Ethnomathematics and Philosophy

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Abstract Any concept of ethnomathematics must eventually meet philosophical debates about the nature of mathematics. In particular neo-realist positions are anathema to the idea that mathematics is culturally based, but even modern quasi-empiricist philosophies are challenged by the fundamental relativity implied in ethnomathematical writing.

A new way of interpreting mathematical history which may allow for a truly relativist mathematics is described, and some evidence is presented to support this view. The kind of studies which would arise from this perspective on mathematics are outlined.

1 The Problems and the Challenge

We, as ethnomathematicians, have a problem.

For more than two thousand years mathematics has been regarded as the epitome of rational truth, the study of the essential features of quantity, relationships and space. There has been argument about how we come to know these things, and about how we can be sure of them, but few working mathematicians have doubted that they were dealing with essential facts of some kind. Mathematicians seem to say "we know that the mathematics we study tells us truths about numbers and points and lines and circles, and that it can be used to build bridges which don't fall down: it works therefore it must be right. Furthermore it is beautiful, and elegant, and has a long history of great thinkers, and... and..." and so on.

However, perhaps this is the world's greatest circle of self-justification. Describing and justifying mathematics on the grounds of our feelings about it and our per-

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ception of its usefulness might be a dangerous activity. But, if it is a circle of self-justification, then this circle has convinced a great number of people for many, many years. Who are we to challenge it now? And if we are to challenge it, then we need to be very sure of our ground, and very convincing in our arguments.

But we *are* challenging it—particularly as ethnomathematicians. Where did this challenge come from? On what grounds can we mount this challenge? And how can we explain the lack of previous challenges?

The problems arose when we started to talk about mathematics and culture in the same sentence. Ethnomathematicians were not the first people to do this: Oswald Spengler (1956) in the early part of this century was one of the first to seriously ask whether the mathematics which developed through various cultural eras was the same mathematics. However when these ideas began to move into the educational arena, and were used to challenge the imperialism of education in non-European countries, then much more was at stake.

And this led to a challenge to the classical view of mathematics. D'Ambrosio's writings (1987, 1990) are now well-quoted as the source of the idea that widespread apparent failure in mathematics was actually a social problem being played out through the filtering mechanism of mathematics education. Furthermore this filtering mechanism acted through the cultural nature of the subject: the social terrorism of mathematics.

D'Ambrosio challenges classical views with a social constructivist conception of "mathematics as a system of codification which allows describing, dealing, understanding and managing reality" (1987, p. 37). Hence (p. 74):

> ...we face a need for alternative epistemologies if we want to explain alternative forms of knowledge. Although derived from the same natural reality, these knowledges are structured differently.

D'Ambrosio appeals (among others) to Bachelard, Kitcher, and Lakatos as possible sources for these epistemologies. What is it that these writers offer? What, indeed, is needed as a philosophical basis for ethnomathematics? This paper explains why these sources are not enough, and describes some starting-points for an alternative relativistic philosophy.

Ethnomathematicians need to be able to discuss the possibility of the simultaneous existence of culturally different mathematics. The challenge for anyone attempting a philosophical basis for ethnomathematics is to ensure that there is an account of the way in which mathematics is structured, understood, and communicated which is consistent with sociological and anthropological descriptions of how mathematics is spread and used. In addition, any account must explain how one mathematics culture has come to be dominant, and, apparently, to be so highly developed compared with other mathematics cultures.

So mathematics must be described in a new way: an alternative philosophical position must establish a position which must be argued on conventional terms. Furthermore part of the task is to show why earlier philosophical positions were plausible and firmly held. It is not enough just to say that mathematics is culturally determined and to go on acting as if this is the case simply because we believe it to be so. We must convince others, particularly mathematicians.

How, then, might we conceive of a culturally relativistic picture of mathematics?

It may seem a waste of effort to start by explaining why traditional and non-relativist philosophies do not fulfil the relativist needs of an ethnomathematical position. However it is by examining those aspects of traditional philosophies which preclude relativism that the shape of a relativistic philosophy may emerge. Elaborated arguments explaining the inadequacy of both traditional and recent philosophies for the degree of relativity required by ethnomathematics are given in Barton (1996), or, in a brief version, in the paper prepared for the ICEM1 conference and available on a CD of the Proceedings. The present paper begins by skimming lightly over the major problems, and explaining why even twentieth century positions which seem to embrace some form of relativism fall short of what is required. In the latter half of the paper an alternative way of thinking about mathematical development is described which may lead to the kind of foundation needed for ethnomathematics.

2 Existing Philosophical Positions

The essential feature of realism which makes it unsuitable as a philosophical basis for a cultural understanding of mathematics is the requirement for a universal, *a priori* basis for truth, i.e. a pre-existing world of mathematics. It could be argued that it is not necessary for a cultural conception of mathematics that realism is rejected. It could be that mathematical objects are absolute, but that they are only knowable through human faculties which depend on various factors, including culture. Cultural views of mathematics are therefore an expression of the inadequacy of human understanding of this ideal world.

However, mathematics would then have to be conceived as a cultural approximation to the truth. One consequence is that any such cultural mathematics can be measured in terms of closeness to the ideal—which allows colonial, ethnocentric categorisations of primitive mathematics or sophisticated mathematics, etc. This, indeed, is exactly what some would argue has happened.

Opposed to a Platonist view is one in which mathematics is regarded as a product of human thought. Aristotle's conception of mathematics as the abstractions that the human mind derives from the physical world, and Kantian notions of mathematics in the organising power of the mind, are precursors to logicism, intuitionism, and formalism positions which change the philosophical orientation from "What is mathematics?" to "How can we be sure about mathematical truths?" (Tymoczko 1986, p. xiv). With respect to relativity, the point of all the efforts to establish these positions has been to secure their foundations so well that there is no room for doubt about mathematics, to eliminate the possibility of more than one (competing) conception. Thus logicism, intuitionism, and formalism call upon, respectively, a universal logic, a universal power of intuition, or a universal understanding of form.

Recent philosophical writing appears to have created room for relativistic notions, and thus, it might be assumed, has created an opportunity for establishing a cultural basis for mathematics. The neo-realists, Lakatos and the emerging quasi-empiricists, and the mathematical sociologists are all concerned more with what it is that mathematicians actually do than the status of the mathematics with which they work. Bringing the human (and therefore the social and cultural) element into the philosophy of mathematics has raised relativity as an issue: a spectre for some, a working assumption for others.

D'Ambrosio (1987, p. 30) mentions Bachelard as one possibility. Bachelard, writing in the 1930's, describes a historically relative notion of objectivity which gives rise to changing conceptions of mathematical objects and of rationality itself (Smith 1982; Tiles 1984). Bachelard's key idea is that objectivity is an ideal rather than a reality. At any time we may think we know how to discover truth or that we understand what makes a proof, but these ideas change over time: the sense of objectivity is illusory. However there is a progression towards a better, and then still better, understanding of what objectivity "really" is.

Thus there are many different historical standpoints from which to view mathematics, and each is correct *at that time*, and each explains previous views. Mathematicians are all aware of the demand for rational thought, and it is this which makes it possible to have a changing mathematics which always retains the objectivity required of the discipline.

Thus mathematics may be historically relative. However problems remain for cultural relativism because the changes are evaluated as progressive, i.e. it is directioned to one increasingly objective conception, against which previous conceptions are seen to be inadequate. Ethnomathematics, on the other hand, requires simultaneous progress in different directions under an assumption of equal validity/objectivity.

Neo-realism (e.g. Maddy 1990; Resnik 1993) similarly falls down when two different mathematical worlds meet: it is assumed that they would need to be resolved into a "best" view, although the criteria are not specified. Fallibilists and quasi-empiricists (Tymoczko 1986) at least specify the criteria in such a situation: for fallibilists two mathematical worlds are assumed to contradict each other, so one (or a new mathematics) must emerge as less contradictory; for quasi-empiricists different mathematical worlds would be measured against experience or useful applications. All of these philosophies require a resolution between different mathematical conceptions: simultaneous mathematics in cognisance of each other is impossible in the long term.

Two sociologists of mathematics are also often appealed to by writers in ethnomathematics. They are Bloor (1976) and Restivo (1993). Unfortunately neither of these elaborate a philosophy of mathematics which will support their sociology (admittedly neither is trying to do that). Bloor retreats into metaphysical belief, and Restivo assumes a position close to that of the quasi-empiricists.

Basically, the problem is that they all imply the existence, or the ideal, of some kind of mathematics (or some ideal criteria with which to judge mathematics) "out there", separate from culture. Whenever different conceptions arise, they must be resolved by appeal to this ideal towards which all development is heading. Cultural relativity may happen temporarily, but there is still the presumption that everyone doing mathematics is trying as hard as they can to approach something which is pure and culture-free.

It is as if the mathematics within each culture is a shadow of the "real" mathematics. As cultures interact, that mathematics which is more developed (closer to the "real" one) will subsume the other, and an illusion of one mathematics developing towards a universal perfection is maintained.

However the idea that cultural relativity in mathematics results from imperfections in the culture does not help to explain the problem which gave rise to ethnomathematics: that of apparent culturally-based mathematics failure. It would allow us to say that some cultures are "seeing" mathematics more truly than others, and the hegemony would continue.

3 An Alternative Model

A much more radical version of mathematical relativity is required. In this version it must make sense to talk about Maori mathematics, or English mathematics, or carpenters' mathematics. This writer has always shied away from such phrases because the use of the word "mathematics" presupposes a whole family of preconceptions which are almost impossible to ignore. D'Ambrosio is ambitious enough to force the use of "mathematics" in this wider meaning. This paper introduces the phrase "QRS system".

A QRS system is a system of meanings by which a group of people make sense of *Q*uantity, *R*elationships, and *S*pace. So the model of mathematics being proposed here is one where each cultural group has its own QRS system.

It is easiest to think about this in an historical way. Imagine two groups who have developed independently of each other. Each has its own way of dealing with quantity, of expressing relationships, and of representing space. As the two cultures begin to interact with each other, their ways of talking and ways of doing things will be mutually translated as far as is possible into each other's systems. Gradually a merging of QRS systems is liable to take place, and, ultimately, it may happen that one will dominate, or that a new system will emerge, probably one which draws more heavily from one system than another. At the end of this process it will seem that both cultures have the same system. The important aspect of this picture is that there is no presumed external "mathematics" or rationality by which one system is judged better than another. This is entirely an internal process, a human process, a cultural process.

One way to think about this picture is to conceive of a range of mountains. Let the QRS system be a range of mountains which circumscribes the landscape of our culture on its horizon. If we are in another culture, then we also see a range of mountains, and they may appear very similar—which is not surprising because they are "about" similar features of our world. It may even seem that it is the same range seen from the other side. If, however, we examine the features of the range, we will see that it is not one range, but two, one behind the other. In silhouette they look the same. If it was possible, we should rise up in the air, and then we would see the two ranges of mountains, and in between we would see a green and fertile valley. As ethnomathematicians we are explorers in that valley, tracing the way the mountains relate to each other in a cross-cultural landscape.

4 Deconstructing the Past

There are huge problems with this picture for mathematicians (and most users of mathematics, and most mathematics teachers). The problems centre around the fixation with "truth" and with "discovery" in relation to mathematics. They will grant that mathematics may be generated by people, but that, for example, once you have a circle and a triangle drawn inside it based on the diameter, then it *must be* a right-angled triangle. Or that $5 + 7$ *must be* 12 no matter what words are used to say it. The modern version is that the beautiful pictures of Chaos theory and Julia Sets were there waiting to be discovered: no-one created them. What is more, much of this mathematics which has been "discovered" is amazingly useful in our physical world—it models that world in fundamental and "true" ways. If one wishes to retain a version of absolute truth, and of a discovery metaphor for doing mathematics, then this culturally relative picture will have to go. But perhaps there is another way of thinking about things.

It is suggested that a philosophy based on Wittgenstein may provide ethnomathematics with the position it needs in order to properly describe the objects of mathematics. The idea being referred to here is the Wittgensteinian idea that we talk mathematics into existence. Shanker's (1987) reading of Wittgenstein proposes that we focus on clarifying what we mean when we talk about mathematics, rather than trying to characterise mathematical knowledge. So, rather than arguing about whether mathematical knowledge is certain or fallible, we should recognise that it is created in our talk. Thus mathematics is neither a description of the world, nor a useful science-like theory. It is a system, the statements of which are "rules" for making sense in that system. (See Barton 1996 for more detail.)

For example, consider a circle. No-one has ever seen, or touched a circle, it is an ideal object. This prompted Plato to hypothesise a world inhabited by such ideal objects, thus, circles exist. Wittgenstein suggests that this is just a "way of talking": circles exist because—and only because—we talk about them. When we do talk about them as if they are real objects then it makes sense to talk as if they had properties, but we should recognise that this is just a convenient figure of speech—literally. When we do not talk about them, they do not exist. In those languages where roundness is embodied as an action, not as an object, circles do not exist. A different QRS system applies. So mathematics is not *about* anything, it *is* away of talking.

Consider negative numbers. For two hundred years many important mathematicians denied the existence of negative numbers. They were right. For mathematicians who would not talk about them (or write them) negative numbers *did not* *exist*. It was only as they began to be used, and their properties discussed, that they were talked into existence.

Similarly, during the development of calculus there was huge debate about infinitesimals. Lakatos' (1978) article on the subject makes it very clear that the protagonists were talking past each other. They were talking about different mathematical worlds—one in which infinitesimals *did* exist, and another different mathematical world, in which they *did not* exist.

Now as well as talking things *into* existence, it is possible to talk things *out of* existence. This, it seems to me, is exactly what imperialist mathematics is all about. Mathematical (let us say QRS) concepts are talked out of existence by being ignored, by being superseded, by not acknowledging that that language is mathematics. And Lo! It isn't mathematics (some examples are discussed below).

Another problem to be dealt with is what has been called "the surprising usefulness of mathematics". If mathematics is simply an arbitrary human creation, then how is it that mathematics corresponds to our world so well, and is so useful in it? Furthermore, how is it that there is one world-wide subject called mathematics? The explanation is parallel to that used to explain why many different types of animal have all evolved with an eye, when their common ancestor had nothing remotely resembling an eye. How can evolution, which thrives on divergence, be so convergent? The answer is that things evolve in response to their surroundings. It is true for all types of animals that, if they have a light-sensitive organ, then they are at an advantage. What's more, if that organ is increasingly sharp and specialised, then that animal has an even greater advantage.

So it is for mathematics. Quantification is a powerful tool in social organisation. Thus cultures which develop the notion of quantification are rewarded for doing so, and tend to develop it further, and develop it in ways which correspond to the world in which they live. As cultures increasingly inhabit the same social world, it is not surprising that the QRS systems they evolve will become like each other.

5 What Is the Evidence?

Well, these are interesting ideas, and are the beginnings of a philosophy, but it would be nice to have some evidence for them—particularly in the face of sceptics who point to a single (almost-) universal mathematical world. Fortunately there is some evidence, if only we are willing to see it.

The first piece of evidence is that embedded in the languages we speak. In everyday English number words are describing words, they act like adjectives. "Three glasses" has the same form as "red glasses" or "tall glasses". In mathematics number words become nouns—they refer to objects. "Three" is a thing in itself.

In New Zealand it has just been realised that, in traditional Maori (but not in the modern Maori spoken today), number words are action words, they act like verbs. In order to say "three glasses" you must say "the glasses are three-ing". This is also true of some American Indian languages (Denny 1986). However Denny also describes how, in Inuktutit (the Inuit language), number words are naming words, they are nouns. In order to say “the three glasses” you must say “the glass set-of-three”. “Glass” is the adjective.

To me this is evidence that those speaking these other languages think about quantity in a fundamentally different way—and that way has been talked out of existence, or, at the least, it has been talked out of existence as mathematics. For example, consider the traditional questions in the philosophy of mathematics: how do we come to know about mathematical objects? But if the way we talk about number or space uses action words, how are we to make sense of a question about mathematical *objects*? Our QRS system will not be admitted into the realm of things being considered by that philosophy.

The second piece of evidence is one in which we can actually see two opposing mathematical worlds in action. That is, within mathematics there is a contemporary example of simultaneous co-existence. In statistics there are two different conceptions of probability: the Frequentist, and the Bayesian. The Frequentist conception has probability as the result of a long run of similar events, the Bayesian one has probability as a unique function of each event, about which we may have some prior information. Each conception gives rise to its own method of dealing with questions involving probability. In the main, these methods coincide in their results. But it is possible to construct problems which one or other method will give a sensible answer to, and the second method gives no result, or a contradictory one.

This demonstrates clearly that probability is not a “real” thing. It is a human construct, and how we construct it affects the results we obtain. Neither of these worlds is right or wrong. They are simply useful or not.

6 Exciting Horizons

Establishing a firm foundation for conceptions of ethnomathematics leaves us with exciting new directions to follow. It also encourages us to pursue depth in our concepts of ethnomathematics. This can be thought about in several ways.

One way is to see ethnomathematical investigations as going *below the surface*. The above language example encourages us to look, not at the different number words, but at the way they function in the language, i.e. not to see the surface feature of the language, but to examine the QRS system that is implied by those features. Thus rather than examine what base system is used to construct these number words (that is to impose a particular concept of quantity on them), we should examine the concept of quantity which they carry.

Similarly, with weaving patterns it is interesting to analyse the patterns evidenced in the art and crafts of different cultures (and this may have some educational uses although this writer regards that as an open question), but it is also important to examine the concept of symmetry employed by the weavers—a concept which may be different from that with which we are familiar.

Another way of pursuing depth is to ask “What if?”.

Much of Gerdes’ work (e.g. 1991) is in this vein: what if Lusona are analysed in the abstract, where will that lead me, what new mathematics might emerge. This is not to imply that Lusona drawers understood this mathematics, rather it is to carry out a QRS extension by whatever means is possible to develop new ideas.

A second example in the "what if" category is that generated by the navigation practices of indigenous Pacific navigators. Amongst their many skills was the ability to sit in their boat and, without watching the water, to sense the size and frequency of swells from eight different directions (Kyselka 1987). Mathematically the resolution of waves in one direction is a well-developed field, but 3-D wave analysis is largely an unsolved problem. What would have happened if the scientific effort and resources which went into developing latitude and longitude as a system of navigation, had been turned instead into the 3-D wave analysis problem. Perhaps we would have a shipboard system which would tell the captain when rocks, shoals, or Titanic ice-bergs were in the boat's path.

A final example is the orientation of geometric thinking. Those of us with conventional mathematical backgrounds tend to think in rectilinear grid systems—our graphs are drawn with axes as verticals and horizontals, our talk is full of "ups and downs". Many weavers of indigenous crafts, however, orient themselves to diagonal systems: weaving on the diagonal is an easier technique in many situations. What mathematical functions would interest us if our graphs were drawn using diagonal axes?

Whether "below the surface" or "what if", I am sure of one thing: ethnomathematical research will continue to surprise us, and at the least expected moments, of course. If we conceive of mathematics in a consistently relativist way, then we may be better able to "see" the hidden valleys between our mathematics'. Ethnomathematical research may then be more interesting, more productive, and more useful to both mathematicians and educators.

References

Barton, B. (1996). *Ethnomathematics: Exploring cultural diversity in mathematics*. Unpublished PhD thesis, The University of Auckland, New Zealand.

Bloor, D. (1976). *Knowledge and social imagery*. London: Routledge & Kegan Paul (page references are to the 2nd ed., Chicago: University of Chicago Press (1991)).

D'Ambrosio, U. (1987). *Etnomatematica: Raizes Socio-Culturais da Arte ou Tecnica de Explicar e Conhecer*. São Paulo: Campinas.

D'Ambrosio, U. (1990). The role of mathematics education in building a democratic and just society. *For the Learning of Mathematics*, *10*(3), 20–23.

Denny, P. (1986). Cultural ecology of mathematics: Ojibway and inuit hunters. In: M. P. Closs (Ed.), *Native American mathematics* (pp. 129–180). Austin: University of Texas Press.

Gerdes, P. (1991). Exploration of the mathematical potential of SONA: An example of stimulating cultural awareness in mathematics teacher education. In *Proceedings 8th symposium of the Southern Africa mathematical sciences association*, Instituto Superior Pedagógico Moçambique, Maputo.

Kyselka, W. (1987). *An ocean in mind*. Honolulu: University of Hawai'i Press.

Lakatos, I. (1978). Mathematics, science and epistemology. In J. Worrall & G. Currie (Eds.), *Philosophical papers*, Vol. 2. Cambridge: Cambridge University Press.

Maddy, P. (1990). *Realism in mathematics*. Oxford: Clarendon Press.

Resnik, M. D. (1993). A naturalized epistemology for a platonist mathematical ontology. In S. Restivo, J. P. Van Bendegem, & R. Fischer (Eds.), *Math worlds: Philosophical and social studies of mathematics and mathematics education*. Albany: State University of New York Press.

Restivo, S. (1993). The social life of mathematics. In S. Restivo, J. P. Van Bendegem, & R. Fischer (Eds.), *Math worlds: Philosophical and social studies of mathematics and mathematics education*. New York: State University of New York Press.

Shanker, S. G. (1987). *Wittgenstein and the turning-point in the philosophy of mathematics*. London: Croom Helm Ltd.

Smith, R. C. (1982). *Gaston Bachelard*. Boston: Twayne Publishers.

Spengler, O. (1956). Meaning of numbers. In *The world of mathematics*, Vol. *IV* (pp. 2315–2347). London: George Allen & Unwin Ltd.

Tiles, M. (1984). *Bachelard: Science and objectivity*. Cambridge: Cambridge University Press.

Tymoczko, T. (Ed.) (1986). *New directions in the philosophy of mathematics: An anthology edited by Thomas Tymoczko*. Boston: Birkhäuser.