Chapter 9
Applied Fuzzy Systems

Data processing not only in physics and engineering, but also in medicine, biology, sociology, economics, sport, art, and military affairs, amounts to the different statements of identification problems. Fuzzy logic is mistakenly perceived by many specialists in mathematical simulation as a mean of only approximate decisions making in medicine, economics, art, sport and other different from physics and engineering humanitarian domains, where the high level of accuracy is not required. Therefore, one of the main goals of the authors is to show that it is possible to reach the accuracy of modeling, which does not yield to strict quantitative correlations, by tuning fuzzy knowledge bases. Only objects with discrete outputs for the direct inference and discrete inputs for the inverse inference were considered in the previous chapters. Such a problem corresponds to the problem of automatic classification arising in particular from medical and technical diagnosis. The main idea which the authors strive to render is that while tuning the fuzzy knowledge base it is possible to identify nonlinear dependencies with the necessary precision.

The use of the fuzzy expert information about the nonlinear object allows us to decrease the volume of experimental researches that gives the significant advantage in comparison with the known methods of identification with the growth of the number of the input variables of the object. Besides that, the fuzzy knowledge base easily interprets the structure of the object, while it is not always possible at the use of known methods.

Numerous examples considered in this chapter testify to wide possibilities of the intellectual technologies of modeling in the different domains of human activity.

9.1 Dynamic System Control

A dynamic system is traditionally considered as one quantitative description of which can be given by the language of differential or other equations [1]. Classical automatic control theory suggests that such equations can be constructed from the laws of physics, mechanics, thermodynamics, and electromagnetism [2]. Construction of dynamic equations requires a deep understanding of the processes and needs good physico-mathematical training [3]. On the other hand, a person can control a complicated object without compiling or solving any equations. We recall for example how easily a driver parks an automobile. Even a novice sitting for the first time in the driver seat can control an automobile by executing the verbal commands from his instructor sitting next to him.

A.P. Rotshtein et al.: Fuzzy Evidence in Identif., Forecast. and Diagn., STUDFUZZ 275, pp. 259–313.
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A unique feature of man is his capacity to learn and to evaluate the observed parameters in natural language: *low* velocity, *large* distance, and so on. Fuzzy set theory makes it possible to formalize natural language statements. Here we show that one can adjust a fuzzy knowledge base and use it to control a dynamic object no less effectively than with classical control theory. This section is written on the basis of work [4].

9.1.1 Control Object

We consider an inverted pendulum (Fig. 9.1), i. e., a rod fixed on a trolley that can oscillate in the longitudinal vertical plane.

The task of the control system is to maintain the inverted pendulum in the vertical position by displacing the trolley. A more ordinary form of this task is to maintain a rod on a finger in the vertical position. In [2] it has been shown that this is the class of problems in simulating the motion of a rocket, a supersonic aircraft, or a set of barges pushed by a tug, all of which are objects in which the centre of mass does not coincide with the point of application of the force.

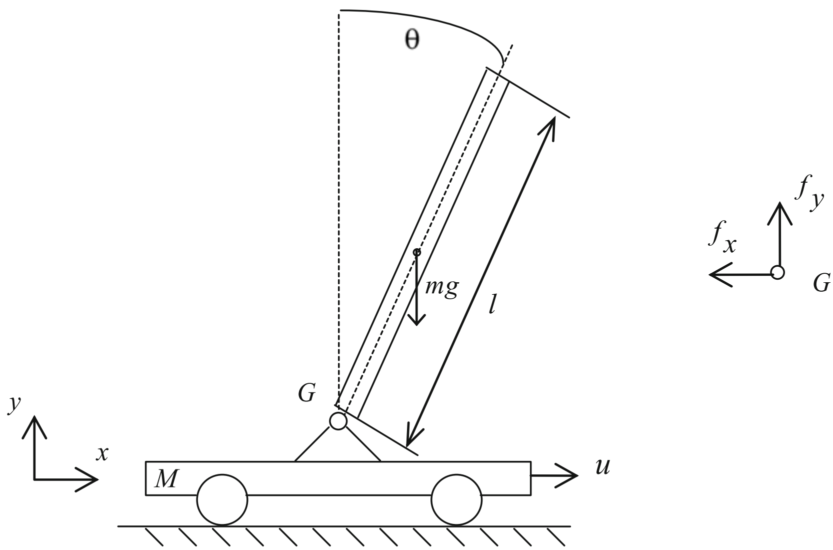


Fig. 9.1. Inverted pendulum

Before we consider the differential equations describing the motion of the pendulum, we note that the rod or the finger is kept vertical by applying simple rules:

If the angle of deviation from the vertical is large, one needs rapid movement in the same direction;

If the angle of deviation is small, one makes a small movement in the same direction;

If the angle of deviation is zero, no movement is made.

9.1.2 Classical Control Model

Following [5], we introduce the following symbols in Fig. 9.1: l – pendulum length, m – pendulum mass, M – trolley mass, g – acceleration due to gravity, u –control for supply to trolley, f_x and f_y – horizontal and vertical components of the forces acting on the pendulum, θ – the angular deviation of the pendulum from vertical, and I – the second moment of the pendulum in the plane of oscillation, which for a rectilinear thin rod is given by $I = \frac{ml^2}{3}$.

The equation of motion for an inverted pendulum as a control object may be written as follows [5]:

turning moment about the point G

$$I\ddot{\theta} = f_x l \cos\theta + f_y l \sin\theta \;;$$

displacement of the projection of G on the y axis

$$f_y - mg = m\frac{d^2}{dt^2}(l\cos\theta) = -ml(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \;;$$

displacement of the projection of G on the x axis

$$f_x = m\frac{d^2}{dt^2}(x - l\sin\theta) = m\ddot{x} - ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) \;;$$

and displacement of the trolley parallel to the x axis

$$u - f_x = M\ddot{x} \;,$$

in which $\dot{\theta}$ is the rate of change in angle θ, $\ddot{\theta}$ is the angular acceleration of the pendulum, and $\ddot{x}$ is the acceleration of the trolley along the x axis.

A linear approximation is used for these equations subject to the condition that θ varies over a fairly narrow range ($\cos\theta \approx 1$, $\sin\theta \approx \theta$, $\theta\dot{\theta} \approx 0$, $\dot{\theta}^2 \approx 0$), which gives us the differential equation of motion as:

$$\ddot{\theta} = \frac{3g(M+m)}{(4M+m)l}\theta + \frac{3u}{(4M+m)l} \quad . \tag{9.1}$$

To maintain the pendulum vertical with an ordinary control system with feedback, we represent the control variable as:

$$u = \alpha\theta + \beta\dot{\theta} \quad , \tag{9.2}$$

which corresponds to a proportional-differential regulator having proportionality coefficients α and β.

To provide stability, we take the coefficients as:

$$\alpha = -10 \;,\; \beta = -2 \,,$$

which gives negative values for the roots

$$\lambda_1 = -2.98 \;,\; \lambda_2 = -16.99$$

in the characteristic equation

$$\lambda^2 - \frac{3\beta}{(4M+m)l}\lambda - \frac{3g(M+m)+3\alpha}{(4M+m)l} = 0,$$

corresponding to (9.1).

To keep it vertical, we can thus use the control input

$$u = -10\theta - 2\dot{\theta} \tag{9.3}$$

in which the equation for the stable motion is:

$$\ddot{\theta} = -\frac{6}{(4M+m)l}\dot{\theta} + \frac{3g(M+m)-30}{(4M+m)l}\theta. \tag{9.4}$$

Table 9.1 gives the behaviour of θ (in rad) and $\dot{\theta}$ (rad/sec) from (9.4) with various initial conditions: γ_1, γ_2 and γ_3. In solving equation (9.4) we have used the following parameter values:

$$m = 0.035 \text{ kg}, \; M = 0.5 \text{ kg}, \; l = 30 \text{ cm}, \; g = 9.8 \text{ m/sec}^2.$$

In what follows, Table 9.1 will be used as the training set for adjusting the fuzzy control model.

Table 9.1. Behavior of an inverted pendulum under regulator control

t	γ_1		γ_2		γ_3	
	θ	$\dot{\theta}$	θ	$\dot{\theta}$	θ	$\dot{\theta}$
0.0	0.175	0.0000	0.105	0.0000	0.035	0.0000
0.1	0.150	-0.3523	0.090	-0.2114	0.030	-0.0705
0.2	0.115	-0.3261	0.069	-0.1957	0.023	-0.0652
0.3	0.086	-0.2540	0.052	-0.1524	0.017	-0.0508
0.4	0.064	-0.1908	0.039	-0.1145	0.013	-0.0382
0.5	0.048	-0.1421	0.029	-0.0852	0.010	-0.0284
0.6	0.035	-0.1056	0.021	-0.0633	0.007	-0.0211
0.7	0.026	-0.0784	0.016	-0.0470	0.005	-0.0157
0.8	0.020	-0.0582	0.012	-0.0349	0.004	-0.0116
0.9	0.015	-0.0432	0.009	-0.0259	0.003	-0.0086
1.0	0.011	-0.0321	0.006	-0.0193	0.002	-0.0064
1.1	0.008	-0.0238	0.005	-0.0143	0.002	-0.0048
1.2	0.006	-0.0177	0.004	-0.0106	0.001	-0.0035
1.3	0.004	-0.0131	0.003	-0.0079	0.001	-0.0026
1.4	0.003	-0.0098	0.002	-0.0059	0.001	-0.0020
1.5	0.002	-0.0072	0.001	-0.0043	0.000	-0.0014
1.6	0.002	-0.0054	0.001	-0.0032	0.000	-0.0011
1.7	0.001	-0.0040	0.001	-0.0024	0.000	-0.0008
1.8	0.001	-0.0030	0.001	-0.0018	0.000	-0.0006
1.9	0.001	-0.0022	0.000	-0.0013	0.000	-0.0004
2.0	0.001	-0.0016	0.000	-0.0010	0.000	-0.0003

9.1.3 Fuzzy Control Model

The dependence of the control u on the variables θ and $\dot{\theta}$ is represented as a knowledge base formed from 25 expert rules as follows:

$$\text{IF } \theta = A_i \text{ AND } \dot{\theta} = B_i \text{, THEN } u = C_j,\ i = \overline{1,5},\ j = \overline{1,7}.$$

These rules form a 5×5 matrix:

Rate of change, $\dot{\theta}$

Deviation angle, θ		*hN*	*N*	*Z*	*P*	*hP*
	hN	*vhN*	*vhN*	*hN*	*N*	*Z*
	N	*vhN*	*hN*	*N*	*Z*	*P*
	Z	*hN*	*N*	*Z*	*P*	*hP*
	P	*N*	*Z*	*P*	*hP*	*vhP*
	hP	*Z*	*P*	*hP*	*vhP*	*vhP*

(9.5)

where variables θ and $\dot{\theta}$ are evaluated by means of five terms:

$A_1 = B_1$ = *high negative (hN)*, $A_2 = B_2$ = *negative (N)*, $A_3 = B_3$ = *zero (Z)*, $A_4 = B_4$ = *positive (P)*, $A_5 = B_5$ = *high positive (hP)*.

and variable u is evaluated by means of seven terms:

C_1 - *very high negative (vhN)*, C_2 - *high negative (hN)*, C_3 - *negative (N)*, C_4 - *zero (Z)*, C_5 - *positive (P)*, C_6 - *high positive (hP)*, C_7 - *very high positive (vhP)*.

As the training set for tuning the control model (9.5), we use the Table 9.1 data and equation (9.3). The task of adjustment consists in selecting parameters for the membership functions in the terms A_i and B_i ($i = \overline{1,5}$) and rule weights in (9.5) such as to produce the minimum discrepancy between the theoretical equations (knowledge base (9.5)) on the one hand and the experimental equations (Table 9.1 and formula (9.3)) on the other.

The adjustment is performed by the method described in Section 3. The obtained membership functions are presented in Fig. 9.2. The weights of the fuzzy rules after adjustment correspond to the elements in the following matrix:

Rate of change, $\dot{\theta}$

Deviation angle, θ		*hN*	*N*	*Z*	*P*	*hP*
	hN	0.9837	0.3490	0.7902	0.8841	0.9015
	N	0.3490	0.9111	0.3901	0.7509	0.2199
	Z	0.7902	0.3901	0.7981	0.6381	0.5594
	P	0.8841	0.7509	0.6381	0.3690	0.5114
	hP	0.9015	0.2199	0.5594	0.5114	0.8708

Fig. 9.3 compares the behaviour of θ for the classical model and the fuzzy model with various initial conditions ($\gamma_1, \gamma_2, \gamma_3$); after the fuzzy control system is adjusted, it provides the same results as a traditional proportional-differential regulator.

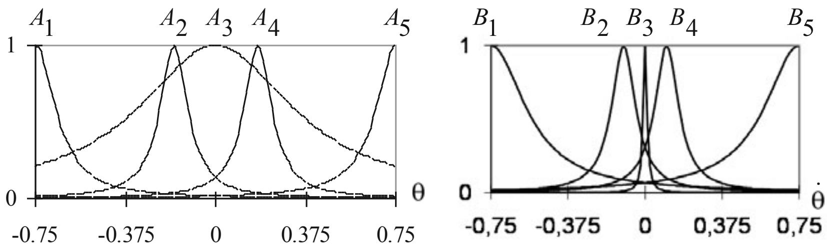


Fig. 9.2. Membership functions for fuzzy levels of variables θ and $\dot{\theta}$ evaluation

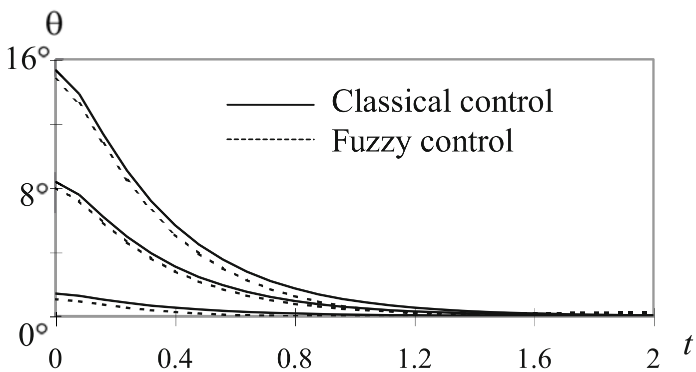


Fig. 9.3. Comparison of fuzzy and classical control systems after tuning

9.1.4 Connection with Lyapunov's Functions

It is shown here, that Lyapunov's functions known in stability theory can be used to synthesize fuzzy rules for control of a dynamic system.

The second or direct Lyapunov's method [3] allows us to study the stability of solutions of the nonlinear differential equations without solving these equations. The stability criterion was developed by Lyapunov on the basis of the following simple physical conception of equilibrium position: equilibrium position of the system is asymptotically stable, if all the trajectories of the process, beginning fairly near from the equilibrium point, stretch in such a way, that a properly defined "energetic" function is converged to the minimum, where position of the local minimum of energy corresponds to this point of equilibrium.

Let us consider the application of this criterion relative to the generalized nonlinear equation:

$$\dot{x} = f(x) \ , \quad x(0) = x_0 \ , \tag{9.6}$$

where x is the vector of the system condition.

We assume, that $f(0)=0$ and function f is continuous in the neighbourhood of the origin of coordinates.

Definition of Lyapunov's function. Function $V(x)$ is called Lyapunov's function (an energetic function) of system (9.6), if:

1) $V(0)=0$,
2) $V(x)>0$ for all $x \neq 0$ in the neighbourhood of the origin of coordinates,
3) $\dfrac{\partial V(x)}{\partial t}<0$ along the trajectory of system (9.6).

The main result, obtained by Lyapunov, was formulated as the theorem of stability.

Lyapunov's Theorem of Stability. The equilibrium position $x=0$ of system (9.6) is asymptotically stable, if Lyapunov's function $V(x)$ of the system exists.

We stress that Lyapunov's method requires derivation of the system dynamics equations. We are interested in the case with a lack of such equations.

Let us consider the inverted pendulum (Fig. 9.1) in the assumption, that only the following a priori information is known:

a) the system condition is defined by the coordinates $x_1=\theta$ and $x_2=\dot{\theta}$;

b) $\dot{x}_2$ is proportional to control *u*, i.e., if *u* increases (decreases), then $\dot{x}_2$ increases (decreases).

To apply Lyapunov's theorem to the inverted pendulum, the following function is selected as a Lyapunov's function candidate:

$$V(x_1,x_2)=\frac{1}{2}(x_1^2+x_2^2) \ . \tag{9.7}$$

If $V(0,0)=0$ and $V(x_1,x_2)>0$ then to assign $V(x_1,x_2)$ as a Lyapunov's function, it is necessary to provide the condition:

$$\frac{\partial V(x_1,x_2)}{\partial t}=x_1\dot{x}_1+x_2\dot{x}_2=x_1x_2+x_2\dot{x}_2<0 \ . \tag{9.8}$$

A fuzzy knowledge base about control $u=u(x_1,x_2)$ can be formulated as the condition of inequality (9.8) implementation. We consider three cases:

IF x_1 and x_2 have the opposite signs, then $x_1x_2<0$ and inequality (9.8) will be implemented for $x_2\dot{x}_2=0$.

IF x_1 and x_2 are positive, then (9.8) will be implemented for $\dot{x}_2<-x_1$.

IF x_1 and x_2 are negative, then (9.8) will be implemented for $\dot{x}_2>-x_1$.

Using the above mentioned reasoning and priori information relative to the fact that x_2 is *proportional* to *u*, we obtain four fuzzy rules for stable control the inverted pendulum:

IF x_1 *positive* AND x_2 *negative*, THEN *u zero*,

IF x_1 *negative* AND x_2 *positive*, THEN *u zero*,

IF x_1 *positive* AND x_2 *positive*, THEN *u high negative*,

IF x_1 *negative* AND x_2 *negative*, THEN *u high positive*.

Adjustment of this knowledge base consists of the selection of membership functions for the corresponding terms.

The essential differences between the classical and fuzzy control systems are given in Table 9.2.

Table 9.2. Control System Comparison

System type	Advantages	Disadvantages
Classical	If there is a model that adequately describes the dynamics, one can operate without adjusting it	Difficult to derive differential equations adequately describing the dynamics in the presence of nonlinear perturbations
Fuzzy	Differential equations not necessary, and dynamic model is readily written in terms of linguistic rules	Requires linguistic model adjustment

9.2 Inventory Control

Minimization of the inventory storage cost in enterprises and trade firms stocks including raw materials, stuffs, supplies, spare parts and products, is the most important problem of management. It is accepted that the theory of inventory control relates to operations research [6]. The models of this theory [7, 8] are built according to the classical scheme of mathematical programming: goal function is minimizing storage cost; controllable variables are time moments needed to order (or distribute) corresponding quantity of the needed stocks. Construction of such models requires definite assumptions, for example, of orders flows, time distribution laws and others. Therefore, complex optimization models may produce solutions that are quite inadequate to the real situation.

On the other hand, experienced managers very often make effective administrative decisions on the common sense and practical reasoning level. Therefore, the approach based on fuzzy logic can be considered as a good alternative to the classical inventory control models. This approach elaborated in works [9 – 12] requires neither complex mathematical models construction nor search for optimal solutions on the basis of such models. It is based on a simple comparison of the demand for the stock of the given item at the actual time moment with the quantity of the stock available in the warehouse. Dependent upon this, inventory action is formed consisting of increasing or decreasing corresponding stocks and materials.

"Quality" of a control fuzzy model strongly depends on the "quality" of fuzzy rules and "quality" of membership functions describing fuzzy terms. The more successfully the fuzzy rules and membership functions are selected, the more adequate the control action will be. However, no one can guarantee that the result of fuzzy logical inference will coincide with the correct (i.e. the most rational) control. Therefore, the problem of the adequate fuzzy rules and membership functions construction should be considered as the most actual one while developing control systems on fuzzy logic.

In this chapter it is suggested to build the fuzzy model of stocks and materials control on the grounds of the general method of nonlinear dependencies identification by means of fuzzy knowledge bases [13]. The proposed method is special due to the tuning stage of the fuzzy inventory control model using "demand - supply" training data. Owing to this tuning stage it is possible to select such fuzzy rules weights and such membership functions forms which provide maximal proximity of the results of fuzzy logical inference to the correct managerial decisions.

To substantiate for the expediency to use this fuzzy approach relative to inventory control, we resort to help of analogy with the classical problem of a dynamic system (turned-over pendulum) control which can be successfully solved using fuzzy logic [4].

9.2.1 Analogy with Turned-Over Pendulum

The approach to inventory control suggested here is similar to turned-over pendulum control with the aim of retaining it in a vertical position by pushing the cart to the left or to the right (Fig. 9.4). A rather habitual version of such a problem is demonstrated by vertically retaining a stick on the finger. The simplest rules for the problem solution can be represented in the following way:

IF the angle of deflection of the stick from the vertical position is *big*,
THEN the finger should *quickly* move in the same direction to keep the stick up;
IF the angle of deflection of the stick from the vertical position is *small*,
THEN the finger should *slowly* move in the same direction to keep the stick up;
IF the angle of deflection of the stick is equal to zero,
THEN the finger *should stay motionless*.

Keeping the speed of the car constant by the driver takes place in analogy to it; if the speedometer needle drops down, then the driver presses the accelerator down; if the speedometer needle goes up, then the driver reduces the speed. It is known that experienced driver retains some given speed (for example, 90 km/hour) in spite of the quickly changing nonlinear road relief.

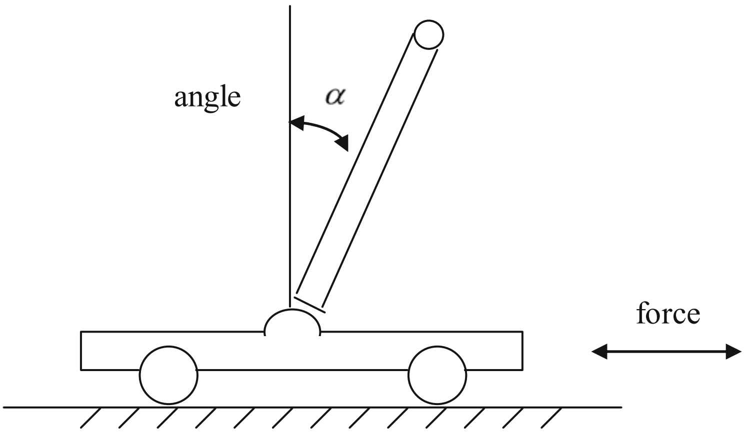


Fig. 9.4. Control system of the turned-over pendulum

Returning to the inventory control system it is not difficult to understand that the actions of the manager must be similar to the actions of the car driver regulating of the vehicle's speed.

9.2.2 Method of Identification

The method of nonlinear objects identification by fuzzy knowledge bases [14] serves as the theoretical basis for the definition of the dependency between control actions and the current state of the control system. The method is based on the principle of fuzzy knowledge bases two-stage tuning. According to this principle the construction of the "inputs – output" object model can be performed in two stages which, in analogy with classical methods [15], can be considered as stages of structural and parametrical identification.

The first stage is traditional for fuzzy expert systems [16]. Formation and rough tuning of the object model by knowledge base construction using available expert information is accomplished at this stage. The higher the professional skill level of an expert, the higher the adequacy of the built fuzzy model at the rough tuning stage will be. However, as was mentioned in the introduction, no one can guarantee the coincidence of the results of fuzzy logic inference (theory) and correct practical decisions (experiment). Therefore, the second stage is needed, at which fine tuning of the model is done by way of training it using experimental data.

The essence of the fine tuning stage consists in finding such fuzzy IF-THEN rules weights and such fuzzy terms membership functions parameters which minimize the difference between desired (experimental) and model (theoretical) behaviour of the object. Fine tuning stage is formulated as nonlinear optimization problem which can be effectively solved by some combination of genetic algorithms and neural networks [14].

9.2.3 Fuzzy Model of Control

Let us present the inventory control system in the form of the object with two inputs ($x_1(t)$, $x_2(t)$) and single output ($y(t)$), where:

$x_1(t)$ is *demand*, i.e. the number of units of the stocks of the given brand, which is needed at time moment t;

$x_2(t)$ is *stock quantity-on-hand*, i.e. the number of units of the stocks of the given brand, which is available in the warehouse at moment t;

$y(t)$ is an *inventory action* at moment t, consisting in increasing – decreasing the stocks of the given brand.

System state parameters $x_1(t)$, $x_2(t)$ and inventory action $y(t)$ are considered as linguistic variables [17], which are estimated with the help of verbal terms on five and seven levels:

$$x_1(t) = \begin{cases} \text{falling } (F) \\ \text{decreased } (D) \\ \text{steady } (S) \\ \text{increased } (I) \\ \text{rising up } (R) \end{cases} \qquad x_2(t) = \begin{cases} \text{minimal } (M) \\ \text{low } (L) \\ \text{adequately sufficient (A)} \\ \text{high } (H) \\ \text{excessive } (E) \end{cases}$$

$$y(t) = \begin{cases} d_1 \text{ – to decrease the stock sharply} \\ d_2 \text{ – to decrease the stock moderately} \\ d_3 \text{ – to decrease the stock minimally} \\ d_4 \text{ – do nothing} \\ d_5 \text{ – to increase the stock minimally} \\ d_6 \text{ – to increase the stock moderately} \\ d_7 \text{ – to increase the stock sharply} \end{cases}$$

Let us note that term “adequately sufficient” in variable $x_2(t)$ estimation depicts the rational quantity of the stock on the common sense level, and does not pretend to be contained within the mathematically strong concept of optimality which envisages the presence of goal function, controllable variables and area of constraints.

Functional dependency

$$y(t) = f(x_1(t), x_2(t)) \tag{9.9}$$

is defined by the table presented in Fig. 9.5.

This table is defined in an expert manner and depicts the complete sorting out of the $(5 \times 5 = 25)$ terms combinations in the triplets $\langle x_1(t),\ x_2(t),\ y(t) \rangle$.

Grouping these triplets by inventory actions types, we shall form a fuzzy knowledge base, presented in Table. 9.3.

This fuzzy knowledge base defines a fuzzy model of the object in the form of the following rules, e.g.:

IF demand is falling AND stock is excessive, OR demand is falling AND stock is high, OR demand is decreased AND stock is excessive,

THEN it is necessary to decrease the stock sharply.

$x_2(t)$ \ $x_1(t)$	F	D	S	I	R
E	d_1	d_1	d_2	d_3	d_4
H	d_1	d_2	d_3	d_4	d_5
A	d_2	d_3	d_4	d_5	d_6
L	d_3	d_4	d_5	d_6	d_7
M	d_4	d_5	d_6	d_7	d_7

Fig. 9.5. Dependency between state parameters and inventory actions

Fuzzy logical equations correspond to the fuzzy knowledge base (Table 9.3). They establish the connection between membership functions of the variables in correlation (9.9). Let $\mu^j(u)$ be membership function of variable u to term j. Let us go on from the fuzzy knowledge base (Table 9.3) to the system of fuzzy logical equations:

$$\mu^{d_1}(y) = \mu^F(x_1)\cdot\mu^E(x_2) \vee \mu^F(x_1)\cdot\mu^H(x_2) \vee \mu^D(x_1)\cdot\mu^E(x_2);$$

$$\mu^{d_2}(y) = \mu^F(x_1)\cdot\mu^A(x_2) \vee \mu^D(x_1)\cdot\mu^H(x_2) \vee \mu^S(x_1)\cdot\mu^E(x_2);$$

$$\mu^{d_3}(y) = \mu^F(x_1)\cdot\mu^L(x_2) \vee \mu^D(x_1)\cdot\mu^A(x_2) \vee \mu^S(x_1)\cdot\mu^H(x_2) \vee \mu^I(x_1)\cdot\mu^E(x_2);$$

$$\mu^{d_4}(y) = \mu^F(x_1)\cdot\mu^M(x_2) \vee \mu^D(x_1)\cdot\mu^L(x_2)$$

$$\vee\mu^S(x_1)\cdot\mu^A(x_2) \vee \mu^I(x_1)\cdot\mu^H(x_2) \vee\mu^R(x_1)\cdot\mu^E(x_2);$$

$$\mu^{d_5}(y) = \mu^D(x_1)\cdot\mu^M(x_2) \vee \mu^S(x_1)\cdot\mu^L(x_2) \vee\mu^I(x_1)\cdot\mu^A(x_2) \vee \mu^R(x_1)\cdot\mu^H(x_2);$$

$$\mu^{d_6}(y) = \mu^S(x_1)\cdot\mu^M(x_2) \vee \mu^I(x_1)\cdot\mu^L(x_2) \vee \mu^R(x_1)\cdot\mu^A(x_2);$$

$$\mu^{d_7}(y) = \mu^I(x_1)\cdot\mu^M(x_2) \vee \mu^R(x_1)\cdot\mu^M(x_2) \vee \mu^R(x_1)\cdot\mu^L(x_2), \qquad (9.10)$$

where (•) is operation AND (*min*); $\vee$ is operation OR (*max*).

The algorithm of decision making on the basis of fuzzy logical equations consists of the following:

1º. To fix the demand $x_1(t)$ and stock quantity-on-hand $x_2(t)$ values at the time moment $t=t_0$.

Table 9.3. Fuzzy knowledge base

IF		THEN
Demand $x_1(t)$	*Stock quantity-on-hand* $x_2(t)$	*Inventory action* $y(t)$
F *F* *D*	*E* *H* *E*	d_1
F *D* *S*	*A* *H* *E*	d_2
F *D* *S* *I*	*L* *A* *H* *E*	d_3
F *D* *S* *I* *R*	*M* *L* *A* *H* *E*	d_4
D *S* *I* *R*	*M* *L* *A* *H*	d_5
S *I* *R*	*M* *L* *A*	d_6
I *R* *R*	*M* *M* *L*	d_7

2^o.To define the membership degrees of $x_1(t)$ and $x_2(t)$ values to the corresponding terms with the help of membership functions.

3^o. To calculate the membership degree of the inventory action $y(t)$ at the time $t = t_0$ to each of the $d_1, d_2, ..., d_7$ decisions classes with the help of fuzzy logical equations.

4^o. The term with maximal membership function, obtained at step 3^o should be considered as inventory action $y(t)$ at the time $t=t_0$. For obtaining the quantitative

$y(t)$ value at the time $t=t_0$ it is necessary to perform the "defuzzification" operation, i.e. to go on from the fuzzy term to a crisp number. According to [14] this operation can be performed as follows. Range $[\underline{y}, \overline{y}]$ of the variable $y(t)$ change is divided into 7 classes:

$$y(t) \in [\underline{y}, \overline{y}] = [\underbrace{\underline{y}, y_1}_{d_1}) \cup [\underbrace{y_1, y_2}_{d_2}) \cup \ldots \cup [\underbrace{y_6, \overline{y}}_{d_7}] .$$

The crisp value of the inventory action $y(t)$ at the time $t=t_0$ is defined by formula:

$$y(t) = \frac{\underline{y}\mu^{d_1}(y) + y_1\mu^{d_2}(y) + \ldots + y_6\mu^{d_7}(y)}{\mu^{d_1}(y) + \mu^{d_2}(y) + \ldots + \mu^{d_7}(y)}. \tag{9.11}$$

9.2.4 Fuzzy Model Tuning

Relations (9.10), (9.11) define the functional dependency (9.9) in the following form

$$y(t) = F(x_1(t), x_2(t), \mathbf{W}, \mathbf{B}_1, \mathbf{C}_1, \mathbf{B}_2, \mathbf{C}_2),$$

where $\mathbf{W} = (w_1, w_2, ..., w_{25})$ is the vector of weights in the fuzzy knowledge base (Table 9.3);

$\mathbf{B}_1 = (b_1^{vD}, b_1^{D}, b_1^{St}, b_1^{I}, b_1^{vI})$, $\mathbf{B}_2 = (b_2^{vL}, b_2^{L}, b_2^{S}, b_2^{B}, b_2^{vB})$ are the vectors of centers for variables $x_1(t)$ and $x_2(t)$ membership functions to the corresponding terms;

$\mathbf{C}_1 = (c_1^{vD}, c_1^{D}, c_1^{St}, c_1^{I}, c_1^{vI})$, $\mathbf{C}_2 = (c_2^{vL}, c_2^{L}, c_2^{S}, c_2^{B}, c_2^{vB})$ are the vectors of concentration parameters for variables $x_1(t)$ and $x_2(t)$ membership functions to the corresponding terms;

F is the operator of "inputs – output" connection corresponding to formulae (9.10), (9.11).

It is assumed that some training data sample in the form of M pairs of experimental data can be obtained on the ground of successful decisions about inventory control

$$\langle \hat{x}_1(t), \hat{x}_2(t), \hat{y}(t) \rangle, \ t = \overline{1, M},$$

where $\langle \hat{x}_1(t), \hat{x}_2(t) \rangle$ are the inventory control system state parameters at time moment t, $\hat{y}(t)$ is the inventory action at time moment t.

The essence of the inventory control model tuning consists of such membership functions parameters (b-, c-) and fuzzy rules weights (w-) finding, which provide for the minimum distance between theoretical and experimental data:

$$\sum_{t=1}^{M} [F(\hat{x}_1(t), \hat{x}_2(t), \mathbf{W}, \mathbf{B}_1, \mathbf{C}_1, \mathbf{B}_2, \mathbf{C}_2) - \hat{y}(t)]^2 = \min_{\mathbf{W}, \mathbf{B}_i, \mathbf{C}_i}, \quad i = 1, 2 . \tag{9.12}$$

It is expedient to solve the nonlinear optimization problem (9.12) by a combination of the genetic algorithm and gradient methods.

9.2.5 Example of Fuzzy Model Tuning

Fuzzy model of inventory control was constructed for the district food-store house, selling some definite kind of agricultural production (buckwheat). The ranges of the input and output variables change consisted of:

$$x_1(t) \in [0, 200]*10^2 \text{ kg}; \quad x_2(t) \in [70, 170]*10^2 \text{ kg}; \quad y(t) \in [-100, 100]*10^2 \text{ kg}.$$

Inventory control at the enterprise is done once per day. Therefore $t \in [1...365]$ days. The triplets $\langle$ demand $x_1(t)$, stock quantity-on-hand $x_2(t)$, inventory action $y(t)$ $\rangle$ values, corresponding to the experienced manager actions, for which the demand for the produce was satisfied while the permissible produce inventory level in store was minimal where taken as training data sample. Training data sample is presented in Fig. 9.6,a-c in the form of the dynamics of the input and output variables change on time t according to 2001 year data. For example, at moments $t = 120$ and $t = 230$ the control consisted of stock quantity-on-hand increasing by $25*10^2$ kg and reducing by $15*10^2$ kg, respectively. Thus the produce remainder in store after control $\varepsilon(t) = x_2(t) + y(t) - x_1(t)$ consists of $2*10^2$ kg and $53*10^2$ kg, respectively. These values do not exceed the permissible inventory level, which is equal to $70*10^2$ kg. The dynamics of the produce remainder after control $\varepsilon(t)$ change, presented in Fig. 9.6,d is indicative of the control stability, i.e. of the tendency of index $\varepsilon(t)$ approaching a zero value. Membership functions of fuzzy terms for variables $x_1(t)$ and $x_2(t)$, and also their parameters (*b*- , *c*-) before and after training are presented in Fig. 9.7, 9.8 and Tables 9.4, 9.5 respectively. Rules weights included in the fuzzy knowledge base before and after training are presented in Table 9.6.

Table 9.6. Rules weights before (after) training

$x_2(t)$ \ $x_1(t)$	*F*	*D*	*S*	*I*	*R*
E	1 (0.954)	1 (0.755)	1 (0.999)	1 (0.967)	1 (0.578)
H	1 (0.986)	1 (0.711)	1 (0.897)	1 (0.679)	1 (0.953)
A	1 (0.695)	1 (0.538)	1 (0.854)	1 (0.968)	1 (0.680)
L	1 (0.842)	1 (0.943)	1 (0.799)	1 (0.869)	1 (0.947)
M	1 (0.857)	1 (0.851)	1 (0.859)	1 (0.995)	1 (0.867)

Comparison of model and reference control before and after fuzzy model training is presented in Fig. 9.9 and 9.10. Comparison of the produce remainder $\varepsilon(t)$ value in store after control before and after fuzzy model training is shown in Fig. 9.11 and 9.12.

The proposed approach can find application in the automated management systems of enterprises and trade firms. Further development of this approach can be done in the direction of creating adaptive inventory control models, which are tuned with the acquisition of new experimental data about successful decisions. Besides that with the help of supplementary fuzzy knowledge bases factors influencing the demand and quantity-on-hand values (seasonal prevalence, purchase and selling praises, delivery cost, plant-supplier power and others) can be taken into account.

Table 9.4. Membership functions parameters of variable $x_1(t)$ fuzzy terms before (after) training

Linguistic assessments of $x_1(t)$ variable	Parameter	
	b	c
falling (*F*)	0 (1.95)	70 (44.11)
decreased (*D*)	50 (30.54)	70 (42.85)
steady (*S*)	100 (105.77)	70 (35.68)
increased (*I*)	150 (170.04)	70 (40.12)
rising up (*R*)	200 (199.43)	70 (47.55)

Table 9.5. Membership functions parameters of variable $x_2(t)$ fuzzy terms before (after) training

Linguistic assessments of $x_2(t)$ variable	Parameter	
	b	c
minimal (*M*)	70 (75.46)	35 (18.76)
low (*L*)	95 (85.12)	35 (22.12)
adequately sufficient (A)	120 (125.15)	35 (16.75)
high (*H*)	145 (157.99)	35 (14.54)
excessive (*E*)	170 (168.63)	35 (12.69)

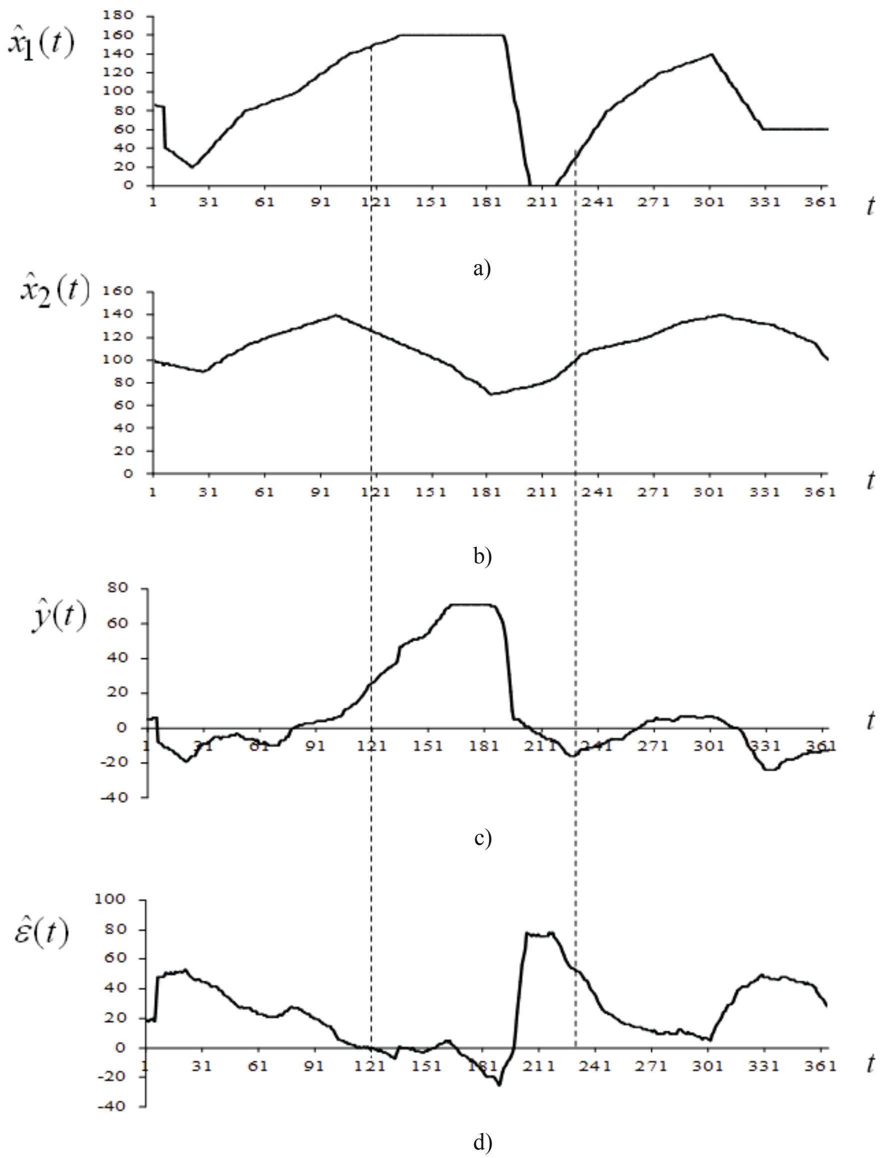


Fig. 9.6. Training data a) change of the demand for the produce in 2001 b) stock quantity-on-hand change in 2001 c) inventory action in 2001 d) change of the produce remainder in store after control in 2001

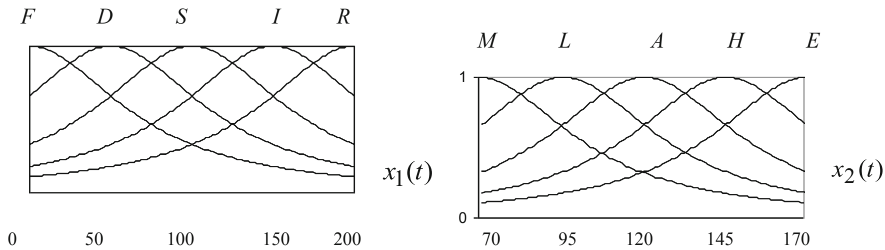


Fig. 9.7. Fuzzy terms membership functions before training

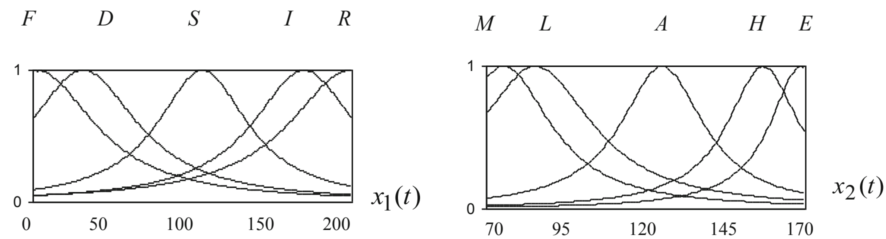


Fig. 9.8. Fuzzy terms membership functions after training

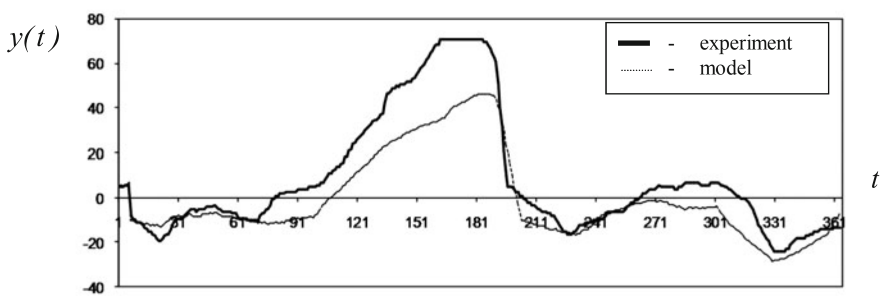


Fig. 9.9. Inventory action generated by fuzzy model before training

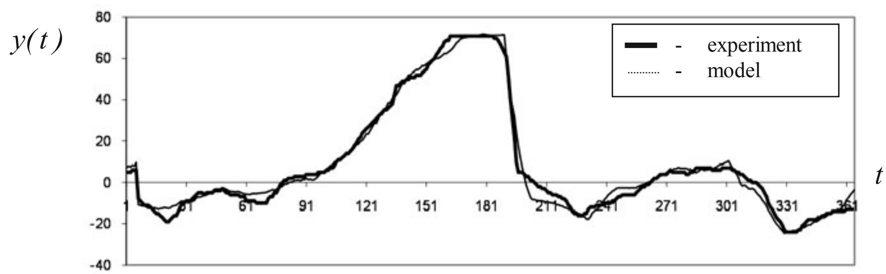


Fig. 9.10. Inventory action generated by fuzzy model after training

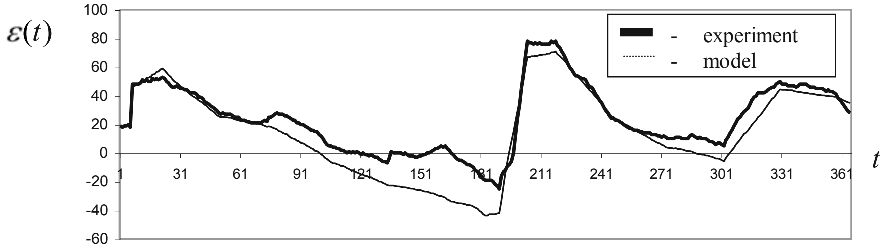


Fig. 9.11. Produce remainder in store after control before fuzzy model training

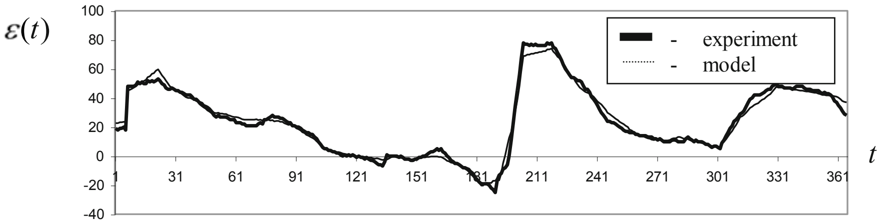


Fig. 9.12. Produce remainder in store after control after fuzzy model training

9.3 Prediction of Football Games Results

The possibilities of the method of non-linear dependencies identification by fuzzy IF-THEN rules [14] are illustrated by an example of the problem of forecasting the results of football games, which is a typical representative of complex forecasting problems that require adaptive model tuning.

Football is a most popular sport attracting hundreds of millions of fans. Prediction of football matches results arouses interest from two points of view: the first one is demonstration of the power of different mathematical methods [18, 19], the second one is the desire of earning money by predicting beforehand any winning result. Models and PC–programs of sport prediction are already being developed for many years (see, for example, http://dmiwww.cs.tut.fi/riku). Most of them use stochastic methods of uncertainty description: regressive and autoregressive analysis [20 – 22], Bayessian approach in combination with Markov chains and the Monte-Carlo method [23 – 26]. The specific features of these models are: sufficiently great complexity, a lot of assumptions, and the need for a great number of statistical data. Besides that, the models cannot always be easily interpreted. Some several years passed before some models using neural networks for the results of football games prediction appeared [27 – 29]. They can be considered as universal approximators of non-linear dependencies trained by experimental data. These models also need a lot of statistical data and do not allow us to define the physical meaning of the weights between neurons after training.

In the practice of prediction making the football experts and fans usually make good decisions using simple reasoning on the common sense level, for example:

IF team T_1 constantly won in previous matches
AND team T_2 constantly lost in previous matches
AND in previous matches between teams T_1 and T_2 team T_1 won,
THEN win of team T_1 should be expected.

Such expressions can be considered as concentration of accumulated experts' experiences and can be formalized using fuzzy logic. That is why it is quite natural to apply such expressions as a support for building a model of prediction.

The process of modeling has two phases. In the first phase we define the fuzzy model structure, which connects the football game result to be found with the results of previous games for both teams. The second phase consists of fuzzy model tuning, i.e., of finding optimal parameters using tournament tables data. For tuning we use a combination of a genetic algorithm and a neural network. The genetic algorithm provides a rough finding of the area of global minimum of distance between model and experimental results. We use the neural approach for the fine model parameters tuning and for their adaptive correction while new experimental data is appearing.

9.3.1 The Structure of the Model

The aim of modeling is to calculate the result of match between teams T_1 and T_2, which is characterized as the difference of scored and lost goals y. We assume that $y \in [\underline{y}, \overline{y}] = [-5, 5]$. For prediction model building we will define the value of y on the following five levels:

d_1 is a big loss (*BL*), $y = -5, -4, -3$;
d_2 is a small loss (*SL*), $y = -2, -1$;
d_3 is a draw (*D*), y=0;
d_4 is a small win (*SW*), $y = 1, 2$;
d_5 is a big win (*BW*), $y = 3, 4, 5$.

Let us suppose that the football game result (y) is influenced by the following factors:

$x_1, x_2, ..., x_5$ are the results of five previous games for team T_1;
$x_6, x_7, ..., x_{10}$ are the results of five previous games for team T_2;
x_{11}, x_{12} are the results of two previous games between teams T_1 and T_2.

It is obvious, that values of factors $x_1, x_2, ..., x_{12}$ are changing in the range from –5 to 5.

The hierarchical interconnection between output variable y and input variables $x_1, x_2, ..., x_{12}$ is represented as a tree shown in Fig. 9.13.

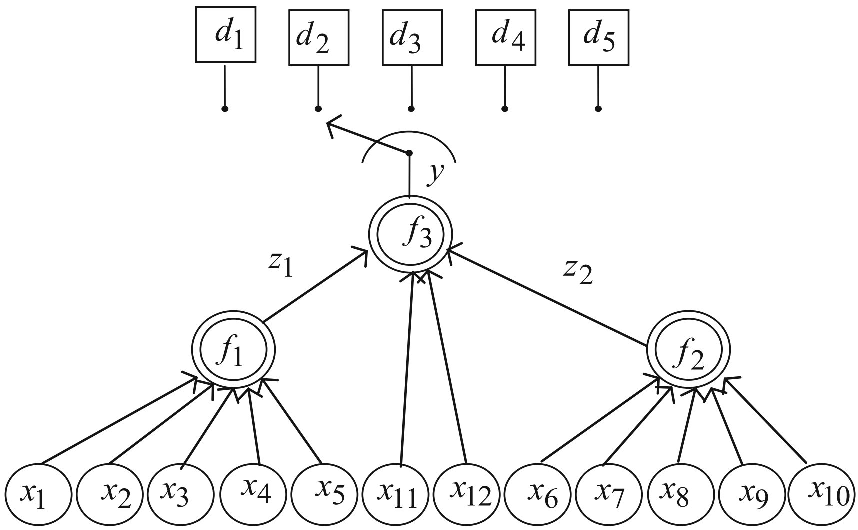


Fig. 9.13. Structure of the Prediction Model

This tree is equal to the system of correlations

$$y = f_3(z_1, z_2, x_{11}, x_{12}), \tag{9.13}$$

$$z_1 = f_1(x_1, x_2, ..., x_5), \tag{9.14}$$

$$z_2 = f_2(x_6, x_7, ..., x_{10}), \tag{9.15}$$

where z_1 (z_2) is the football game prediction for team T_1 (T_2) based on the previous results $x_1, x_2, ..., x_5$ ($x_6, x_7, ..., x_{10}$).

The variables $x_1, x_2, ..., x_{12}$, as well as z_1 (z_2) will be considered as linguistic variables [17], which can be evaluated using above mentioned fuzzy terms: *BL, SL, D, SW* and *BW*.

To describe the correlations (9.13) - (9.15) we shall use the expert matrices of knowledge (Tables 9.7, 9.8). These matrices correspond to fuzzy IF-THEN rules received on the common sense and practical reasoning level. An example of one of these rules for Table 9.7 is given below:

IF (x_{11}=*BW*) AND (x_{12}=*BW*) AND (z_1=*BW*) AND (z_2=*BL*)
OR (x_{11}=*SW*) AND (x_{12}=*BW*) AND (z_1=*SW*) AND (z_2=*D*)
OR (x_{11}=*BW*) AND (x_{12}=*D*) AND (z_1=*BW*) AND (z_2=*SL*)
THEN $y = d_5$.

Table 9.7. Knowledge about correlations (9.14) and (9.15)

$x_1(x_6)$	$x_2(x_7)$	$x_3(x_8)$	$x_4(x_9)$	$x_5(x_{10})$	$z_1(z_2)$
BL	*BL*	*BL*	*BL*	*BL*	*BL*
BW	*SL*	*BL*	*SL*	*BW*	
SW	*BL*	*SL*	*SL*	*SW*	
SL	*SL*	*SL*	*SL*	*SL*	*SL*
D	*SL*	*SL*	*D*	*D*	
SW	*D*	*SL*	*SL*	*SW*	
D	*D*	*D*	*D*	*D*	*D*
SL	*SW*	*SW*	*D*	*SL*	
D	*D*	*SW*	*SW*	*D*	
SW	*SW*	*SW*	*SW*	*SW*	*SW*
D	*BW*	*BW*	*SW*	*D*	
SL	*SW*	*SW*	*BW*	*SL*	
BW	*BW*	*BW*	*BW*	*BW*	*BW*
SL	*BW*	*SW*	*BW*	*SL*	
BL	*SW*	*BW*	*SW*	*BL*	

Table 9.8. Knowledge about correlation (9.13)

x_{11}	x_{12}	z_1	z_2	y
BL	*BL*	*BL*	*BW*	d_1
BW	*D*	*BL*	*D*	
SW	*BL*	*SL*	*SL*	
SW	*SL*	*D*	*SL*	d_2
D	*SL*	*SL*	*D*	
SW	*D*	*SL*	*SL*	
D	*D*	*D*	*D*	d_3
SL	*SW*	*SW*	*D*	
SL	*D*	*SW*	*SW*	
SL	*SW*	*SW*	*BW*	d_4
D	*BW*	*BW*	*SW*	
SL	*SW*	*SW*	*BW*	
BW	*BW*	*BW*	*BL*	d_5
SW	*BW*	*SW*	*D*	
BW	*D*	*BW*	*SL*	

9.3.2 *Fuzzy Model of Prediction*

Using the generalized fuzzy approximator [14] and the tree of evidence (Fig. 9.13), the prediction model can be described in the following form:

$$y = F_y(x_1, x_2, ..., x_{12}, \mathbf{W}_1, \mathbf{B}_1, \mathbf{C}_1, \mathbf{W}_2, \mathbf{B}_2, \mathbf{C}_2, \mathbf{W}_3, \mathbf{B}_3, \mathbf{C}_3), \qquad (9.16)$$

where F_y is the operator of inputs-output connection, corresponding to correlations (9.13) – (9.15),

$$\mathbf{W}_1 = ((w_1^{11},...,w_1^{13}),...,(w_1^{51},...,w_1^{53}))\,,\ \mathbf{W}_2 = ((w_2^{11},...,w_2^{13}),...,(w_2^{51},...,w_2^{53}))\,,$$

$\mathbf{W}_3 = ((w_3^{11},...,w_3^{13}),...,(w_3^{51},...,w_3^{53}))$ are the vectors of rules weights in the correlations (9.13), (9.14), (9.15), respectively;

$$\mathbf{B}_1 = (b_{1-5}^{BL}, b_{1-5}^{SL}, b_{1-5}^{D}, b_{1-5}^{SW}, b_{1-5}^{BW})\ ,\ \mathbf{B}_2 = (b_{6-10}^{BL}, b_{6-10}^{SL}, b_{6-10}^{D}, b_{6-10}^{SW}, b_{6-10}^{BW})\ ,$$

$\mathbf{B}_3 = (b_{11,12}^{BL}, b_{11,12}^{SL}, b_{11,12}^{D}, b_{11,12}^{SW}, b_{11,12}^{BW})$ are the vectors of centres for variables $x_1, x_2, ..., x_5$, $x_6, x_7, ..., x_{10}$ and x_{11}, x_{12} membership functions to terms *BL, SL,…, BW;*

$$\mathbf{C}_1 = (c_{1-5}^{BL}, c_{1-5}^{SL}, c_{1-5}^{D}, c_{1-5}^{SW}, c_{1-5}^{BW})\,,\ \mathbf{C}_2 = (c_{6-10}^{BL}, c_{6-10}^{SL}, c_{6-10}^{D}, c_{6-10}^{SW}, c_{6-10}^{BW})\,,$$

$\mathbf{C}_3 = (c_{11,12}^{BL}, c_{11,12}^{SL}, c_{11,12}^{D}, c_{11,12}^{SW}, c_{11,12}^{BW})$ are the vectors of concentration parameters for variables $x_1, x_2, ..., x_5$, $x_6, x_7, ..., x_{10}$ and x_{11}, x_{12} membership functions to terms *BL, SL,…, BW*.

In model (9.16) we assume that for all of variables $x_1, x_2, ..., x_5$ fuzzy terms *BL, SL,…, BW* have the same membership functions. Same assumption we made for variables $x_6, x_7, ..., x_{10}$ and variables x_{11}, x_{12} (See. Fig. 9.14).

9.3.3 Genetic and Neuro Tuning

The reasonable results of simulation can be reached by fuzzy rules tuning using tournament tables data. Training data in the form of M pairs of experimental data assumed to be obtained with use of tournament tables

$$\left\langle \hat{\mathbf{X}}_l, \hat{y}_l \right\rangle,\ l = \overline{1, M}\,,$$

where $\hat{\mathbf{X}}_l = \{(x_1^l, x_2^l, ...x_5^l),\ \ (x_6^l, x_7^l, ...x_{10}^l),\ \ (x_{11}^l, x_{12}^l)\}$ are the previous matches results for teams T_1 and T_2 in the experiment number l,

$\hat{y}_l$ is the game result between teams T_1 and T_2 in experiment number l.

The essence of the prediction model tuning consists of such membership functions parameters (*b*-, *c*-) and fuzzy rules weights (*w*-) finding, which provide for the minimum distance between theoretical and experimental results:

$$\sum_{l=1}^{M} (F_y(\hat{x}_1^l, \hat{x}_2^l, ..., \hat{x}_{12}^l, \mathbf{W}_i, \mathbf{B}_i, \mathbf{C}_i) - \hat{y}_l)^2 = \min_{\mathbf{W}_i, \mathbf{B}_i, \mathbf{C}_i}\ ,\quad i = 1,2,3\,.$$

To solve this non-linear optimization problem we propose a genetic algorithm and neural network combination. The genetic algorithm provides for a rough off-line finding of the area of global minimum, while the neural network is used for on-line improvement of unknown parameters values.

For the fuzzy model tuning we used the results from tournament tables of the Finland Football Championship characterized by a minimal number of sensations. Our training data included results of 1056 matches for the last 8 years from 1994 to 2001. The results of the fuzzy model tuning are given in Tables 9.9 – 9.12 and in Fig. 9.14.

Table 9.9. Fuzzy rules weights in correlation (9.13)

Genetic algorithm	Neuro-fuzzy network
1.0	0.989
1.0	1.000
1.0	1.000
0.8	0.902
0.5	0.561
0.8	0.505
0.6	0.580
1.0	0.613
0.5	0.948
1.0	0.793
0.9	0.868
0.6	0.510
0.6	0.752
0.5	0.500
0.5	0.500

Table 9.10. Fuzzy rules weights in correlation (9.14)

Genetic algorithm	Neuro-fuzzy network
0.7	0.926
0.9	0.900
0.7	0.700
0.9	0.954
0.7	0.700
1.0	1.000
0.9	0.900
1.0	1.000
0.6	0.600
1.0	1.000
0.7	0.700
1.0	1.000
0.8	0.990
0.5	0.500
0.6	0.600

Table 9.11. Fuzzy rules weights in correlation (9.15)

Genetic algorithm	Neuro-fuzzy network
0.7	0.713
0.8	0.782
1.0	0.996
0.5	0.500
0.5	0.541
0.5	0.500
0.5	0.500
0.5	0.522
0.6	0.814
1.0	0.903
0.6	0.503
1.0	0.677
1.0	0.515
0.5	0.514
1.0	0.999

Table 9.12. *b*- and *c*- parameters of membership functions after tuning

Terms	Genetic Algorithm						Neuro-Fuzzy Network					
	$x_1, x_2, ..., x_5$		$x_6, x_7, ..., x_{10}$		x_{11}, x_{12}		$x_1, x_2, ..., x_5$		$x_6, x_7, ..., x_{10}$		x_{11}, x_{12}	
	b-	*c*-	*b*-	*c*-	*b*-	*c*-	*b*-	*c*-	*b*-	*c*-	*b*-	*c*-
BL	-4.160	9	-5.153	9	-5.037	3	-4.244	7.772	-4.524	9.303	-4.306	1.593
SL	-2.503	1	-2.212	5	-3.405	1	-1.468	0.911	-1.450	5.467	-2.563	0.555
D	-0.817	1	0.487	7	0.807	1	-0.331	0.434	0.488	7.000	0.050	0.399
SW	2.471	3	2.781	9	2.749	7	1.790	1.300	2.781	9.000	2.750	7.000
BW	4.069	5	5.749	9	5.238	3	3.000	4.511	5.750	9.000	3.992	1.234

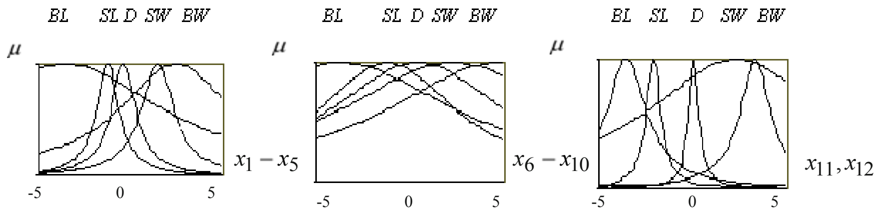


Fig. 9.14. Membership functions after tuning

To test the prediction model we used the results of 350 matches from 1991 to 1993. The fragment of testing data and prediction results are shown in Table 9.13, where:

T_1 , T_2 are teams' names,

$\hat{y}$, $\hat{d}$ are real (experimental) results,

y_G , d_G are results of prediction after genetic tuning of the fuzzy model,

y_N , d_N are results of prediction after neural tuning of the fuzzy model.

Symbol * shows no coincidences of theoretical and experimental results.

The efficiency characteristics of fuzzy model tuning algorithms for the testing data are shown in Table. 9.14.

Table 9.14. Tuning algorithms efficiency characteristics

Efficiency characteristics		Genetic Tuning	Neural Tuning
Tuning Time		52 min	7 min
Number of iterations		25000	5000
Probability of correct prediction for different decisions	d_1 – big loss	30 / 35 = 0.857	32 / 35 =0.914
	d_2 – small loss	64 / 84 = 0.762	70 / 84 = 0.833
	d_3 – draw	38 / 49 = 0.775	43 / 49 = 0.877
	d_4 – small win	97 / 126 = 0.770	106 / 126 = 0.841
	d_5 – big win	49 / 56 = 0.875	53 / 56 = 0.946

Table 9.14 shows, that the best prediction results we can receive for the marginal decision classes (the loss and win with big score d_1 and d_5), and the worst results of prediction we can receive for the small loss and small win (d_2 and d_4).

The future improvement of fuzzy prediction model can be done by taking into account some additional factors in fuzzy rules such as: the game on host/guest field, number of injured players, different psychological effects.

Table 9.13. Fragment of the prediction results

№	T_1	T_2	Year	$x1$	$x2$	$x3$	$x4$	$x5$	$x6$	$x7$	$x8$	$x9$	$x10$	$x11$	$x12$	Score	$\hat{y}$	$\hat{d}$	y_G	d_G	y_N	d_N
1	Kuusysi	Reipas	1991	**2**	**1**	**2**	**0**	**1**	**-1**	**0**	**1**	**-2**	**-3**	**2**	**1**	2-0	**2**	d4	**1**	d4	**1**	d4
2	Ilves	PPT	1991	**1**	**3**	**-1**	**1**	**0**	**0**	**2**	**-1**	**-2**	**0**	**0**	**0**	2-1	**1**	d4	**0**	d3*	**0**	d3*
3	Haka	Jaro	1991	**-1**	**2**	**0**	**-1**	**1**	**1**	**0**	**-2**	**-1**	**-2**	**-1**	**1**	1-1	**0**	d3	**0**	d3	**0**	d3
4	MP	OTP	1991	**3**	**1**	**2**	**0**	**2**	**-1**	**-2**	**1**	**-2**	**-3**	**1**	**3**	4-0	**4**	d5	**3**	d5	**3**	d5
5	KuPS	HJK	1991	**-1**	**-3**	**-4**	**1**	**-3**	**1**	**0**	**2**	**0**	**0**	**-2**	**0**	1-3	**-2**	d2	**-1**	d2	**-1**	d2
6	TPS	RoPS	1991	**3**	**1**	**2**	**-2**	**0**	**2**	**0**	**1**	**-1**	**1**	**0**	**-1**	1-0	**1**	d4	**0**	d3*	**0**	d3*
7	PPT	Jaro	1991	**0**	**-5**	**-1**	**0**	**1**	**1**	**2**	**-2**	**-1**	**1**	**1**	**-3**	0-1	**-1**	d2	**-1**	d2	**-1**	d2
8	Haka	Reipas	1991	**2**	**-1**	**3**	**1**	**4**	**2**	**-2**	**0**	**-1**	**0**	**-1**	**2**	3-0	**3**	d5	**2**	d4*	**2**	d4*
9	OTP	Kuusysi	1991	**-1**	**-2**	**-3**	**-2**	**0**	**1**	**3**	**4**	**-1**	**2**	**-2**	**-1**	1-4	**-3**	d1	**-3**	d1	**-3**	d1
10	HJK	TPS	1991	**1**	**1**	**1**	**0**	**2**	**0**	**1**	**-1**	**2**	**-3**	**0**	**2**	2-0	**2**	d4	**2**	d4	**2**	d4
11	MyPa	Jaro	1992	**-3**	**1**	**2**	**1**	**0**	**2**	**1**	**-2**	**-1**	**0**	**-2**	**0**	0-0	**0**	d3	**0**	d3	**0**	d3
12	Jazz	Ilves	1992	**2**	**2**	**1**	**-1**	**0**	**3**	**4**	**-1**	**0**	**1**	**1**	**-1**	2-1	**1**	d4	**0**	d3*	**1**	d4
13	Haka	RoPS	1992	**-2**	**-2**	**0**	**1**	**1**	**-1**	**1**	**1**	**1**	**0**	**1**	**3**	1-1	**0**	d3	**1**	d4*	**1**	d4*
14	HJK	Oulu	1992	**2**	**3**	**0**	**0**	**1**	**0**	**-5**	**1**	**-2**	**-1**	**-1**	**2**	4-0	**4**	d5	**2**	d4*	**3**	d5
15	MP	Kuusysi	1992	**0**	**1**	**-2**	**-1**	**-1**	**3**	**1**	**2**	**0**	**1**	**0**	**-2**	0-3	**-3**	d1	**-3**	d1	**-3**	d1
16	KuPS	HJK	1992	**-2**	**-1**	**-3**	**1**	**-2**	**4**	**2**	**1**	**2**	**1**	**-2**	**-3**	0-5	**-5**	d1	**-4**	d1	**-4**	d1
17	Kuusysi	MP	1992	**0**	**-1**	**3**	**2**	**-1**	**-3**	**2**	**-1**	**-2**	**0**	**1**	**0**	3-1	**2**	d4	**1**	d4	**1**	d4
18	TPS	Haka	1992	**-1**	**2**	**3**	**-1**	**-2**	**0**	**-1**	**0**	**3**	**1**	**-1**	**1**	2-2	**0**	d3	**0**	d3	**0**	d3
19	RoPS	MyPa	1992	**-2**	**-1**	**2**	**0**	**-1**	**1**	**-1**	**1**	**1**	**-2**	**1**	**-1**	1-2	**-1**	d2	**0**	d3*	**0**	d3*
20	Jazz	Ilves	1992	**-2**	**1**	**-3**	**5**	**-1**	**1**	**1**	**-2**	**0**	**-1**	**2**	**0**	1-0	**1**	d4	**1**	d4	**1**	d4
21	TPS	Jaro	1992	**-2**	**-1**	**2**	**-1**	**-3**	**1**	**0**	**2**	**-1**	**3**	**1**	**-2**	0-2	**-2**	d2	**-1**	d2	**-1**	d2
22	Haka	MyPa	1992	**1**	**1**	**-1**	**0**	**1**	**0**	**3**	**2**	**1**	**-1**	**-1**	**-3**	0-1	**-1**	d2	**-2**	d2	**-2**	d2
23	HJK	RoPS	1992	**1**	**2**	**0**	**-1**	**1**	**-1**	**2**	**2**	**-1**	**1**	**0**	**0**	2-1	**1**	d4	**0**	d3*	**0**	d3*
24	MP	Kuusysi	1992	**1**	**-1**	**-2**	**-3**	**1**	**1**	**-1**	**-2**	**2**	**3**	**-2**	**1**	0-2	**-2**	d2	**-1**	d2	**-1**	d2
25	Ilves	Kups	1992	**3**	**0**	**-2**	**2**	**-2**	**1**	**1**	**-1**	**0**	**-2**	**1**	**0**	1-0	**1**	d4	**1**	d4	**1**	d4
26	Haka	HJK	1992	**0**	**-2**	**-1**	**-1**	**0**	**2**	**3**	**-1**	**0**	**3**	**-1**	**-2**	0-3	**-3**	d1	**-3**	d1	**-3**	d1
27	Jaro	MyPa	1992	**-1**	**-1**	**1**	**2**	**1**	**-3**	**1**	**2**	**1**	**0**	**1**	**1**	1-1	**0**	d3	**1**	d4*	**0**	d3
28	RoPS	TPS	1992	**-1**	**1**	**-1**	**1**	**4**	**-5**	**-2**	**3**	**-1**	**-2**	**5**	**1**	2-0	**2**	d4	**2**	d4	**1**	d4
29	MP	Ilves	1992	**1**	**2**	**-1**	**1**	**0**	**0**	**1**	**0**	**0**	**-1**	**1**	**-2**	2-3	**-1**	d2	**-1**	d2	**-1**	d2
30	Kuusysi	KuPS	1992	**2**	**2**	**0**	**3**	**1**	**-1**	**-1**	**1**	**-3**	**0**	**2**	**3**	4-1	**3**	d5	**3**	d5	**3**	d5
31	Jazz	MP	1993	**2**	**2**	**2**	**0**	**3**	**-2**	**-1**	**0**	**-1**	**-3**	**4**	**3**	5-0	**5**	d5	**4**	d5	**4**	d5
32	Kuusysi	TPS	1993	**1**	**-1**	**0**	**-1**	**1**	**-2**	**2**	**0**	**-1**	**1**	**0**	**1**	0-0	**0**	d3	**0**	d3	**0**	d3
33	MyPa	RoPS	1993	**-1**	**-1**	**2**	**2**	**3**	**2**	**-1**	**1**	**2**	**-2**	**3**	**-1**	2-0	**2**	d4	**1**	d4	**1**	d4
34	Haka	HJK	1993	**-3**	**-1**	**-2**	**1**	**0**	**1**	**4**	**1**	**2**	**0**	**-1**	**-2**	1-3	**-2**	d2	**-1**	d2	**-1**	d2
35	Jaro	Ilves	1993	**2**	**0**	**-1**	**0**	**-1**	**-2**	**-1**	**-2**	**2**	**1**	**2**	**0**	2-1	**1**	d4	**1**	d4	**1**	d4
36	Ilves	HJK	1993	**1**	**-2**	**-1**	**-1**	**1**	**3**	**1**	**2**	**0**	**1**	**-1**	**-1**	0-2	**-2**	d2	**-1**	d2	**-1**	d2
37	Jazz	Jaro	1993	**2**	**1**	**0**	**1**	**5**	**-1**	**-2**	**-2**	**1**	**-1**	**2**	**1**	3-0	**3**	d5	**2**	d4*	**2**	d4*
38	MyPa	MP	1993	**1**	**3**	**1**	**-1**	**1**	**-1**	**0**	**2**	**-1**	**1**	**1**	**0**	1-0	**1**	d4	**1**	d4	**1**	d4
39	Kuusysi	Haka	1993	**-1**	**-2**	**1**	**1**	**2**	**-1**	**-3**	**1**	**-5**	**2**	**3**	**-1**	3-1	**2**	d4	**1**	d4	**1**	d4
40	TPS	RoPS	1993	**-1**	**1**	**-2**	**1**	**2**	**1**	**2**	**-1**	**1**	**-2**	**1**	**1**	1-0	**1**	d4	**1**	d4	**1**	d4
41	MP	HJK	1993	**-1**	**-1**	**0**	**2**	**-1**	**2**	**3**	**1**	**-1**	**1**	**-2**	**1**	1-2	**-1**	d2	**0**	d3*	**0**	d3*
42	Kuusysi	Jaro	1993	**2**	**2**	**-2**	**1**	**2**	**0**	**-1**	**2**	**-2**	**0**	**1**	**2**	2-1	**1**	d4	**1**	d4	**1**	d4
43	Jazz	Haka	1993	**2**	**3**	**2**	**-1**	**1**	**-1**	**-3**	**-4**	**-2**	**0**	**2**	**2**	4-0	**4**	d5	**3**	d5	**3**	d5
44	FinnPa	MyPa	1993	**-1**	**1**	**-2**	**-1**	**2**	**1**	**-2**	**-1**	**1**	**0**	**-1**	**-1**	1-2	**-1**	d2	**-1**	d2	**-1**	d2
45	TPS	Ilves	1993	**2**	**1**	**2**	**1**	**-1**	**2**	**2**	**-2**	**1**	**-3**	**0**	**2**	2-0	**2**	d4	**1**	d4	**1**	d4
46	RoPS	Jazz	1993	**-1**	**-1**	**2**	**-2**	**-1**	**4**	**1**	**5**	**0**	**2**	**1**	**-3**	2-5	**-3**	d1	**-3**	d1	**-3**	d1
47	MyPa	Ilves	1993	**5**	**0**	**2**	**1**	**1**	**-3**	**-1**	**-2**	**1**	**-2**	**3**	**0**	5-1	**4**	d5	**3**	d5	**3**	d5
48	TPV	Kuusysi	1993	**-2**	**-1**	**0**	**1**	**0**	**-1**	**0**	**2**	**-1**	**0**	**0**	**1**	0-0	**0**	d3	**0**	d3	**0**	d3
49	RoPS	HJK	1993	**-1**	**-1**	**1**	**-2**	**0**	**3**	**1**	**-2**	**1**	**1**	**-2**	**1**	0-2	**-2**	d2	**0**	d3*	**-1**	d2
50	TPS	Jaro	1993	**-1**	**-1**	**1**	**2**	**2**	**-2**	**-1**	**1**	**-2**	**1**	**3**	**1**	1-0	**1**	d4	**1**	d4	**1**	d4

9.4 Identification of Car Wheels Adhesion Factor with a Road Surface

The task of car wheels adhesion factor (AF) evaluation with a road surface arises with an execution of a technical expert's examination during an investigation of traffic accidents (TA). The objectivity of decision making relative to guilt or innocence of the driver who caused the TA depends on the precision of the AF definition (for example, run over a pedestrian). The existing technique [30, 31] allows determining of only some range of possible AF values depending upon a series of the influencing factors. Therefore, its final evaluation is determined by the auto engineering expert, subjectively taking into account the additional factors and conditions which are not involved in this technique.

Decision making relative to the cause of the accident is very sensitive to the value of AF: the subjective choice of the lower or upper value of AF can decide the fate of the accident participants.

The purpose of this research, the results of which are presented in this chapter, is to develop a mathematical model of AF evaluation taking into account all accessible information about the influencing factors, and at the expense of the AF magnitude improvement to raise a solution's objectivity.

This section is based on materials of [32].

9.4.1 Technique of Identification

The model of AF evaluation was developed on the basis of fuzzy rule-based methodology of identification described in [14]. The model was created in two stages: first – structural identification; second – parametrical identification. At the first stage the structure of the AF dependence upon the influencing factors was built by expert IF-THEN rules. At the second stage we selected such parameters of membership functions and such weights of fuzzy rules which allow us to minimize the difference between model and experimental results.

9.4.2 Structural Identification

The structure of the suggested model is shown in Fig. 9.15 in the form of a tree, whose trailing tops are the factors influencing AF.

The characteristic of the model consists of the fact that it takes into account both of the traditional factors, which are generalized by the integrated index Q, and additionally entered factors: S, H, P, N, V. All the influencing factors shown in Table 9.15 are considered as linguistic variables given using the appropriate universal sets and are estimated by fuzzy terms.

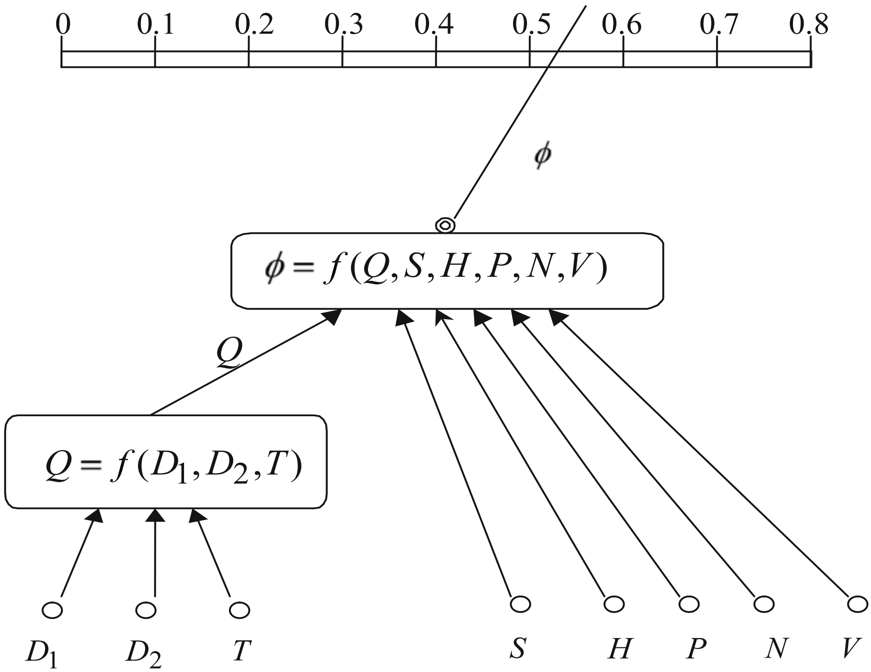


Fig. 9.15. The model structure for AF definition

The integrated index Q included in Table 9.15 depends on the factors: D_1 – road surface type; D_2 – road surface condition; T – tires type. The recommendations for the evaluation of the integrated index Q are given in Table 9.16 according to the known technique [30].

The expert knowledge base necessary for AF evaluation is shown in Table 9.17 (experts V. Rebedailo, A. Kashkanov). The application of the model of fuzzy logic inference to the knowledge base (Table 9.17) allows us to predict AF in some practical range of its modification. However, the exact evaluation of this factor depends on the choice of parameters for the model tuning.

9.4.3 Parametrical Identification

The tuning of the model was realized using training data, which represents the population of pairs "influencing factors – adhesion factor". To provide this training data a specially organized experiment with the automobile "Moskvich – 412" was carried out. In this experiment we used the car braking with different motion speeds on the horizontal road. Values of the factors which influence on AF were registered together with the values of car brake distances and values of brake initial velocity [31].

Table 9.15. The factors influencing AF

Factor	Universal set	Terms for estimations
Q – Integrated index "type of tires – road"	(0 – 9) conditional unit	Low (Q_1), Below average (Q_2), Average (Q_3), Above average (Q_4), High (Q_5)
S – Degree of tires slip	(0 – 100)%	Rolling with slip (S_1), Skid (S_2)
H – Wear of tires	(0 – 100)%	New (H_1), Within admissible range (H_2), Worn tire (H_3)
P – Pressure in tires	(0.1 – 0.325) MPa	Reduced (P_1), Normal (P_2), Higher than normal (P_3)
N – Load on a wheel	(0 – 100)%	Without load (N_1), Average (N_2), Full load (N_3)
V – Velocity of the car	(0 – 130) kms/h	Low (V_1), Below average (V_2), Average (V_3), Above average (V_4), High (V_5)

Table 9.16. Recommendations for evaluation of the integrated index Q

Road surface		Index Q for a type of tires (T)		
Type (D_1)	Condition (D_2)	High pressure	Low pressure	High permeability
Asphalt, Bitumen	Dry	5.63 – 7.88	7.88 – 9	7.88 – 9
	Rain moisture	3.1 – 4.33	4.33 – 4.95	4.33 – 4.95
	Wet	3.94 – 5.06	5.06 – 6.19	5.63 – 6.75
	Covered with a dirt	2.81 – 5.06	2.81 – 4.5	2.81 – 5.06
	Wet snow (t>0°C)	2.1 – 3.4	2.1 – 4.2	2.1 – 4.2
	Ice (t<0°C)	0.9 – 1.69	1.13 – 2.25	0.56 – 1.13
Cobble	Dry	4.5 – 5.63	5.63 – 6.19	6.75 – 7.88
	Wet	2.7 – 3.75	3.75 – 4.43	4.5 – 6.19
Metal	Dry	5.63 – 6.75	6.75 – 7.88	6.75 – 7.88
	Wet	3.38 – 4.5	4.5 – 5.63	4.5 – 6.19
Ground road	Dry	4.5 – 5.63	5.63 – 6.75	5.63 – 6.75
	Rain moisture	2.25 – 4.5	3.38 – 5.06	3.94 – 5.63
	Time of bad roads	1.68 – 2.81	1.68 – 2.81	2.25 – 3.38
Virgin soil in summer: Sand	Dry	2.25 – 3.38	2.48 – 4.5	2.25 – 3.38
	Damp	3.94 – 4.5	4.5 – 5.63	4.5 – 5.63
Clayed soil	Dry	4.5 – 5.63	5.06 – 6.19	4.5 – 5.63
	Humidified up to a plastic state	2.25 – 4.5	2.81 – 4.5	3.38 – 5.06
	Humidified up to a fluid state	1.69 – 2.25	1.69 – 2.81	1.69 – 2.81
Virgin soil in winter: Snow	Mellow	2.25 – 3.38	2.25 – 4.5	2.25 – 4.5
	Smooth	1.69 – 2.25	2.25 – 2.81	3.38 – 5.63

The total volume of the training sample included 60 pairs of “influencing factors – AF” data.

After tuning we received the membership functions shown in Fig. 9.16. Parameters of centres (b) and concentration (c) of the tuned membership functions presented in Table 9.18. Weights of the fuzzy rules obtained after tuning are given in the right side of Table 9.17.

Table 9.17. Fuzzy knowledge base

Q	S	H	P	N	V	ϕ	Weight
Q_1	S_2	H_2	P_2	N_1	V_1		1.000
Q_1	S_1	H_1	P_1	N_3	V_1	ϕ_1	0.700
Q_1	S_1	H_3	P_3	N_2	V_2		0.999
Q_2	S_2	H_2	P_2	N_2	V_3		0.700
Q_1	S_1	H_2	P_1	N_2	V_2	ϕ_2	0.700
Q_2	S_1	H_1	P_3	N_3	V_3		0.998
Q_2	S_1	H_2	P_2	N_3	V_5		0.700
Q_2	S_1	H_1	P_3	N_2	V_3	ϕ_3	0.400
Q_2	S_2	H_2	P_3	N_1	V_2		0.300
Q_2	S_1	H_2	P_2	N_1	V_2		0.400
Q_3	S_2	H_2	P_2	N_2	V_3	ϕ_4	0.997
Q_3	S_1	H_1	P_1	N_1	V_5		0.400
Q_4	S_2	H_1	P_2	N_3	V_2		0.999
Q_3	S_1	H_1	P_3	N_1	V_1	ϕ_5	1.000
Q_4	S_2	H_3	P_2	N_1	V_3		0.400
Q_4	S_2	H_2	P_2	N_1	V_1		0.999
Q_4	S_1	H_2	P_1	N_3	V_2	ϕ_6	0.400
Q_4	S_2	H_1	P_2	N_1	V_3		0.400
Q_4	S_1	H_1	P_2	N_1	V_2		0.699
Q_5	S_1	H_1	P_2	N_3	V_5	ϕ_7	1.000
Q_5	S_2	H_2	P_1	N_2	V_4		1.000
Q_5	S_2	H_2	P_2	N_3	V_2		1.000
Q_5	S_2	H_2	P_2	N_1	V_3	ϕ_8	1.000
Q_5	S_1	H_1	P_2	N_1	V_4		0.600

Table 9.18. Parameters of membership functions after tuning

Term	b	c	Term	b	c	Term	b	c
Q_1	0.90	0.97	H_1	21.36	24.33	N_2	64.48	28.92
Q_2	2.50	0.40	H_2	57.15	38.68	N_3	85.92	20.31
Q_3	4.63	0.59	H_3	90.21	26.55	V_1	10.40	14.74
Q_4	6.23	0.42	P_1	0.14	0.04	V_2	10.40	30.06
Q_5	8.58	0.75	P_2	0.20	0.04	V_3	14.07	42.26
S_1	24.88	41.76	P_3	0.32	0.07	V_4	64.65	5.82
S_2	98.93	41.95	N_1	0.10	38.98	V_5	119.99	13.48

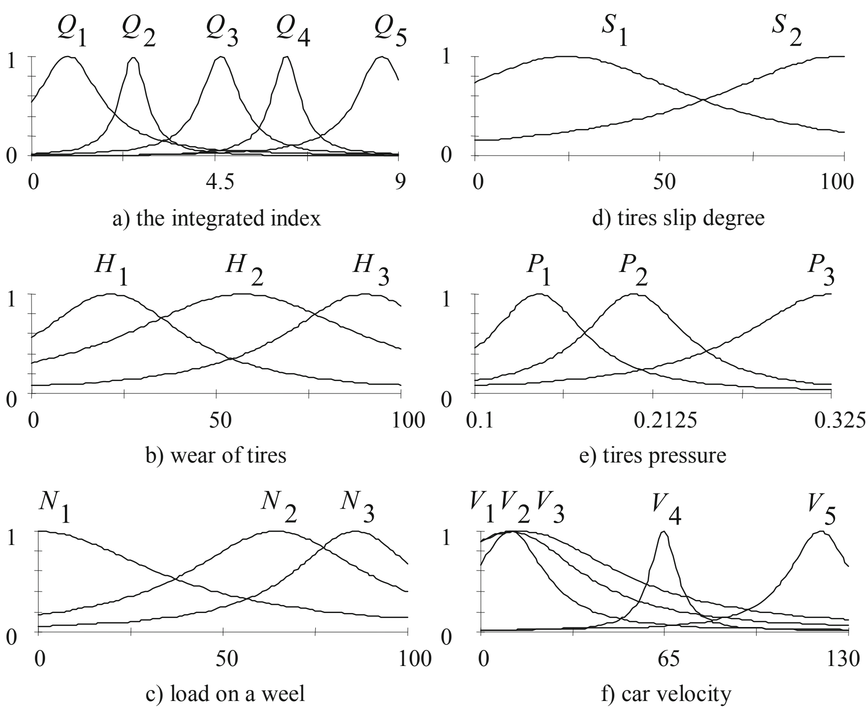


Fig. 9.16. Fuzzy terms membership functions after tuning

Table 9.19. Comparison of decisions

Factors						Adhesion factor		
Q	*S*	*H*	*P*	*N*	*V*	Tabular	1)	2)
6,15	100	62	0,2	15	20	0.45 – 0.55	0,55	0,54
4,45	100	65	0,2	15	60	0.25 – 0.4	0,33	0,35
4,7	100	65	0,18	20	40	0.30 – 0.45	0,39	0,39
3,4	90	45	0,17	95	120	0.22 – 0.40	0,26	0,26
3,7	64	95	0,25	45	72	0.20 – 0.40	0,28	0,29
3,9	84	81	0,27	67	65	0.25 – 0.45	0,32	0,31
8,1	67	72	0,25	20	58	0.60 – 0.70	0,68	0,68
3,4	65	80	0,14	15	15	0.25 – 0.40	0,27	0,28
3,6	40	75	0,18	20	45	0.30 – 0.45	0,34	0,31
3,9	100	35	0,29	45	110	0.20 – 0.40	0,29	0,29
7,4	35	70	0,19	60	90	0.60 – 0.70	0,62	0,62
5,3	30	5	0,26	90	35	0.40 – 0.50	0,45	0,45
8,6	100	60	0,2	15	20	0.70 – 0.80	0,76	0,75
6,15	100	62	0,2	15	40	0.45 – 0.55	0,52	0,52
6,3	100	65	0,18	20	20	0.50 – 0.60	0,56	0,54
4,7	100	65	0,18	20	60	0.30 – 0.45	0,36	0,38
4,8	15	55	0,21	62	32	0.40 – 0.50	0,42	0,41
5	37	15	0,18	17	25	0.40 – 0.50	0,44	0,42
6,8	70	28	0,16	90	52	0.50 – 0.70	0,55	0,54
7,3	41	37	0,2	50	65	0.60 – 0.70	0,62	0,62
6,7	80	55	0,12	56	62	0.50 – 0.60	0,52	0,54
4,8	100	20	0,23	10	80	0.35 – 0.50	0,39	0,38
3,3	50	90	0,3	50	85	0.25 – 0.40	0,24	0,24
2,1	20	55	0,23	70	40	0.15 – 0.20	0,16	0,15
8,6	100	60	0,2	15	40	0.70 – 0.80	0,74	0,74
6,15	100	62	0,2	15	60	0.45 – 0.55	0,48	0,51
6,3	100	65	0,18	20	40	0.50 – 0.60	0,53	0,52
7,2	70	70	0,19	15	60	0.60 – 0.70	0,63	0,62
1,7	35	30	0,16	74	34	0.10 – 0.20	0,16	0,15
1,3	72	35	0,15	70	33	0.08 – 0.15	0,12	0,13
2,25	62	21	0,31	85	64	0.20 – 0.25	0,17	0,18
4,5	32	75	0,19	90	80	0.35 – 0.50	0,35	0,36
7,5	75	25	0,18	71	67	0.60 – 0.70	0,64	0,63
2,6	65	50	0,16	60	55	0.20 – 0.30	0,22	0,20
5	70	20	0,17	100	25	0.40 – 0.50	0,39	0,40
0,7	100	75	0,18	20	10	0.05 – 0.10	0,06	0,06
8,6	100	60	0,2	15	60	0.70 – 0.80	0,70	0,70
4,45	100	65	0,2	15	20	0.25 – 0.40	0,40	0,38
6,3	100	65	0,18	20	60	0.50 – 0.60	0,51	0,52
5,6	100	75	0,2	25	100	0.45 – 0.55	0,46	0,44
2,9	48	25	0,24	51	68	0.20 – 0.40	0,22	0,21
2,85	56	75	0,29	40	40	0.20 – 0.30	0,20	0,22
5,5	53	98	0,18	100	35	0.40 – 0.50	0,42	0,43
5,2	78	20	0,17	38	129	0.40 – 0.55	0,41	0,41
8,2	15	10	0,2	100	115	0.70 – 0.80	0,67	0,66
8,3	100	30	0,17	80	40	0.70 – 0.80	0,71	0,71
4,3	90	10	0,13	10	120	0.35 – 0.40	0,33	0,33
8,6	100	62	0,2	15	80	0.70 – 0.80	0,67	0,68
4,45	100	65	0,2	15	40	0.25 – 0.40	0,36	0,38

1) Experimental
2) On suggested models

The comparison of the model with the experimental results of the AF evaluation shown in Table 9.19 testifies the adequacy of the obtained model for practical use.

9.4.4 Example and Comparison with the Technique in Use Now

The case of the run over a pedestrian by the automobile "GAZ-24" is discussed. The traffic accident protocol information:

- type of road surface (D_1) – asphalt;
- condition of road surface (D_2) – covered by dirt;
- type of tires (T) – low pressure;
- tires slip degree (S) – rolling with slip;
- wear of tires (H) – in admissible limits (about 50%);
- pressure in tires (P) – normal (0.2MPa);
- load on a wheel (N) – low (about 10%);
- car velocity (V) – 55 km/h.

We consider the horizontal road strip. After the run-over and up to the full stoppage automobile GAZ – 24 in the state of employed brakes run the distance of 9.2 m. From the moment when the motion barrier occurred and up to the moment of the pedestrian run-over he walked 5 m with the velocity of 4.5 km/h. The pedestrian was knocked-down by the front part of the car.

The results of the AF calculations as follows:

a) using the conventional technique [30]: $\varphi = 0.25 - 0.4$;

b) using the suggested technique: $\varphi = 0.35$.

The results using all the known information are presented in Table 9.20. The last column of this table shows the significance of the exact AF knowledge for the relevant decision making.

Table 9.20. Calculation results for decision making

Technique	Adhesion factor	Car braking distance	Distance up to the obstacle at the moment of dangerous situation	Decision making about the possibility to avoid collision
In use	0.25	68.8 m	46.2 m	Impossible
	0.4	51.0 m	55.3 m	Possible
Suggested	0.35	55.3 m	53.3 m	Impossible

9.5 Innovative Projects Creditworthiness Evaluation

Estimation of innovation project quality level is an important task of any investment firm. An instant and correct solution of this problem that can generally be accomplished only by specialist economists allows one to manage financial resources optimally. In this connection it is necessary to design computer based information system providing intelligent support for investment firm's personnel in decision making.

The expert system suggested here was developed to the order of Ukraine Innovation Fund. Expert IF-THEN rules were obtained from a group of analysts under the leadership of Vinnitsa Chapter of Ukraine Innovation Fund Director Prof. N. Petrenko.

This chapter is written on the basis of the work [33].

9.5.1 Types of Decisions and Partial Figures of Quality

Innovation project quality estimation is used for making one of the following decisions: d_1 - to finance, d_2 - to finance after retrofit, d_3 - to finance when means are available, d_4 - to reject.

Let us use letter D to designate the integral figure of innovation project quality. To estimate this figure we will use the following information:

X - level of the enterprise-applicant, which is estimated using the following partial figures: x_1 - level of enterprise leader, x_2 - enterprise assets, x_3 - enterprise liabilities, x_4 - enterprise balance profit, x_5 - enterprise debt receivables, x_6 - enterprise indebtedness under credits. To estimate enterprise leader level we take into account the following figures: a_1 - sociability, a_2 - fidelity, a_3 - education, a_4 - leader work experience, a_5 - comfort;

Y - technical economic level of the project, in point for which estimation the following partial figures are used: y_1 - project scale, y_2 - project novelty, y_3 - development trend priority, y_4 - degree of perfection, y_5 - juridical protection, y_6 - ecology level;

V - expected sales level;

Z - financial level of the enterprise-applicant, which is estimated using the following partial figures: z_1 - ratio of internal funds to innovation funds, z_2 - innovation fund means return.

The task of estimation is in bringing one of the decisions $d_1 \div d_4$ into correspondence with some innovation project with known partial figures.

9.5.2 Fuzzy Knowledge Bases

A hierarchy diagram of accepted innovation project quality figures is shown in Fig. 9.17 in the form of a fuzzy logic inference tree, to which this system of relations corresponds:

$$D = f_D(X, Y, V, Z) \ , \tag{9.17}$$

$$X = f_X(x_1, x_2, x_3, x_4, x_5, x_6) \ , \tag{9.18}$$

$$x_1 = f_{x_1}(a_1, a_2, a_3, a_4, a_5) \ , \tag{9.19}$$

$$Y = f_Y(y_1, y_2, y_3, y_4, y_5, y_6) \ , \tag{9.20}$$

$$Z = f_Z(z_1, z_2) \ . \tag{9.21}$$

Partial figures in point $x_1 \div x_6$, $a_1 \div a_5$, $y_1 \div y_6$, V, z_1 and z_2, and also enlarged figures X, Y, Z are considered as linguistic variables. To estimate the introduced linguistic variables we will use the unitary scale of qualitative terms: *vL* – very Low, *L* - Low, *lA* – lower than average, *A* - average, *hA* – higher than average, *H* - High, *vH* – very high.

Each of these terms represents some fuzzy set preset using the following membership function model. Using introduced quality terms let us represent relations (9.17) - (9.21), in the knowledge base form by Tables 9.21-9.25.

Table 9.21. Knowledge about relation (9.17)

X	*Y*	*V*	*Z*	*D*
H	*H*	*H*	*H*	
hA	*H*	*H*	*H*	d_1
H	*H*	*H*	*hA*	
hA	*hA*	*hA*	*hA*	
hA	*H*	*H*	*hA*	d_2
hA	*hA*	*H*	*A*	
H	*H*	*A*	*A*	
H	*A*	*A*	*A*	d_3
H	*A*	*hA*	*A*	
L	*L*	*L*	*L*	d_4
A	*L*	*L*	*L*	

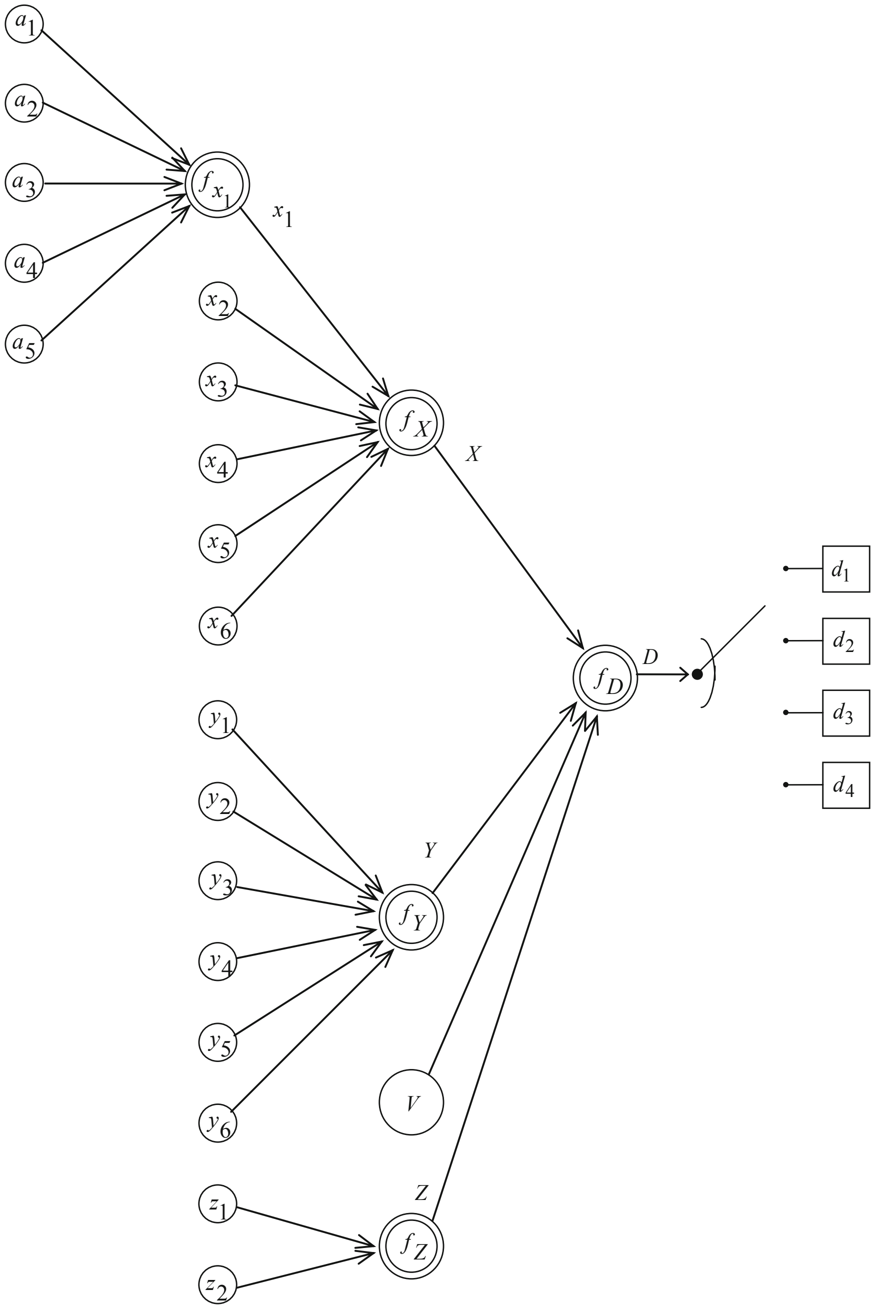


Fig. 9.17. Fuzzy logic evidence tree

Table 9.22. Knowledge about relation (9.18)

x_1	x_2	x_3	x_4	x_5	x_6	X
L	*L*	*L*	*L*	*L*	*H*	*L*
H	*H*	*H*	*lA*	*lA*	*lA*	*lA*
H	*H*	*H*	*A*	*A*	*lA*	*A*
H	*H*	*H*	*hA*	*hA*	*A*	*hA*
H	*H*	*H*	*H*	*H*	*L*	*H*
H	*H*	*H*	*A*	*hA*	*L*	

Table 9.23. Knowledge about relation (9.19)

a_1	a_2	a_3	a_4	a_5	x_1
vL	*vL*	*vL*	*vL*	*vL*	*vL*
L	*L*	*L*	*L*	*L*	*L*
lA	*A*	*lA*	*A*	*lA*	*lA*
A	*A*	*A*	*A*	*A*	*A*
hA	*H*	*hA*	*H*	*A*	*hA*
H	*H*	*H*	*H*	*H*	*H*
vH	*vH*	*vH*	*vH*	*vH*	*vH*

Table 9.24. Knowledge about relation (9.20)

y_1	y_2	y_3	y_4	y_5	y_6	Y
vL	*vL*	*vL*	*vL*	*vL*	*vL*	*L*
L	*L*	*L*	*L*	*L*	*L*	
A	*A*	*L*	*L*	*L*	*A*	*lA*
A	*A*	*A*	*A*	*A*	*A*	*A*
H	*H*	*H*	*H*	*H*	*H*	*hA*
vH	*vH*	*vH*	*vH*	*vH*	*vH*	*H*

Table 9.25. Knowledge about relation (9.21)

z_1	z_2	Z
vL	*vL*	*L*
A	*L*	*lA*
A	*A*	*A*
hA	*H*	*hA*
vH	*vH*	*H*

9.5.3 Evaluation Examples

Some of the partial figures have a qualitative character; that is, they have no precise quantitative measurement. Therefore, while making estimations of the same figure by several experts there can be various opinions. In addition, the expert is not always capable of making an estimation of the partial figure using words though he intuitively feels its level. To overcome these difficulties we can estimate partial figures using the thermometer principle [14]. Convenience of such an approach is in the fact that various sense partial figures are defined as linguistic variables given on the unitary universal set $U = [0, 100]$, which is the scale of a thermometer. Parameters (b) and (c) of membership functions are introduced in Table 9.26.

Table 9.26. Membership functions parameters

Term	vL	L	lA	A	hA	H	vH
b	0.0	16.7	33.3	50.0	66.7	83.3	100
c	15	15	15	15	15	15	15

Examples of three innovation projects' estimations by the suggested fuzzy model are represented in Table 9.27. Results of decision making are well in accordance with expert assessments of quality.

Table 9.27. Examples of innovation projects quality estimation

Partial figure	Project 1	Project 2	Project 3
a_1			
a_2			
a_3			
a_4			
a_5			
x_2			
x_3			
x_4			
x_5			
x_6			
y_1			
y_2			
y_3			
y_4			
y_5			
y_6			
z_1			
z_2			
V			
Decision	To finance with means available	To finance	To finance after retrofit

9.6 System Reliability Analysis

Probabilistic models of reliability of technological processes and systems were considered in [34 – 39]. The application of these models presumes the availability of statistical data on probabilities of correct execution of elements of algorithmic process, i. e., technological operations. To take into account influencing factors, it is expedient to use experiment planning theory and regression models. It is very difficult to provide the equal conditions of experiment reiteration necessary for the statistical methods' correct application while evaluating the probabilities of correct (noncorrect) performance of the system and its elements' functioning process. From the other side an experiment and statistical data processing is too complicated because of the many factors influencing the reliability such as environment task conditions; psychological stress and the degree of fatigue of an operator etc. It is relatively easy and natural to take into account such a factor's influence linguistically, e.g., "if the degree of fatigue of an operator is low, environment task conditions is good, psychological stress is low then human reliability is high".

The active research on fuzzy logic using in reliability theory began in the 10th decade of the last century. The first approaches to the fuzzy reliability theory creation have been proposed in monographs [40, 41]. The overwhelming majority of known works uses for the system reliability analysis the descriptive possibilities of fuzzy logic in combination with probability theory and descriptive possibilities of Boolean algebra [42 – 45]. In this chapter, we consider basic principles, mathematical models and the example of application of the new method of complex systems reliability analysis on the basis of algebra of algorithms [46, 47] and fuzzy logic [48]. Here we present the results of simulation of the bioconversion technological process reliability. In reliability modeling of a technological system it is necessary to take into account not only the structure of the technological process but also the influencing factors, connected with the quality of raw material, the technological equipment and the operator, controlling the process.

This chapter is written on the basis of the works [49, 50].

9.6.1 Basic Principles

The approach proposed in [49, 50] is based on the following principles:

1. *Principle of algorithmization*

This principle, adopted from theory of reliability of man-machine systems [35], envisages construction of the reliability model on the basis of the algorithmic description of the events, connected with the occurring, detecting and removal of the failures (faults, defects, errors) in the system. To depict the algorithm, we use graph-schemes or the language of V.M. Glushkov's algorithmic algebra [46, 47], in which any regular algorithm can be built with the help of the three structures:

a) *linear* (B *-structure*): $A_1 A_2 = B$, producing the operator B , which is equivalent to the consecutive performance of the operators A_1 and A_2 ;

b) *alternative* (C -*structure*): $\underset{\alpha}{(A_1 \vee A_2)} = C$, producing the operator C , such that

$$C = \begin{cases} A_1, \ if \ \ condition \ \alpha \ is \ \ true \ (\alpha = 1) \\ A_2, \ if \ \ condition \ \alpha \ is \ \ fault \ (\alpha = 0) \end{cases} ;$$

c) *iterative* (D -*structure*): $\underset{\alpha}{\{A\}} = D$, producing the operator D , which is equivalent to repeated implementation of operator A till the condition α has become true $(\alpha = 1)$.

2. *Principle of fuzzy correctness*

Conception of the crisp boarder between "correct" (1) and "noncorrect" (0) results of the system and its elements functioning lacking underlies this principle. For the formal evaluation of the level of operator A correct performance, we use the multidimensional membership function $\mu_A^1(x_1, x_2, ..., x_n)$, which depends on the measured parameters (input variables). Correctness of each of the parameters is defined by the membership function $\mu^1(x_i)$, which can be interpreted as a parameter x_i values' correctness distribution.

3. *Principle of linguistic evaluation of control quality*

The system functioning process control is accomplished with the help of the checking and correction operations. If the checking operation is performed by a human, then the 1st type error (false alarm or rejection of "good" result) can be connected with the level of "objectivity – preconception" of the inspector, and the 2nd type error (acceptance of defective goods), – with the level of "vigilance – negligence" of the inspector.

This principle envisages the possibility of evaluation of the checking and correction operations using verbal terms: low (average, high) tendency of man-operator to commit the 1st and 2nd type errors; low (average, high) repair quality, etc. Membership functions, necessary for these terms formalization, are formed with the help of extension-compression operations [48], which underlie the idea of Soft Computing – computing with words.

4. *Principle of fuzzy identification*

This principle emphasizes that the problem of system reliability evaluation amounts to the problem of "multiple inputs – single output" object identification with the help of fuzzy knowledge bases [14].

Inputs of the object are the measured parameters of the quality of raw material, equipment and man-operator. Output of the object is the discrete double-throw switch: 1 - correct; 0 - noncorrect. Because of the lack of the crisp border between 1 and 0 results, the degrees of membership of the vector of input parameters to the levels 1 and 0 are calculated during system reliability modeling.

Fuzzy knowledge bases, i.e., IF-THEN rules, necessary for solving the identification problem, are determined by B -, C - and D - structures, from which the algorithmic model of reliability is built.

9.6.2 Fuzzy-Algorithmic Elements

The fuzzy-algorithmic model of system reliability is built using the following elements (Fig. 9.18):

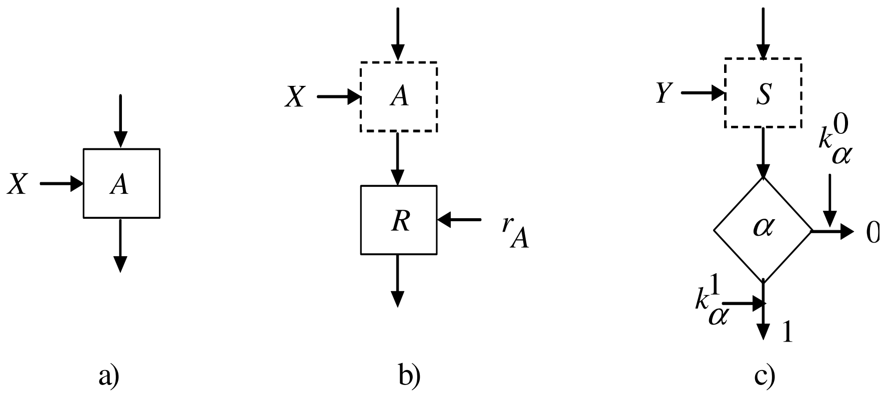


Fig. 9.18. Elements of reliability model

Working operator A (Fig. 9.18a) is the element of the model, describing occurring abnormalities in the system functioning process. Quality of the working operator A performance depends on the vector of measured parameters $\mathbf{X} = (x_1, x_2, ..., x_n)$, where $x_i = x_i(t)$, i.e., parameters values depend on time.

Correctness of the working operator A performance is defined by the formula:

$$\mu_A^1(\mathbf{X}) = \prod_{i=1}^{n} \mu^1(x_i), \tag{9.22}$$

where $\mu_A^1(\mathbf{X})$ is the multidimensional membership function of the vector of parameters $\mathbf{X}$ to the term "correct performance of the operator A",

$\mu^1(x_i)$ is the membership function, which describes the distribution of parameter x_i, $i = 1, 2, ..., n$, values' correctness.

Correction operator R (Fig. 9.18b) is the element of the model, which describes removal of abnormalities, occurred while performing the working operator A.

Different kinds of repair and updating included in the system functioning algorithm can be described by the correcting operator R.

Correctness of the correction operator R performance is defined by the formula:

$$\mu_R^1(\mathbf{X}) = 1 - [1 - \mu_A^1(\mathbf{X})]^{r_A}, \tag{9.23}$$

where $\mu_A^1(\mathbf{X})$ is defined by formula (9.22),

r_A is the parameter, which characterizes the quality of correction:

$r_A = 1, 3, 5, 7, 9$, if the quality of correction is low (1), lower than average (3), average (5), higher than average (7), high (9).

If, for example, a working operator A has correctness $\mu_A^1(\mathbf{X}) = 0.5$, then the correctness of the correcting operator R is increased with the growth of parameter r_A:

r_A	1	3	5	7	9
$\mu_R^1(X)$	0.5	0.875	0.967	0.992	0.9998

Correctness of algorithm AR, i.e., "work (A) – correction (R)", performance is defined by formula:

$$\mu_{AR}^1(\mathbf{X}) = \mu_A^1(\mathbf{X}) + [1 - \mu_A^1(\mathbf{X})] \cdot \mu_R^1(\mathbf{X}), \tag{9.24}$$

from which it is shown, that if $\mu_R^1(\mathbf{X}) = 1$, then $\mu_{AR}^1(\mathbf{X}) = 1$.

Logical condition α (Fig. 9.18c) is the element of the model, which describes correctness checking for the vector of parameters $\mathbf{Y} = (y_1, y_2, ..., y_l)$. This vector of parameters can correspond to the condition of the system components: the raw material, the equipment, the man-operator or the results of functioning process implementation. In particular, the diagnostic and functional checking which are used in reliability theory of man-machine systems [35] can be described by logical condition α.

While performing condition α the two results are possible:

$\alpha = 1$, if all the parameters of vector $\mathbf{Y}$ are correct,

$\alpha = 0$, if at least one of the parameters of vector $\mathbf{Y}$ is noncorrect.

Correctness of condition α performance is defined as follows:

$\mu_\alpha^{11}(\mathbf{Y})$ is the possibility distribution of the condition α performance for result 1, i.e., without 1st type errors, when real correctness (1) is subjectively recognized as true (1),

$\mu_\alpha^{00}(\mathbf{Y})$ is the possibility distribution of the condition α performance for result 0, i.e., without 2nd type errors, when false (0) is subjectively recognized as false (0).

These distributions are defined by the formulae:

$$\mu_\alpha^{11}(\mathbf{Y}) = [\mu_\alpha^1(\mathbf{Y})]^{k_\alpha^1}, \tag{9.25}$$

$$\mu_\alpha^{00}(\mathbf{Y}) = [1 - \mu_\alpha^1(\mathbf{Y})]^{k_\alpha^0}, \tag{9.26}$$

$$\mu_\alpha^1(\mathbf{Y}) = \prod_{i=1}^{l} \mu^1(y_i), \tag{9.27}$$

where $\mu^1(y_i)$ is the correctness distribution of the parameter y_i, $i = 1, 2, ..., l$.

k_α^1 and k_α^0 are the coefficients, describing the tendency of the checking operation α to the 1st and 2nd type errors, respectively ($k_\alpha^1 \geq 1, k_\alpha^0 \geq 1$).

If $k_{\alpha}^{1}=1$ and $k_{\alpha}^{0}=1$, then the 1st and 2nd type errors are absent. The increase of these coefficients results in compression of the membership functions in (9.25) and (9.26), and, respectively, lowering down of the level of correctness of checking condition α performance for results 1 and 0. This is equivalent to the growth of the 1st and 2nd type errors levels.

For calculations on the basis of linguistic assessments one can use:

$k_{\alpha}^{1}=1$, if the 1st type errors are absent (if the inspector is objective),

$k_{\alpha}^{1}=2$, for small tendency to the 1st type errors (if the inspector is somewhat preconceived),

$k_{\alpha}^{1}=3$, for sufficient tendency to the 1st type errors (if the inspector is preconceived),

i.e. with the growth of the inspector preconception (or with the lowering down of his/her objectivity) the possibility of the 1st type error is increased.

For the 2nd type errors:

$k_{\alpha}^{0}=1$, if the 2nd type errors are absent (if the inspector is vigilant),

$k_{\alpha}^{0}=2$, for small tendency to the 2nd type errors (if the inspector is somewhat negligent),

$k_{\alpha}^{0}=3$, for sufficient tendency to the 2nd type errors (if the inspector is negligent),

i.e., with the lowering down of the inspector vigilance (or with the growth of his/her negligence) the possibility of the 2nd type error is increased.

9.6.3 Fuzzy-Algorithmic Structures

Each of the algorithmic structures produces the mathematical model, which allows us to calculate the correctness of this structure implementation depending on the correctness of the included operators and conditions implementation. Such models are obtained in [50] on the basis of the graphs of events, taking place while performing each of the structures (Fig. 9.19). Necessary formulae are given below.

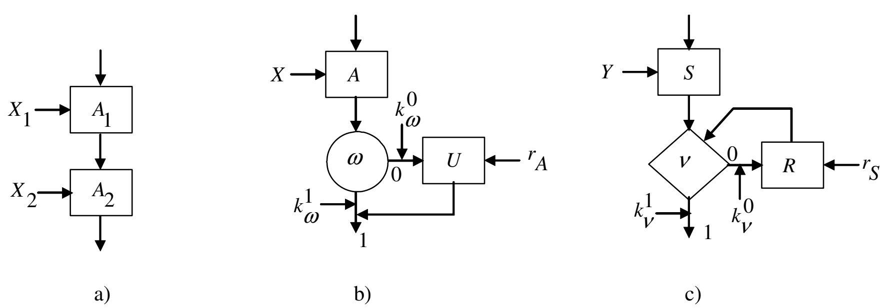


Fig. 9.19. Algorithmic structures

Linear structure (Fig. 9.19a) is given by algorithm

$$B = A_1 A_2 \,, \tag{9.28}$$

in which the working operators A_1 and A_2 depend on the vectors of parameters $\mathbf{X}_1 = (x_1^1, x_2^1, ..., x_n^1)$ and $\mathbf{X}_2 = (x_1^2, x_2^2, ..., x_n^2)$, respectively.

Fuzzy correctness of the equivalent operator B performance in (9.28) is defined by the formula:

$$\mu_B^1(\mathbf{X}_1, \mathbf{X}_2) = \mu^1(\mathbf{X}_1) \cdot \mu^1(\mathbf{X}_2) \,, \tag{9.29}$$

where $\mu^1(\mathbf{X}_1) = \prod_{i=1}^{n_1} \mu^1(x_i^1)$, $\mu^1(\mathbf{X}_2) = \prod_{j=1}^{n_2} \mu^1(x_j^2)$.

Alternative structure (Fig. 9.19b) is given by algorithm

$$C = A \underset{\omega}{(} E \vee U) \,, \tag{9.30}$$

in which ω is the logical condition, verifying during the checking of the correctness of the working operator A implementation, where

$$\omega = \begin{cases} 1, \textit{if vector of parameters } \mathbf{X} \textit{ is normal}, \\ 0, \textit{otherwise.} \end{cases}$$

E is the identical operator, corresponding to the checking operation ω results fixation,

U is the operator correcting the parameters of the working operator A,

k_ω^1, k_ω^0 and r_A are the parameters of condition ω and operator U implementation quality, respectively.

Structure (9.30) corresponds to the process "work – checking – correction without feedback" [35].

Fuzzy correctness of the equivalent operator C performance in (9.30) is defined by the formula:

$$\mu_C^1(\mathbf{X}, k_\omega^1, k_\omega^0, r_A) = \mu_\omega^1 \cdot \mu_\omega^{11} + [\mu_\omega^1 (1 - \mu_\omega^{11}) + (1 - \mu_\omega^1) \mu_\omega^{00}] \mu_U^1 \,, \tag{9.31}$$

where $\mu_\omega^1 = \prod_{i=1}^{n} \mu^1(x_i)$, $\mu_\omega^{11} = (\mu_A^1)^{k_\omega^1}$, $\mu_\omega^{00} = (1 - \mu_A^0)^{k_\omega^0}$,

$\mu_U^1 = 1 - (1 - \mu_A^1)^{r_A}$,

$\mu^1(x_i)$ is the correctness distribution of the parameter x_i $i = 1, 2, ..., n$,

k_ω^1, k_ω^0 and r_A are the numbers (1,2,3,…), which define the quality of checking ω and correcting U operators, respectively.

Iterative structure (Fig. 9.19c) is given by algorithm

$$D = \underset{\nu}{S\{R\}}, \tag{9.32}$$

in which ν is the logical condition, verifying during the checking of the parameters of the working operator S, where

$$\nu = \begin{cases} 1, \text{ if vector of parameters } \mathbf{Y} \text{ is normal}, \\ 0, \text{ otherwise.} \end{cases}$$

R is the operator correcting parameters of the working operator S,

k_ν^1, k_ν^0 and r_S are the parameters of condition ν and operator R implementation quality, respectively.

Structure (9.32) describes the process “diagnostics – repair with feedback” [35] when the equipment is diagnosed.

In the general case, operator S corresponds to the equipment functioning, the raw material preparation or the man-operator work.

Fuzzy correctness of the equivalent operator D performance in (9.32) is defined by the formula:

$$\mu_D^1(\mathbf{Y}) = a + ba_1 \cdot \frac{1}{1-b_1}, \tag{9.33}$$

where $a = \mu_\nu^1 \cdot \mu_\nu^{11}$, $a_1 = \mu_R^1 \cdot \mu_\nu^{11}$,

$b = \mu_\nu^1\left(1-\mu_\nu^{11}\right) + \left(1-\mu_\nu^1\right)\mu_\nu^{00}$, $b_1 = \mu_R^1\left(1-\mu_\nu^{11}\right) + \left(1-\mu_R^1\right)\cdot\mu_\nu^{00}$,

$\mu_\nu^1 = \prod_{l=1}^{m} \mu^1(y_l)$, $\mu_\nu^{11} = \left(\mu_\nu^1\right)^{k_\nu^1}$, $\mu_\nu^{00} = \left(1-\mu_\nu^1\right)^{k_\nu^0}$,

$\mu_R^1 = 1 - \left(1-\mu_\nu^1\right)^{r_\nu}$,

$\mu^1(y_l)$ is the correctness distribution of the parameter y_l, $l = 1,2,...,m$,

k_ν^1, k_ν^0 and r_S are the numbers (1,2,3,...), which define the quality of checking ν and correcting R operators, respectively.

9.6.4 Example of Technological System Reliability Analysis

Let us consider the bioconversion technological process (BCTP), the algorithmic model of which (Fig. 9.20) is defined by the formula:

$$F = \underset{\nu}{S\{R\}}\underset{\omega}{A(E \vee U)}, \tag{9.34}$$

where S is the working operator, corresponding to the raw material preparation;

ν is the raw material parameters checking (ho - homogeneity, Ph - hydrogen factor, hu - humidity);

R is the operator of the raw material parameters correction;

A is the working operator, corresponding to the process performance;

ω is the process parameters checking (V – rate of mixing, t^0 -temperature);

E is the identical operator, corresponding to the checking operation ω results fixation;

k_V^1, k_V^0, r_S, k_ω^1, k_ω^0 and r_A are the parameters of the process control quality, shown in Fig. 9.20.

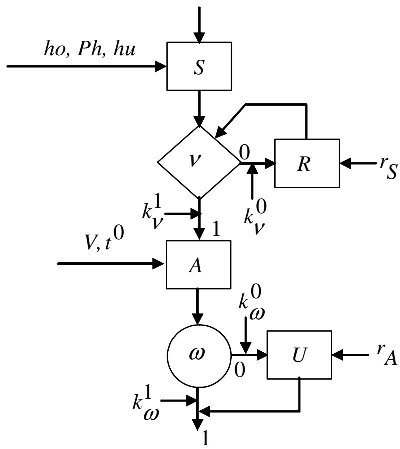


Fig. 9.20. Algorithmic model of the bioconversion process reliability

The parameters correctness distributions are presented in Table 9.28.

Algorithm (9.34) is presented as follows

$$F = D \cdot C\,, \quad D = \underset{V}{S}\{R\}, \quad C = A \underset{\omega}{\ }(E \vee U)\,.$$

Therefore, the problem of reliability analysis is reduced to the consecutive application of the models of *B*-, *C*- and *D*- structures:

$$\mu_F^1(hu, Ph, ho, V, t) = \mu_D^1(hu, Ph, ho) \cdot \mu_C^1(V, t)\,,$$

where $\mu_F^1(...)$ is the process (9.34) performance correctness distribution;

$\mu_D^1(...)$ is the operator D performance correctness distribution calculated by formula (9.33) for $\mathbf{Y} = (hu, Ph, ho)$,

$\mu_C^1(...)$ is the operator C performance correctness distribution calculated by formula (9.31) for $\mathbf{X} = (V, t)$.

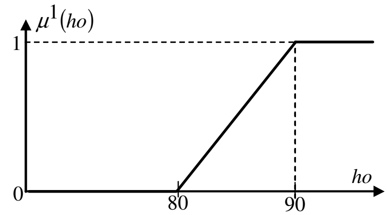
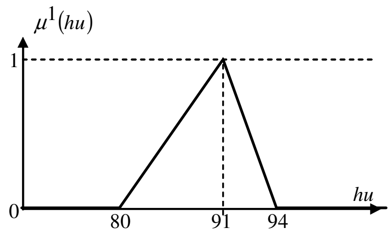
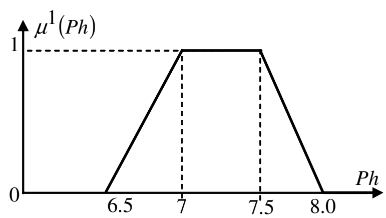
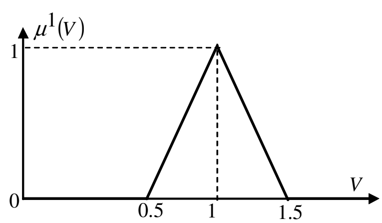
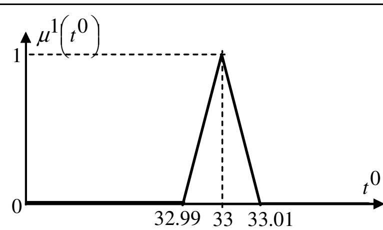
The tree of inference, which defines the interconnection of the fuzzy-algorithmic structures in identifying the process (9.34) reliability level, is shown in Fig. 9.21, where *double circles* are the models of *B*-, *C*- and *D*- structures;

single circles are the operators and conditions, appearing in algorithm (9.34);

output arrow is the process performance correctness level, which is defined by the membership function μ_F^1;

input arrows are the variables, influencing the correctness level μ_F^1.

Table 9.28. Parameters correctness distributions

Parameter	Membership function
Homogeneity (*ho, %*)	
Humidity (*hu, %*)	
Hydrogen factor (*Ph,* c.u.)	
Rate of mixing (*V*, rpm)	
Temperature (t^0 C)	

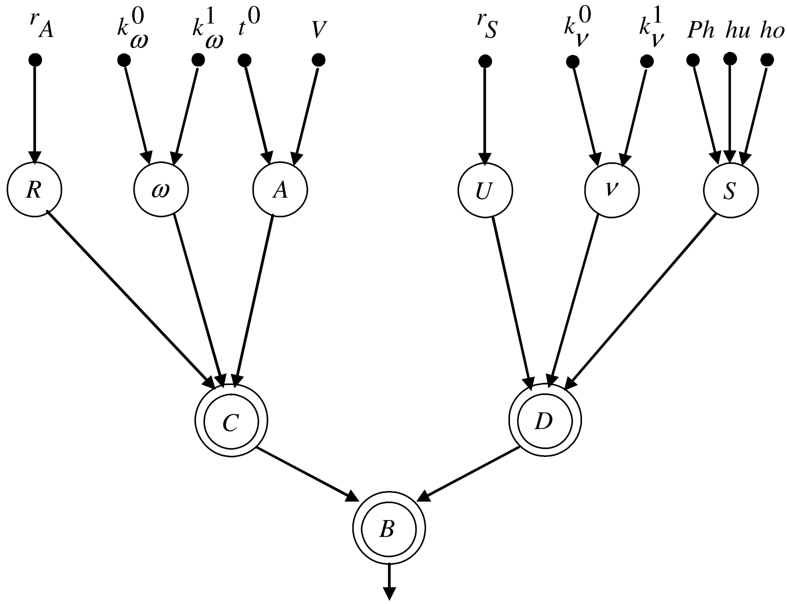


Fig. 9.21. Tree of inference

The aim of simulation consisted of the construction of three-dimensional correctness distributions $\mu_F^1\left(V,t^0\right)$ for different combinations of the raw material quality levels (Table 9.29) and the process control quality levels (Table 9.30). The nine three-dimensional distributions were obtained.

Table 9.29. Values of the raw material parameters

Raw material parameters	Quality levels		
	Low	Average	High
ho (%)	83	87	91
hu (%)	83	87	91
Ph (c.u.)	6.6	6.8	7.1

The three distributions $\mu_F^1(V,t^0)$, which correspond to high level of raw material quality and low (a), average (b) and high (c) levels of the process control quality are shown in Fig. 9.22.

Table 9.30. Processes control parameters values

Control element	Parameters of checking and correcting operations	Control quality levels		
		Low	Average	High
V	k_V^1	5	3	1
	k_V^0	9	5	1
R	r_S	1	5	9
ω	k_ω^1	5	3	1
	k_ω^0	9	5	1
U	r_A	1	5	9

The correctness distributions $\mu_F^1(V,t^0)$ allow us to obtain the regions of parameters change (V and t^0), which provide the required level of the process performance correctness. Let call them zones (cross-sections) of μ - working capacity, $\mu \in [0,1]$. Such zones for levels $\mu_F^1(V,t^0) = 0.9, 0.8, 0.7, 0.6$ are presented in Table 9.31. The obtained zones of μ - working capacity provide the possibility of optimization of the system reliability with taking into account the restrictions of the region of permissible parameters change [51].

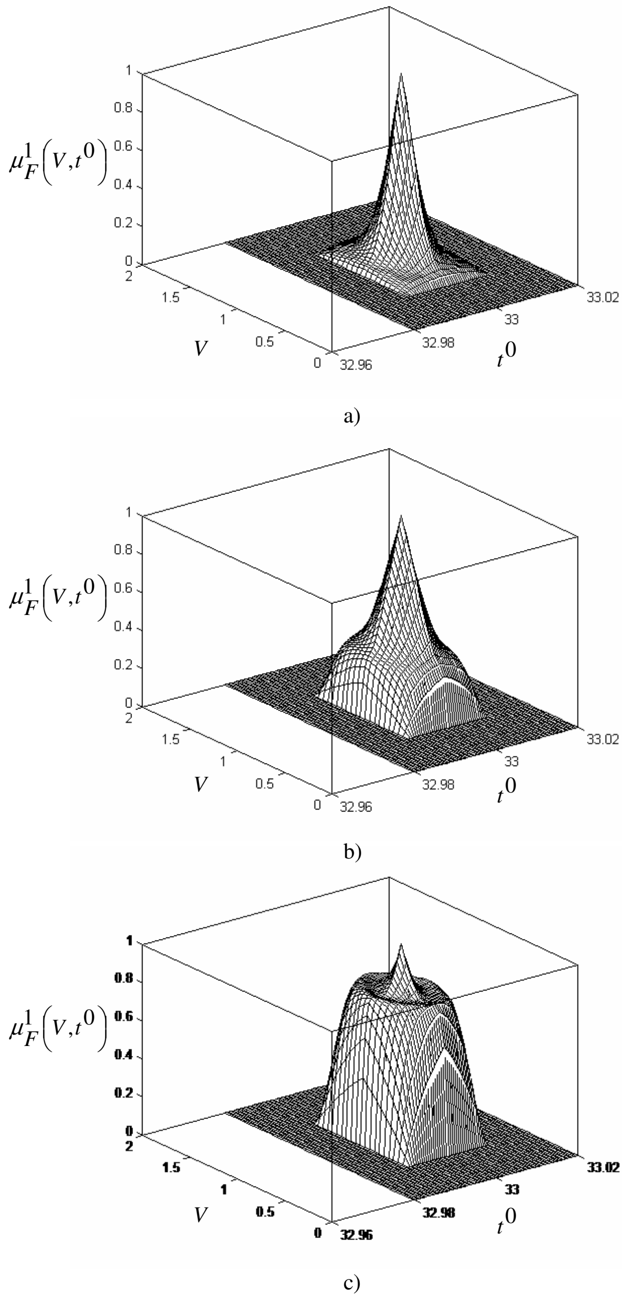
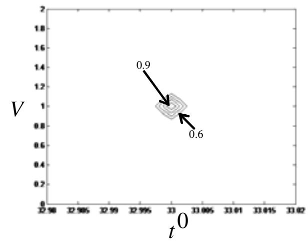
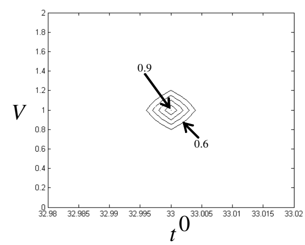
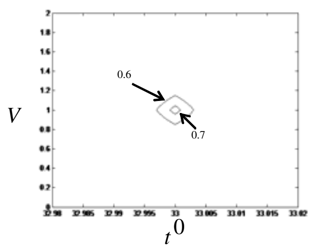
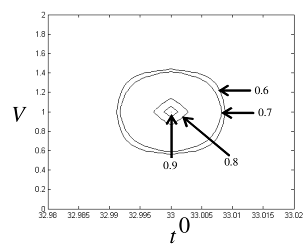


Fig. 9.22. Process performance correctness distributions for low (a), average (b) and high (c) control quality levels

Table 9.31. Working capacity zones for process performance (0.6, 0.7, 0.8, 0.9)-correctness levels

Control quality	Raw material quality		
	Low	Average	High
Low	Zone is absent	Zone is absent	
Average	Zone is absent	Zone is absent	
High	Zone is absent		

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