised in terms of centripetal and centrifugal forces. Duranti (1998) explains these terms as follows:

> The centripetal forces include the political and institutional forces that try to impose one variety of code over others [...] These are centripetal because they try to force speakers toward adopting a unified linguistic identity. The centrifugal forces instead push speakers away from a common center and toward differentiation. These are the forces that tend to be represented by the people (geographically, numerically, economically, and metaphorically) at the periphery of the social system. (Duranti 1998, p. 76)

The struggle between these two sets of forces (for me 'force' is metaphorical) is present each time we speak and, moreover, shapes what we say:

> Every concrete utterance of a speaking subject serves as a point where centrifugal as well as centripetal forces are brought to bear. The processes of centralization and decentralization, of unification and disunification, intersect in the utterance; the utterance not only answers the requirements of its own language as an individualized embodiment of a speech act, but it answers the requirements of heteroglossia as well; it is in fact an active participant in such speech diversity. And this active participation of every utterance in living heteroglossia determines the linguistic profile and style of the utterance to no less a degree than its inclusion in any normative centralizing system of a unitary language. (Bakhtin 1981, p. 272)

On the one hand, then, every utterance must conform to some recognisable pattern of language (or it would be incomprehensible). On the other hand, every utterance contributes to the continuous variation and reinvention of human communication. These forces are apparent in the classrooms described above. The heteroglossia in the mathematics classrooms I have described is resisted by the requirement to speak French or English and the goal of learning to communicate a fixed version of mathematics in a recognisable way. This sense of a struggle between unifying and stratifying forces of language (and of mathematics) makes possible a deeper analysis of some of the issues that have arisen in research in second language or multilingual mathematics classrooms. In the next part of this chapter, I describe and analyse four such issues, which I present as a set of tensions. By *tension*, I am drawing on Bakhtin's sense that each utterance simultaneously represents both diversity and uniformity: there is a kind of struggle between the uniqueness of each utterance and its conformity to the patterns of language. I have synthesised four tensions from the existing literature on teaching and learning mathematics in linguistically diverse classrooms. Following Bakhtin, each tension involves a unitary pole and a heteroglossia pole. They are illustrated with examples from relevant studies. Of course, these tensions are simplications: they interact and overlap and in many ways and can be seen as different lights cast on the deeper underlying tension inherent in diversity itself.

3 Tension Between School and Home Languages

Perhaps the most explicit manifestation of unitary language in mathematics classrooms is the stipulation of an official language of schooling. In the classe d'accueil, for example, French is the language of teaching and learning. The curriculum, classroom texts, classwork and the teacher are almost always presented exclusively in French. The expectation is that students will become more proficient in the classroom language over time. In practice, Spanish, for example, is heard, sometimes directly in conversations between students, sometimes indirectly in the accents or interpretations of French words. On one occasion, I observed a student ask if he could explain in Spanish how he knew a shape was convex; he was asked to try in French. There is an inevitable tension, then, between the languages that students use in or out of the classroom and the requirements to use a particular language to talk about mathematics.

The research literature provides many examples of the tension between home and school languages, examining the presence and use of students' home languages, or practices such as code-switching in which students switch between two or more different languages. Adler (1995, 2001), for example, observed secondary school mathematics lessons and recorded interviews with teachers in multilingual South Africa, a nation with 11 official languages. Adler (1995) reports the following remarks from interviews with mathematics teachers in two different types of secondary school:

> …in Std 7, where I asked a question and one answered in Tswana. And I said: "Can you please try answer that in English—I don't understand that?" and he said, crossly "No, mam, but you are Tswana—you are not white!" He was angry.
>
> Sometimes you find that you get stuck because students cannot communicate—then, though not much, you resort to Tswana. You are careful because if you do that then they want you to do it all the time.
>
> …there are Xhosa speakers in the class—so if I am speaking Tswana then they complain I am favouring them.
>
> The problem (in group work—with discussion in any language, report back in English) is… if all your discussion is in Zulu you get to the concept then you can't report back in English so you can't talk about it in English 'cos you never developed it in English. (All from Adler 1995, pp. 268–269)

Both centripetal and centrifugal forces are much in evidence in these comments. The enforcement of English is in tension with the heteroglossia of the students. The school, the curriculum and the teacher are pushing for the use of a single, unitary language, while the students and at some points the teacher are used to using several different languages. This tension leads to what Adler (e.g. 2001) calls 'teaching dilemmas'—for example, the dilemma of whether it is better to use English, knowing that students may not always understand, or to use students' home languages, knowing that they will not develop proficiency in the English of mathematics. The notion of 'dilemma' corresponds closely to my use of the word 'tension' in this chapter: the dilemmas identified in Adler's research represent the expression of these tensions by teachers. Some teachers may not see these situations as dilemmatic: the tension, however, would still be present, since it is part of human communication.

Adler's work shows how the tension between home and school languages can be experienced by teachers. Clarkson has investigated students' perspectives, seeking to understand when and how multilingual students 'switch' between languages when working on mathematics in Australia (e.g.: Clarkson 2007; Clarkson and

Dawe 1997; Parvanehnezhad and Clarkson 2008). This work has led to recognition that 'home' languages can have a role in understanding and learning mathematics:

> some students appeared to rely in part on what they had learnt from parents, related some of the mathematics items to their lived experiences embedded in L1 contexts in their own community, and used some Persian mathematical words which they habitually use because of their home background. [...] For bilingual students, some of what they know, and indeed are, is embedded in a L1 social context, and some of their ideas are clearly more easily expressed in their L1. (Parvanehnezhad and Clarkson 2008, p. 77)

Essentially, what Parvanehnezhad and Clarkson have uncovered is an often hidden heteroglossia that maintains a level of language diversity in mathematics. In the Australian context, in which there is a strong ideology around the (monolingual) use of English for teaching and learning, Clarkson's work illustrates the tension for students; if multiple language use were accepted in Australian classrooms, there would be little need for Parvanehnezhad and Clarkson to argue for the kind of position set out in the above quote.

One of the challenges of researching the tension between home and school languages in mathematics classrooms is that home languages are often marginalised. I spent a full year visiting the classroom in the UK described at the start of this chapter but rarely heard use or acknowledgement of students' home languages. Their 'other' languages must have been present for these students—heteroglossia does not simply disappear in the face of the centripetal forces of a unitary language ideology. Indeed, in my research in this classroom, I did find evidence of their home languages and experiences informing their work in mathematics (e.g. Barwell 2005d). The danger is that in such classrooms, performance in mathematics is assessed from a unitary language perspective, as Moschkovich (2009) has suggested:

> Too often, descriptions of bilingual students focus on the obstacles they face in understanding text or utterances in English and these misunderstandings are invariably ascribed to their lack of proficiency in their second language. (Moschkovich 2009, p. 79)

There is a danger, then, that our research contributes to the centripetal forces that serve to marginalise the kind of diversity that is, for the most part, ever present. Students are used to using a mixture of languages, regularly incorporating bits of one language into fragments of another in creative and entirely comprehensible ways. School, including in mathematics, requires them to use just one language. My purpose here is not to argue for more heteroglossia, nor for that matter to defend a unitary language approach. At this point, I want merely to highlight the tension that arises—although it is worth noting that it arises more strongly for second language learners of the classroom language, who are by and large, as Duranti (1998) observed, at the periphery of the system.

4 Tension Between Formal and Informal Language in Mathematics

The second recurring tension observable in studies of mathematics learning in multilingual settings concerns students' informal expression of their mathematical thinking and the need for students to learn to use more formal mathematical language.

In classrooms in which multiple languages are actively used, such as the classroom in Pakistan that I described above, it is common for formal mathematical terms to be presented and used in the official language. For example, in the classroom in Pakistan, words like ‘algebra’, ‘divide’ or ‘axis’ would commonly be in English, whether as part of a discussion in English or incorporated into a discussion in Urdu or Burushaski. Furthermore, much of the English used in mathematics lessons quoted the textbook, either through reading it aloud or through subsequent reproduction of the text. Students’ informal discussion of mathematical ideas were, however, more likely to be in Urdu or Burushaski. This kind of phenomenon has been more systematically documented by Setati (2005) who in primary school mathematics classrooms in South Africa, found exactly this division. Moreover, she shows how discussion of mathematics in English tended to be more procedural in nature, while discussion of students’ thinking or mathematical ideas was more likely to be in their home languages. This work shows how centripetal forces, in this case in the form of a unitary language view of the mathematics register, are in tension with the centrifugal forces of students’ ‘informal’ ways of expressing their mathematical thinking. In Bakhtin’s terms, these informal forms of expression reflect the many social languages the students bring with them into the mathematics classroom, whether they are social languages within the official language of the classroom or within the other languages (and combinations of languages) they speak.

The tension for teaching and learning arises from a need for students to develop recognisable ways of communicating mathematics set against a desire on the part of teachers that their students should meaningfully discuss mathematics. This tension is often seen as a trade-off, in which teachers allow a degree of informal expression to ensure that students understand the mathematics, while seeking to gradually enhance students’ use of more formal or standard forms of expression (Adler 2001; Khisty 1995). Indeed both Setati and Adler (2000) and Clarkson (2009) develop explicit models to deal with this tension. In each case, the approach consists of recognising and building from students’ informal expression of mathematics in any language, towards more formal expression of mathematics in any language, with perhaps the ultimate goal that students are proficient users of formal mathematical English. These models encapsulate the underlying tension between the centripetal forces of standard mathematical language and the centrifugal forces of language diversity, where diversity includes both natural and social languages.

In settings in which students are seen as second language learners, rather than as multilingual, the same kinds of tension are present. Khisty (1995), for example, compared three second-grade and two fifth-grade mathematics classes containing some Spanish-speaking bilingual students. The class teachers were all English-Spanish bilingual to some degree, although Spanish was not used much for mathematical discussion. In one classroom considered to be an effective learning environment, mathematics was negotiated through discussion, challenge and debate. This environment frequently required students to explain their ideas and to draw on previous experience to make sense of new situations. For Khisty, the culture of this classroom led to students making mathematical meaning for themselves through interacting with both the teacher and other students. Teachers in the study that seemed to be more effective paid more attention to the language of mathematics as well as to the mathematics itself. Through this attention, as in the models described above, students' informal expressions of their thinking are shaped into more formal mathematical expression. The same kinds of forces are at work, then, in various settings, with students' social languages seen as a starting point for developing a more recognisably standardised use of mathematical language. Again, I do not see this tension as easily resolvable; the purpose of a fairly standard mathematical register is to facilitate communication of mathematical ideas. Equally, however, there is a danger that this same mathematical register marginalises students at the peripheries.

5 Tension Between Language Policy and Mathematics Classroom Practice

Language policy is a specific manifestation of Bakhtin's centripetal forces in language diverse mathematics classrooms. The purpose of such policy, after all, is, in most cases, to mandate some form of unifying order. By language policy in the context of this chapter, I am referring to any official statement of how language or languages should be used in mathematics classrooms. Such statements range from general language policies set out a national level to school-level policies. Such policy may arise within mathematics curriculum or other policy documents. Government guidance for primary school teachers in the UK, for example, includes statements about how teachers should work with learners of English as an additional language. Such statements have tended to emphasise the precise and the unambiguous nature of mathematical language and the importance of learning to do mathematics in English (cf. Barwell 2005c). More recent statements have highlighted the value of students' home languages in learning mathematics, but only as a useful strategy in developing proficiency in English (e.g. DCSF 2007). Such guidance therefore generally reflects a unifying perspective on language and mathematics. In particular, diversity tends to be presented as something to be accommodated, rather than an intrinsic and valuable aspect of mathematics classroom life (Barwell 2005e, p. 318). Such policies represent an idealised view of language use in mathematics classrooms; what actually transpires is inevitably rather messier. In the UK classroom described at the start of this chapter, for example, while languages other than English were rarely heard, their presence was nevertheless felt in the form of non-standard pronunciations or discussions of linguistic features of English (see, e.g., Barwell 2005d). While in unitary language terms English was the only language used, the nature of this English was diverse and clearly interacted with how students' interpreted mathematical texts such as word problems (Barwell 2005d).

The same kind of tension arises in officially multilingual societies. Farrugia's (2009a, 2009b) research provides a good example: she investigated the use of English and Maltese in mathematics classrooms in Malta. Official policy requires only English to be used, despite the use of a mixture of English and Maltese being widespread in Maltese society. Farrugia found that mathematics classroom interaction reflected this practice, with students drawing on both languages. Her account of this situation clearly illustrates this tension:

> Although codeswitching in many classrooms is a common practice in Malta, the writers of the NMC [the national curriculum] appeared not to agree with it. Hence, they suggested that mathematics [...] be taught through English. Codeswitching was only acceptable when the use of English caused 'great pedagogical problems' (Ministry of Education 1999, p. 82). I cannot exclude the fact that some of the writers may have had a personal perception of English as being in some way 'superior' to Maltese, or, as Baker (2001) implies, that the recommendation is motivated by an ulterior agenda to offer an advantage to certain social groups. However, from my personal acquaintance with some of the writers, and my interpretation of the document, the apparent reasons for the recommendation were to find a way to improve students' competence in English and a disapproval of codeswitching as a pattern of language. The medium of instruction issue for mathematics is a hotly debated topic in Malta. Those in favour of English argue in a similar way to the NMC writers, while those in favour of Maltese (or rather codeswitching) tend to present arguments that prioritise mathematical understanding. (Farrugia 2009b, p. 101)

As a 'hotly debated' subject, the tension is apparently very real in Malta. The terms of the debate can be traced to Bakhtin's centripetal and centrifugal forces. The former are represented in several forms including the supposed superiority of English or the disapproval of code-switching as degenerate, despite its prevalence in the language practices of Maltese society. Similar issues have been reported elsewhere including in Pakistan (Halai 2009), Swaziland (Dlamini 2008) and in Malaysia (Lim and Ellerton 2009). More generally, decisions about the language of textbooks or debates about the suitability of some languages for doing mathematics also reflect this tension. It is noticeable in all these examples that the tension between language policy and mathematics classroom practice is closely interrelated with the preceding tensions—with concerns about code-switching and informal language practices. Language policy generally seems to seek to suppress such practices, or to see them at best as steps on the road to monolingual proficiency in a given high status language. It is rare (at least in my experience) to find policies that explicitly advocate the growth of students' home languages through their work in mathematics classrooms.

6 Tension Between a Language for Learning Mathematics and a Language for Getting On in the World

The final tension I will discuss highlights the political role of language in society. It concerns two, sometimes conflicting roles of language. First, language is used to learn. In many cases, learning mathematics would be more effective if conducted using a language repertoire familiar to students. In some cases, this would mean using a single language (perhaps Urdu, in Pakistan). In other cases, it might mean using a mixture of languages (perhaps French and Spanish for some students in the classe d'accueil). Second, however, language has a socio-political role. The reason for the existence of the classe d'accueil, for example, is, in part, to protect and maintain the French-speaking nature of Québec society. French-medium education is mandatory for the majority of immigrants to Québec, regardless of their language background. At the national level, Canada is an officially bilingual country in which the roles of

French, English and bilingualism are highly politicised—to the extent that consideration of aboriginal languages or immigrant languages is somewhat overshadowed, despite their prevalence.

The appearance of this tension in relation to mathematics classrooms has been investigated by Setati (2008), who drew on interviews with teachers and high-school students in South Africa. The majority of teachers and students in her study expressed a preference for the learning and teaching of mathematics to be conducted in English. Setati attributes their position to an implicit view of English as international and as providing access to higher education, jobs and generally getting on in the world. This preference must, however, be set against the effectiveness of students' learning of mathematics. The two issues are in tension, however, as illustrated by one of Setati's interviews with high-school students:

> Nhlanhla, however, had conflicting cultural models. While she acknowledged the power of English, she also accepted the fact that if she focused on wanting to understand mathematics she would choose her home language as the language of learning and teaching.
>
> Nhlanhla: ...is the way it is supposed to be because English is the standardized and international language.
> MS: Okay, if you had a choice what language would you choose to learn maths in?
> Nhlanhla: For the sake of understanding it, I would choose my language. But I wouldn't like that [English as language of learning and teaching] to be changed because somewhere somehow you would not understand what the word 'transpose' mean, ukhithi uchinchela ngale [that you change to the other side], some people won't understand. They would not understand what it means to change the sign and change the whole equation.
>
> (Setati 2008, pp. 110–111)

In most cases, students and teachers in South Africa appear to prioritise learning English over understanding mathematics. Although Setati's work does not appear to have been replicated elsewhere in the world, the picture it presents is likely to be widespread. In many societies, the possibility of choosing the language of mathematics teaching and learning does not even arise. In England, for example, state schooling is in English: speakers of other languages have little choice but to accept this position. In countries where there is a choice, the tendency is for high-status languages to be preferred. Halai (2009) reports similar issues from Pakistan, where English-medium education is highly valued, to the extent that national policy has shifted towards teaching some high-school subjects, including mathematics, in English rather than Urdu (p. 48). In Pakistan, as in South Africa, there is a trend towards the use of English and away from the official use of national or local languages. This trend and the related tension is again traceable to Bakhtin's centripetal and centrifugal forces. The role of language in getting on in the world seems to lead to a unitary perspective in which high status languages are seen as more suitable for mathematics teaching and learning due to their socio-political value, rather than their educational value. The centripetal force towards a unitary language is not, however, simply an issue of bureaucratic policy-making; it reflects widespread pressure from students, families and communities. The terms of the debate, however, also reflect a unitary language perspective—observation of classrooms in South Africa, Pakistan or elsewhere suggest that language use is much more diverse than Nhlanhla's comments suggest. In reality, the choice is not so clear cut, as illustrated by Nhlanhla's natural use of a mixture of languages in her reply.

7 Discussion: Heteroglossia and Equity

Teaching and learning mathematics in culturally and linguistically diverse classrooms has been shown to influence the teaching and learning of mathematics through a range of tensions. What I have argued in this chapter is that these tensions all arise from Bakhtin's centripetal and centrifugal forces. The centripetal forces are often in the form of language policies or assumptions on the part of teachers, parents and communities that it is better to learn mathematics in one unitary language. Specifically, the pressure comes from a preference for a single school language, a standardised mathematical register, national and institutional language policies and the high status accorded to some natural languages (e.g. English) and social languages (e.g. the language of the middle classes, the mathematics register). The centrifugal forces arise from the heteroglossia found in all mathematics classrooms, but particularly in culturally and linguistically diverse mathematics classrooms. This heteroglossia is both collective and individual. Each classroom includes speakers of multiple natural languages and multiple social languages, including the languages of class, race, gender and so on. Just as significantly, each individual speaks in diverse ways drawing on a repertoire of natural and social languages to do mathematics. Indeed Bakhtin goes further, arguing that the tension is present in each utterance, which conforms to a relatively rigid language system, while at the same time drawing on multiple natural or social languages and invoking multiple possible meanings (cf. Holquist 2002, p. 48). The tension between these two sets of forces is inherent in human communication, both in general and in mathematics. Without common ways of talking or representing ideas, such communication would be impossible. Without variation, new things could never be said. My presentation of these four tensions is necessarily a simplification. These tensions are abstractions discernible in the heteroglossia of linguistically diverse mathematics classrooms. No classroom can be fully or neatly described in terms of these four tensions alone: the tensions are not things in themselves. Each classroom will swirl with various facets of the tensions I have identified, as well as others, as yet undocumented or unremarked. It is therefore useful to represent these tensions as occupying a wider space, which I have attempted to do in Fig. 1.

For me, such a diagram is an aid for thinking, not a reification. In thinking about the classroom in which I worked in Pakistan, for example, I can readily note the tension arising around the use of English as a medium of instruction for mathematics. How this tension plays out in specific utterances is, however, highly situated. It depends, for example, on the English proficiency of the students and of the teacher, as well as the nature of the English used in the textbooks and so on.

At the start of this chapter, I pointed out that language diversity in mathematics classrooms is widespread but is itself diverse. I suggested several questions that

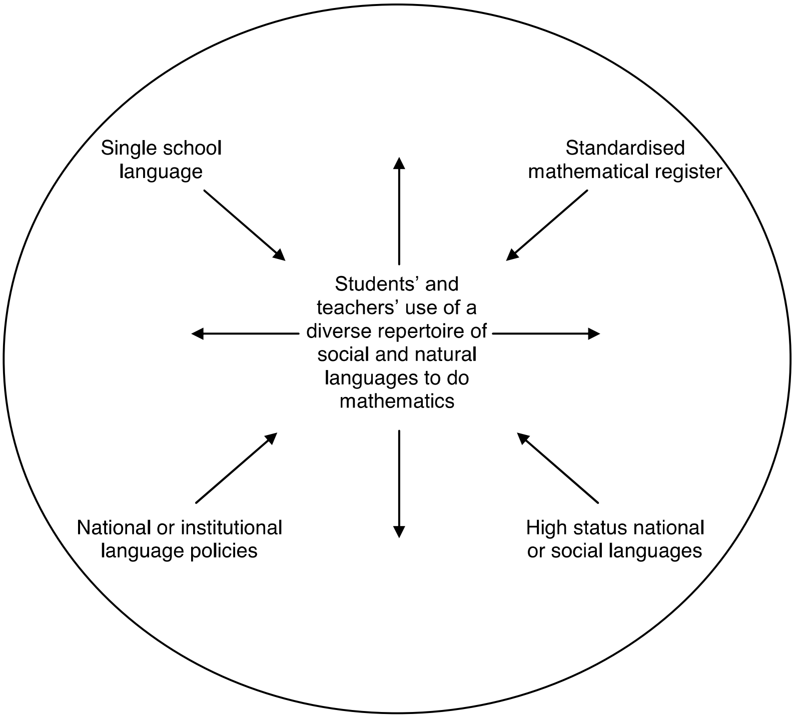

Fig. 1 Centripetal and centrifugal forces in language use in mathematics classrooms

arise in relation to this observation: In terms of mathematics teaching and learning, do these situations and others like or unlike them have anything in common? Are there common issues or challenges that arise? How are language, diversity and mathematics related? In the analysis set out above, I have addressed these questions. Despite the diverse nature of language diversity, teaching and learning mathematics in the context of language diversity does involve various tensions (and associated issues and challenges) that arise in a range of different situations. Four of these tensions are discussed in this chapter. Furthermore, these tensions are all related to the underlying push and pull of the centripetal and centrifugal forces inherent in language itself (indeed, in human communication more generally). This tension is the key nexus in which language, diversity and mathematics interact.

There is one remaining question: How can teachers deal equitably with such diversity? I have argued, following Bakhtin, that the kind of tensions I have described are inherent in human communication. The four tensions I have discussed are unlikely to be the only such tensions that arise; the tension between unitary language and heteroglossia will manifest itself in mathematics classrooms in a variety of different ways. But the underlying tension is ever-present. These tensions are likely to have different impacts on different students. Students whose repertoire of natural and social languages is well aligned with prevailing unitary languages are at an advantage: they do not need to 'crack the code' (Zevenbergen 2000); they already have the key. Students from the peripheries must either submit to the prevailing unitary language approach or find ways to resist it and assert their own repertoires. In reality, there is no perfect alignment between a given student's language repertoire and the language of the mathematics classroom, even if some students' repertoires are more closely aligned than others. To a greater or lesser extent, all students must live in the tension between heteroglossia and unitary language, between centrifugal and centripetal forces. This situation makes consideration of equity somewhat complex. In particular, heteroglossia cannot be eliminated; difference and diversity is unavoidable. Indeed, diversity accounts for some students' success as much as other students' underachievement. The challenge, therefore, is to find ways to decouple diversity from disequity.

8 Conclusion: Shifting the Tension

If the kind of tensions I have described in this chapter are inherent in human communication, they cannot be eliminated by a change in policy or classroom practice. Nevertheless, my analysis does suggest some ideas that have perhaps not been considered very widely. In particular, much concern with teaching and learning mathematics in diverse classrooms tends to assume a unitary perspective—debates about whether to use a particular language of instruction, for example. It is apparent from the research I have referred to and research in linguistics more generally that human communication is not naturally unitary. I suggest, therefore, that rather than seeking to eliminate the tensions I have described, a more productive approach would be to shift the tension more towards heteroglossia and away from a unitary perspective. I am not advocating a move towards absolute heteroglossia (whatever that would be), which would be as unhelpful as an approach that is too strongly unitary. But greater recognition of the heteroglossia of human communication in mathematics classrooms would better reflect the lives and experiences of students and teachers. The following two brief examples illustrate what such a move might look like.

In the research I conducted in the classroom in the UK described at the start of this chapter, I analysed how learners of English engaged with a task set by their teacher to write arithmetic word problems. My analysis of students' interaction as they worked together on this task suggests that the task itself has some important affordances (cf. Barwell 2009). In particular, the task did not require students to interpret an unfamiliar context, as is the case with solving a problem set by the teacher or a textbook. Rather, the students were able to draw on contexts that were familiar to them and to draw on their language repertoires to express these contexts. Students wrote word problems about shopping, buying presents for their families, earning pocket money, going to concerts or about monsters and morgues. The task also allowed students to engage with the language of word problems, infusing them with their own forms of expression. And writing word problems provided opportunities for the students to learn English and to make connections between the mathematics, context and language of word problems. The word problem task does not eliminate the tensions discussed in this chapter, but it does allow diversity of expression and so takes greater account of the heteroglossia of the classroom. In particular, a key feature of the task is that it allows students to bring their own experiences and ways of talking into their mathematics.

Setati et al. (2008) report another kind of task that involves greater recognition of classroom heteroglossia. Their work was conducted in a classroom in South Africa in which there were 36 students, including speakers of Setswana, Xitsonga, IsiZulu and Tshivenda. The official language was English. During work on a complex page-long problem about electricity prices, students were organised into language groups. They were provided with both the English version of the problem and a version of the problem in their home language (e.g. Setswana). Setati et al. report that students drew on both versions as they worked in groups on the task. It is notable that they did not use a single version—they referred to both and drew on a mixture of languages to think about the task. As I have argued above, these kinds of language practices are more typical of students' language use; the difference with this task is that it explicitly recognises these practices. Setati et al. highlight that the use of these multiple languages did not interfere with their work and argue that it enabled a greater focus on the mathematics of the task. Again, this kind of approach does not eliminate the tensions discussed in this chapter, but it does go some way towards recognising and incorporating the heteroglossia of the students and of society into mathematics classroom interaction.

These two examples suggest some possible directions for future work on equity in mathematics classrooms. To develop such strategies requires an awareness of the kinds of tensions I have discussed in this chapter. Of course, such an awareness does not eliminate the tensions: indeed awareness is likely to result in the kind of teaching dilemmas identified by Adler (2001). It is, however, better to work with the tensions than to work in ignorance of them. Whether the setting is broadly characterised in terms of second language learners or of bilingual or multilingual learners, there is a common underlying principle: that more recognition be given to the heteroglossia of the class and of each individual student. To make a start, we simply need to start listening to the diversity around us.

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