Chapter 4
Physical Principles of Wind Energy Conversion

The primary component of a wind turbine is the energy converter which transforms the kinetic energy contained in the moving air, into mechanical energy. For the initial discussions of principles, the exact nature of the energy converter is irrelevant. The extraction of mechanical energy from a stream of moving air with the help of a disk-shaped, rotating wind energy converter follows its own basic rules.

The credit for having applied this principle to windmills is owed to Albert Betz. Between 1922 and 1925, Betz published writings in which he was able to show that, by applying elementary physical laws, the mechanical energy extractable from an air stream passing through a given cross-sectional area is restricted to a certain fixed proportion of the energy or power contained in the air stream [1]. Moreover, he found that optimal power extraction could only be realised at a certain ratio between the flow velocity of air in front of the energy converter and the flow velocity behind the converter. Similar approaches have been made by Rankine and Froude on the example of ship propellers in the nineteenth century.

Although Betz's "momentum theory", which assumes an energy converter working without losses in a frictionless airflow, contains simplifications, its results are quite usable for performing rough calculations in practical engineering. But its true significance is founded in the fact that it provides a common physical basis for the understanding and operation of wind energy converters of various designs. For this reason, the following sections will provide a summarised mathematical derivation of the elementary "momentum theory" by Betz.

4.1 Betz's Elementary Momentum Theory

The kinetic energy of an air mass m moving at a velocity v can be expressed as:

$$E = \frac{1}{2} m v^2 \qquad (Nm)$$

Considering a certain cross-sectional area A, through which the air passes at velocity v, the volume V flowing through during a certain time unit, the so-called "volume flow", is:

$$\dot{V} = v\,A \qquad (m^3/s)$$

and the mass flow with the air density ϱ is:

$$\dot{m} = \varrho\, v\, A \qquad (kg/s)$$

The equations expressing the kinetic energy of the moving air and the mass flow yield the amount of energy passing through cross-section A per unit time. This energy is physically identical to the power *P:*

$$P = \frac{1}{2}\,\varrho\, v^3\, A \qquad (W)$$

The question is how much mechanical energy can be extracted from the free-stream airflow by an energy converter. As mechanical energy can only be extracted at the cost of the kinetic energy contained in the wind stream, this means that, with an unchanged mass flow, the flow velocity behind the wind energy converter must decrease. Reduced velocity, however, means at the same time a widening of the cross-section, as the same mass flow must pass through it. It is thus necessary to consider the conditions in front of and behind the converter (Fig. 4.1).

Here, v_1 is the undelayed free-stream velocity, the wind velocity, before it reaches the converter, whereas v_2 is the flow velocity behind the converter.

The mechanical energy which the disk-shaped converter extracts from the airflow corresponds to the power difference of the air stream before and after the converter:

$$P = \frac{1}{2}\,\varrho\, A_1\, v_1^3 - \frac{1}{2}\,\varrho\, A_2\, v_2^3 = \frac{1}{2}\,\varrho\,(A_1\, v_1^3 - A_2\, v_2^3) \qquad (W)$$

Maintaining the mass flow (continuity equation) requires that:

$$\varrho\, v_1\, A_1 = \varrho\, v_2\, A_2 \qquad (kg/s)$$

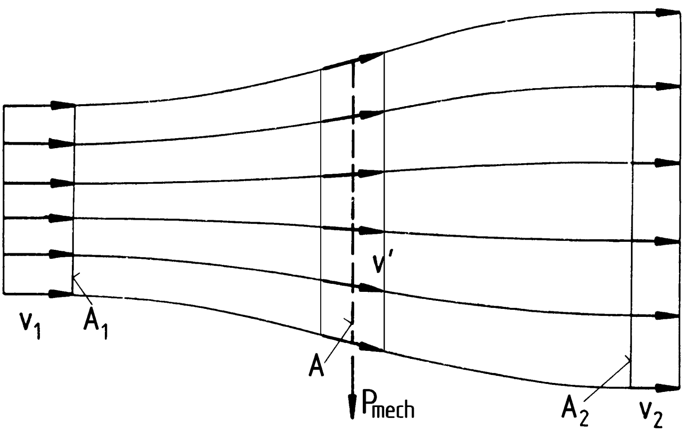

Fig. 4.1. Flow conditions due to the extraction of mechanical energy from a free-stream air flow, according to the elementary momentum theory

Thus,

$$P = \frac{1}{2}\,\varrho\, v_1\, A_1 (v_1^2 - v_2^2) \qquad (W)$$

or

$$P = \frac{1}{2}\,\dot{m}\,(v_1^2 - v_2^2) \qquad (W)$$

From this equation it follows that, in purely formal terms, power would have to be at its maximum when v_2 is zero, namely when the air is brought to a complete standstill by the converter. However, this result does not make sense physically. If the outflow velocity v_2 behind the converter is zero, then the inflow velocity before the converter must also become zero, implying that there would be no more flow through the converter at all. As could be expected, a physically meaningful result consists in a certain numerical ratio of v_1/v_2 where the extractable power reaches its maximum.

This requires another equation expressing the mechanical power of the converter. Using the law of conservation of momentum, the force which the air exerts on the converter can be expressed as:

$$F = \dot{m}\,(v_1 - v_2) \qquad (N)$$

According to the principle of "action equals reaction", this force, the thrust, must be counteracted by an equal force exerted by the converter on the airflow. The thrust, so to speak, pushes the air mass at air velocity v', present in the plane of flow of the converter. The power required for this is:

$$P = F\,v' = \dot{m}\,(v_1 - v_2)\,v' \qquad (W)$$

Thus, the mechanical power extracted from the air flow can be derived from the energy or power difference before and after the converter, on the one hand, and, on the other hand, from the thrust and the flow velocity. Equating these two expressions yields the relationship for the flow velocity v':

$$\frac{1}{2}\,\dot{m}\,(v_1^2 - v_2^2) = \dot{m}\,(v_1 - v_2)\,v' \qquad (W)$$

$$v' = \frac{1}{2}\,(v_1 - v_2) \qquad (m/s)$$

Thus the flow velocity through the converter is equal to the arithmetic mean of v_1 and v_2:

$$v' = \frac{v_1 - v_2}{2} \qquad (m/s)$$

The mass flow thus becomes:

$$\dot{m} = \varrho\, A\, v' = \frac{1}{2}\,\varrho\, A\,(v_1 + v_2) \qquad (kg/s)$$

The mechanical power output of the converter can be expressed as:

$$P = \frac{1}{4}\,\varrho\,A\,(v_1^2 - v_2^2)\,(v_1 + v_2) \qquad (W)$$

In order to provide a reference for this power output, it is compared with the power of the free-air stream which flows through the same cross-sectional area A, without mechanical power being extracted from it. This power is:

$$P_0 = \frac{1}{2}\,\varrho\,v_1^3\,A \qquad (W)$$

The ratio between the mechanical power extracted by the converter and that of the undisturbed air stream is called the "power coefficient" c_P:

$$c_P = \frac{P}{P_0} = \frac{\frac{1}{4}\,\varrho\,A\,(v_1^2 - v_2^2)\,(v_1 + v_2)}{\frac{1}{2}\,\varrho\,A\,v_1^3} \qquad (-)$$

After some re-arrangement, the power coefficient can be specified directly as a function of the velocity ratio v_2/v_1:

$$c_P = \frac{P}{P_0} = \frac{1}{2}\left[1 - \left(\frac{v_2}{v_1}\right)^2\right]\left[1 + \frac{v_2}{v_1}\right] \qquad (-)$$

The power coefficient, i.e. the ratio of the extractable mechanical power to the power contained in the air stream, therefore, now only depends on the ratio of the air velocities

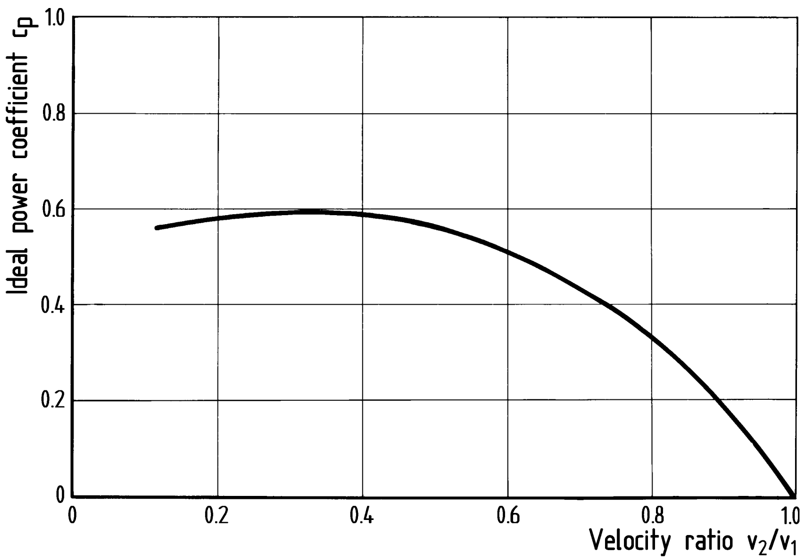

Fig. 4.2. Power coefficient versus the flow velocity ratio of the flow before and after the energy converter

before and after the converter. If this interrelationship is plotted graphically - naturally, an analytical solution can also be found easily - it can be seen that the power coefficient reaches a maximum at a certain velocity ratio (Fig. 4.2).

With v_2/v_1 = 1/3, the maximum "ideal power coefficient" c_P becomes

$$c_P = \frac{16}{27} = 0.593$$

Betz was the first to derive this important value and it is, therefore, frequently called the "Betz factor".

Knowing that the maximum, ideal power coefficient is reached at v_2/v_1 = 1/3, the flow velocity v'

$$v' = \frac{2}{3}\, v_1 \qquad (m/s)$$

and the required reduced velocity v_2 behind the converter can be calculated:

$$v_2 = \frac{1}{3}\, v_1 \qquad (m/s)$$

Fig. 4.3 shows the flow conditions through the wind energy converter once again, in greater detail. In addition to the flow lines, the variations of the associated flow velocity and of the static pressure are indicated. When approaching the converter plane the air is

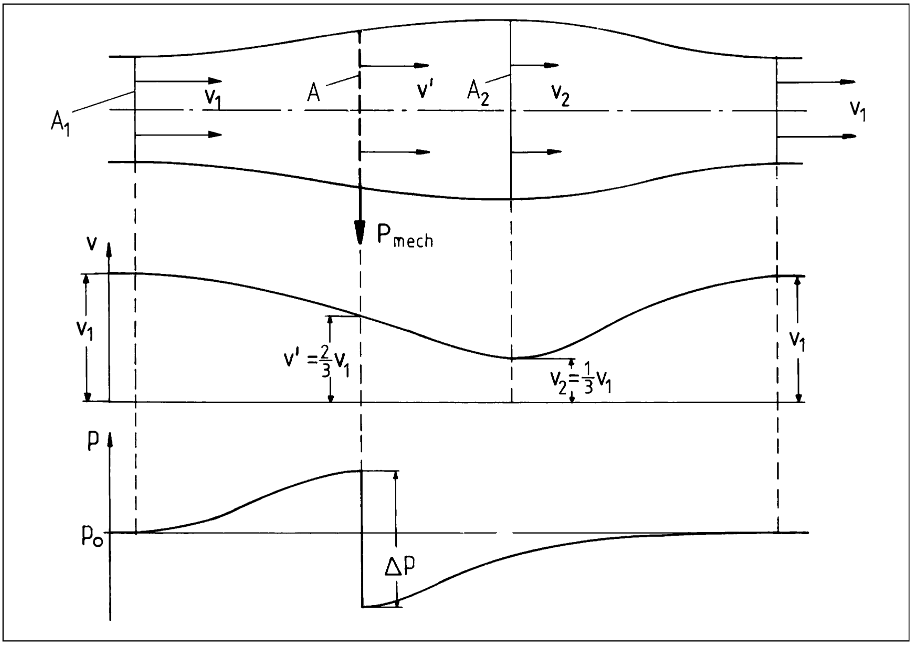

Fig. 4.3. Flow conditions of the stream through an ideal disk-shaped energy converter with the maximum possible extraction of mechanical power

retarded, it flows through and is then slowed down further to a minimum value behind the turbine. The flow lines show a widening of the stream tube to a maximum diameter at the point of lowest air velocity. Approaching the turbine, the static pressure increases, and then jumps to a lower value, to level out again at the ambient pressure behind the converter due to pressure equalisation. The flow velocity then also increases again to its initial value far behind the converter and the widening of the stream tube disappears.

It is worthwhile to recall that these basic relationships were derived for an ideal, frictionless flow, and that the result was obviously derived without having a close look at the wind energy converter. In real cases, the power coefficient will always be smaller than the ideal Betz value. The essential findings derived from the momentum theory can be summarised in words as follows:

- The mechanical power which can be extracted from a free-stream airflow by an energy converter increases with the third power of the wind velocity.
- The power increases linearly with the cross-sectional area of the converter traversed, it thus increases with the square of its diameter.
- Even with an ideal airflow and lossless conversion, the ratio of extractable mechanical work to the power contained in the wind is limited to a value of 0.593. Hence, only about 60 % of the wind energy of a certain cross-section can be converted into mechanical power.
- When the ideal power coefficient achieves its maximum value $c_P = 0.593$, the wind velocity in the plane of flow of the converter amounts to two thirds of the undisturbed wind velocity and is reduced to one third behind the converter.

4.2 Wind Energy Converters Using Aerodynamic Drag or Lift

The momentum theory by Betz indicates the physically based, ideal limit value for the extraction of mechanical power from a free-stream airflow without considering the design of the energy converter. However, the power which can be achieved under real conditions cannot be independent of the characteristics of the energy converter.

The first fundamental difference which considerably influences the actual power depends on which aerodynamic forces are utilised for producing mechanical power. All bodies exposed to an airflow experience an aerodynamic force the components of which are defined as aerodynamic drag in the direction of flow, and as aerodynamic lift at a right angle to the direction of flow. The real power coefficients obtained vary greatly in dependence on whether aerodynamic drag or aerodynamic lift is used [2].

Drag devices

The simplest type of wind energy conversion can be achieved by means of pure drag surfaces (Fig. 4.4). The air impinges on the surface A with velocity v_W, the power capture P of which can be calculated from the aerodynamic drag D, the area A and the velocity v_r with which it moves:

$$P = D\, v_r \qquad (W)$$

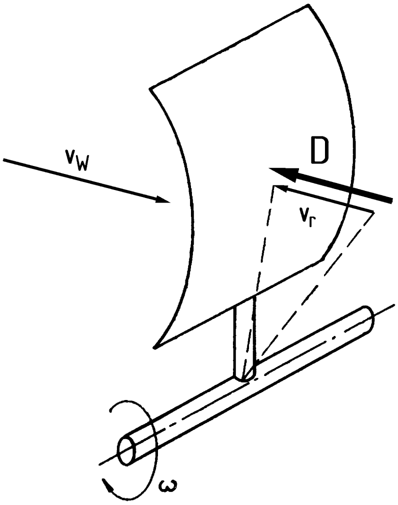

Fig. 4.4. Flow conditions and aerodynamic forces with a drag device

The relative velocity $v_{res} = v_W - v_r$, which effectively impinges on the drag area, is decisive for its aerodynamic drag. Using the common aerodynamic drag coefficient c_D, the aerodynamic drag can be expressed as:

$$D = c_D \frac{\varrho}{2} (v_W - v_r)^2 A \qquad (N)$$

The resultant power is

$$P = c_D \frac{\varrho}{2} (v_W - v_r)^2 A \, v_r \qquad (W)$$

If power is expressed again in terms of the power contained in the free-stream airflow, the following power coefficient is obtained:

$$c_P = \frac{P}{P_0} = \frac{c_D \frac{\varrho}{2} (v_W - v_r)^2 A \, v_r}{\frac{\varrho}{2} v_W^3 A} \qquad (-)$$

Analogously to the approach described in Chapter 4.1, it can be shown that c_P reaches a maximum value with a velocity ratio of $v_r/v_W = 1/3$. The maximum value is then

$$c_{Pmax} = \frac{4}{27} c_D \qquad (-)$$

The order of magnitude of the result becomes clear if it is taken into consideration that the aerodynamic drag coefficient of a concave surface curved against the wind direction can hardly exceed a value of 1.3. Thus, the maximum power coefficient of a pure drag-type rotor becomes:

$$c_{Pmax} \approx 0.2$$

It thus achieves only one third of Betz's ideal c_p value of 0.593. It must be pointed out that, strictly speaking, this derivation only applies to a translatory motion of the drag surface. Figure 4.4 shows a rotating motion, in order to provide a more obvious relationship with the wind rotor.

Rotors using aerodynamic lift

If the rotor blade shape permits utilisation of aerodynamic lift, much higher power coefficients can be achieved. Analogously to the conditions existing in the case of an aircraft airfoil, utilisation of aerodynamic lift considerably increases the efficiency (Fig. 4.5).

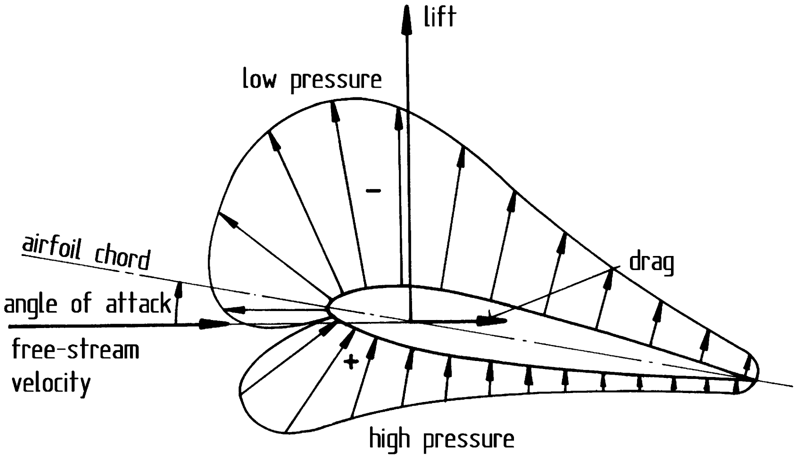

Fig. 4.5. Aerodynamic forces acting on an airfoil exposed to an airstream

All modern wind rotor types are designed for utilising this effect and the type best suited for this purpose is the propeller type with a horizontal rotational axis (Fig. 4.6). The wind velocity v_W is vectorially combined with the peripheral velocity u of the rotor blade. When the rotor blade is rotating, this is the peripheral velocity at a blade cross-section at a certain distance from the axis of rotation. Together with the airfoil chord the resultant free-stream velocity v_r forms the aerodynamic angle of attack. The aerodynamic force created is resolved into a component in the direction of the free-stream velocity, the drag D, and a component perpendicular to the free-stream velocity, the lift L. The lift force L, in turn, can be resolved into a component L_{torque} in the plane of rotation of the rotor, and a second component perpendicular to its plane of rotation. The tangential component L_{torque} constitutes the driving torque of the rotor, whereas L_{thrust} is responsible for the rotor thrust.

Modern airfoils developed for aircraft wings and which also found application in wind rotors, have an extremely favourable lift-to-drag ratio (E). This ratio can reach values of up to 200. This fact alone shows qualitatively how much more effective the utilisation of aerodynamic lift as a driving force must be. At this stage, however, it is no longer possible to calculate the achievable power coefficients of lift-type rotors quantitatively with the aid of elementary physical relationships alone. More sophisticated theoretical modelling concepts are now required as will be described in the next chapter.

One more note: Some rotor types, for example the Savonius rotor, can be built both as pure drag-type rotors and, with the appropriate aerodynamic shape, as rotors which partly utilise lift. This is one reason for the frequently greatly varying figures quoted for the power coefficient.

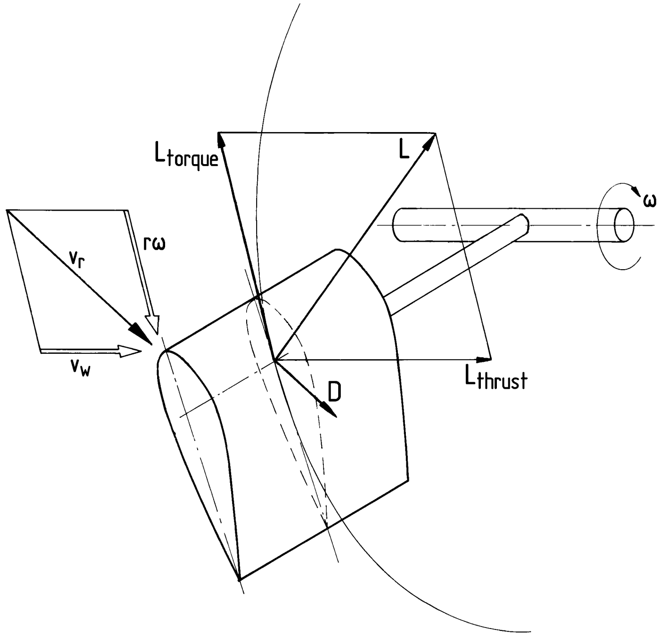

Fig. 4.6. Flow velocities and aerodynamic forces acting on a propeller-like rotor

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2. Molly, J.P.: Windenergie in Theorie und Praxis. C. F. Müller-Verlag, Karlsruhe (1978)