Chapter (23)

Fuzzy Logic Control for Dialysis Application

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CHAPTER OUTLINES

- Introduction
- Modeling paradigms
- Fundamental of fuzzy control
- Fuzzy logic models
- Fuzzy control of dialysis session
- Comparison with other approaches
- Clinical results
- Diffusion of dialysis control system
- Future directions
- Conclusion

CHAPTER OBJECTIVES

- To understand the principles of fuzzy control.
- To understand hemodynamic instability in dialysis.
- To apply fuzzy control to prevent dialysis hypotensive episodes.

KEY TERMS

- Modeling paradigms
- Fuzzy logic
- MIMO system
- Fuzzy logic controller
- Hemodialysis
- Blood pressure
- Blood volume
- Ultrafiltration rate
- Conductivity
- Plasma refilling

ABSTRACT

This chapter introduces the fuzzy control approach for a dialysis session. Due to the complexity of the human system, the classical control methods, like PID, can fail to reach the target, mainly for what it concerns the stabilization of the system, which can induce sudden and undesired hypotensive collapses. To this purpose, a heuristic strategy based on expert rules, as fuzzy logic control, can help to reach the desired performances, reducing undesired collateral effects and increasing the potentiality of the dialysis session.

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23.1 INTRODUCTION

Automate control of many manual activities is nowadays possible and advisable, thanks both to great progress in hardware evolution and the relative cost fall-down, and to the availability of powerful algorithms for optimization and control of complex multi-dimensional systems, included the human body and many clinical operations that can be applied. To this purpose, in the following paragraphs, an innovative controller which has been applied by some Medical Center to optimize the dialysis parameters will be described. This system is based on fuzzy logic, a “new way” of thinking and managing with non probabilistic uncertainty (Bellazzi et al. 1994; Degani and Pacini 1980; Kageyama et al. 1990; Linkens et al. 1996; Mitra 1994; Moller 1993; Roy and Biswas 1992). After a brief introduction on fuzzy logic based control, the fuzzy logic hierarchical controller for hemodialysis will be described. This system will automatically optimize time by time the values of the therapeutic variables used during a dialysis session.

23.2 MODELING PARADIGMS

Models are developed to facilitate the solving of real world problems. Modeling is also a part of the learning process. It is an iterative, continual process of formulating hypotheses, testing and revision, of both formal and mental models (Sterman 2000). Modeling is not an individual effort. It requires active involvement of the decision makers and the individuals who are familiar with the system. Inputs from these subject matter experts ensure the integrity of the model structure with the actual system structure. The modeled interrelationships among variables depict mental models of the decision makers on which the decisions are based in the real world. Modeling activity has to follow a systematic step by step approach else our efforts can very easily digress from the main problem under consideration. Predictive modeling, also called predictive learning, consists in estimating an unknown dependency from known observations. Once the dependency has been estimated, it can be used to predict the response for future input data. Therefore, the basic objective of a predictive modeling procedure is to seek a model, using only a finite set of resources, with low prediction error. The prediction error is usually called generalization error, since it measures the capacity of the model to generalize, i.e. to return a good prediction of the output for input values not used during the modeling process. Inferring a predictive model from data is inherently an "ill-posed" problem due to the lack of knowledge about the underlying dependency and the finiteness of available data. This means that many models can often fit a given finite data set, and yet these models might generalize very differently on new data drawn from the same distribution. To make the modeling problem well-posed, one needs to somehow calibrate the complexity of the model to the amount and quality of available sample data. This is described by the bias/variance tradeoff which states that the number of free parameters should be kept to a minimum in the model, thus reducing the fit to the noise on the data. To find the best balance between accuracy and complexity of the model, a search is necessary in the space of all possible models belonging to a fixed family. This search for the best model involves three main tasks: choosing the best structure, choosing the best set of parameters given the structure, and validating the resulting model. Actually, the generalization capability is heavily reliant on the model's structure, and hence structure identification is arguably the most important task in a modeling process. Instances of structure identification include the problem of choosing the degree of a polynomial model or the problem of determining the best number of hidden nodes in a neural network.

Finding the best structure requires a search over all possible structures. However, exhaustive search over the space of model structures is computationally infeasible and motivates the use of heuristic strategies that dramatically reduce the search complexity by employing directed search algorithms. Examples are pruning/growing algorithms that start with a large/small structure and then prune/grow it to obtain a smaller/larger one. The second modeling task, i.e. finding a good set of parameters, is typically accomplished by minimizing an objective function. Examples of parametric identification procedures are linear least-squares for linear models and gradient-descent techniques. After the apparently best model has been found, its quality must be evaluated through a model validation procedure which returns the estimate of the generalization error on the basis of the finite training set. Examples of validation techniques are resampling methods, such as the holdout, the cross-validation and the bootstrap. Alternatively, the ability to generalize can be measured through complexity-based criteria that penalize the accuracy of the model by its size, hence adhering to the principle of parsimony. Therefore, if the model assessed on the basis of the estimate produced by the validation step is found inadequate, then the selected structure should be revised and the modeling process is repeated until a valid model is found. Hence the model should also be interpretable, so that the user can gain insight and improved understanding about the process that produced the data. To this end, the modeling process should attempt to induce a parsimonious representation of the available input-output data. However, accurate (and hopefully interpretable) predictive models are not necessarily simple to identify. Model construction is typically a non-trivial task especially for complex high-dimensional problems. Models of phenomena found in everyday life are usually derived from two fundamental sources: numerical data acquired from observation and a priori knowledge about the phenomenon. These two knowledge sources are invaluable in any modeling process, where all available a priori knowledge should be utilized, while inadequacies found in this knowledge can be compensated by the ability to learn from data. Depending on the extent to which these two kinds of knowledge are exploited, three basic levels of model synthesis can be defined (Ljung 1999):

- **White Box.** The model is completely constructed from a priori knowledge and physical insight. Here, empirical data are not used during model identification and are only used for validation. Complete a-priori knowledge of this kind is very rare, because usually some aspects of the distribution of the data are unknown.
- **Gray Box.** An incomplete model is constructed from a priori knowledge and physical insight, then available empirical data are used to adapt the model by finding several specific unknown parameters.
- **Black Box.** No a priori knowledge is used to construct the model. The model is chosen as a flexible parameterized function, which is used to fit the data.

The different modeling paradigms are summarized in Table 23.1 (Babuska 1998). In such a case, the most suitable approach is expected to be the gray-box one, even though none of the three modeling approaches can be easily applied, due to the lack of knowledge.

Table 23.1 Different modeling paradigms (Babuska 1998).

Modeling Approach	Source of Information	Method of Acquisition	Example	Deficiency
Mechanistic (white-box)	Formal knowledge and data	Mathematical	Differential equations	cannot use "soft" knowledge
Black-box	Data	Optimization (learning)	Regression, neural network	cannot at all use knowledge
Fuzzy (grey box)	Various knowledge and data	Knowledge-based + learning	Rule-based model	“curse” of dimensionality

Actually, these three modeling paradigms are not as distinct as this classification suggests, and a general rule of thumb is to employ both qualitative a priori knowledge and empirical data to derive a model. Thus, the important requirement for any modeling technique is the ability to exploit available a priori knowledge. When such knowledge relies on some physical information describing some input-output behavior, an expert can often describe such behavior using natural language. A linguistic model is a knowledge-based representation of information; its rules and input-output variables are described in a linguistic form which can be easily understood and handled by humans. The fuzzy set theory formulated by Zadeh in 1965 provides an appropriate method for handling linguistic terms and human concepts (Zadeh 1965). Zadeh's proposal to build models based on such linguistic descriptions through fuzzy values (fuzzy sets) rather than crisp numbers led to fuzzy systems. In recent years, fuzzy logic based modeling, and more generally linguistic modeling of complex processes, as a complement to conventional modeling techniques, has become an active research topic and found successful applications in many areas (Harris 2006; Ross 2004; Terano et al. 1994). The main advantage of fuzzy systems is that they can provide simple intuitive for interpretation and prediction in the form of fuzzy rules. However, due to vagueness and subjectivity of natural language statements, fuzzy rules based on qualitative knowledge alone can adequately model only very simple processes. Fuzzy models consist of a series of linguistic rules, which can easily be understood and constructed by humans.

23.3 FUNDAMENTAL OF FUZZY CONTROL

The basis for proposing fuzzy logic was that humans often rely on imprecise expressions like big, expensive or far. But the "comprehension" of a computer is limited to black-white, everything-or-nothing, or true-false modes of thinking. In this context, Lofti Zadeh emphasises that humans easily let themselves be dragged along by a desire to attain the highest possible precision without paying attention to the imprecise character of reality (Zadeh 1973). The basic idea of fuzzy sets introduced by Lofti Zadeh in 1965 is quite easy to comprehend. In a classical set, this is a collection of distinct objects in which dichotomize the elements of the universe of discourse into two groups, then:

$\mu_A(x) = 1$, if x is an element of the set A, and
$\mu_A(x) = 0$, if x is not an element of the set A

On the other hand, fuzzy sets eliminate the sharp boundaries that divide members from nonmembers in a group. In this case, the transition between full membership and nonmembership is gradual (a fuzzy membership function) and an object can belong to a set partially. The degree of membership is defined through a generalized characterized function called the membership function. The membership degree allows obtaining a desired level of smoothness about the threshold (set-points). Mathematically, a fuzzy set A is represented by a membership function defined on a domain X, called universe of discourse, given by:

$$\mu_A(x): X \rightarrow [0, 1]$$

Where A is the fuzzy label or linguistic (value) term describing the variable x. The values of the membership function are real numbers in the interval [0,1], where 0 means that the object is not a member of the set and 1 means that it belongs entirely to the set. Each value of the function is called a membership degree. Consider the fuzzy variable temperature, which can be described by many different adjectives each with its own fuzzy set. A typical partition of the universe of discourse, 0-40°C, is shown in Fig. 23.1, where the fuzzy sets cold, warm and hot are defined. In this example, the crisp temperature 20°C has a grade of membership of 0.5 for both the cold and the warm fuzzy sets i.e. $\mu_{cold}(20°C) = \mu_{warm}(20°C) = 0.5$. It is clear that the definition of fuzzy sets is non-unique for the nature of language, but it is very context-dependent and user specific (e.g. this definition may seem inappropriate to an Eskimo!). On specifying a membership function $\mu_A(x)$ in its present context the vague fuzzy label A is precisely defined.

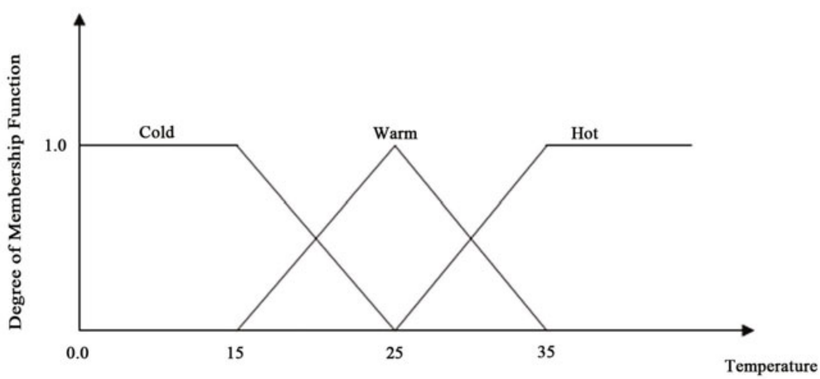

Fig. 23.1 Typical fuzzy sets defined for a variable.

Hence fuzzy sets can be thought as measuring the inherent vagueness of language precisely. The properties of these fuzzy sets play an important role in the modeling capabilities of the fuzzy system, and for a model to be truly transparent these sets should sensibly represent terms that describe the input and output variables. It is up to the system designer to determine the shape of the fuzzy sets. In most cases, however, the semantics captured by fuzzy sets is not too sensitive to variations in the shape; hence it is convenient to use simple membership functions. The membership function choice is the subjective aspect of fuzzy logic, it allows the desired values to be interpereted appropriatly. The most common membership functions are the triangular, the trapezoidal, and the Gaussian function. Fig. 23.2 shows some typical shapes of membership functions and described in detail in Appendix A. Fuzzy sets form a key methodology for representing and processing uncertainty.

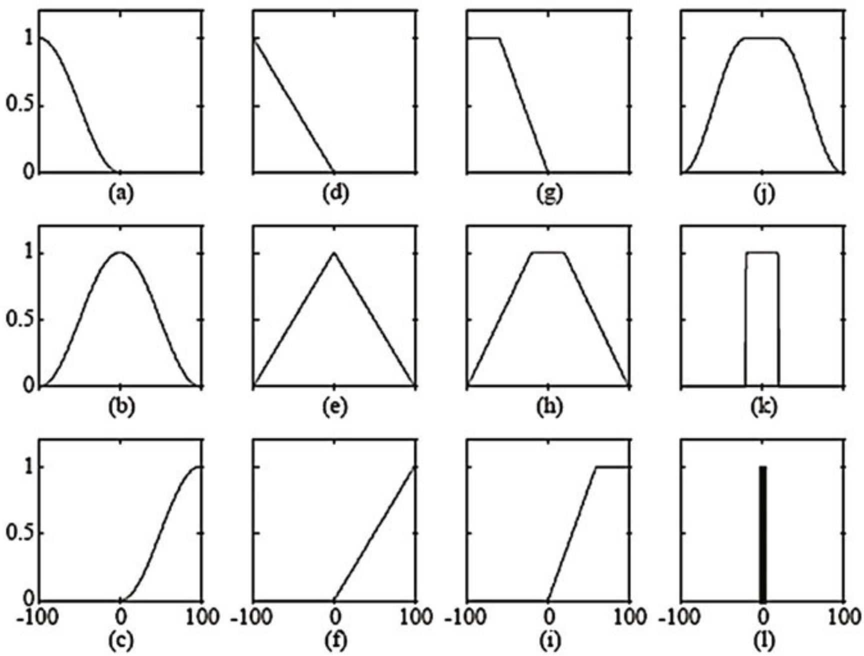

Fig. 23.2 Examples of membership functions (Fuller 2000). Read from top to bottom, left to right: (a) S-function, (b) gaussian-function, (c) Z-function, (d-f) triangular versions, (g-i) trapezoidal versions, (j) gbell-function, (k) rectangle, (l) singleton.

As such, fuzzy sets constitute a powerful approach not only to deal with incomplete, noisy or imprecise data but also to develop models of the data that provide smarter and smoother performance than traditional modeling techniques. The popularity and practicality of fuzzy systems derives from their ability to express complex relations in terms of linguistic rules. Thus, fuzzy systems have advantages of excellent capabilities to describe a given input-output mapping. The second important property to be considered concerns function approximation: fuzzy systems have been proved to be universal approximators (Kreinovich et al. 1998; Castro and Delgado 1996; Castro 1995), i.e. they are able to uniformly approximate continuous functions to any degree of accuracy on closed and bounded (compact) sets, a property they share with feed-forward neural networks. Moreover, in contrast with other universal approximators (e.g. neural networks) fuzzy systems are uniquely suited to incorporate linguistic information in a natural and systematic way. These two important properties qualify fuzzy systems as excellent candidates for predictive modeling tasks.

With these properties, the main disadvantage of fuzzy systems, however, is that they do not have much learning capability to tune their fuzzy rules and membership functions. Normally, fuzzy rules are decided by experts or operators according to their knowledge or experiences. However, when the fuzzy system model is designed, it is often too difficult (sometimes impossible) for human beings to define all the desired fuzzy rules or membership functions in an optimized way, due to the ambiguity, uncertainty or complexity of the identifying system. Also, fuzzy systems do not have any learning capability in which their fuzzy rules, along with their corresponding membership function, could be automatically tuned in order to reach the desired optimal fuzzy rules and membership functions.

23.3.1 Design of Fuzzy Logic Controller

Fuzzy Control System (FCS) is based on *heuristic* rules, acting on a set of input variables by means of *linguistic variables* to produce one or more control variables (as for Multiple Input – Multiple Output systems, MIMO for brevity). For this reason, it is not so easy to obtain some general rules for stability, overshoot, response time, etc., as for linear control systems. Some attempts were done in the past, and some results were obtained only for particular situation. Moreover, not only the fuzzy control is strongly not linear, but, it depends on many design choice, like the form of the fuzzy sets, which represent the linguistic variables, the choice of the aggregation operators for the rules, the number of the rules, and so on. Again, the mathematical analysis of a fuzzy control system is not easy in general, and in most cases it can be carried on with numerical simulations.

The basic structure of a fuzzy system, as described by Mamdani and Assillian (1975), is shown in Fig. 23.3 (Tanaka and Wang 2001). There are four main building blocks of the fuzzy logic system: the fuzzification interface, fuzzy logic rule base, inference engine and the defuzzification interface. A fuzzy system processes crisp data at the input and produces crisp data at the output through inference from a fuzzy rule base. Therefore a fuzzifier is used at the front of the system to convert crisp data to fuzzy sets, and a defuzzifier is used at the output of the system to convert fuzzy sets into crisp values. A rule-base is a set of If-Then rules, which contains a fuzzy logic quantification of the expert's linguistic description of how to achieve good control. The fuzzy inference engine combines the rules in the rule base according to approximate reasoning theory to produce a mapping from fuzzy sets in the input space to fuzzy sets in the output space. Hence a fuzzy system provides a computational scheme describing how rules must be evaluated and combined to compute a crisp output value (vector) for any input crisp value. One can therefore think of a fuzzy system simply as a parameterized function that maps real vectors to real vectors.

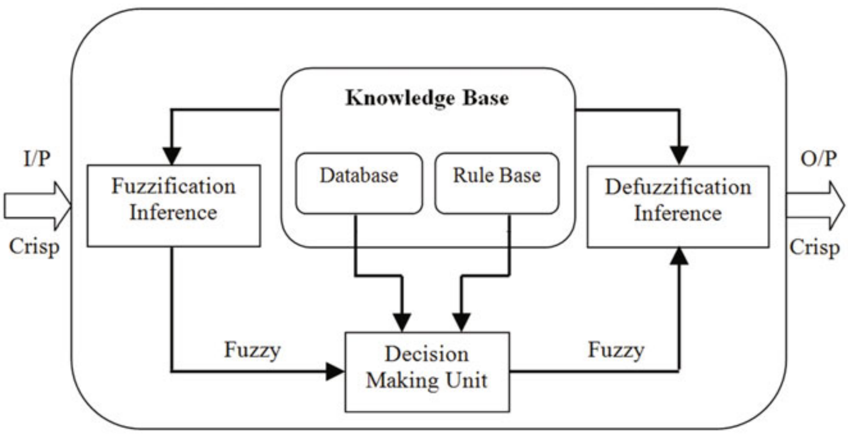

Fig. 23.3 The basic structure of a fuzzy system.

Unlike conventional control which uses a model of the controlled process and applies specific techniques to satisfy as best as possible the desired closed-loop behavior, fuzzy control tries to replicate an expert strategy, as usually done by human operator. As a matter of fact, if the model is unavoidable, or too much expensive to be obtained, or if the model itself is strongly not nonlinear, a mathematical analysis can fail, and it is preferable to use expert knowledge. At the same time, a linear control, like a classical PID controller, is not appropriate for a non linear plant. Moreover, for MIMO control the aggregation of different types of information and sources can be easily performed by a human operator, but the same cannot easily be done using a mathematical tool. Thus the idea of *fuzzy* control relies on this main reason: to capture in an efficient way the human expertise, from one or more process operators, and to use this knowledge to implement a performing control law. Technically, this task can be achieved by defining a set of rules over the logical combination of input variables, using some linguistic terms (*term-sets* like LOW, MEDIUM, HIGH, BAD, GOOD, etc.) in such a way that, for each combination of input variables, some rules fire with different degree. The firing degree will depend on the true degree of the antecedent, and on the logical combination of the linguistic degrees of true. Subsequently the outputs of each rule are combined, weighting more the ones for which the membership degree is higher, thus obtaining the values of the output variables. In this sense, we can speak of *rule-based fuzzy* control. Thus every FRCS (Fuzzy Rule Control System) is characterized by a suitable set of (fuzzy) *if-then* rules. For what above pointed out, the antecedent of each rule is formed by the logical disjunction of some elementary proposition, each of them referring to a single input variable. Such elementary propositions are in the following form: "X_i is $A_{i,j}$", where X_i is the numerical value of the i-th input variable, and $A_{i,j}$ is the term-set for the i-th variable in the j-th rule. Every term-set, which is linguistically expressed in the form of an attribute of the natural language, is represented by a suitable *fuzzy set* (Klir and Yuan 1995; Coletti and Scozzafava 2004) which defines its meaning. In the case of a control system, the input variables can be the error, its derivative or its integral. The same is done for the control variables. Each rule can be written as an inference condition, where in the antecedent part the logical conjunction of the input variables appears, while in the consequent part the control actions are described. For instance, a rule of a MIMO FCS with 3 input variables and 2 control variables can be written in the following way:

$$\text{IF } X_1 \text{ is } A_{1,j} \text{ AND } X_2 \text{ is } A_{2,j} \text{ AND } X_3 \text{ is } A_{3,j} \text{ THEN } C_1 \text{ is } B_{1,j} \text{ AND } C_2 \text{ is } B_{2,j}$$

where X_i, i=1,2,3 is the i-th input variables, $A_{i,j}$, i=1,2,3, is the linguistic term-set referring to the i-th input variable in the j-th rule, C_i , i=1,2 is the i-th control variables, and $B_{i,j}$, i=1,2, is the linguistic term-set referring to the i-th output variable. The complete FCS can be represented by a set of such rules: the Rule Data Set (RDS for brevity in the sequel). More in general, a RDS can be formally represented as follows:

$$\left\{ \bigcap_{i=1,..,n} A_{i,j}(x_i) \rightarrow \bigcup_{k=1,..,m} B_{k,j}(y_k) \right\}, j=1,..,NR$$

where NR is the number of inference rules. $A_{i,j}(x_i), B_{k,j}(y_k)$ are the values of true of the propositions x_i is $A_{i,j}$, y_k is $B_{k,j}$, and are computed by means of suitable pre-defined functions (*membership degree*) assigned by the user.

23.4 FUZZY LOGIC MODELS

Depending on the types of fuzzy reasoning and fuzzy if-then rules employed, most fuzzy inference systems can be classified into three types (Babuska 1998). From a technical point of view, the two most used FCS implementations are based on the *Mamdani* FCS, usually used as a *direct* closed-loop controller, and the *Takagi-Sugeno-Kang (TSK)* fuzzy controllers, more often used as a *supervisory* controller (Yager and Filev 1994)[1].

23.4.1 Mamdani Fuzzy Models

This method was introduced by Mamdani and Assilian in 1975 as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators (Mamdani and Assillian 1975). This is first vision of fuzzy models, and by far the most innovating one, assumes to represent an input/output mapping by means of a collections of IF-THEN rules whose antecedents and consequences utilize fuzzy values, i.e.:

$$R_k : \text{IF } (x \text{ is } A^k) \text{ THEN } (y_1 \text{ is } B_1^k) \text{ AND....AND } (y_m \text{ is } B_m^k)$$

The use of linguistic terms in consequent parts makes these models very intuitive and understandable. This class of fuzzy models uses fuzzy reasoning and forms the basis for qualitative modeling, which describes an input-output mapping by using a natural language. The Mamdani model form falls into this category. When adopting this perspective, which pursuits the ultimate goal of fuzzy logic, i.e. "computing with words", the emphasis is put essentially on the readability of the model, rather than on computational cost and accuracy of the model (i.e. fine quality of approximation, classification or control). The advantages of the Mamdani fuzzy inference system: it’s intuitive, it has widespread acceptance and it’s well suited to human cognition. However, fuzzy models of this class tend to become complex, requiring too many parameters, hence they can become heavy to

[1] Mamdani and TSK, are not only used for implementing a FCS, but more in general to represent a fuzzy-rules data base.

run, to maintain and to manually tune. This leads to the prepositions of other types of fuzzy inference systems. The fuzzy reasoning procedure for Mamdani model is shown in Fig. 23.4.

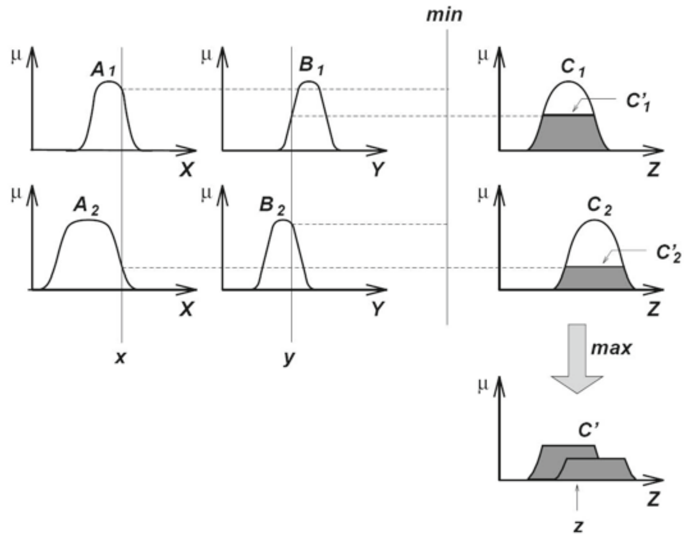

Fig. 23.4 Mamdani fuzzy inference system using the min and max operators (Adapted from Jang JSR, Sun CT (1995) Neuro-Fuzzy Modeling and Control. Proceedings of IEEE; 83(3): 378-406, with permission from IEEE)

For a MIMO controller, the i-th rule of the rule data base appears as follows:

IF $x_1$is $A_{i,1}$AND $x_2$is $A_{i,2}$.... AND $x_n$is $A_{i,n}$THEN $y_1$is $B_{i,1}$... AND $y_m$is $B_{i,m}$

where $i = 1, 2, ..., R$; A_{ij} is the term-set of the j-th input variable for the i-th rule; $B_{k,i}$ is the term-set of the k-th output variable and R is number of inference rules.

Both A_{ij} and $B_{k,i}$ are represented by suitable fuzzy numbers. To obtain the value of the crisp output $y_1 \dots y_m$, the steps are the following:

1) Compute the *degree of truth* of the elementary propositions: x_j is A_{ij}; if $\mu_{ij}(z)$ is the membership function concerning the linguistic term set A_{ij} , it is sufficient to compute $\mu_{ij}(x_j)$,

2) Compute τ_i, the degree of truth of the compounded proposition x_1 is A_{i1} ..... AND x_n is A_{in}, that is the logical conjunction of n elementary propositions; this task is performed by a suitable aggregation operator $\otimes$ (see the comments below):

$$\tau_i = \mu_{i,1}(x_1) \otimes \mu_{i,2}(x_2) \otimes \ldots \otimes \mu_{i,n}(x_n),$$

3) Compute the fuzzy quantity $\eta_{k,i} = \tau_i \wedge B_{k,i}$, the minimum between the crisp number τ_i and the fuzzy number $B_{k,i}$,

4) For each k, compute the fuzzy quantity $\xi_k = \eta_{1,k} \oplus \eta_{2,k} \oplus \ldots \oplus \eta_{R,k}$, that is the logical disjunction, or other aggregation operator, as the generalized mean. This is the fuzzy output for the k-th output variable,

5) Finally, to obtain a crisp number for all the m output variables, the *defuzzification* task is activated, obtaining a crisp value (a real number) from the fuzzy output, which encompasses all the available information obtained by the data and by the inference mechanism. To this purpose, many methods were proposed in the literature, the most commonly used in the "center of gravity" approach (Yager and Filev 1994).

Some comments are in order:

a) The conjunction $\otimes$ is a logical operator which can be extended from the *boolean* case to the continuous one through the so called *triangular norm* operators, T-norm for brevity (Klement et al. 2000), a monotonic function defined in $[0,1]^n \rightarrow [0,1]$, which is also commutative and associative, with one as the neutral element. The upper bound of the class of all the T-norm is the minimum operator, but other choices are possible, like the product or the Łukasiewicz T-norm, see the above quoted reference;

b) Similarly, the logical disjunction $\oplus$ can be represented by a triangular co-norm (S-norm for brevity), with the same property as for the T-norm, except the last one, since the neutral element is zero. The lower bound of the class of all the T-conorm is the maximum operator, but other possibilities exist, like the probabilistic sum, or the bounded sum[2]. Every S-norm is the dual operator of a T-norm, and namely the biunivocal relationship is:

$$S(x, y) = 1 - T(1 - x, 1 - y);$$

c) An important family of T-norm (S-norm) is formed by the *Hamacher* T-norm, which depends continuously on a tuning parameters which permit to obtain a wide range of T-norm (S-norm).

Note that for a Mamdani FCS, the following data need to be assigned:

i. Membership functions ,

ii. Rules (rule data base),

iii. T-norm and the S-norm used for the conjunction and disjunction respectively (sometimes other aggregation operators can be used).

Even if many, possibly, infinite, choices exist for the T-norm (S-norm) the most common are the min and the max operators respectively. The type of the inference rules, on the other side, is usually performed with an Expert (or more) of the application domain. Together with the memberships functions, the assignment of a T-norm (S-norm) and of the rule data base can be a long and stressing phase; sometimes a neuro-fuzzy algorithm could help if many input-output data are available, i.e. a collection of instances (we have in this case a non linear regression problem to be solved).

The Mamdani controller can be used to implement a fuzzy version of a classical PID controller, in this case, the input variables are the error, its derivative, and its integral.

23.4.2 Takagi-Sugeno-Kang (TSK) Fuzzy Models

The "Sugeno fuzzy model" (also known as the "TSK fuzzy model") was proposed by Takagi, Sugeno and Kang in an effort to develop a systematic approach to generate fuzzy rules from a given input-output data set (Takagi and Sugeno 1985). These models use fuzzy rules with fuzzy antecedents and functional consequent parts, thereby qualifying them as mixed fuzzy or non-fuzzy models. Such models can represent a general class of static or dynamic nonlinear mappings via a combination of several linear models. The whole input space is decomposed into several partial fuzzy spaces and

[2] The averaging operators, as the weighted mean, the generalized mean, the OWA operators, the Choquet integral with non additive measures are in between the min and the max operators; the family of the T-norm is a contiguous class of operators with furnishes lower values, while the family of the S-norm is a contiguous class of operators with furnishes higher values (w.r.t. an averaging operators).

each output space is represented with a linear equation. The resulting models are referred to as TSK models and are represented by a series of fuzzy rules of the form:

$$R_k : \text{IF } (x \text{ is } A^k) \text{ THEN } (y_1 = h_1^k(x)) \text{ AND....AND } (y_m = h_m^k(x))$$

Where $h_j^k(x)$; j =1,…,m are polynomial functions of the inputs and represent local models used to approximate the response of the system in the region of the input space represented by the antecedent $\mathbf{A}^k$. When $h_j^k(x)$; is a first order polynomial, the resulting fuzzy inference system is called a "first order Sugeno fuzzy model". When y is constant, the resulting model is called "zero-order Sugeno fuzzy model", which can be viewed either as a special case of the Mamdani inference system, in which each rule's consequent is specified by a fuzzy singleton, or a special case of the Tsukamoto fuzzy model (to be introduced next) in which each rule's consequent is specified by a MF of a step function center at the constant. The Sugeno fuzzy type of knowledge representation does not allow the output variables to be described in linguistic terms, which is one of the drawbacks of this approach. Hence, this class of fuzzy models should be used when only performance is the ultimate goal of predictive modeling. Each of these fuzzy models has inherent drawbacks. For Mamdani fuzzy models the defuzzification process may be time-consuming, and systematic fine tuning of the parameters is not easy.

For TS fuzzy models it is hard to assign appropriate linguistic terms to the rule consequence part, which does not use fuzzy values. Readability and performance thus appear as antagonist objectives in fuzzy rule-based systems. Some form of compromise can be found by using simplified fuzzy rules of the form:

$$R_k : \text{IF } (x \text{ is } A^k) \text{ THEN } (y_1 \text{ is } b_1^k) \text{ AND....AND } (y_m \text{ is } b_m^k)$$

Where b_j^k are fuzzy singletons (see def. A.6). Fuzzy models relying on such rules are referred to as singleton fuzzy models. This class of fuzzy models can employ all the other types of fuzzy reasoning mechanisms because they represent a special case of each of the above-described fuzzy models. More specifically, the consequent part of this simplified fuzzy rule can be seen either as a singleton fuzzy set in the Mamdani model or as a constant output function in TS models. Thus the two fuzzy models are unified under this simplified fuzzy model. Another interesting aspect of the simplified fuzzy model is its functional equivalence to the Radial Basis Function (RBF) network. This equivalence is established when Gaussian membership functions are used to describe the antecedents of rules. Sugeno system is suited for modeling nonlinear systems by interpolating between multiple linear models. Because it is a more compact and computationally efficient representation than a Mamdani system, the Sugeno system lends itself to the use of adaptive techniques for constructing fuzzy models. These adaptive techniques can be used to customize the membership functions so that the fuzzy system best models the data. Figure 3.9 shows the fuzzy reasoning procedure for a first-order Sugeno model. Since each rule has a numeric output, the overall output is obtained via "weighted average", thus avoiding the time-consuming process of defuzzification required in a Mamdani model. In practice, the weighted average operator is sometimes replaced with the "weighted sum" operator (that is, $w_1z_1 + w_2z_2$ in Fig. 23.5) to reduce computation further specially, in the training of a fuzzy inference system (Castillo and Melin 2001). However, this simplification could lead to the loss of MF linguistic meaning unless the sum of firing strengths (that is, $\sum w_i$) is close to unity.

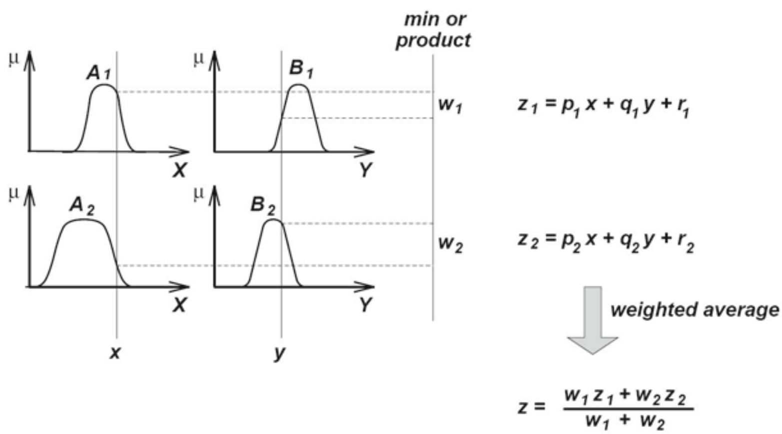

Fig. 23.5 First-order Sugeno fuzzy model (Adapted from Jang JSR, Sun CT (1995) Neuro-Fuzzy Modeling and Control. Proceedings of IEEE; 83(3): 378-406, with permission from IEEE)

Thus, the main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant. Also the difference lies in the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedures differ suitably. The number of the input fuzzy sets and fuzzy rules needed by the Sugeno fuzzy systems depend on the number and locations of the extrema of the function to be approximated. In Sugeno method a large number of fuzzy rules must be employed to approximate periodic or highly oscillatory functions. The minimal configuration of the TS fuzzy systems can be reduced and becomes smaller than that of the Mamdani fuzzy systems if nontrapezoidal or nontriangular input fuzzy sets are used. Sugeno controllers usually have far more adjustable parameters in the rule consequent and the number of the parameters grows exponentially with the increase of the number of input variables. Far fewer mathematical results exist for TS fuzzy controllers than do for Mamdani fuzzy controllers, notably those on TS fuzzy control system stability.

23.4.3 Tsukamoto Fuzzy Models

In the "Tsukamoto fuzzy models", the consequent of each fuzzy if-then rule is represented by a fuzzy set with a monotonical MF, as shown in Fig. 23.6 (Tsukamoto 1979; Castillo and Melin 2001). As a result, the inferred output of each rule is defined as a numeric value induced by the rule firing strength. The overall output is taken as the weighted average of each rule's output. Figure 23.6 illustrates the reasoning procedure for a two-input two-rule system. Since each rule infers a numeric output, the Tsukamoto fuzzy model aggregates each rule's output by the method of weighted average and thus avoids the time-consuming process of defuzzification. However, the Tsukamoto fuzzy model is not used often since it is not as transparent as either the Mamdani or Sugeno fuzzy models (Castillo and Melin 2001).

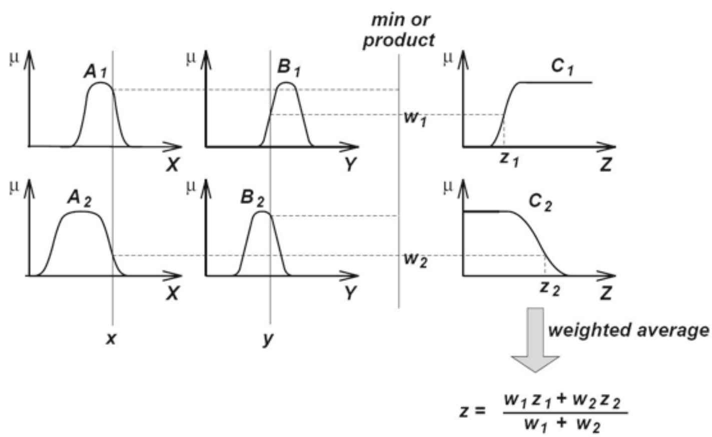

Fig. 23.6 Tsukamoto fuzzy model (Adapted from Jang JSR, Sun CT (1995) Neuro-Fuzzy Modeling and Control. Proceedings of IEEE; 83(3): 378-406, with permission from IEEE)

23.5 FUZZY CONTROL OF DIALYSIS

23.5.1 Automatic Control of Dialysis Session

A dialysis session lasts more or less four hours, in this period the removal of toxic substances must be sufficient to obtain a grade of renal insufficiency compatible with life; hyperkaliemia, metabolic acidosis and water overload must be corrected. This means that in few hours urea concentration drops of about 65%, serum potassium halves, bicarbonate rises of about 25% and at least 2 or 3 liters of water are removed. The impact of these alterations on body physiology is not negligible; as a matter of fact a hemodynamic instability often occurs during the dialysis session.

Water is primarily removed from blood, determining a reduction in circulating blood volume. This process is counteracted by the refilling from interstitial fluid that transfers water from the extracellular compartment to the intravascular one, thus restoring blood volume. This flow is primary determined by the increased osmotic pressure due to hemoconcentration. A pivotal role is due to serum proteins and sodium concentration that are the two most important osmotic agents. The physiologic response to hypovolemia involves also an increase in heart rate and a constriction of the arterial bed that causes an increase in vascular resistance. When these mechanisms fail, hypotension occurs.

In general mean arterial pressure is determined by cardiac output and systemic vascular resistance according to the law:

$$MAP = CO \times SVR \tag{23.1}$$

where MAP is mean arterial pressure, CO is cardiac output and SVR means systemic vascular resistance. Cardiac output reduction induced by hypovolemia may be corrected by fluid infusion or tempered by increasing Na concentration that enhances plasma refilling and/or slowdown of ultrafiltration rate. Systemic vascular resistance may be increased by the reduction of body temperature or by eliminating any vasodilators in the dialysis bath (acetate free dialysis bath) or by administering vasoconstrictor agents (rarely used).

Owing to the possibility of the described side effects, dialysis procedure usually requires a strict medical supervision, since it often produces severe undesired side effects. Hypotensive episodes are mainly caused by the unbalanced flow between ultrafiltration (UFR) and plasma refilling rate (PRR). An excessive sodium extraction or excessive UFR cannot be compensated by a proportional PRR, thus the blood volume (BV) is reduced, determining hypotension. Conversely, a too high sodium load or a too low UFR can originate a fluid overload followed by heart failure. Hence, it is necessary to obtain the best compromise. Usually, a doctor adjusts the two control variables, UFR and the *sodium conductivity* (Na) in the dialysate, aiming to satisfy some therapy targets, the most important of which are:

- The decrease of body weight to a pre-defined value, i.e. the removal of a predetermined excess of water,
- The desired sodium balance during and at the end of the session
- To avoid at the same time severe hypotension episodes, cramps and other undesired effects, while guaranteeing the removal of some toxic substances.

To avoid the fluid unbalance, the doctor tries to achieve a sufficient PRR (Churchill 1996). In so doing, medical experience is quite important to obtain the fulfillment of the objectives, on the other side, the manual control is resource consuming, and subject to personal consideration brought to human errors, due for instance to inattention or tiredness. To this aim, some model-based tools were developed to control the process, but the results were not completely satisfactory, because the models cannot account for the physiological processes involved in hemodynamic stability not directly measurable (Daugirdas 1991). Conversely an approach based on a fuzzy rule based control system is different and more suitable for real clinical applications (Giove 1998; Giove et al. 1993; Nordio et al. 1995, 1999). The most important feature of this method is that even the trend can be taken into account, because a frequent sampling is possible, thus predictive action can be implemented to avoid sudden changes.

The advantages of a fuzzy based approach are the following:

- instead of using a model-based approach, this method captures, implements, and replays the medical experience,
- it is almost completely model free, because only a predetermined weight reduction and sodium removal are fixed,
- the clinical knowledge is implemented using a MIMO fuzzy rule data base,
- it is based on a *hierarchical* strategy, and on a *heuristic* knowledge.

The diagram of the fuzzy logic support system for the dialysis session is depicted in Fig. 23.7.

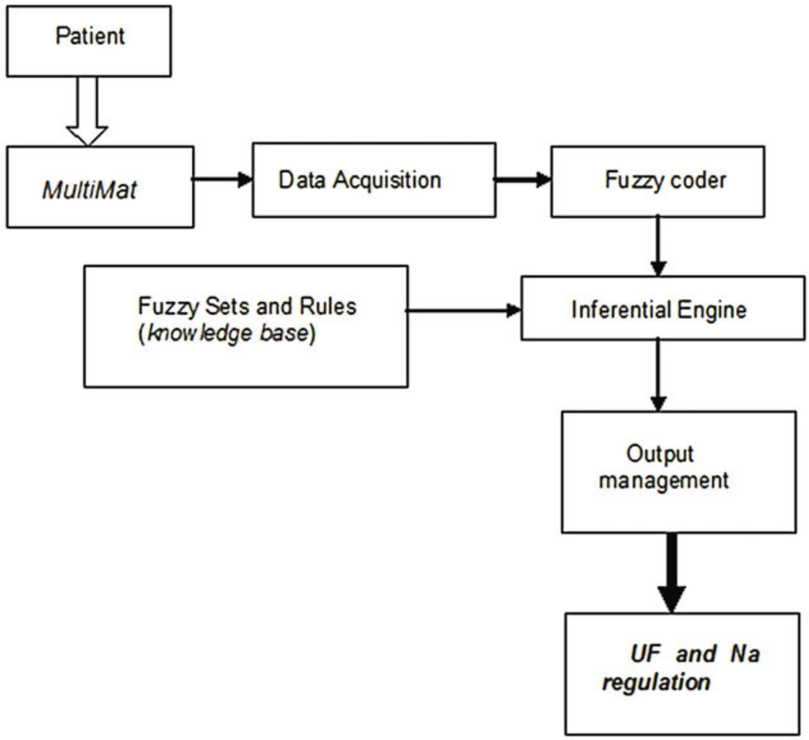

Fig. 23.7 Diagram of the UF and Na fuzzy regulation. MultiMat is a dialysis machine by Bellco (Mirandola, Italy) used in the Nineties to perform paired filtration dialysis.

23.5.2 Data Acquisition

The fuzzy control described in this section was performed using a particular technique of hemodiafiltration named "paired filtration dialysis" (PFD) in which the convective component occurs in a little hemofilter separated from the dialysis filter. The plasmatic water collected during the convective process was used to measure continuously the conductivity (and hence plasma sodium concentration). In a classical hemodialysis procedure the measurement of plasmatic sodium conductivity may be substituted by ionic dialysance with little changes in the rules.

A dialysis PFD machine has been used to collect the sampled data, namely blood pressure, blood volume, sodium conductivity, body weight, recorded in a PC every minute (except for blood pressure, acquired every 10 minutes), and processed every 10 min. The on-line sodium balance is computed by the measurements of plasma conductivity and dialysate conductivity, using a well-known mathematical model (Di Filippo et al 1996). A pre-elaboration procedure computes the trend using different methods (least squares, or the average on the last observed values, and other ones), and fuzzify the variables. The trend measures the tendency to increasing or decreasing. For instance, Fig. 23.8 reports the interpolating line of the sampled blood volume values; the trend is nothing else but the linear coefficient of the interpolating line.

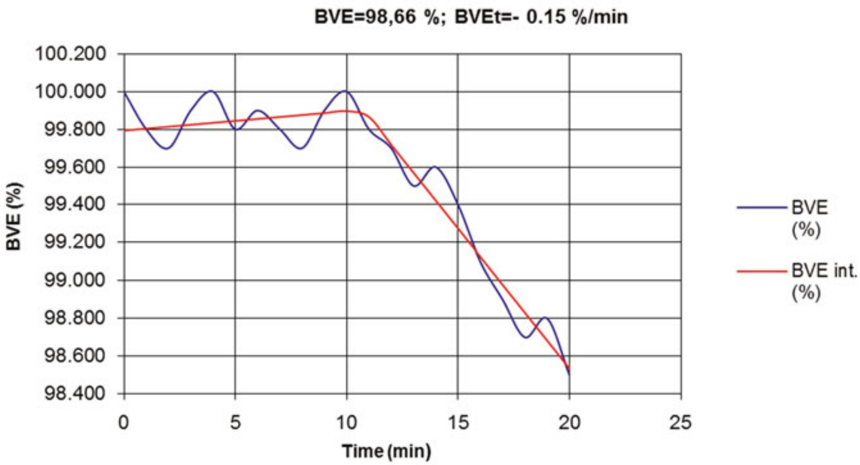

Fig. 23.8 Blood Volume Trend (BVE = blood volume error).

The variables and the relative trends are fuzzyfied. To this purpose, *trapezoidal* fuzzy sets were used, since they are easy to be obtained by expert's judgments using only 4 points; in fact a trapezoidal fuzzy sets [a,b,c,d] splits the real line into subsets where the linguistic term is completely true, completely false or uncertain. In particular, in all the points below a, and in all the points above d, the membership is null (the linguistic term in false), in all the points in between b and c the linguistic term in completely true, while in the two intervals [a,b] and [c,d] there is uncertainty, that is, the considered value belongs to the fuzzy set with membership linearly increasing from a to b, and linearly decreasing from c to d[3].

[3] The trapezoidal fuzzy number are particular case of L-R type fuzzy numbers (Klement et al. 2000), that are characterized by an increasing behaviour, followed by a constant interval, the *core* (possibly degenerating into a single point) and finally by a decreasing part. Triangular fuzzy numbers are trapezoidal numbers whose core is formed by a single point. Many other L-R type fuzzy numbers exist in the literature, like the Gaussian type, etc. Anywise, from many simulations it can be verified that the performances of a FIS do not significantly change if Gaussian type are used instead that trapezoidal, mainly if many variables and rules are involved.

This formalism is easy to explain to the Clinician and he/she can fulfill a sheet where the four points representing all the fuzzy sets, i.e. for each linguistic terms, can be easily be inserted.

Moreover, in this heuristic approach, there is not the necessity to introduce a tuning strategy, as in neuro-fuzzy controller (even if it could be done of course), since the required knowledge is directly extracted from the Clinician's expertise, and tested by off-line simulations.

Fig. 23.9 reports the fuzzy sets for the variables SBP (systolic blood pressure) and SBPt, its trend; the labels "VL, L, G, H, VH", and "N, Z, P", stand for "Very Low", "Low", "Good", "High", Very High" and "Negative", "Zero", "Positive" respectively.

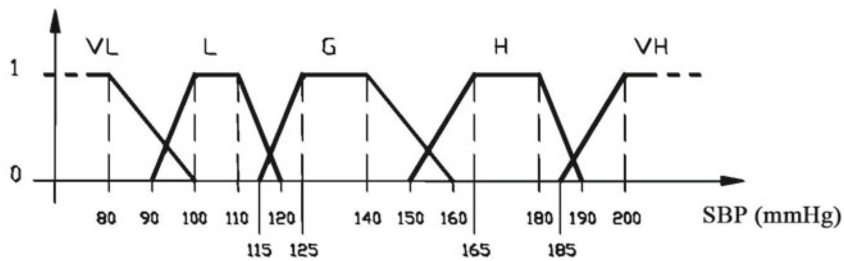

Fig. 23.9 (a) Fuzzy sets for systolic blood pressure

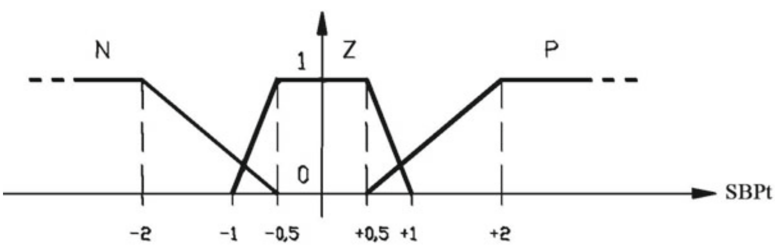

Fig. 23.9 (b) Fuzzy sets for systolic blood pressure trend

The input variables are (with the number of corresponding fuzzy sets within parenthesis): systolic blood pressure SBP (5) and its trend SBPt (3), blood volume changes BVC (3) and its trend BVCt (3), sodium balance error NaEr (3) and body weight error BWEr (3)

The output variables are the change of UFR, ΔUFR, and the change of dialysate conductivity, ΔDC, computed by the fuzzy engine, in order to achieve the desired performances.

23.5.3 The Fuzzy Logic Controller

The fuzzy rule system is a TSK-type inference MIMO controller, because it acts on the error and the error trend of several state variables to determine the optimal strategy[4]. It uses a *hierarchical* approach, defined by a fuzzy *meta*-rule data base. To this purpose, 4 sub-systems are defined; one for each controlled variable, and then the supervisory control determines the optimal mix of each sub-module proposed control values, taking the most important control objective into account. The control action is applied every 10 min., when all the state variables are collected, and the control values remains constant within each sampling interval $[T_i, T_{i+1}]$. The outputs of each rule are crisp singletons, like the ones in Fig. 23.10 for the variable ΔUFR.

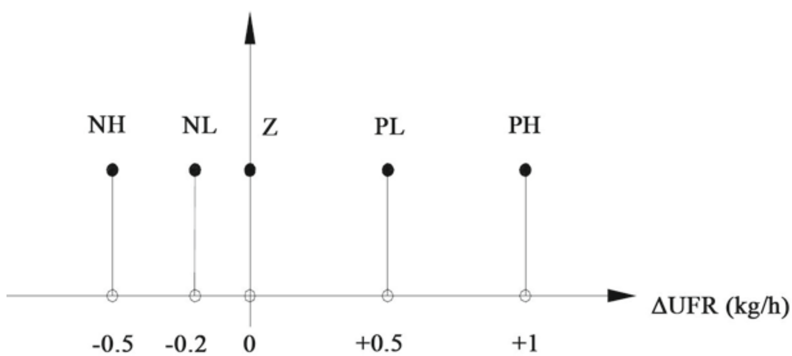

Fig. 23.10 ΔUFR crisp consequents

All the rules, in the form of an inference "if.., then" rule are collected in a tabular form, and can be modified by the user (he can also select all other optional parameters and items, like the T-norm). Table 23.2 reports the rules for pressure control. For instance, the four column-third row rule means "IF SBP is High AND SBPt is Positive THEN ΔUFR is Positive Low AND ΔCD is Negative Low". The outputs of each subsystem are subsequently aggregated by a super-visor rule block, using a defined priority strategy.

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[4] In this sense, it is not properly a PID controller, but it is very similar in the conceptual philosophy.

Table 23.2 SBP and SBPt Rule Table

SBP→ SBPt ↓	Very Low	Low	Good	High	Very High
Negative	ΔUFR=NH ΔCD=PH	ΔUFR=NH ΔCD=PL	ΔUFR=NL ΔCD=Z	ΔUFR=PL ΔCD=Z	ΔUFR=PL ΔCD=NL
Zero	ΔUFR=NH ΔCD=PH	ΔUFR=NL ΔCD=PL	ΔUFR=Z ΔCD=Z	ΔUFR=PL ΔCD=Z	ΔUFR=PH ΔCD=NL
Positive	ΔUFR=NH ΔCD=PL	ΔUFR=NL ΔCD=Z	ΔUFR=PL ΔCD=Z	ΔUFR=PL ΔCD=NL	ΔUFR=PH ΔCD=NH

23.5.4 The Priorities and the Integrated System

The priority of the actions to be performed was given by a set of weights computed using, again, a decision table of fuzzy rules. When both SBP and BVE were satisfactory, the tables for NaEr and BWEr were used to gain the objectives of the correct Na balance (predetermined by the doctor) and the dry weight of the patient. If SBP or BVE were not good, their proper actions were preferred. Anywise SBP had the maximal priority, followed, in order, by BVE, NaEr, and BWEr. For instance, referring to BWEr (characterized by the lowest priority), its weight is negligible if at least one of the other variables (SBP, BVE, NaEr) has a high weight. The aggregation of the outputs was obtained using a weighted sum:

$$\Delta\text{UFR} = \sum_{i=1}^{4} w_i \Delta\text{UFR}(i), \qquad \Delta\text{DC} = \sum_{i=1}^{4} w_i \Delta\text{CD}(i) \tag{23.2}$$

where ΔUFR(i) and ΔCD(i) are the output of each rule table and w_i are the weights calculated by the priority table (see Table 23.3). The values SBPB, SBPG, BVEB, BVEG (SBPB stand for “SBP is Bad”, SBPG stands for “SBP is Good”, and so on) are computed from the relative sub-module decision tables. For instance, the value SBPB is the maximum activation degree of the rules implemented in the Table 23.2, except the element in second row and third column (corresponding to “SBP is Good, SBPt is Zero”). The system was tested in 10 sessions, and all the patients gained the prescribed dry weight with a correct sodium balance. No hypotension episode was observed. In four cases the dry weight and a correct sodium balance were obtained reducing the dialysis time without significant changes in blood pressure.

Table 23.3 Priority Management Table

Pressure	Volemia	Output weighs	Output
SBPB		W_1 = SBPB	TabP
SBPG	BVEB	W_2 = min{SBPG,VVEB}	TabV
SBPG	BVEG	W_3 = min{SBPG,VVEG}	TabNa
SBPG	BVEG	W_4 = min{SBPG,VVEG}	TabDBW

TabP, TabV, TabNa and TabDBW are the outputs from the rule tables respectively of blood pressure, blood volume, conductivity and blood volume change.

23.5.5 Other Fuzzy Controllers

An alternative fuzzy controller was developed by Schmidt and others (Schmidt et al. 2001). In this case the input parameter is blood pressure, measured as relative difference of systolic pressure and pre-adjusted set point pressure, short term pressure trend (15 min) and long term pressure trend (25 min). The numeric values are fuzzified into linguistic variables described by trapezoidal or triangular fuzzy sets. Specific rules are used to obtained the corresponding fuzzy sets of UFR and the infusion rate of hypertonic saline (the output variables), finally converted into crisp outputs for adaptation of the variables to patient's actual blood pressure behavior. The control is provided by the transmission of these outputs both to the dialysis machine for UFR change and to a programmable infusion pump for hypertonic saline infusion. The underlying strategy is to establish a set point systolic blood pressure (90 – 100 mmHg) and a starting UFR (150% of the average UFR) in order to avoid excessive loss of water in the last part of the session. If the actual systolic blood pressure comes close to the set point, the system reacts first with injection of 20% saline, if blood pressure does not stabilize, then UFR is lowered. In the final phase the main reaction is to decrease UFR and sodium injection is avoided, to avoid a positive Na balance. This system was subsequently improved by substituting the hypertonic saline infusion pump with dialysate conductivity changes (Hickstein et al. 2009).

This system is conceptually simple: it looks for the signs of an incipient collapse and it reacts by increasing plasma refilling with an osmotic solution, if this maneuver fails, then UFR is considered excessive and consequently decreased. The system proposed by Nordio et al (1995) is more complex because both systolic blood pressure and blood volume changes are considered and weighted with rules that give the maximal priority to heavy blood pressure changes. The underlying strategy is to look for the best UFR profile for each patient. As a matter of fact some hypotensive-prone patients show a drop in blood pressure at the beginning of the dialysis session, in this case the problem is given by a inappropriate sympathetic response to UFR that corresponds to inadequate vasoconstriction. In this situation sodium is given and UFR is reduced in the meanwhile hemodynamic stabilization is obtained, at this point, with a stable blood pressure (SBPt near zero), UFR restarts. If blood pressure holds in the first and medium parts of the dialysis session the two fuzzy control systems are equivalent.

23.6 COMPARISON WITH OTHER APPROACHES

In order to better understand the flexibility and the advantages of a fuzzy controller, a comparison with a classical adaptive controller is proposed.

The controller targets prescribed are total body weight loss, equivalent dialysate conductivity and relative BV change (Santoro et al. 1996). The input parameters for the controller are the monitored discrepancies between the instantaneous actual value and the instantaneous desired target for BV change, dialysate conductivity and weight loss. The control variables (output parameters) are the instantaneous dialysate conductivity and weight loss rate, which can vary from instant to instant to reach the desired targets. This system aims at driving the BV reduction curve over time along a pre-set trajectory balancing the classical goals of removal of sodium and water excess (total body weight and equivalent conductivity targets) with the new goal of (relative BV change target).

The core of this closed-loop bio-feedback software is an error-based mathematical model. The net result produced by this system is that the three target parameters (relating to water balance, sodium balance and desired BV reduction curve), decided by the operator at the beginning of the sessions, are smoothly driven, during the HD session, through a precise three-dimensional curve that represents the best compromise among the targets themselves. This operation is performed with a given range of tolerances for each parameter, decided by the operator as a safety feature.

The control system acts as a BV controller that continuously modifies the instantaneous UF rate and dialysate conductivity, while guaranteeing sodium and water balance. The rationale of this system is to smooth out the acute and sudden reductions in BV that can appear during HD sessions, consequent to a transient imbalance in the patient's vascular refilling capacity, in order to try to reduce the incidence of intra-HD hypotension episodes.

Both the fuzzy and the classical system just described are MIMO controller. The input variables of the fuzzy system are systolic blood pressure SBP and its trend, blood volume, blood volume changes, sodium balance error and weight error. The classical adaptive system input variables are the discrepancies between the instantaneous actual value and the instantaneous desired target for BV change, dialysate conductivity and weight loss. The true target of the adaptive control, that is blood pressure, cannot be treated, because BP is not easily modeled. Thus, only hypotensive episodes due to BV changes may be predicted and corrected, and nothing can be made for hypotension due to vasodilatation. The fuzzy controller works better with hypotension due to BV reduction, but it is able to account also for hypotensive episodes due to other causes, since SBP is an input variable.

23.7 CLINICAL RESULTS

In the clinical practice both systems works, allowing for a reduction in hypotensive episodes in a similar percentage (Santoro et al. 2002; Mancini et al. 2007; Hickstein et al. 2009). This means that the policy to control dialysis related hypotension by modulation of dialysate sodium conductivity and ultrafiltration rate is only partially effective. These results may be someway expected. In all the control systems described (and available), the response to blood pressure or blood volume reduction is given by sodium infusion (given directly or indirectly by an increase in sodium dialysate) or by UFR reduction. These responses are proper if the cause of the hemodynamic instability is only inadequate refilling, but when the problem is inadequate vascular reaction to blood volume reduction, no correct response is available. In this case a particular attention should be devoted to temperature control system (Schneditz et al. 2003) that aims to maintain blood pressure by minimizing vasodilatation due to thermal unbalance.

An integration of BV/ultrafiltration/conductivity and temperature control system should be desirable. This integration is problematic using model based controller, while may be feasible with a fuzzy logic controller since tables of rules may be easily performed.

23.8 DIFFUSION OF DIALYSIS CONTROL SYSTEMS

Three dialysis companies implemented in their dialysis machines the dialysis control systems. In particular, a company implemented thermal balance, another one the classical adaptive controller and the last one the fuzzy controller. This policy is due to two main reasons: first of all each dialysis company promoted research, engineering and trade, thus linking a single system to a particular machine with the aim to increase the sale of the final product owing to the additional benefit implemented; second, since the control systems strictly interacts with the machine hardware (UFR and conductivity control), the software is integral part of the machine. The market of dialysis machines is only partially linked to innovative technologies, since it is governed in part by private companies that are often the same producers of the machines and thus they use their products and in part by public health systems that pay attention to a balance between costs and innovation.

By an epidemiologic point of view, symptomatic intradialytic hypotension is an important but not overwhelming problem, as a matter of fact it occurs in about 20% of diabetic patients and 15% of non-diabetic patients (Davenport et al. 2008). Other researches suggest that a correct definition of the dry body weight (Agarwal et al. 2010), and sodium and/or ultrafiltration profiling are able to avoid a large part of hypotensive episodes without having recourse to sophisticated tools.

These considerations explain why these systems are not so widespread in clinical practice.

23.9 FUTURE DIRECTIONS

All available control systems are probably not effective in optimizing sodium removal, especially in patients highly hypotension prone. No study gives evidence that biofeedback works better than conductivity and UFR profiling and randomized studies are lacking. Biofeedback systems are tightly linked to machines, profiling is independent on machines. An interesting alternative should be offline profiling optimization. This objective requires various steps: pattern recognition of blood pressure and/or blood volume during the dialysis session, adjustment of conductivity an UFR profiling according to specific patterns, implementation of an expert system able to perform this task.

Expert driven fuzzy systems or, better, neuro-fuzzy system should be suitable methods to give the better conductivity and UFR profile for each patient's blood pressure pattern. Both patterns and profiles could be stored in servers linked to the machine, now widespread in many dialysis facilities, thus this decision support system should be independent on machines.

23.10 CONCLUSION

Hemodialysis is often complicated by hypotensive episodes that lead to chronic hyperhydration and determine suffering for the patients. Excessive decrease of blood volume is probably the most important cause of dialysis induced hypotension. It is due to the unbalance between water removal and plasma refilling. Mathematical models that links blood volume changes to ultrafiltration rate and plasma conductivity are available; as a consequence control systems that adapt these variables to obtain a desired slope of blood volume decrease may be performed. Unfortunately, the physiology of blood pressure is much more complex, since it includes not only factors related to blood volume, but also to cardiac performance and to vascular reactivity. A quantitative model that links all these variables is not available owing to the rough knowledge of the whole systems, but a qualitative knowledge, expressed in linguistic terms, is possible. Fuzzy logic is suitable to solve this kind of problems.

The two fuzzy control systems use directly blood pressure and its trend as input variable, this is not possible with a classical PID controller since the quantitative relationships between blood pressure and the control variables are not known. In the fuzzy control system the relationships are described in tables that link the linguistic description of the input variables with the appropriate adaptive response. One may expects that fuzzy controllers work better than classical PID controllers, but this has not been demonstrated, also if a direct comparison between the two methods has not been performed. This result is probably explained by the fact that the adaptive control is assigned to the same variables: changes in ultrafiltration rate and in dialysate conductivity. These variables can only act on plasma refilling.

Fuzzy controllers allows future improvements because they allow to implement other control variables such as temperature, infusion of drugs that acts on heart performance or vascular reactivity, concentration of K or Ca in the dialysate and so on. It is sufficient to create tables with the corresponding rules and the hierarchy of intervention.

APPENDIX A: FUZZY SETS

A.1 MEMBERSHIP FUNCTIONS

Although the underlying purpose of the membership functions is to represent vague linguistic terms, they should also possess other important properties making the fuzzy system more suitable for predictive modeling. For example, the output of membership functions should be computationally cheap to be evaluated as this is one of the most frequently performed operations in fuzzy systems. Common choices fall into the following families of parameterized functions (Pedrycz and Gomide 1999):

- Triangular function:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{c-a} & \text{if } x \in [a,c] \\ \dfrac{b-x}{b-c} & \text{if } x \in [c,b] \\ 0 & \text{if } x \ge b \end{cases}$$

Where c is a modal value, and a and b denote the lower and upper bounds, respectively, for nonzero values of $\mu_A(x)$. Sometimes it is more convenient to use the notation explicitly highlighting the function's parameters:

$$\mu_A(x;a,b,c) = \max\left\{0, \min\left[\frac{(x-a)}{(c-a)}, \frac{(b-x)}{(b-c)}\right]\right\}$$

- Trapezoidal function:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{c-a} & \text{if } x \in [a,c] \\ 1 & \text{if } x \in [c,d] \\ \dfrac{b-x}{b-c} & \text{if } x \in [d,b] \\ 0 & \text{if } x > b \end{cases}$$

$$\mu_A(x;a,b,c,d) = \max\left\{0, \min\left[\frac{(x-a)}{(c-a)}, \frac{(b-x)}{(b-d)}\right]\right\}$$

- Gaussian function:

$$\mu_A(x) = e^{-k\,(x-c)^2}$$

where k > 0. Typically $k = 1/a^2$ where a^2 reflects the width (spread) of $\mu_A(x)$.

- Bell-shaped function:

$$\mu_A(x) = \frac{1}{1 + k\,(x-c)^2} \quad \text{with } k > 1$$

A.2 CHARACTERISTICS OF FUZZY SETS

Fuzzy sets are characterized in more detail by referring to the concepts of support, core, normality, convexity, etc. Let $\mathcal{F}$(X) denote the set of all possible fuzzy sets defined on the universe of discourse *X*. For a fuzzy set $A \in \mathcal{F}(X)$ we can give the following definition (Pedrycz and Gomide 1999).

Definition A.1 (*support*) *The support of A, denoted by supp(A), is a fuzzy set of X whose elements all belong to A with nonzero degree:*

$$\text{supp}(A) = \{x \in X | \mu_A(x) > 0\}$$

Definition A.2 *(core) The core of A, denoted by core(A), is a fuzzy set of X whose elements exhibit a full membership degree in A:*

$$\text{core}(A) = \{x \in X | \mu_A(x) = 1.0\}$$

Definition A.3 *(α-cut) The α-cut of A, denoted by $[A]^\alpha$ with α > 0:0, is a non-fuzzy set of X consisting of those elements whose membership values exceed the threshold level α:*

$$[A]^\alpha = \{x \in X | \mu_A(x) \geq \alpha\}$$

Definition A.4 *(empty fuzzy set) A is an empty fuzzy set if $\mu_A(x)$= 0.0,* $\forall x \in X$ *that is if Supp(A) = Φ.*

Definition A.5 *(universal fuzzy set) A is a universal fuzzy set if $\mu_A(x)$=* 1.0, $\forall x \in X$ *, that is if Core(A) = X.*

Definition A.6 *(fuzzy singleton) A is a fuzzy singleton if supp(A) = {x_0} and we use the notation A =* $\overline{x}_0$.

Definition A.7 *(normal fuzzy set) A is said normal if its membership function attains 1, that is* $\sup_x \mu_A(x) = 1.0$. *The supremum is the height of A. Hence A is normal if its height is equal to 1. If the height is less than 1, then A is called subnormal.*

Definition A.8 *(convex fuzzy set) A is said convex if its membership functions is such that:*

$$\mu_A(\lambda x_1 + (1-\lambda)\, x_2 \geq \min\left[\mu_A(x_1), \mu_A(x_2)\right]$$

For any x_1, $x_2 \in X$, *and* $\lambda \in [0,1]$. *Equivalently, it can be said that A is normal if* $[A]^\alpha$ *is a convex subset of* $X \, \forall \alpha \in [0,1]$.

Definition A.9 *(Compact fuzzy set) A is said compact if supp(A)* $\subset$ *X.*

A.2.1 Basic Relationships between Fuzzy Sets

As in set theory, we can define generic relations between two fuzzy sets, such as inclusion and equality. Let $A \in \mathcal{F}(X)$ and $B \in \mathcal{F}(X)$ be fuzzy sets (Pedrycz and Gomide 1999).

Definition A.10 *(inclusion) We say that A is included in B, denoted by A* $\subseteq$ *B ,iff*

$$\mu_A(x) \leq \mu_B(x), \; \forall x \in X .$$

Definition A.11 *(equality) A and B are said to be equal, denoted by A = B, iff*

$$A \subset B \text{ and } B \subset A. \text{ Of course } A = B \text{ iff } \mu_A(x) = \mu_B(x), \; \forall x \in X .$$

Another relationships between two fuzzy sets is similarity expressed in terms of a distance function between their membership functions which is treated as an indicator of their closeness. The more similar the two fuzzy sets, the lower distance between them. For this reason, it can be convenient to normalize the distance function so as to obtain a similarity measure by straight complementation of the normalized distance. In general, the distance between A and B can be defined as:

$$d(A,B)=\left[\int_X |\mu_A(x)-\mu_B(x)|^p\,dx\right]^{1/P}$$

Where P ≥1. Specific cases typically encountered in applications are:

$$\text{Hamming distance } (p=1)\text{: } d(A,B)=\left[\int_X |\mu_A(x)-\mu_B(x)|\,dx\right]$$

$$\text{Euclidean distance } (p=2)\text{: } d(A,B)=\left[\int_X |\mu_A(x)-\mu_B(x)|^2\,dx\right]^{1/2}$$

A.2.2 Operations on Fuzzy Sets

In this section the operations on fuzzy sets are defined, which come from an extension of the classical set theoretic operations from ordinary sets. Logical fuzzy operations are defined through triangular norms and triangular conorms, referred to as the T-norm and T-conorm (or S-norm) in the literature and here denoted by $\mathrm{T}(\cdot)$ and $\sigma(\cdot)$, respectively.

Table A.1 The most popular choices for the T-norm and T-conorm (Pedrycz and Gomide 1999).

norm	T-norm	T-conorm
truncation	T (a,b) = min{a, b}	σ (a,b) = max{a, b}
algebraic	T (a,b) = ab	σ (a,b) = a + b - ab
bounded	T (a,b) = max {a + b - 1, 0}	σ (a,b) = min {a + b , 1}

Definition A.12 *(Triangular norm) A t-norm is a binary mapping*

$$\mathrm{T} : [0,1] \times [0,1] \rightarrow [0,1]$$

*Satisfying the following properties: commutativity, associativity, monotonicity and one-*identity (i.e. $\mathrm{T}(x,1)=x, \forall x \in [0,1]$)

Definition A.13 *(Triangular conorm) A s-norm is a binary mapping*

$$\sigma : [0,1] \times [0,1] \rightarrow [0,1]$$

satisfying the following properties: commutativity, associativity, monotonicity and zero-identity (i.e. $\sigma(x,0)=x,\ \forall x \in [0,1]$*)*

These norms provide a wide range of suitable operators but the most popular are the algebraic and truncation operators as summarized in table (A.1). that the truncation operators may produce model outputs with discontinuous derivatives, while the algebraic operators produce smooth outputs ideal for modeling and control. For this reason algebraic operators are preferred.

Let $A \in \mathcal{F}(X)$ and $B \in \mathcal{F}X)$ be fuzzy sets.

Definition A.14 *(Complement) The complement of A, denoted by* $\neg A$*, is a fuzzy set of X with membership function:*

$$\mu_{\neg A}(x) = 1 - \mu_A(x), \forall x \in X$$

Definition A.15 *(Intersection) The intersection of A and B is a fuzzy set of X with membership function:*

$$\mu_{A\bullet B}(x) = \top(\mu_A(x), \mu_B(x)),\ \forall x \in X$$

Definition A.16 *(Union) The union of A and B is a fuzzy set of X with membership function:*

$$\mu_{A\cup B}(x) = \sigma(\mu_A(x), \mu_B(x)),\ \forall x \in X$$

Definition A.17 *(Possibility measure) The possibility measure of A with respect to B, denoted by Poss (A,B) is defined as*

$$Poss(A,B) = \sup_{x\in X} [\top(\mu_A(x), \mu_B(x))]$$

It quantifies the extent to which A and B overlap

Definition A.18 *(Necessity measure) The necessity measure of A with respect to B, denoted by Nec(A,B) is defined as*

$$\mathrm{Nec}(A,B) = \inf_{x\in X} [\sigma(\mu_A(x), 1-\mu_B(x))]$$

It quantifies the degree to which B is included in A

Definition A.19 (*Fuzzy Relation) Let X and Y be nonempty sets. A fuzzy relation is a fuzzy subset of* $X \times Y$ *i.e.* $R \in \mathcal{F}(X \times Y)$

Definition A.20 *(Composition of Fuzzy Relations) If R ∊ $\mathcal{F}(X \times Y)$* and S ∊ *$\mathcal{F}(Y \times Z)$ are fuzzy relations, the composition of R and S is a fuzzy relation in X × Z denoted by R ∘ S and is defined by:*

$$\mu_{R \circ S}(x,z) = \sup_{y \in Y} [\top(\mu_R(x,y), \mu_S(y,z))]$$

Definition A.21 (Cartesian Product) If A and B are fuzzy sets defined on X and Y, respectively, the Cartesian product A × B is a fuzzy relation in the product space X × Y with membership function

$$\mu_{A \times B}(x,y) = \top(\mu_A(x), \mu_B(y))$$

Definition A.22 *(Fuzzy Implication) Let A ∊ $\mathcal{F}(X)$ and B ∊ $\mathcal{F}(Y)$) be fuzzy sets. The fuzzy implication A →B is a fuzzy relation A × B.*

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ESSAY QUESTIONS

1. Compare between the basic levels of model synthesis.
2. What is fuzzy logic?
3. What does fuzzy set mean?
4. List some applications of fuzzy logic systems
5. List the main building blocks of the fuzzy logic system
6. What is a fuzzy inference rule?
7. Define the defuzzification process

8. Is FCS a linear controller?
9. Describe the main differences between Mamdani and Sugeno fuzzy models
10. What is Fuzzy Control System (FCS)?
11. What is the difference between fuzzy logic and probabilities?
12. Can we affirm that the proposed approach for the fuzzy logic controller is a Decision Support System, rather than a fuzzy control system for a dynamic system?
13. Describe the mechanisms involved in maintaining hemodynamic stability during hemodialysis.
14. Describe the main differences between a fuzzy blood pressure controller and a classical blood volume adaptive controller in dialysis.

MULTIPLE CHOICE QUESTIONS

Choose the best answer

1. Who is the founder of fuzzy logic?
   A. Mamdani
   B. Sugeno
   C. Lotfi Zadeh
   D. Tsukamoto

2. What are the following sequence of steps taken in designing a fuzzy logic machine?
   A. Fuzzification→Rule evaluation→Defuzzification
   B. Rule evaluation→Fuzzification→Defuzzification
   C. Fuzzy Sets→Defuzzification→Rule evaluation
   D. Defuzzification→Rule evaluation→Fuzzification

3. Fuzzy logic has rapidly become one of the most successful of today's technologies for developing sophisticated control systems. The reason for this is…
   i. Fuzzy logic resembles the human way of thinking.
   ii. Fuzzy logic enables the ability to generate precise solutions from certain or approximate information.
   iii. Fuzzy logic is easy to implement.

A. i & ii & iii
B. i & ii
C. ii & iii only
D. none of the above

4. “Fuzzy logic” is…
A. A “new type” of logic
B. A different name of probability calculus
C. A logic with incomplete information
D. A way to reason with non-probabilistic uncertainty

5. A fuzzy control system is…
A. A linear control law
B. A control law which uses empirical knowledge
C. An optimal control law
D. A control law based on zero-pole allocation

6. A FIS is useful…
A. To implement expert knowledge for many application fields
B. To solve nonlinear differential equations
C. To build a neural net
D. To solve logistic problems

7. Defuzzification consists of…
A. Process reasoning to infer a conclusion from a rule data set
B. Computing the membership degree of the antecedent part in a FIS
C. Obtain a crisp number from a fuzzy number
D. Computing a T-norm between two membership degree

8. The proposed FCS for dialysis control…
A. if formed by a Mamdani control
B. it is an adaptive PID controller
C. it is a DSS for determining the optimal values of UFR and Na
D. it is a SISO control (Single Input-Single Output)

9. Trapezoidal fuzzy numers are…
   A. A different representation of complex numbers
   B. Particular probability distributions
   C. Linearly increasing and decreasing fuzzy numbers (with possibly a constant part)
   D. A way to represent S-norms

10. The priorities in the integrated FCS systems serve to…
   A. Introduce a feedback action
   B. Analyse the stability
   C. Select the "best" rules to be applied
   D. Modify the Sodium Balance

11. A T-norm is…
   A. A particular type of membership function
   B. A conjunction logical operator
   C. A disjunction logical operator
   D. A method to defuzzify a fuzzy number

12. In the fuzzy controller described in the chapter, the input variable with the higher priority was…
   A. Blood pressure
   B. Blood volume changes
   C. Blood pressure trend
   D. Blood volume trend

13. The fuzzy controller and the classical adaptive controller share as input variables…
   A. Blood pressure
   B. Blood volume changes
   C. A and B
   D. other

14. Fuzzy rules are…
   A. rules for synapsis computation in neural nets
   B. an algorithm to optimize an adaptive control
   C. a way to represent not probabilistic uncertain inference
   D. other

15. The proposed fuzzy support system for dialysis regulation is…
    A. a PID controller
    B. a dynamic adaptive control
    C. an optimal control system
    D. an inference rule based system

16. Triangular fuzzy numbers are…
    A. Probability density
    B. Fuzzy numbers with infinite support
    C. Gaussian type fuzzy numbers
    D. L-R type fuzzy numbers

17. A TSK controller is…
    A. a non linear controller
    B. a discrete time stable controller
    C. a predictive controller
    D. a Mamdani type controller

18. A FCS for the dialysis session…
    A. Is based on the medical experience
    B. uses a model-based approach
    C. is necessarily stable
    D. can be applied only for SISO systems

19. A FIS is…
    A. A dynamic system identifier
    B. An inference system based on fuzzy rules
    C. A way to check if a dynamic system is stable
    D. A stable control rule

20. Fuzzy identification from a…modeling point of view
    A. White-box
    B. Grey-Box
    C. Black Box