Chapter 7
Vibration Characteristics

Vibration problems in wind turbines are an old phenomenon. Even in the Middle Ages, the post windmill of that time was also called a "rocking mill", as the mounting of the entire millhouse on a trestle led to a rocking motion. This drawback then became the stimulus to continued development, from which evolved the more stable Dutch windmill which ran more smoothly.

Modern wind turbines are of slender and elastic construction, above all the rotor blades and the tower. They are, therefore, structures which are extremely prone to vibration. In addition, there is no lack of excitations as the discussion of cyclically alternating rotor forces has shown. These forces can excite certain subsystems or even the entire turbine to vibrate dangerously. Wind turbines must, therefore, undergo a painstaking analysis of their natural vibrational modes and possible resonance problems, even at the design stage.

This vibration analysis has the aim of verifying the dynamic stability and the absence of resonances within the permissible operating range. Unstable speed ranges, for example with regard to a bending vibration of the tower, or resonances of bending or torsional vibrations of other important structural components, must be avoided, at least in steady-state operating phases. For this reason, the natural frequencies of rotor blades, tower and mechanical drive train components must not be too close to each other, and their clearance from the possible excitation frequencies must not be too small.

This objective of verifying the dynamic stability must not be confused with the task of calculating the increase in dynamic loads resulting from the elastic response characteristics of the structure. It is true that the mathematical approaches and calculation methods are similar, even identical in part, but the formulation of the task is different and hence also the procedure. Vibration problems in wind turbines are essentially concentrated in four areas:

- The slender rotor blades are subject to aeroelastic influences. To avoid hazardous vibrations, certain criteria of stability must be met.
- The mechanical-electrical drive train is prone to torsional vibrations which can be excited both by aerodynamic influences and by electrical influences.

- The yaw system has its own dynamics which can also lead to undesirable vibrational behaviour.
- Not least, the entire wind turbine, i.e. the rotor/tower-system, can start vibrating. This is caused by the periodic forces caused by the rotor resonating with the tower's natural bending frequency.

Theoretically, these four areas are not independent of one another. But a joint treatment with a comprehensive mathematical model would be both impractical and unnecessary. Generally, the vibrational coupling of these individual processes is not so strong that an independent treatment would be impossible.

7.1 Exciting Forces and Vibrational Degrees of Freedom

Vibrations of the entire wind turbine or of its subsystems are essentially triggered by the periodically alternating and stochastically occurring aerodynamic forces. In principle, actions of force from the "interior" of the components, e.g. from the gearbox, or from the grid, can act to excite vibrations. However, such phenomena generally play only a minor role. Depending on the number of subsystems, a multiplicity of vibrational degrees of freedom are possible which can also influence each other (Fig. 7.1).

In view of this situation, it could be assumed that a vast number of the most varied types of vibrations occurs. This is so, in theory, but the vibrational coupling of the individual degrees of freedom in the overall system manifests itself in many different ways so that only a few coupled vibrations are of practical significance whereas most of the other ones are only of rather academic interest.

From a practical point of view, the vibration characteristics of the wind turbine can be reduced to a limited number of states of vibration with certain degrees of freedom. The periodic rotor forces primarily excite the flexural vibrations of the tower. In the rotor blades, it is the flapwise and chordwise movement and the torsional movement with their corresponding natural frequencies which are of significance. The first natural flapwise bending frequency of the blades can resonate with the tower bending whereas the second natural bending frequency of the blades is in most cases so high that it no longer interferes. The chordwise movement, i.e. the antimetric chordwise movement of the blades, is associated with the vibrational behaviour of the drive train (s. Chapt. 7.2.3).

Preventing the exciting rotor forces from resonating with the natural frequencies of the components is the first and most important requirement for keeping the vibrational behaviour of the entire system under control. For this reason, the most important natural frequencies of the components must already be placed correctly with respect to the exciting rotor frequencies during the design process. In this context, the fact is of significance that, with increasing size of the components, the natural frequencies become lower and the areas of resonance with the external excitations thus shift. The exciting forces of the rotor can be assigned to two categories:

- Exciting forces occurring with the rotor's rotational frequency. These are primarily forces from mass imbalances.

- Exciting forces occurring with the rotor's rotational frequency multiplied by the number of rotor blades. Among these are the "aerodynamic imbalances", i.e. forces developing as a result of an asymmetrical air flow against the rotor (tower shadow effect, vertical wind shear).

The aerodynamically caused exciting forces are the critical ones, since they cannot be avoided, in contrast to mass imbalances. The position of the tower's first natural bending frequency relative to these exciting frequencies characterises the design of the turbine with respect to its vibrational behaviour. The situation differs depending on the number of rotor blades.

In a turbine with a two-bladed rotor, the aerodynamic frequency of excitation occurs at twice the rotational frequency of the rotor (2 P). According to American literature, the frequencies of excitation are called 1 P, 2 P and 3 P (per revolution). Plotted against the rotor speed, they are located along straight lines (Fig. 7.2).

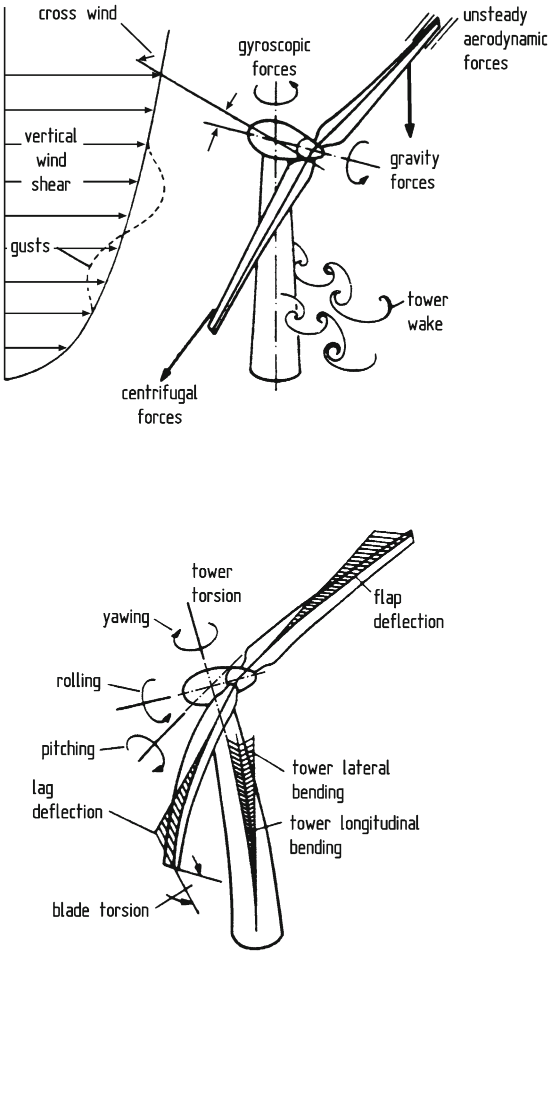


Fig. 7.1. Exciting forces and dgrees of vibrational freedom of a wind turbine [1]

The same considerations and definitions basically apply to three-bladed rotors. The main difference is that the critical aerodynamic rotor excitation occurs at 3 P instead of 2 P. For the soft tower design, there is yet another option, namely between 3 P and 2 P, between 2 P and 1 P and below 1 P.

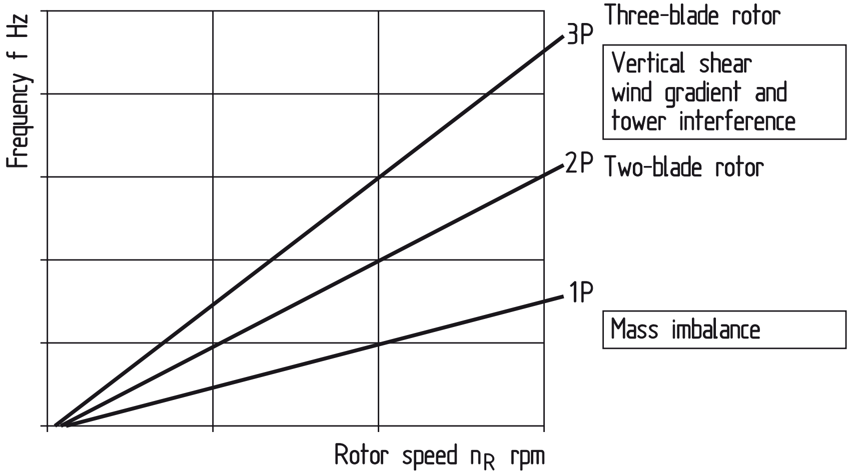


Fig. 7.2. Exciting frequencies as a function of rotor speed

A fundamental factor in the vibrational characteristics of a wind turbine is the position of the natural frequencies of the critical components of the wind turbine with respect to these exciting frequencies. This applies especially to the natural bending frequency of the tower (s.a. Chapt. 7.5).

7.2 Aeroelastic Stability of Rotor Blades

One of the first prerequisites for avoiding unwanted vibrations and structural failure is that the rotor blades are aeroelastically stable. Aeroelastic instabilities arise when a cumulative interaction develops between the deformations of the elastic structure and the resultant aerodynamic forces. Elements creating lift, such as aircraft wings, are especially prone to this. Calculation methods for detecting aeroelastic instabilities have, therefore, been developed above all in aeronautical engineering and can be applied directly to the rotor blades [2]. Basically, the most varied types of aeroelastic instability phenomena exist, but in this book only the most important ones can be discussed.

7.2.1 Static Divergence

A phenomenon known from the behaviour of aircraft wings is the torsional instability of the wing at a certain flying speed. This effect depends on the relative position of the so-called *elastic axis*, which is the imaginary axis around which the wing twists free of moments, and of the *aerodynamic centre*. In almost all airfoils, the aerodynamic centre, the point of attack of the lift forces, is located at approximately a quarter of the chord length (Fig. 7.3). If the aerodynamic centre is located in front of the elastic axis, the lift creates a torsional moment which increases the angle of attack. This moment increases with the square of the free-stream velocity. However, the restoring moment resulting from the wing's torsional stiffness is independent of the speed, so that at a certain speed, the "speed of divergence", a torsional instability develops.

In most wind turbine rotor blades, this static divergence does not pose a problem. It should nevertheless be checked in all real cases. It must be noted here that a torsional moment in the sense of a greater angle of attack is not only caused by the aerodynamic lift force, but additionally by a component of the centrifugal force, if the rotor blade is twisted out of the plane of rotation, or when the rotor has a coning angle.

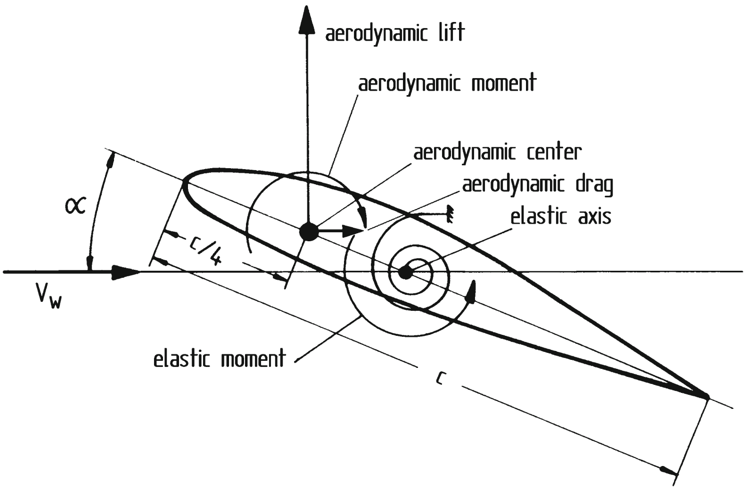


Fig. 7.3. Aerodynamic and elastic moments in a wing or rotor blade cross-section

7.2.2 Natural Frequencies and Vibration Modes

In order to detect aeroelastic instabilities which can lead to vibrations, the natural frequencies and vibration modes of the rotor blade must first be determined (Fig. 7.4). The mathematical calculation is generally based on the isolated model of a stationary, fixed rotor blade. This can be used for determining the natural frequencies and eigenmodes with sufficient accuracy. Other influences such as, for example, the stiffening effect of the centrifugal force with a rotating rotor are comparatively small.

The lowest natural frequency is generally the first natural flapwise frequency, followed by the first natural bending frequency in the chordwise direction. As already mentioned in Chapter 7.2.1, the first natural torsional frequency is comparatively high so that the static divergence does not present a problem.

The resonance diagram (Fig. 7.5) shows how the natural frequencies of the rotor blade behave with respect to possible vibrational excitations. In this example, the wind turbine has a nominal rotational speed of 16 rpm in the full-load range. At this speed, there are no resonances with the first flapwise frequency from the mass imbalance (1P excitation) and from the aerodynamic "imbalance", the vertical wind shear and the tower influence (3P). A resonance with the 3P excitation would only occur at 18 rpm. The other excitations of higher harmonics have little energy and, do not, therefore, lead to hazardous resonance phenomena. Considering the entire speed range, i.e. also the partial-load range in which the rotor is operated with variable speed, there are several points of resonance with the higher harmonics. Since, on the one hand, the turbine does not stay at a certain speed for very long and these exciting frequencies are also not very rich in energy, these resonances do not present any danger.

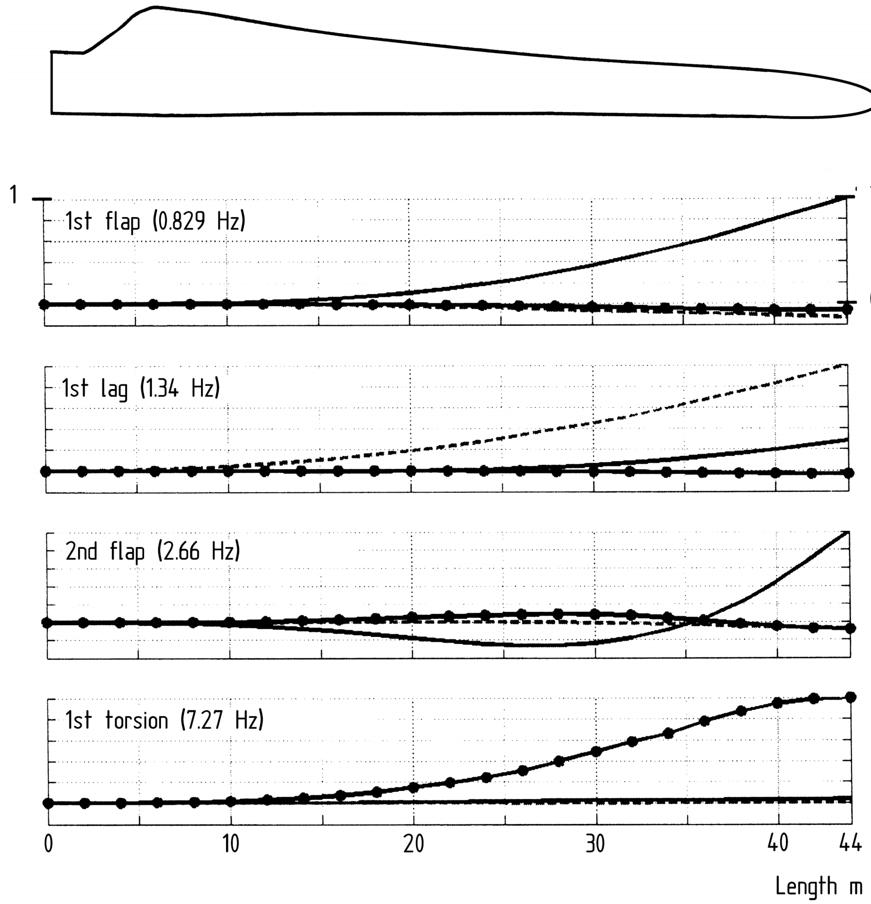


Fig. 7.4. Natural frequencies and eigenmodes of an Euros rotor blade of 44 m length [3]

If necessary, the vibrational behaviour of the rotor blades must also be considered from an additional point of view, apart from considering the aeroelastic structural instabilities. In the case of rotor blades which are adjusted about their longitudinal axis, i.e. in their pitch angle, attention must be paid not only to the torsion of the blade per se but also to the torsional moments about the axis of blade rotation. The position of the rotational axis, which is established in most cases for constructional design reasons, must, therefore, be included in these considerations. The pitch mechanism, too, acting in conjunction with the control system, has its own dynamic characteristic. Interacting with the structurally elastic torsional characteristic of the rotor blades, it can lead to instabilities and states of vibration. Hydraulic actuators, in particular, can form an elastic and thus vibrating element. In contrast, the electric-motor drives normally used today exhibit very stiff characteristics so that the model concept of a tightly clamped rotor blade is justified in this case.

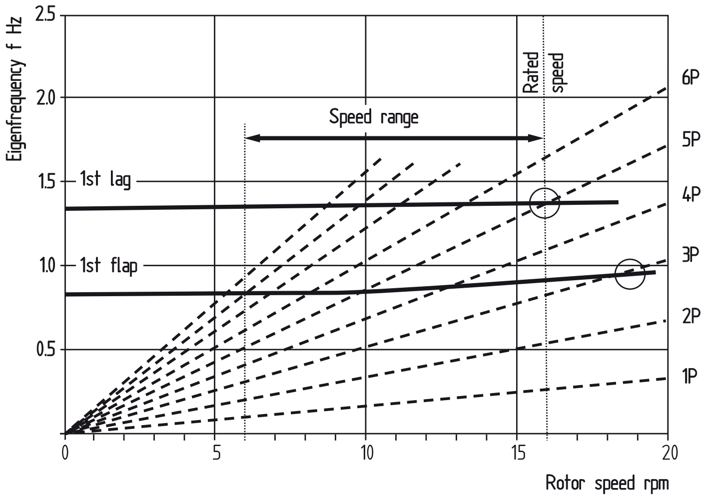


Fig. 7.5. Natural frequencies of the Euros rotor blade in the resonance diagram of a wind turbine [3]

7.2.3 Typical Rotor Blade Vibrations

When the natural frequencies are correctly placed, i.e. with the correct stiffness design, unwanted vibrations of the rotor blades can be largely avoided. Nevertheless, there are some specific vibrations which do not so much have their cause in the overall vibrational behaviour of the unit but are rather a consequence of the stiffness of the rotor blades and of the aerodynamic design of the rotor. These vibrational phenomena can occur especially in the case of large stall-controlled rotors.

Flutter

A certain type of aeroelastic instability of a wing or a rotor blade is called "flutter". If, for whatever reason, the airfoil is excited into an oscillating motion, a mutual excitation of aerodynamic forces, elastic forces and mass forces can occur. In particular, it is the combined bending-torsional vibration, which is virtually unavoidable in a twisted wing or rotor blade, which represents the classical case of flutter. Since this directly involves the aerodynamic angle of attack and thus the lift forces, this flutter is especially dangerous and can lead to destruction in the shortest period of time.

An effective counter-force is, above all, *aerodynamic damping*. Aerodynamic damping is understood to be the speed-related force resulting from the change in the angle of attack, acting counter to the direction of movement. It is proportional to the speed of the vibrational deflections and is not to be confused with the aerodynamic drag in the direction of the free stream velocity. The aerodynamic damping is much greater in the flapwise direction than in the chordwise direction of the blade. Despite the presence of aerodynamic damping, the vibrational mechanism of the flutter can absorb energy under certain boundary conditions, thus becoming hazardous.

A special case of flutter is the so-called *stall flutter* which is characterised by a periodic change between flow separation and normal flow on the airfoil when the angle of attack is high and close to the critical angle of attack. This stall flutter can represent a danger to rotors which are deliberately operated close to aerodynamic stall at higher wind speeds. As stall is approached, the lift coefficient develops a negative gradient at increasing angles of attack. The aerodynamic damping then also becomes negative and the rotor blades can resonate with exciting frequencies both in the flapwise and the chordwise direction. In particular, chordwise rotor blade vibration was observed in some larger stall-controlled turbines [2].

The tendency of the rotor blades to flutter is determined by a multitude of parameters. The most important ones are the natural frequencies of the blades as regards the direction of flapping, chordwise and torsion motion, the coning angle, the twist, as well as the relative positions of the aerodynamic centre with respect to the centre of mass, and of the elastic axis and the plane of rotation. A first criterion for a possible susceptibility to flutter is the relationship between torsional stiffness and the distance of the elastic axis from the centre of mass. This makes it possible to specify "stability limits" (Fig. 7.6).

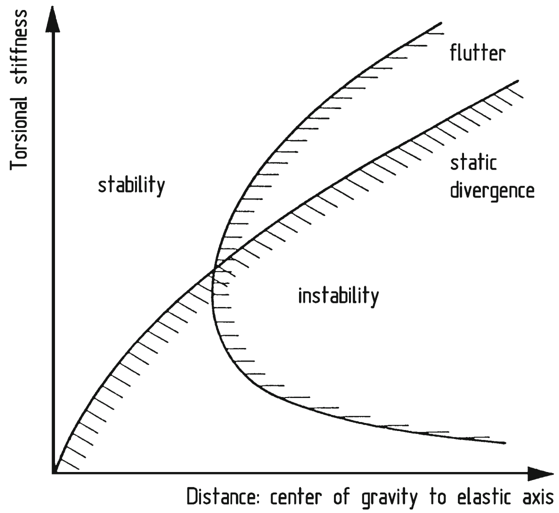


Fig. 7.6. Stability limits for the "static divergence" and "flutter" of wings and rotor blades [2]

Chordwise vibrations

The chordwise movement of the rotor blades, i.e. the movement of the blades in the direction of the airfoil chord and thus, in a normal operating position, in the plane of rotor rotation, is not very damped aerodynamically, in contrast to the flapwise movement. In the plane of rotor rotation, the periodically alternating bending moment from the natural weight of the blades comes into action. In addition, the chordwise movement of the rotor blades is associated with the dynamics of the drive train (s. Chapt. 7.3). To these must be added also irregular and alternating forces arising during flow separation when the rotor is operated in the stall region.

The last-mentioned effect, in particular can cause a chordwise bending vibration of the rotor blades. This phenomenon was observed especially in large stall-controlled rotors when they are operated at high wind velocities, i.e. within stalling range. The rotor blade manufacturers have attempted to suppress these vibrations by means of various measures, for example by changing the structural damping which, however, is only very little, in any case, or by means of aerodynamic aids such as stall strips (s.a. Chapt. 5.3.4).

One large blade manufacturer (LM) has provided special hydraulic dampers in the blade tips of some blades in order to suppress the chordwise vibrations (Fig. 7.7). In the meantime, chordwise vibrations of the rotor blades have become a rather rare phenomenon since large stall-controlled rotors are no longer being built and the problem is solved in this way. But on the other hand stall can also occur at pitch controlled rotors. At least under certain operational conditions there is the possibility that flutter vibrations can cause problems.

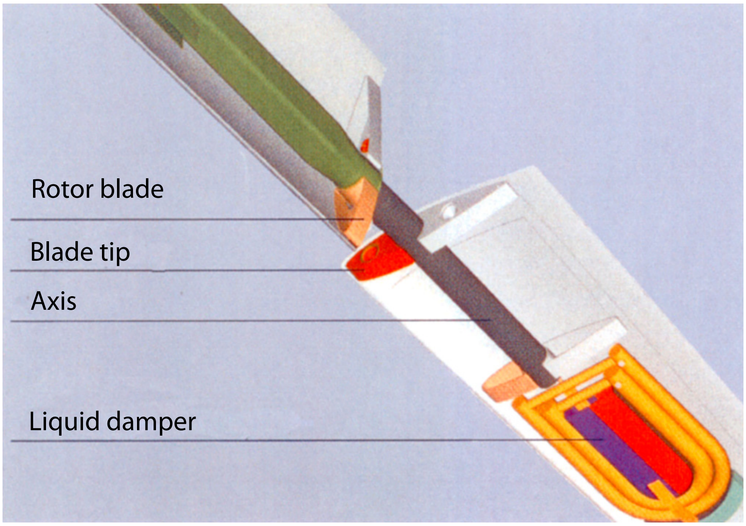


Fig. 7.7. Hydraulic damper in the rotor blade tip for suppressing chordwise vibrations (LM)

7.3 Torsional Vibration of the Drive Train

The drive train of a turbine with its rotating masses and torsionally elastic components is a subsystem capable of vibrating. Vibrational modes can also be excited by external influences at both ends of the energy transmission chain. Apart from the stochastic fluctuations of the rotor torque caused by wind turbulence, the rotor also generates cyclic torque variations which represent an ideal source of excitation. At the other end there is the electrical generator with its connection to the grid or to a particular load. In the discussion of the characteristic properties of electrical generators, it has been pointed out that, in particular, the synchronous generator coupled directly to the grid tends to vibrate. However, the problem also occurs in other types of generators.

Against this background, it is absolutely mandatory to deal with the phenomenon of *drive train vibrations* in wind turbines. The most important natural frequencies and modes of vibration must be analysed and tuned to the possible exciting frequencies in such a way so as to avoid resonances. Vibrational resonances in the drive train can exert a considerable influence on the dynamic load of the components, on the quality of the power output and even on the mechanical noise.

The term electrical-mechanical drive train usually includes the elements of the energy transmission chain, without the rotor blades. For dynamic considerations, however, the rotor blades must be included as their share in the rotating masses is by far the largest and, in addition, they have a decisive part in determining the dynamic behaviour due to their bending behaviour in the chordwise direction.

The series-connected components of the drive train such as rotor hub, rotor shaft, gearbox, high-speed shaft, brake and clutches have such diverging dimensions, mass distributions and material properties, that an accurate analysis of vibrations can only be carried out to a limited extent. The most important parameters can, nevertheless, be calculated by means of comparatively simple equivalent mechanical models.

7.3.1 Mathematical Model

The mathematical simulation of the vibrational behaviour of rotating multi-mass or multi-body systems is widely used in the field of engineering, so that only the basic approaches will be called to mind here [3], [4]. Apart from these numerical simulation techniques an analytical approach can be used. Firstly, the most important natural frequencies and modes of inherent vibrations (modal analysis) are calculated with the aid of an equivalent mathematical vibration model. In a second step, the reactions of the drive train to excitations are examined and critical resonances found.

In a method developed by Lagrange, the kinetic and potential energies of the multi-mass system consisting of torsional masses and torsionally-elastic shaft elements are balanced and the equations of motion are derived by differentiation with respect to time. To solve these equations, all mechanical parameters are "reduced" to a uniform rotational speed. The multi-mass system with different rotational speeds becomes an equivalent system with a uniform speed, taking into consideration the requirement that the total energy be conserved. The vibration equation for this equivalent system can be solved. The solutions of the so-called "characteristic equation" provide the natural frequencies. Inserted into the general solution of the system of differential equations, these, in turn, yield the associated natural vibration frequencies.

The vibration response of a torsional vibration system is determined by three elasto-mechanical parameters:

- the polar moment of inertia of the rotating masses,
- the torsional stiffness of the elastic shafts and connecting elements,
- the torsional damper constants.

These three parameters must be determined from the design and material properties of the drive train components involved. This is where the main difficulty arises.

Apart from the torsional stiffness of the drive train itself, the chordwise bending behaviour of the rotor blades also plays a role, as already mentioned. The antimetrical chordwise elastic deformation of the rotor blades is directly related to the torsional dynamics of the drive train. An equivalent torsional stiffness can be calculated if the first natural bending frequency of the blades in chordwise direction is known.

The torsional damping constants of the mechanical components are generally small. This applies both to structural damping and damping caused by bearing friction. Hence an equivalent mathematical model without damping can be used (conservative system), as long as no special damping elements are used for damping the vibrations in the drive train.

The drive train of a horizontal-axis wind turbine can be composed essentially of two masses: the rotor and the generator rotor. A "two-mass model", therefore, provides a first overview. Occasionally, the rotor hub in connection with the blade roots represents a relatively "soft" link, so that a three-mass model consisting of rotor blades, hub, generator rotor with gearbox and "the rest" represent a suitable equivalent model by means of which the most important natural frequencies and resonances can be recognised. The proportion of the inertial moments of these subsystems contributing to the overall inertial moment of the drive train of wind turbines with widely differing sizes and technical concepts are shown in Table 7.8.

Table 7.8. Proportions of the components contributing to the overall inertia

Component / Wind turbine	Blades	Hub	Generator	Rest
Aeroman	87 %	2 %	9 %	2 %
WKA-60	91 %	1 %	7 %	1 %
Growian	85 %	8 %	5 %	2 %

In practice, the mathematical treatment is based on an idealised concept of the drive train (Fig. 7.9), According to this, the polar mass moment of inertia and the torsional stiffness are referred to the selected "reference speed" for all components involved in the torsion al vibration, and are clearly plotted to scale along the drive train (Fig. 7.10).

In this way, an idea of the vibration-related influence of the components involved is obtained, and the suitable "multi-body model" can be selected for the vibration calculation.

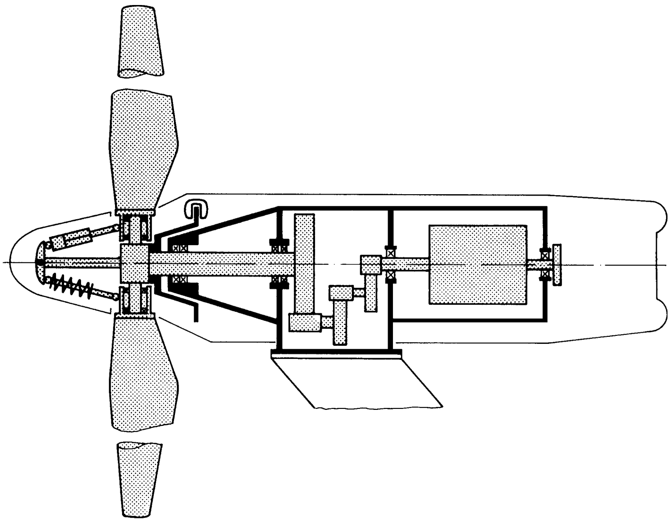


Fig. 7.9. Idealised drive train of a small turbine of the Aeroman type

In some cases the investigation of the vibrational behaviour of the drive train requires a simulation of a comprehensive system simulation including rotor blades, rotor hub, rotor shaft, gearbox with support structure and electrical generator. The resulting values of such a multi-body simulation in the time domain method is particularly helpful to detect undesirable vibration modes of the higher order. This kind of simulation is also a tool in the design optimisation process of the gearbox regarding bearings, shafts and gears as well as the stiffness of the gearbox housing and support structure.

Simulation in the time domain mode also offers the possibility of investigating the influence of different operational modes and external loads, for example the loading in the case of an emergency rotor stop, the influence of heavy wind gusts or a sudden loss of the load due to a grid failure. Changing speeds and torques in the gearbox can have a significant influence on the dynamic behaviour and on the loading on the drive train. These affects only can be found by the aid of a simulation.

Apart from the gearbox the other components of the mechanical drive train, rotor shaft, the hub with regard to stiffness and last but not least the bearings are affected by the vibrational behavior of the drive train. In particular the bearings can be affected by vibrations at a certain operational status of the wind turbine. Torsional vibrations of the drive train together with low loading on the bearings (idling of the turbine) can cause “slipping” movements of the bearings. This effect has very negative consequences for the life time of the bearings (s. Chapt. 9.6.1).

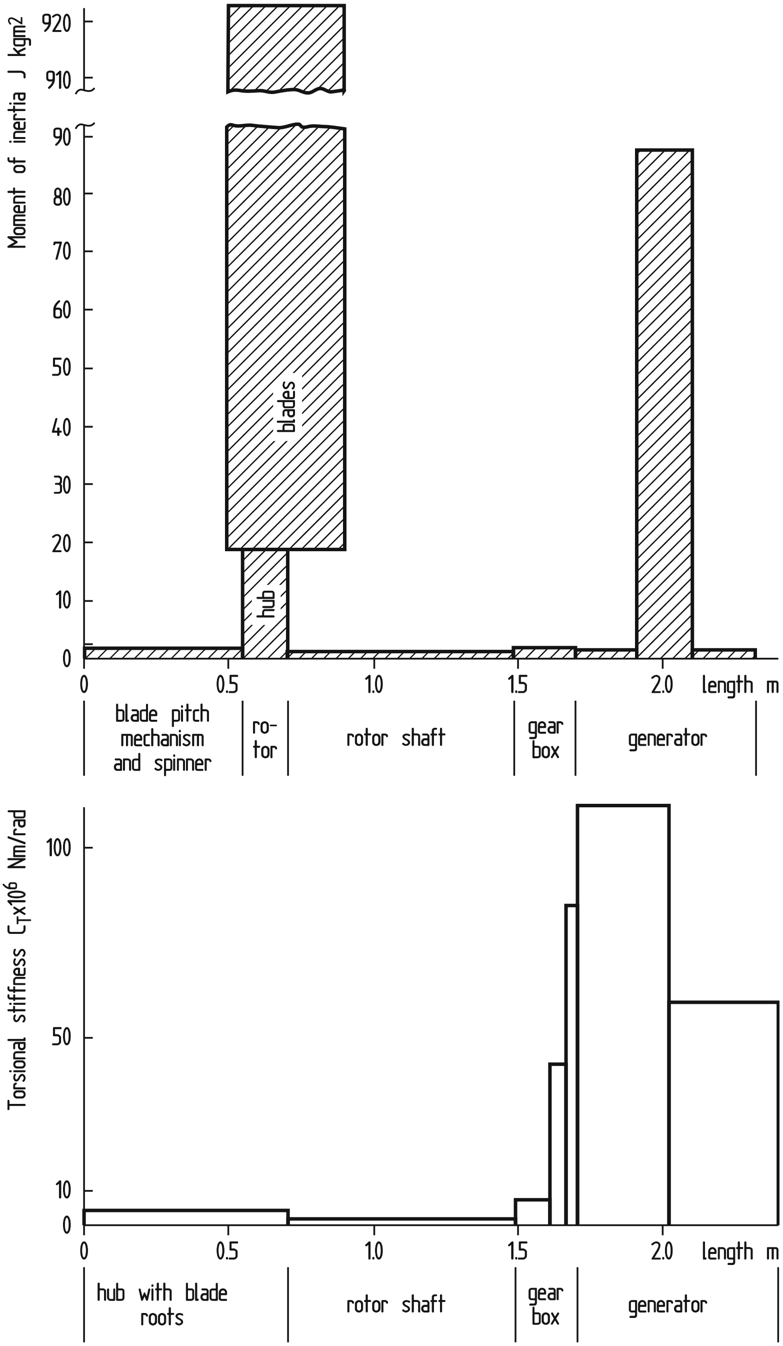


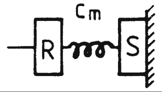
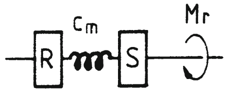
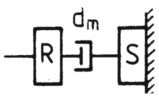
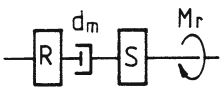
Fig. 7.10. Distribution of the polar mass moment of inertia and torsional stiffness of the drive train of a small wind turbine

7.3.2 Equivalent Mechanical Models for the Electrical Grid Coupling

Apart from elastomechanical properties, the dynamics of the drive train are also determined by the electrical aspects. In the discussion of the generator characteristics it was pointed out that the various types of generator behave very differently as far as their dynamic coupling to the grid or the load are concerned. The electrical characteristics can be represented by analogous, equivalent mechanical models (Fig. 7.11). These equivalent models are only valid as far as the vibrational behaviour is concerned, namely for very small deviations around a steady-state point of operation. Any speed variability which may exist is of no consequence.

The synchronous generator is characterised by the dominant torsionally-elastic behaviour. The magnetic coupling between rotor and stator (grid) can be described by a mechanical torsion spring. The damping is so slight that it can be virtually neglected. If the generator operates on a fixed-frequency grid, the torsion spring is clamped to a "solid wall", as it were. In isolated operation the frequency is determined by the instantaneous generator speed. The generator only loads the drive train with the moment of resistance corresponding to the power delivered. In contrast to the induction generator, the magnetic coupling of the synchronous generator is weak when idling, due to the grid-independent excitation of its rotor.

In the induction generator, the slip existing between rotor and stator under load acts as torsional damping, whereas the elasticity is virtually zero. During idling or after a load shedding, the coupling between rotor and stator disappears completely.

generator type	operational mode	analogous mechanical model
synchronous generator	grid coupled operation	
	isolated operation	
asynchronous generator	grid coupled operation	
	isolated operation	

c_m : magnetic torsional stiffness — R : rotor
d_m : magnetic damping factor — S : stator
M_r : resistance moment of the load

Fig. 7.11. Equivalent mechanical models for the electrical coupling of the generator to the grid or the load

The equivalent mechanical models for the electrical grid coupling of the drive train show that, in addition to the generator type, the operational mode, too, must be taken into consideration. This results in different natural frequencies and vibration modes, depending on the load condition.

7.3.3 Natural Frequencies and Vibration Modes

The models and calculation method outlined above provide the natural torsional frequencies and with them the vibration modes (eigenmodes) of the drive train as the most important results. Depending on the type of electrical grid coupling or, in isolated operation, on the type of load characteristics, typical vibration modes are obtained. These are shown in Fig. 7.12 calculated by means of a "three-body model".

Synchronous generator coupled to the grid

The rotational speed of the generator rotor fluctuates around the grid frequency, the drive train masses vibrate in opposition to one another. Given the usual mass conditions of a horizontal-axis turbine, the following characteristic vibration modes are obtained:

- At the first natural frequency, the entire drive train vibrates in opposition to the fixed-frequency grid.
- The second eigenmode is characterised by the vibration of the second largest partial mass, the generator rotor, around the other parts of the drive train.
- At the third natural frequency, the third largest partial mass, the hub, vibrates between the adjacent larger masses.

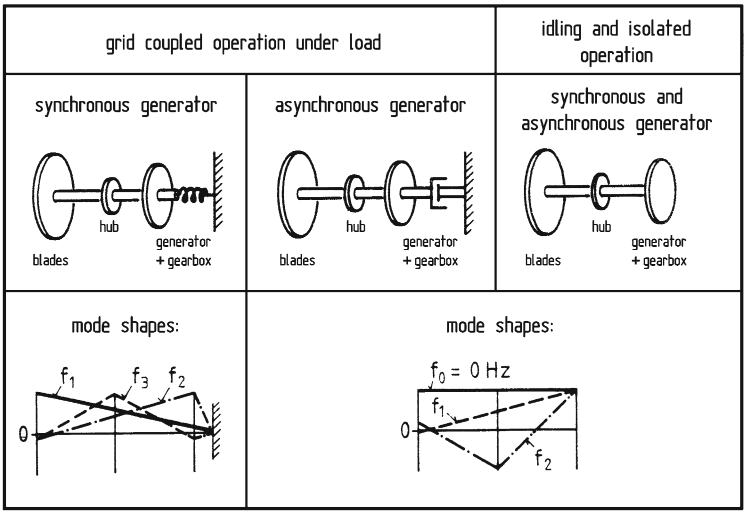


Fig. 7.12. Natural vibration modes of the drive train with grid-coupled generator, and with isolated operation

The magnetic coupling of the generator rotor to the grid frequency is dependent on power, the same as the natural frequencies. This is shown clearly by the example of the American MOD-0 test turbine (Fig. 7.13). This turbine was equipped with a synchronous generator directly coupled to the grid. Strong vibrational resonances, a consequence of the tower shadow excitation, among others, necessitated the subsequent installation of a damping hydraulic coupling in the high-speed shaft (Fig. 7.14).

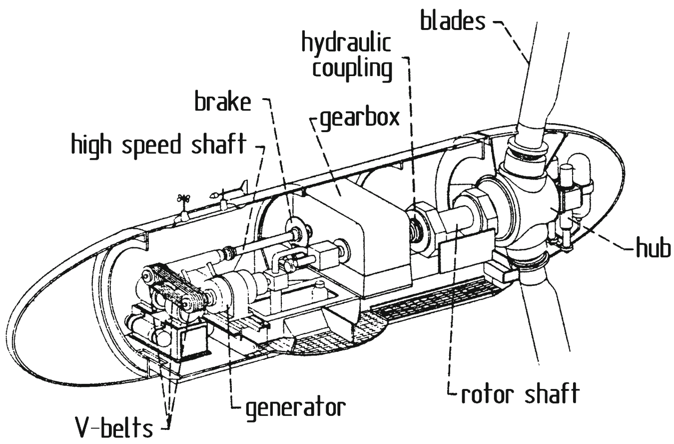


Fig. 7.13. Drive train of the experimental MOD-O [5]

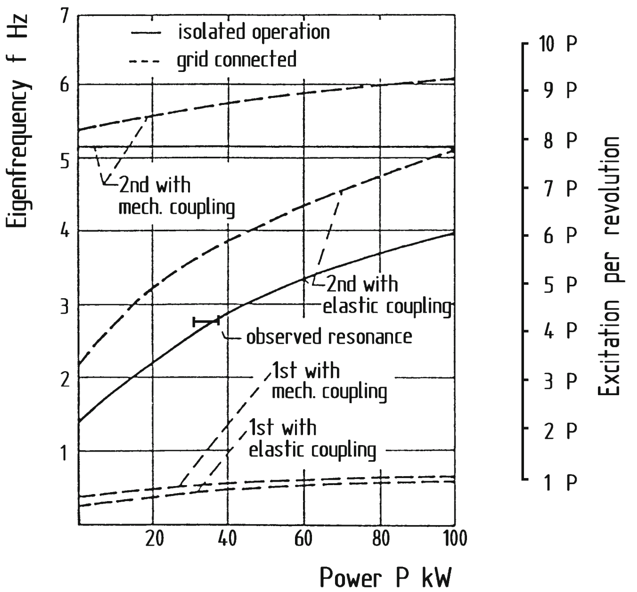


Fig. 7.14. Natural frequencies of the drive train of the MOD-0 with a synchronous generator directly coupled to the grid [5]

Grid-coupled induction generator

The slip of grid-coupled induction generators is the reason why the grid does not exert a reversing spring force on the drive train, but merely a damping force. The "zeroth natural frequency" resulting from these conditions is the total rotation of the drive train corresponding to the rotational generator speed, with a zero vibrational frequency. In contrast to the synchronous generator with direct grid coupling, the natural frequencies are not or only slightly dependent on power.

Synchronous and induction generators in idling and isolated operation

When idling or in isolated operation, both generator types behave in the same way. The 0th natural frequency is again the total rotation with zero vibrational frequency. At the first natural frequency, the drive train vibrates around its largest mass, the rotor blades, whereas the second mode includes the vibration of the partial mass next in size. As already mentioned in Chapter 7.2.3, the eigenmodes of the drive train can be influenced by the chordwise bending characteristics of the rotor blades. In very large units, the first natural chordwise bending frequency of the rotor blades is often located close to the first natural torsional frequency of the drive train. This results in a coupling so that it is no longer permissible to consider the drive train in isolation, assuming that the rotor blades are rigid. Calculating the coupled vibrational behaviour then becomes much more complicated since aerodynamic characteristics also play a role.

7.3.4 Excitations and Resonances

In the second step, the dynamic responses to various sources of excitations can be examined on the basis of the results of the modal analysis, and hazardous resonance points can then be identified. The excitation of vibrations in the drive train of a wind turbine can have its origin in many areas: External excitations can affect the drive train by way of the rotor. This applies mainly to cyclically alternating forces (Chapt. 6.2):

- tower wind shadow or tower dam,
- vertical wind shear,
- cross wind at the rotor due to yaw misalignments or an inclined rotor axis,
- mass imbalance of the rotor blades.

The external excitations occur with multiples of the rotor speed and are therefore characterised by 1 P, 2 P etc. (Chapt. 7.1). On the generator side, attention has to be paid to the following:

- electrical grid oscillations when grid feed lines are excessively long,
- oscillations of the inverters and AC-DC-AC links,
- control influences,
- load feedback in isolated operation.

Apart from these external influences, the drive train vibrations can also have "internal" origins. Possible causes are "mass imbalances" of the rotating parts, and "meshing frequencies" of the gearbox. Which of the excitation sources does indeed lead to hazardous resonances naturally depends on the actual numerical values of the natural and excitation frequencies. Small turbines with drive trains of relatively high stiffness react readily to internal excitation sources (Fig. 7.15). In the example shown, the

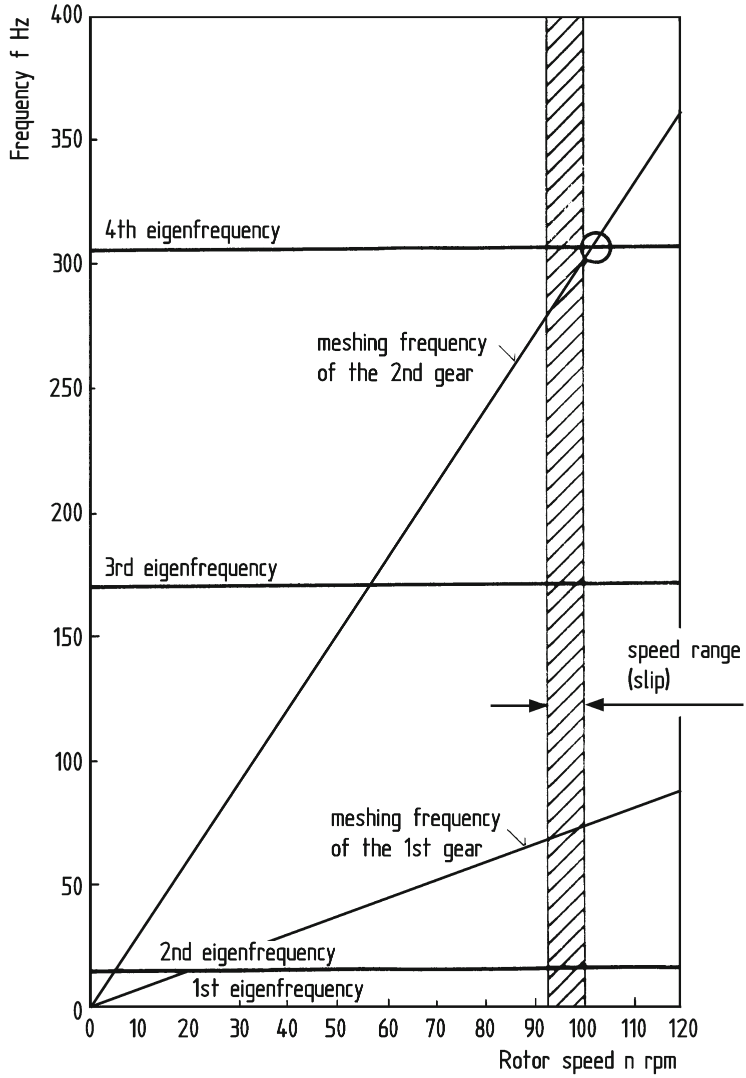


Fig. 7.15. Resonance diagram (Campbell diagram) of the Aeroman drive train with a resonant point at the fourth natural harmonic of the drive train, caused by the meshing frequency of the second gear stage close to rated operational speed

meshing frequency of the second gear stage excites the fourth natural harmonic of the drive train. In this real example, this fourth natural harmonic is the result of the vibration of the relatively large mechanical centrifugal switch mounted on the high-speed shaft. Strong resonances occurred here during a test operation.

In large wind turbines, the first natural frequencies of the drive train are lower by almost one order of magnitude and range around "a few Hertz". This is the range where the cyclically alternating aerodynamic forces from the rotor, for example the tower shadow interferences or the influence of the vertical wind shear, are located. Hence, there is a greater risk that the excitation frequencies emanating from the rotor will resonate with the drive train torsion, as is clearly shown by Figure 7.16.

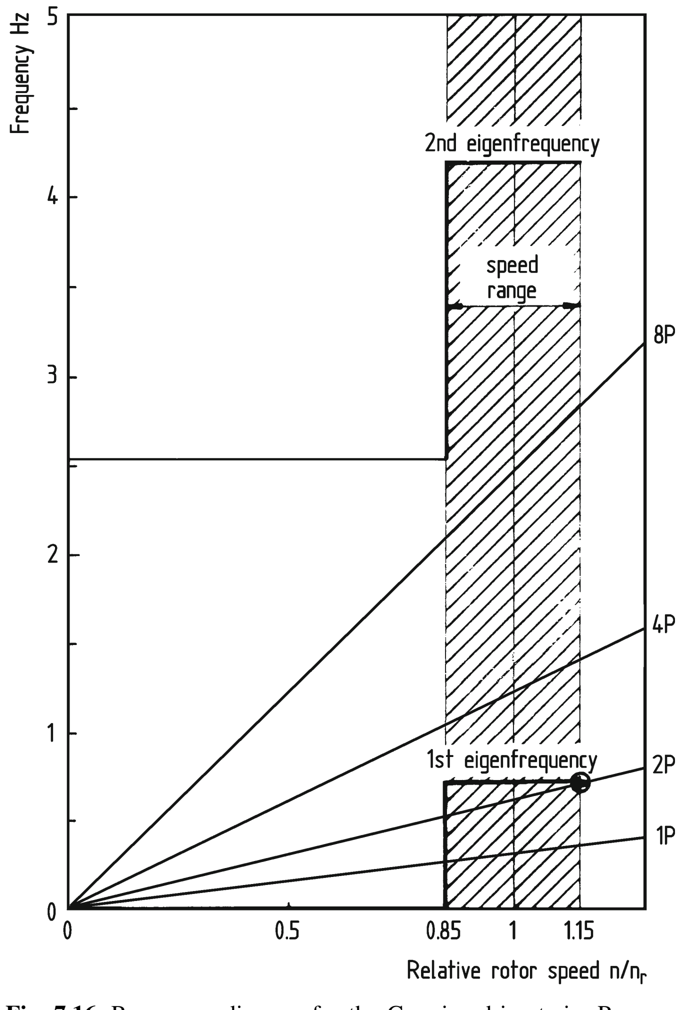


Fig. 7.16. Resonance diagram for the Growian drive train. Resonance of the first natural drive train frequency with tower shadow interference (2 P) at a rotor overspeed of 115 %

In the last years much more complex mathematical models have been developed for evaluating the drive train dynamics [6]. By means of these tools the influence of the elastic bearing systems of the gearbox and the generator are considered. Further more the influence of the electric control of the generator, for example including a special damper winding in the rotor can be evaluated.

7.4 Dynamics of the Yaw System

The failure statistics of wind turbines display a conspicuous accumulation in the "yaw system" component. The smaller turbines, in particular, frequently have problems with the life span of the yaw drive system. The reason for this is that the dynamic loads acting on the yaw system are often underestimated. It is, therefore, absolutely imperative that the dynamic load situation and the vibrational behaviour of the yaw drive be analysed. Just like the drive train, the azimuth drive has certain natural frequencies with respect to the yaw oscillation of the tower head. If resonances develop with the cyclically alternating rotational forces of the rotor, destruction of the components is only a matter of time.

7.4.1 Modelling and Moments Around the Yaw Axis

In principle, the mathematical model of the yaw system is very simple (Fig. 7.17). If the tower may be considered as being torsionally stiff, a "one-mass model" with an equivalent mass for the rotor and the nacelle is sufficient in the simplest case. As a rule, this assumption applies to older tubular steel towers of stiff design. Recent, more flexible towers require more accurate calculations which take into account the tower's torsional elasticity. In this case a two-mass model is necessary.

There is some difficulty in determining the torsional stiffness of the yaw drive from a real example. On the other hand, it is easier to determine the damping moments of friction. With some experience, it is nevertheless possible to calculate the numerical values of the most important first natural frequencies of the yaw vibration. On the basis of this and including the external excitation sources, the analysis of the vibrational behaviour can then be carried out.

The load situation for the yaw system varies greatly depending on the existing design features of the turbine, particularly on the number of rotor blades, the rotor position relative to the tower, the hub type and the distance of the rotor plane from the tower axis. In any case, the changing yaw moments can trigger undesirable torsional vibrations in the yaw system. Hence, the system must be designed with sufficient torsional stiffness. In addition, there must be sufficient frictional damping during the yawing process and, in a fixed azimuth position, suitable arresting forces. For damping the motion during yawing one or two yaw brakes can be activated.

The external excitation forces and moments vary, depending on whether the tower head is standing still or yawing is taking place. When the rotor is rotating during a yawing motion, the following moments are active around the yaw axis:

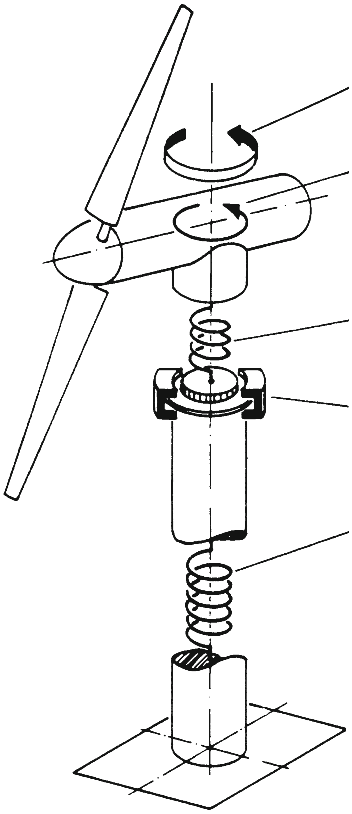


Fig. 7.17. Model of the vibrational behaviour of the yaw system

Aerodynamic moments

Depending on whether the rotor is a downwind rotor or an upwind rotor, the aerodynamic yaw moment of the rotor either has an assisting effect or an opposing effect. In both cases, the aerodynamically caused moments around the yaw axis are particularly undesirable, as these cyclic loads fluctuate intensely or even alternate. This is especially true for two-bladed rotors with a hingeless hub. This problem is solved almost completely by a teetering hub.

Gyroscopic moments

Yawing of the rotating rotor causes gyroscopic moments around the pitch axis of the nacelle. In turbines with active yaw drives, these moments only play a subordinate role, since the yawing rate is, as a rule, so slow that only small gyroscopic moments are developed. But problems will arise in smaller turbines with free yawing systems (Chapt. 6). Abrupt changes in wind direction, producing fast yawing movements, can lead to destructive gyroscopic moments.

Components of the rotor torque

If the rotor axis is tilted, a component of the rotor torque develops around the yaw axis. This moment must be taken into account in the balance of moments.

Moment of friction of the yaw bearing

The friction in the yaw bearing and brakes naturally also enters into the balance of moments. This moment is relatively small with the usual roller bearings. However, some turbines which do not have any separate yaw brakes have frictional sliding bearings (so-called “sliding blocks”) or roller bearings with special damping elements. Electric yaw drive motors with integrated brakes are also used (see Chapt. 9.13).

Moment of friction of the yaw brakes

Large turbines which have a number of active yaw brakes will use one or two brakes which are engaged during yawing in order to suppress unwanted yaw vibrations.

7.4.2 Excitation and Resonances

Yaw vibrations of the rotor and nacelle are, above all, excited by aerodynamic forces and moments. The main causes are cyclically fluctuating forces from wind shear or the tower shadow. They represent an ideal source of excitation for the tower head yaw vibration, mainly in combination with the dynamic mass effects of a two-bladed rotor with hingeless hub. The moment of inertia of a two-bladed rotor, which changes with respect to the pitch and yaw axes during one revolution, represents an additional so-called "parametric excitation" (Chapt. 6.8.2).

The natural modes of vibration of the yaw system have a characteristic feature which needs special attention. The yaw drive, i.e. either the driving pinion acting on a gear ring on the nacelle or tower, or the transmission gears of the drive motor, always has some play. If this play comes into effect, for example if the friction brakes are too weak, it will have considerable consequences with respect to the system’s natural frequency. The treatment of play-related vibrations is described in the relevant specialist literature [3].

Yawing systems with aerodynamically driven fantail wheels, commonly in use in the past, were particularly subject to this hazard. The small Aeroman turbine had initially been equipped with aerodynamic yawing using a fantail wheel acting on a worm gear (Chapt. 5.6). To ensure that the worm gear was running smoothly, a certain amount of play was necessary which, moreover, increased with increasing operating time. There was also the turbine design with a hingeless two-bladed rotor and the associated large rotor yaw moments around the vertical axis.

In the resonance diagram, the natural frequency range was outside the area of critical excitation, not taking into account the play in the worm gear (Fig. 7.18). As soon as the play in the gears became perceptible after some hundreds of hours of operation, the natural frequency dropped drastically, and a resonance with the exciting aerodynamic moments of the rotor developed. Without considerable frictional damping or a yaw brake, such a vibration will destroy the yaw system after a short period of time. Passive aerodynamic yaw systems with fantail wheels and without yaw damping or brake are, therefore, seldom used today. The small Aeroman turbine, too, had to be retrofitted with an active yaw drive with yaw damping in the azimuth bearing. Large turbines generally have fewer problems with the dynamic behaviour of the yaw system.

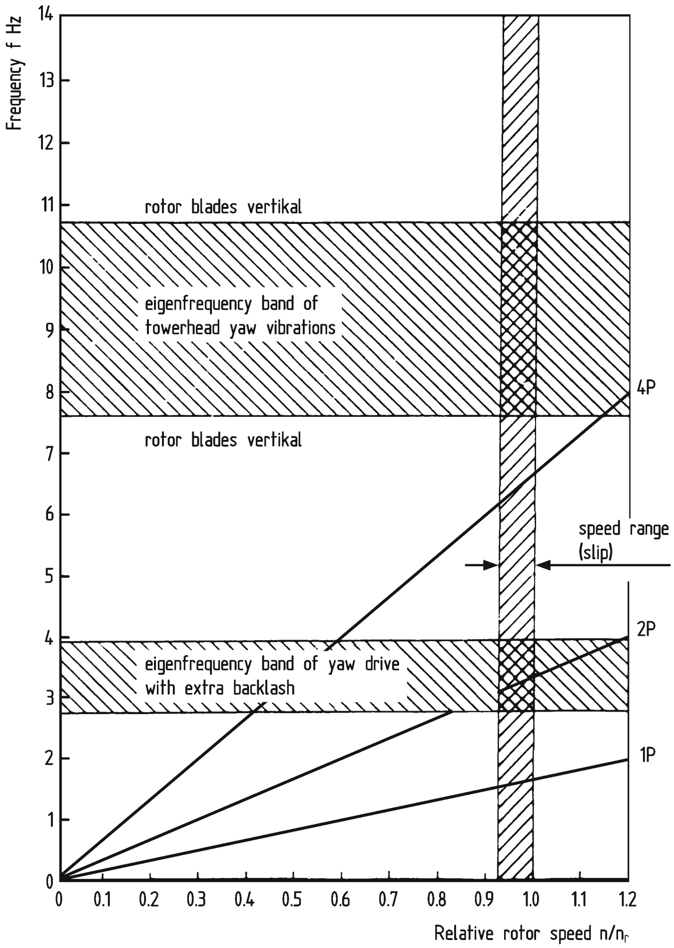


Fig. 7.18. Resonance diagram of the yaw motion of Aeroman. Natural frequency ranges with and without play in the worm gear

The active yaw system, with servo-controlled yaw drives and a number of yaw brakes, can be controlled more precisely and is, above all, protected from resonances by having the brakes applied when it is in the stand-still position. The risk of resonance is, therefore, restricted to the yaw movement. During yawing, however, there is still a risk of an inadmissible vibration. In this condition, the yaw systems natural frequencies must, therefore, be taken into consideration.

7.5 Vibration of the Whole Wind Turbine

When speaking of the vibrational behaviour of the turbine as a whole, this refers primarily to the vibrational coupling between the rotor and the tower. The rotor-tower system is continuously subject to the risk of self-excitation (s.a. Fig. 7.1).

The natural frequencies of the tower are here the natural frequencies of "tower plus head mass". In the usual tower designs there is a risk that the first natural bending frequency of the tower will resonate with the vibration-exciting rotor forces. The position of the first natural bending frequency of the tower in relation to the exciting frequencies from the rotor is, therefore, a decisive criterion for the vibrational behaviour of the wind turbine system.

7.5.1 Tower Stiffness

The tower's first natural bending frequency must not under any circumstances coincide with the critical exciting forces. Moreover, care must be taken to ensure that a certain distance from the remaining multiples of the rotor frequency is maintained. This distance cannot be generally specified. The distances between frequencies at which excessive vibrations will occur are determined by system damping, i.e. both structural as well as aerodynamic damping. Experience from existing turbines indicates that a safety distance of at least 10 % from the dominant frequency of excitation is a good guide value.

It has become accepted practice to designate the tower as being "stiff" or "soft" corresponding to the position of the tower's first natural bending frequency relative to the dominant excitation frequency of the rotor (s. Figs. 7.19 and 7.20). In a tower of stiff design, the natural tower frequency is not encountered during the start-up or shut-down procedures, thus eliminating the resonance hazard. Conversely, this "encounter problem" with a risk of resonance does exist in the soft tower. In some cases, the tower stiffness (first natural tower bending frequency) is set below the 1P exciting force. This case is occasionally referred to as the "double soft" or "soft-soft" design.

However, tower torsion must not be entirely disregarded, either, even if in most towers the first natural torsion frequency is distinctly higher than the first bending frequency. Above all, it is the yaw moment of the two-bladed rotor, which has already been mentioned numerous times, which can excite a torsional vibration in the tower. As far as vibrations are concerned, a turbine with a hingeless rotor and flexible tower design would, therefore, be an extremely hazardous concept. In the teetering rotor, the rotor yaw and pitch moments are largely decoupled from tower torsion or bending, thus permitting a less stiff and hence more cost-effective tower design to be realised.

In most cases, towers of the first generation were stiff. Resonances developing during the start-up sequence of the rotor were feared. In the course of development, however, almost all manufacturers changed to increasingly more flexible designs. For reasons of economy, the material saved in the process almost became a critical requirement (Chapt. 12.4).

Apart from the terms "soft" and "stiff" used for the tower design, the terms "supercritical" and "subcritical" design are occasionally mentioned in the literature. These terms are normally used in steam- or gasturbine design where similar problems exist. The natural frequency of a turbine wheel with flexurally elastic characteristics on a shaft (a so-called "Laval rotor") has a "critical value" at a certain rotational speed. Turbines running at less than the critical speed are said to run "subcritically". If their speed is above that, they are called "supercritical" runners. As these designations are much less graphic in their descriptive function and as, moreover, it is often not clear which component is super- or subcritical with respect to what, these designations are not used here [7].

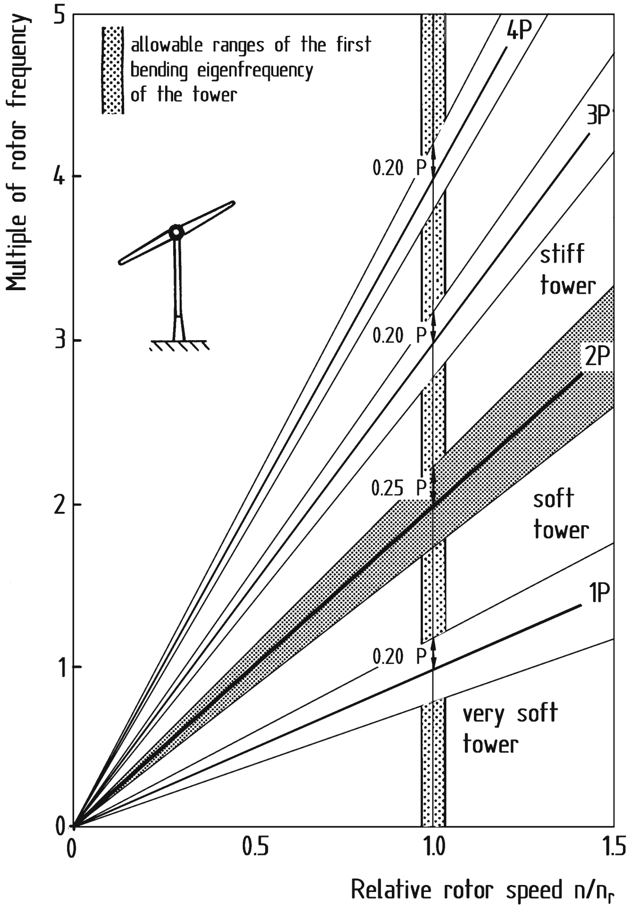


Fig. 7.19. Tower stiffness in the resonance diagram of a wind turbine with a two-bladed rotor (Campbell diagram)

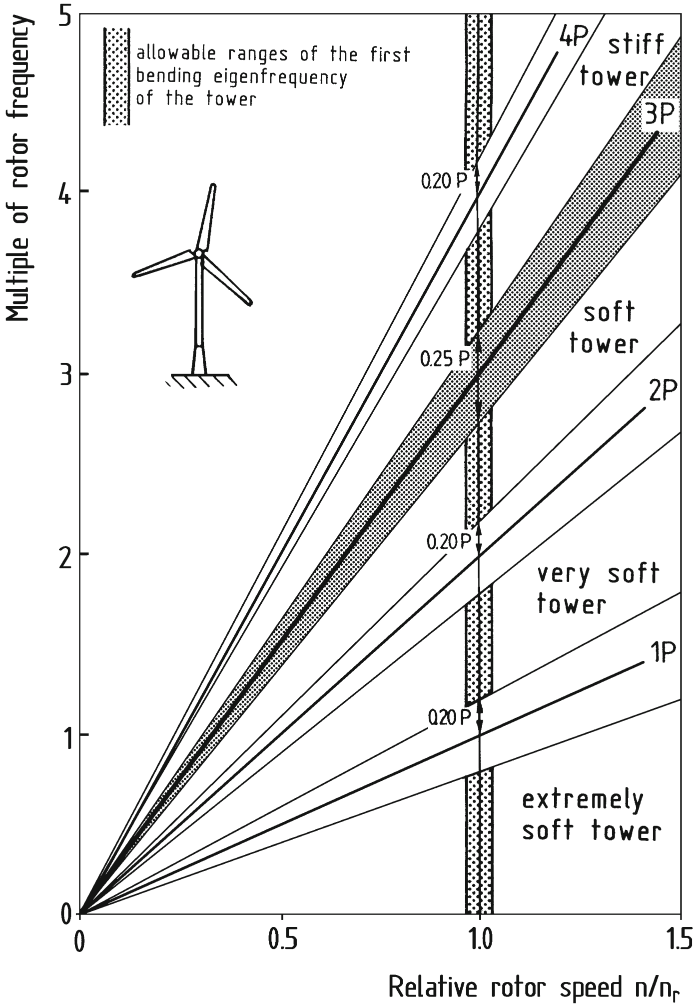


Fig. 7.20. Tower stiffness in the resonance diagram of a wind turbine with a three-bladed rotor

7.5.2 Vibrational Characteristics of Existing Wind Turbines

Designers initially achieved control over the vibrational behaviour of wind turbines by means of various designs. Proponents of the stiff design stood in opposition to those favouring the "soft line". It only emerged in the course of the development that the problematic and risky flexible line became the preferred design. It was found that, particularly with the increasing size of turbines, the weight saving in the tower which could be achieved by the flexible design represents a considerable economic benefit. Since then, the soft tower concept is generally considered to be the more advanced design and is, therefore, almost the only one implemented today. A soft tower design is almost mandatory for large turbines for these reasons.

The following resonance diagrams (called *Campbell diagrams* in English-language literature) can be described as the "dynamic calling cards" of the wind turbines. The numerical values indicated are not highly accurate. The data published by the individual manufacturers are not complete and are also based on not fully comparable preconditions. Partly, the figures may be calculated results from the design stage determined, on the one hand, for isolated components, i.e. rotor blades, and, on the other hand, for subsystems, i.e. the rotating rotor. These values are set against measurement results from operational turbines. Their accuracy is, nevertheless, sufficient to provide an overview of the vibrational characteristics of the individual turbines.

The "first generation" American MOD-0 and MOD-1 experimental turbines (Fig. 7.21) were representatives of the stiff turbine design [7]. The first natural bending frequency of the tower was located well above the rotor's 2 P excitation frequency. The lattice towers, which had been used at that time, had the appropriate pre-conditions for this layout. Moreover, the lattice steel towers had extraordinary torsional stiffness, a characteristic which was definitely necessary in view of the hingeless two-bladed rotors.

The "second-generation" MOD-2 (Fig. 7.22) represented the transition to the flexible design. The first natural bending frequency of the tower was placed between the 1 P and 2 P excitation frequencies. The rotor incorporated a teetering hub avoiding the extreme yaw moments of the stiff rotor.

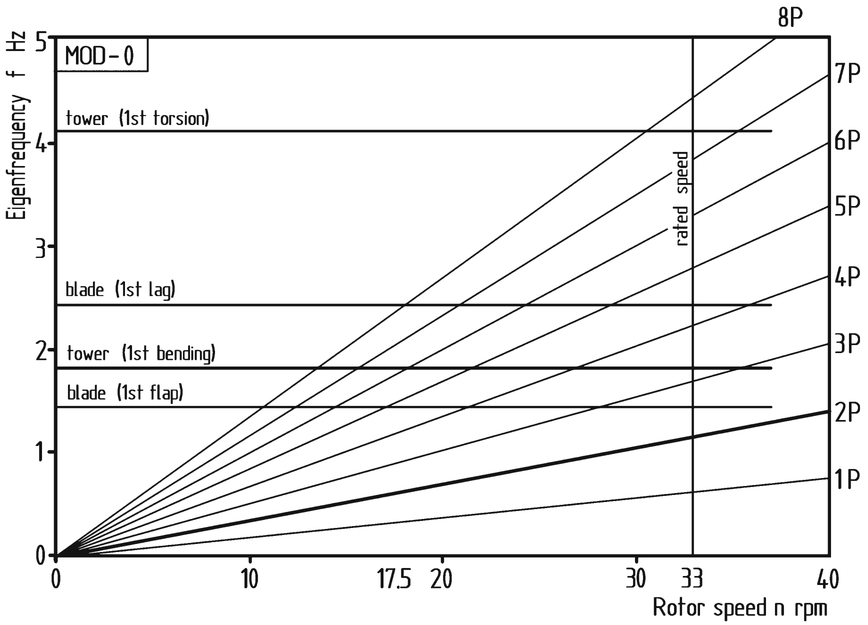


Fig. 7.21. Resonance diagram of the MOD-0 with hingeless two-bladed rotor on the downwind side and lattice tower of stiff design

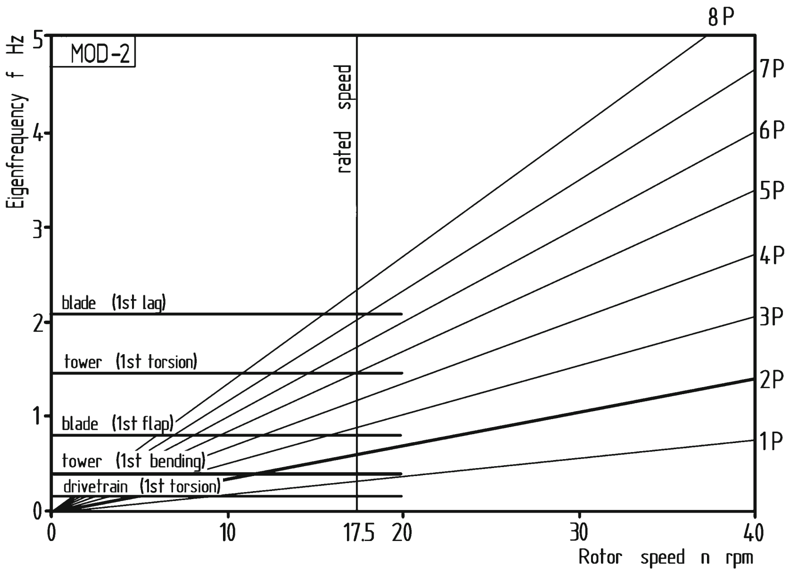


Fig. 7.22. Resonance diagram of the MOD-2 with teetering rotor on the upwind side and steel tube tower of soft design

A comparison with the large Swedish WTS-75 and WTS-3 turbines and the American WTS-4 sister model is particularly interesting. In a way, they represented the key elements in the wide range of vibrational design.

The WTS-75 stood for the stiff design (Fig. 7.23). However, the consequences were obvious: the concrete tower weighed a colossal 1500 tonnes! The concept of the WTS-3/4 is the exact opposite. In combination with a two-bladed teetering rotor, a soft tower between 1 P and 2 P was chosen for the Swedish WTS-3 (Fig. 7.24). The American version WTS-4 even had a "double-soft" tower with a first natural bending frequency below 1 P. Experience with the vibrational characteristics of the WTS-4 showed, however, that these extremely flexible tower designs lead to problems at least with this large two-bladed turbine. The tower head's vibration amplitudes were described as extremely unpleasant.

With regard to their vibrational behaviour, turbines with three-bladed rotors are more unproblematic. Above all, there is no strong excitation of the yawing movement with respect to tower torsion, typical of the two-bladed rotor. An example of a turbine with a three-bladed rotor is the Tjaereborg turbine (Fig. 7.25). It is representative of the traditional Danish line. The concrete tower has a "soft" design with respect to the critical 1 P excitation, i.e. the rotor passes through the tower resonance during the start-up sequence.

More recent three-bladed turbines with steel towers all have flexible towers with a natural bending frequency between 1 P and 2 P or even below 1 P. As the size of wind turbines increases and they are being economically optimised, the reduction in tower mass thus achieved has clearly become a factor which is no longer negligible. In addition, the vibrational characteristics of wind turbines are increasingly being brought

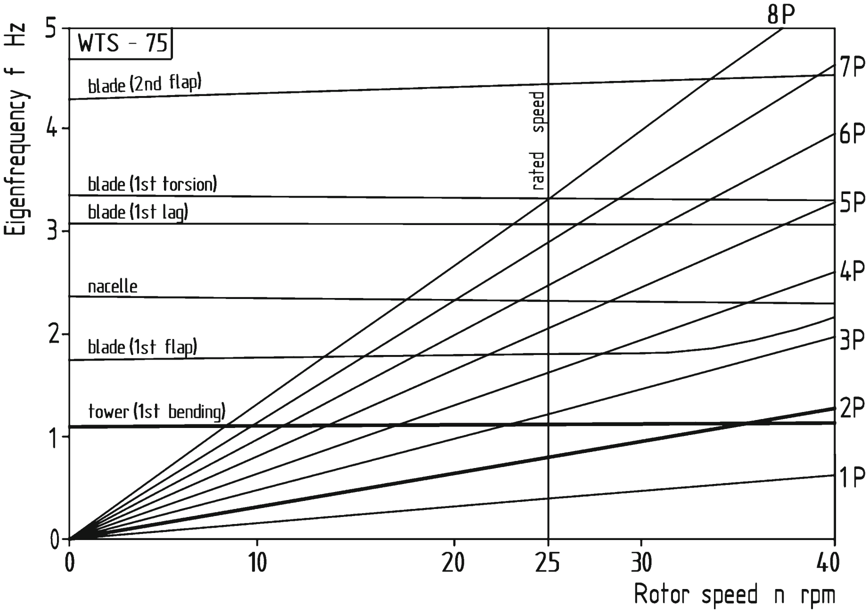


Fig. 7.23. Resonance diagram of the WTS-75 with hingeless rotor on the upwind side and a stiff concrete tower [8]

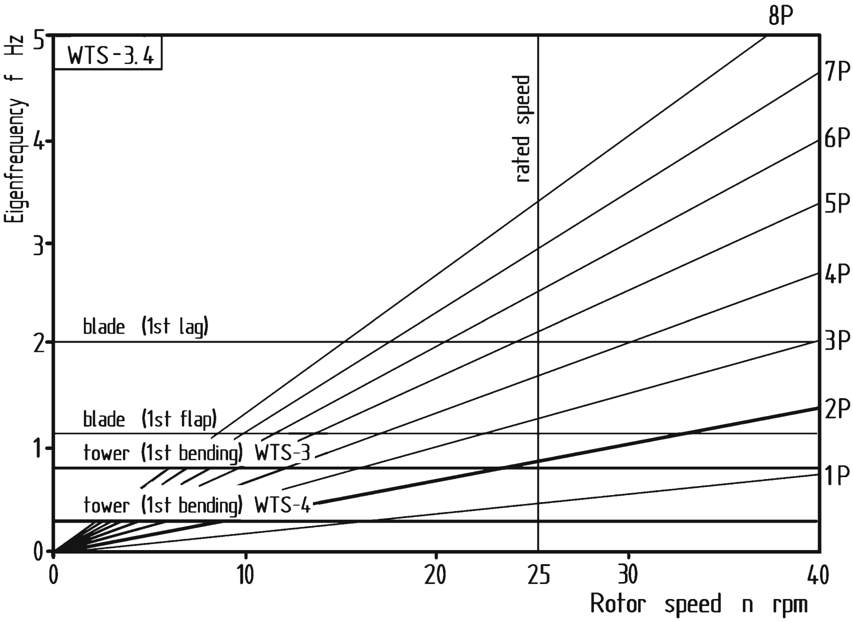


Fig. 7.24. Resonance diagram of the WTS-3/4 with two-bladed teetering rotor on the downwind side and soft tower design (WTS-3) as well as double-soft tower design (WTS-4)

under control so that it is becoming possible to push the design closer to the technical limits.

Avoiding resonances becomes considerably more difficult when the rotor is of the variable-speed type. Either the speed range is restricted by the locations of the relevant natural frequencies, or a critical speed section has to be bridged by the speed control system passing through it quickly and not allowing steady-state operation within it. The first of the two options was chosen for the WKA-60 (Fig. 7.26).

In this turbine, however, the first natural bending frequency of the concrete tower coincided precisely with the 2 P excitation at rated speed. Even though these multiples of 1 *P* excitation are basically considered to be less critical in a three-bladed rotor, unpleasant resonances did, nevertheless, develop in practical operation. The *dynamic amplification factor,* i.e. the ratio of the maximum vibrational response to the amplitude of the excitation, shows how the 2 P excitation affects the tower's transverse vibration (Fig. 7.27). If resonance occurs, the cyclic transverse force is magnified by an amplification factor of 4, a fact which must be taken into consideration with respect to fatigue life. A peculiarity associated with concrete towers must be mentioned in this context. It is no rare occurrence that the actual natural frequencies of concrete towers differ considerably from the calculated values. This may be due to a lack of appropriate care and attention during construction.

The same considerations and criteria basically apply to the dynamic behaviour of vertical-axis rotors as do to horizontal-axis rotors (Fig. 7.28) calculation methods for dynamics analysis are available from published literature [9]. The special characteristics of vertical-axis rotors are based on the fact that the free-stream velocity and the angle of attack of the rotor blades oscillate during the cycle of rotation, resulting in special excitation situations for the vibrational behaviour of the rotor blades. This is influenced mainly by the symmetrical and asymmetrical vibration modes in the rotor plane (in two-bladed rotors).

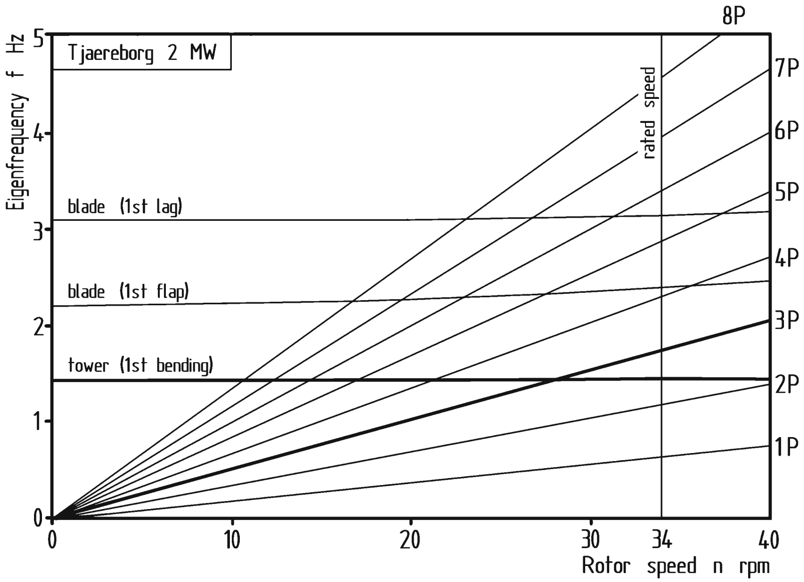


Fig. 7.25. Resonance diagram of the Tjaereborg turbine with three-bladed rotor on the upwind side and soft concrete tower

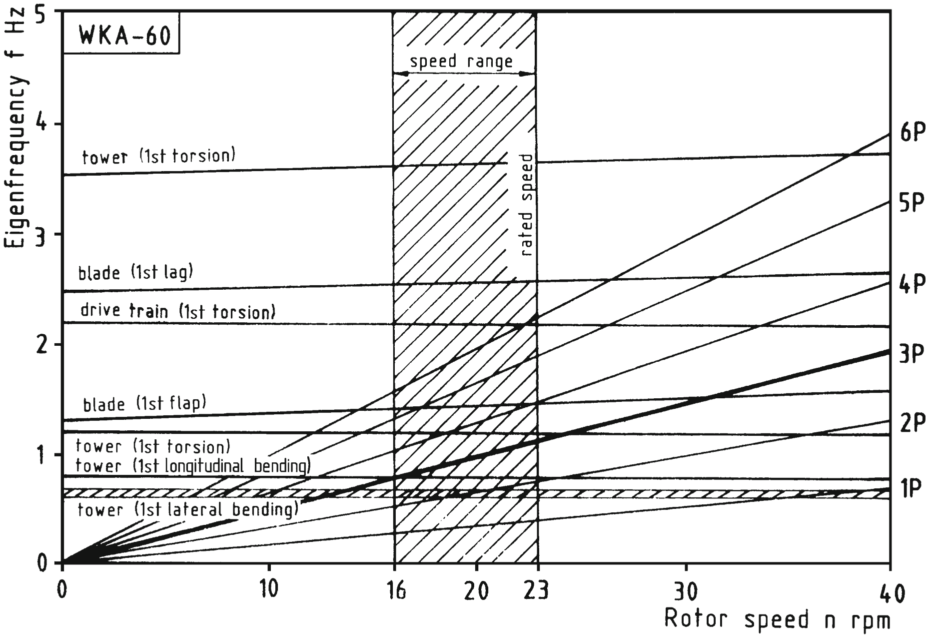


Fig. 7.26. Resonance diagram of the WKA-60 with variable speed three-bladed rotor on the up-wind side and soft tower

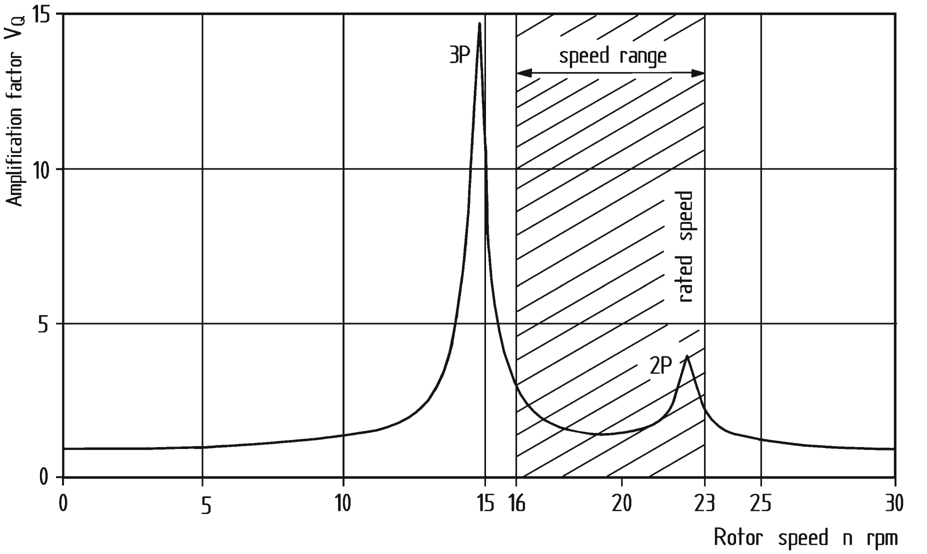


Fig. 7.27. Amplification of cyclic transverse force acting on the tower of the WKA-60 in a case of resonance

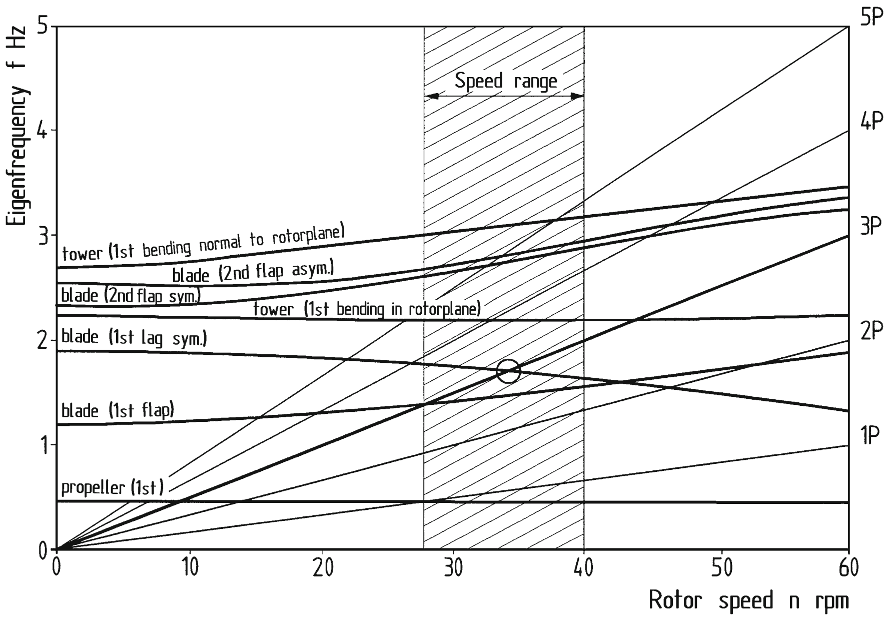


Fig. 7.28. Resonance diagram of a Darrieus rotor with two rotor blades [9]

7.6 Mathematical Simulation

Strictly speaking, the vibrational behaviour of a system with several degrees of freedom can only be treated as a total system. This is true, above all, when the dynamic coupling of the excited degrees of freedom is so strong, that complex vibrational coupling modes are produced the natural frequencies of which deviate distinctly from the separate natural frequencies of the components involved. This is basically the situation found in wind turbines. In addition, aerodynamics, gravitational forces, structural and aerodynamic damping and, not least, control characteristics must also be included in the calculation.

Before beginning with a mathematical simulation of such an overall system, it is helpful to find out as much as possible about the basic vibrational character of the turbine or of its design so that the critical vibration modes can be recognised. In most cases, isolated mathematical treatment of the components or of specific subsystems of the turbine is feasible. For this purpose, the first and some higher natural frequencies and the vibration modes of the most important components are isolated and then calculated for a stand-still condition.

The natural frequencies of the subsystem "tower with tower head" can always be calculated with sufficient accuracy when it is assumed that the rotor is at stand-still. The influence of the rotating rotor is small. This only applies to a limited extent to the rotor blades. When rotating, the rotor blades behave differently compared to the standstill condition. The centrifugal forces exert a "stiffening" influence. Precise determination of the natural rotor blade frequencies therefore requires a mathematical simulation of the rotating rotor, taking into consideration the degrees of freedom and the stiffness of the hub construction. The resonance diagram determined on this basis already provides reliable information about the resonant behaviour of any reasonable design. In most cases, the "drive train" and "yaw control" subsystems are decoupled from the overall system to such an extent that they can be considered in isolation.

If the vibrational behaviour is assessed as being particularly complex, or even critical, and if shifting of the natural component frequencies is no longer feasible by constructional measures, a mathematical simulation of the coupled total system is unavoidable. Considering the fact that this mathematical simulation of the vibrational behaviour plays an important role in the development at least of large experimental turbines, the basic principles of mathematical simulation techniques will be explained here briefly.

As has been described above, the first step is to determine the natural frequencies of the main components. Based on this, the subsystems "tower with nacelle and rotor mass" are coupled mathematically to the subsystem "rotating rotor", taking into account the kinematic and kinetic constraints. As is common practice in the case of multiple mass systems, further treatment is carried out according to a theory by Lagrange. The kinetic and potential energy, as well as the energy of the external forces (aerodynamic forces), are determined for the subsystems and then coupled according to a modal condition of compatibility, i.e. one relating to the vibration mode. Differentiation of the energy equations yields differential equations (equations of motion) for the variation of the vibrations with time.

For mass-symmetrical rotors, i.e. rotors with three and more blades, these equations are comparatively easy to solve. The mathematical treatment of rotors with two or even only one rotor blade which are not gravitationally symmetrical is much more difficult. Due to the fact that the inertial moment alternates with rotation in relation to the fixed system of axes, the equations of motion contain time-dependent periodic coefficients. This makes the matrix operations applied for solving the equations very complex. The solution is achieved with the aid of the so-called “Floquet theory”.

As a consequence of the periodic coefficients, the result contains non-sinusoidal natural vibrations of the overall system with components of higher harmonics. To every degree of freedom, several natural frequencies can be assigned. In practice, one natural frequency or vibration mode is almost always distinctly dominant and, moreover, usually lies close to the natural frequency of the subsystem treated in isolation. This holds true at least as long as no resonance occurs (Fig. 7.29).

Apart from the phenomena explained here, the vibrational simulation of the entire system can reveal a number of other aeroelastic effects. Such as the so-called *nacelle whirl vibration* in which the centre of the rotor performs an elliptical movement, or *transverse tower-nacelle vibrations*. Axial tower-nacelle vibrations can affect the blade pitch control and manifest themselves as *control system vibrations*. In practice, these effects rarely play a role and are only mentioned here, therefore.

One of the first mathematical simulation techniques for the vibrational behaviour and the dynamic loads of wind turbines with horizontal-axis rotors has been developed by the Paragon Pacific Institute in the US under the name of MOSTAS (Modular Stability Derivative Program) [10]. This collection of programs had originally been developed for the treatment of aeroelastic effects of aircraft structures and helicopter rotors and was then adapted for the mathematical treatment of the vibrational behaviour of wind rotors and wind turbines. The theoretical results were verified mainly with the experimental MOD-0 wind turbine. After several stages of development of the theory and the computer programs, consistency with the measured results seems to be satisfactory.

Very comprehensive multi-body simulation codes have been developed and are also been used in recent years (s. Chapt. 6.7.3). Today, with the computer performance available, it is possible to simulate very complex systems including a great number of details [11]. The computing capacity seems to be no longer a limit. However, some further critical remarks with respect to the mathematical simulation of the vibrational behaviour of wind turbines are necessary. Under the pretext of a comprehensive simulation of vibrational behaviour and dynamic loads or instabilities to be derived from these, veritable "computer orgies" are staged, the practical value of which is frequently inversely proportional to the number of degrees of freedom taken into consideration and of the coefficients in the differential equations.

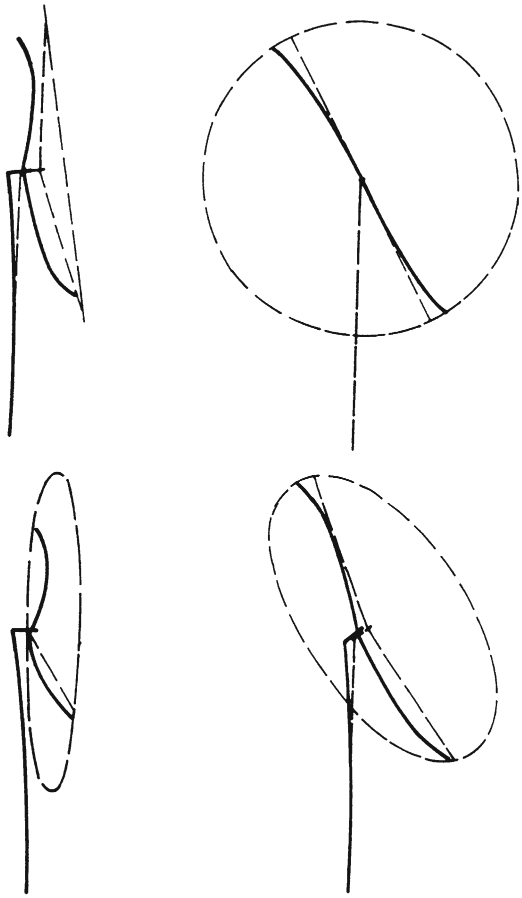


Fig. 7.29. Vibration modes of the coupled "rotor-tower" system as a result of a mathematical simulation [12]

The more complex the simulation technique, the more input data are required. But this is exactly what is lacking. At the design stage, detailed stiffness and damping parameters of the complex mechanical structures are almost never available. Without reliable input data, however, simulation becomes a pointless formality.

If the technical concept of the wind turbine is “reasonable”, the vibrational coupling between rotor and tower will not be as drastic as may be feared, and if it is, the design must be changed. A vibrationally safe overall technical concept with sensible determination of rotor design and tower stiffness and correct placement of the natural frequencies of the critical components is the only decisive prerequisite for managing the vibrational behaviour of a wind turbine. Mathematical simulation has its justification as a checking tool, but it is no substitute for a vibrationally safe design.

The uncertainties remaining in the mathematical simulation of vibration response have led to occasional efforts of trying to experimentally determine vibrational behaviour by means of model tests in the wind tunnel. These attempts almost always meet with insurmountable difficulties as to observing the necessary laws of similarity which is required for guaranteeing that the model results can be transferred to the original. The problem is that both aerodynamic model laws and model laws concerning structural elasticity must be observed, something which is not possible in practice with small model scales in the available wind tunnels. Nevertheless, these types of experimental investigations can be of use if they are carried out with limited goals and if the results are interpreted correctly, as they improve the basic understanding of the mechanisms at work.

During the development of the Growian turbine, the Institute of Aeroelastics of the DLR in Göttingen, Germany, carried out qualitative investigations on an aeroelastic model with a scale of 1:66 [12]. A similar test program was carried out in the US for the MOD-2 turbine, using a model with a rotor diameter of 3.8 m.

References

1. Försching, H.W.: Grundlagen der Aeroelastik. Springer, Heidelberg (1974)
2. Eggle, D.M., Stoddard, E.S.: Wind Turbine Engineering Design. Van Nostrand and Reinhold Company, New York (1987)
3. Magnus, K.: Schwingungen; B.G. Teubner, Stuttgart (1969)
4. Gasch, R., Nordmann, R., Pfützner, H.: Rotordynamik. Springer-Verlag, Berlin (2002)
5. Sullivan, T.L., Miller, D.R., Spera, D.A.: Drive Train Normal Modes Analysis for the ERDA/NASA 100-Kilowatt Wind Turbine Generator, NASA TM-73718 (1977)
6. Schlecht, B., et al.: Analysen zu Schwingungen in Windenergieanlagen mittels Mehrkörpersimulationen und Finite-Elemente-Methode, Haus der Technik, Essen (March 16-17, 2006)
7. Bradford, S.L., Glasgow, J.: Experimental Data and Theoretical Analysis of an Operating 100 kW Wind Turbine. NASA TM-73883 (1978)
8. Boving-KMW Turbin, AB: Wind Turbine System Näsudden, Technical Documentation, Kristineham, Schweden (1985)
9. Vollan, A.: Structural Dynamic Problems of Darrieus Rotors, Wind Resources and Darrieus Rotor, Paris (April 1986)

10. Hoffmann, J.A.: Coupled Dynamics Analysis of Wind Energy Systems. NASA CR-135152 (1977)
11. Schlecht, B., Rosenlöcher, T.: Determining Reliable Load Assumptions in Wind Turbines using SIMPACK, Technische Universität Dresden, SIMPACK News (September 2010)
12. Kießling, E.: Modellierung des aeroelastischen Gesamtsystems einer Windturbine mit Hilfe symbolischer Programmierung. DFVLR Research Report DFVLR-FB 84-10 (1984)