Chapter 8
Fuzzy Relations Extraction from Experimental Data

In this chapter, a problem of fuzzy genetic object identification expressed mathematically in terms of fuzzy relational equations is considered.

Fuzzy relational calculus [1, 2] provides a powerful theoretical background for knowledge extraction from data. Some fuzzy rule base is modelled by a fuzzy relational matrix, discovering the structure of the data set [3 – 5]. Fuzzy relational equations, which connect membership functions of input and output variables, are built on the basis of a fuzzy relational matrix and Zadeh's compositional rule of inference [6, 7]. The identification problem consists of extraction of an unknown relational matrix which can be translated as a set of fuzzy IF-THEN rules. In fuzzy relational calculus this type of problem relates to inverse problem resolution for the composite fuzzy relational equations [2]. Solvability and approximate solvability conditions of the composite fuzzy relational equations are considered in [2, 8, 9]. While the theoretical foundations of fuzzy relational equations are well developed, they call for more efficient use of their potential in system modeling. The non-optimizing approach [10] is widely used for fuzzy relational identification. Such adaptive recursive techniques are of interest for the most of on-line applications [11 – 13]. Under general conditions, an optimization environment is the convenient tool for fuzzy relational identification [14]. An approach for identification of fuzzy relational models by fuzzy neural networks is proposed in [15 – 17].

The genetic algorithm as a tool to solve the fuzzy relational equations was proposed in [18]. The genetic algorithm [19 – 21] allows us to solve the inverse problem which consists of the restoration of the unknown values of the vector of the unobserved parameters through the known values of the vector of the observed parameters and the known fuzzy relational matrix. In this chapter, the genetic algorithm [19 – 21] is adapted to identify the relational matrix for the given inputs-outputs data set. The algorithm for fuzzy relation matrix identification is accomplished in two stages. At the first stage, parameters of membership functions included in the fuzzy knowledge base and rules weights are defined using the genetic algorithm [22]. In this case, proximity of linguistic approximation results and experimental data is the criterion of extracted relations quality. It is shown here that in comparison with [22] the non-unique set of IF-THEN rules can be extracted from the given data. Following [18 – 21], at the

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second stage the obtained null solution allows us to arrange the genetic search for the complete solution set, which is determined by the unique maximum matrix and a set of minimum matrices. After linguistic interpretation the resulting solution can be represented as a set of possible rules collections, discovering the structure of the given data.

The approach proposed is illustrated by the computer experiment and the example of medical diagnosis. This chapter is written on the basis of [23].

8.1 "Multiple Inputs – Multiple Outputs" Object

Let us consider an object

$$\mathbf{Y} = f(\mathbf{X}) \tag{8.1}$$

with n inputs $\mathbf{X} = (x_1, x_2, ..., x_n)$ and m outputs $\mathbf{Y} = (y_1, y_2, ..., y_m)$, for which the following is known:

- intervals of inputs and outputs change

$$x_i \in [\underline{x}_i, \overline{x}_i],\ i = \overline{1,n};\ y_j \in [\underline{y}_j, \overline{y}_j],\ j = \overline{1,m};$$

- classes of decisions e_{jp} for evaluation of output variable y_j, $j = \overline{1,m}$, formed by digitizing the range $[\underline{y}_j, \overline{y}_j]$ into q_j levels

$$[\underline{y}_j, \overline{y}_j] = \underbrace{[\underline{y}_j, \overline{y}_{j1})}_{e_{j1}} \cup \ldots \cup \underbrace{[\underline{y}_{jp}, \overline{y}_{jp})}_{e_{jp}} \cup \ldots \cup \underbrace{[\underline{y}_{jq_j}, \overline{y}_j]}_{e_{jq_j}};$$

- training data in the form of L pairs of "inputs-outputs" experimental data

$$\left\langle \hat{\mathbf{X}}_s, \hat{\mathbf{Y}}_s \right\rangle, \quad s = \overline{1,L},$$

where $\hat{\mathbf{X}}_s = (\hat{x}_1^s, \hat{x}_2^s, ..., \hat{x}_n^s)$ and $\hat{\mathbf{Y}}_s = (\hat{y}_1^s, \hat{y}_2^s, ..., \hat{y}_m^s)$ are the vectors of the values of the input and output variables in the experiment number s.

It is necessary to transfer the available training data into the following system of IF-THEN rules [7]:

$$\begin{aligned} \text{Rule } l: \quad & \text{IF} \quad x_1 = a_{1l} \ \text{ AND} \ldots\ x_i = a_{il} \ \text{ AND} \ldots x_n = a_{nl} \\ & \text{THEN } \ y_1 = b_{1l} \ \text{ AND} \ldots\ y_j = b_{jl} \ \text{ AND} \ldots\ y_m = b_{ml},\ l = \overline{1,N}, \end{aligned} \tag{8.2}$$

where a_{il} is the fuzzy term describing a variable x_i in rule l, $i = \overline{1,n}$;

b_{jl} is the fuzzy term describing a variable y_j in rule l, $j = \overline{1,m}$;

N is the number of rules.

8.2 Fuzzy Rules, Relations and Relational Equations

This fuzzy rule base is modelled by the fuzzy relational matrix presented in Table 8.1.

Table 8.1. Fuzzy knowledge base

IF inputs						THEN outputs										
						y_1			…	y_j			…	y_m		
	x_1	…	x_i	…	x_n	e_{11}	…	e_{1q_1}	…	e_{j1}	…	e_{jq_j}	…	e_{m1}	…	e_{mq_m}
						E_1	…				E_k	…				E_M
C_1	a_{11}	…	a_{i1}	…	a_{n1}	r_{11}	…				r_{1k}	…				r_{1M}
…	…	…		…	…	…	…				…	…				…
C_l	a_{1l}	…	a_{il}	…	a_{nl}	r_{l1}	…				r_{lk}	…				r_{lM}
…	…	…		…	…	…	…				…	…				…
C_N	a_{1N}	…	a_{iN}	…	a_{nN}	r_{N1}	…				r_{Nk}	…				r_{NM}

This relational matrix can be translated as a set of fuzzy IF-THEN rules

$$\text{Rule } l: \text{IF } \mathbf{X} = C_l \text{ THEN } y_j = e_{jp} \text{ with weight } r_{l,jp}, \tag{8.3}$$

where C_l is the combination of input terms in rule l, $l = \overline{1,N}$;

$r_{l,jp}$ is the relation $C_l \times e_{jp}$, $j = \overline{1,m}$, $p = \overline{1,q_j}$, interpreted as the rule weight.

We shall redenote the set of classes of output variables as $\{E_1, E_2, ..., E_M\} = \{e_{11}, e_{12}, ..., e_{1q_1}, ..., e_{m1}, e_{m2}, ..., e_{mq_m}\}$, where $M = q_1 + q_2 + ... + q_m$.

In the presence of relational matrix

$$\mathbf{R} \subseteq C_l \times E_k = [r_{lk},\ l = \overline{1,N},\ k = \overline{1,M}]$$

the "inputs-outputs" dependency can be described with the help of Zadeh's compositional rule of inference [6]

$$\boldsymbol{\mu}^E(\mathbf{Y}) = \boldsymbol{\mu}^C(\mathbf{X}) \circ \mathbf{R}, \tag{8.4}$$

where $\boldsymbol{\mu}^C(\mathbf{X}) = (\mu^{C_1}, \mu^{C_2}, ..., \mu^{C_N})$ is the vector of membership degrees of vector $\mathbf{X}$ to input combinations C_l;

$\boldsymbol{\mu}^E(\mathbf{Y}) = (\mu^{E_1}, \mu^{E_2}, ..., \mu^{E_M})$ is the vector of membership degrees of variables y_j to classes e_{jp};

$\circ$ is the operation of *max-min* composition [6].

The system of fuzzy relational equations is derived from relation (8.4):

$$\mu^{e_{jp}}(y_j) = (\mu^{C_1}(\mathbf{X}) \wedge r_{1,jp}) \vee (\mu^{C_2}(\mathbf{X}) \wedge r_{2,jp}) \vee ... \vee (\mu^{C_N}(\mathbf{X}) \wedge r_{N,jp}),$$

where

$$\mu^{C_l}(\mathbf{X}) = \mu^{a_{1l}}(x_1) \wedge \mu^{a_{2l}}(x_2) \wedge ... \wedge \mu^{a_{nl}}(x_n), \quad l = \overline{1,N};$$

or

$$\mu^{e_{jp}}(y_j) = \underset{l=\overline{1,N}}{\vee}((\underset{i=\overline{1,n}}{\wedge} \mu^{a_{il}}(x_i)) \wedge r_{l,jp}). \tag{8.5}$$

Here

$\mu^{a_{il}}(x_i)$ is a membership function of a variable x_i to the fuzzy term a_{il};

$\mu^{e_{jp}}(y_j)$ is a membership function of a variable y_j to the class e_{jp}.

Taking into account the fact that operations $\vee$ and $\wedge$ are replaced by *max* and *min* in fuzzy set theory, system (8.5) is rewritten in the form

$$\mu^{e_{jp}}(y_j) = \max_{l=\overline{1,N}}\left(\min\left(\min_{i=\overline{1,n}}[\mu^{a_{il}}(x_i)], r_{l,jp} \right) \right). \tag{8.6}$$

We use a bell-shaped membership function model of variable u to arbitrary term T in the form [22]:

$$\mu^T(u) = \frac{1}{1 + \left(\frac{u - \beta}{\sigma} \right)^2}, \tag{8.7}$$

where β is a coordinate of function maximum, $\mu^T(\beta) = 1$; σ is a parameter of concentration.

The operation of defuzzification is defined in [22] as follows:

$$y_j = \frac{\sum_{p=1}^{q_j} \underline{y}_{jp} \cdot \mu^{e_{jp}}(y_j)}{\sum_{p=1}^{q_j} \mu^{e_{jp}}(y_j)}. \tag{8.8}$$

Relationships (8.6) – (8.8) define the generalized fuzzy model of an object (8.1) as follows:

$$\mathbf{Y} = F_R(\mathbf{X}, \mathbf{R}, \mathbf{B}, \mathbf{\Omega}), \tag{8.9}$$

where $\mathbf{B} = (\beta_1, \beta_2, .., \beta_K)$ and $\mathbf{\Omega} = (\sigma_1, \sigma_2, ..., \sigma_K)$ are the vectors of β - and σ - parameters for fuzzy terms membership functions in (8.3);

K is the total number of fuzzy terms;

F_R is the operator of inputs-outputs connection, corresponding to formulae (8.6)–(8.8).

8.3 Optimization Problem for Fuzzy Relations Extraction

Let us impose limitations on the knowledge base (8.2) volume in the following form:

$$N \le \overline{N},$$

where $\overline{N}$ is the maximum permissible total number of rules.

So as content and number of linguistic terms a_{il} ($i = \overline{1,n}$, $l = \overline{1,N}$) used in fuzzy knowledge base (8.2) are not known beforehand then we suggest to interpret them on the basis of membership functions (8.7) parameter values ($\beta^{a_{il}}, \sigma^{a_{il}}$). Therefore, knowledge base (8.2) synthesis is reduced to obtaining the matrix of parameters shown in Table 8.2 [22].

Table 8.2. Knowledge base parameters matrix

IF inputs				THEN outputs											
	x_1	…	x_n	y_1			…	y_j			…	y_m			
				e_{11}	…	e_{1q_1}	…	e_{j1}	…	e_{jq_j}	…	e_{m1}	…	e_{mq_m}	
				E_1	…			E_k	…					E_M	
C_1	$(\beta^{a_{11}}, \sigma^{a_{11}})$	…	$(\beta^{a_{n1}}, \sigma^{a_{n1}})$	r_{11}	…			r_{1k}	…					r_{1M}	
…	…	…	…	…	…			…	…					…	
C_l	$(\beta^{a_{1l}}, \sigma^{a_{1l}})$	…	$(\beta^{a_{nl}}, \sigma^{a_{nl}})$	r_{l1}	…			r_{lk}	…					r_{lM}	
…	…	…	…	…	…			…	…					…	
C_N	$(\beta^{a_{1N}}, \sigma^{a_{1N}})$	…	$(\beta^{a_{nN}}, \sigma^{a_{nN}})$	r_{N1}	…			r_{Nk}	…					r_{NM}	

This problem can be formulated as follows. It is necessary to find such a matrix (Table 8.2), which satisfies the limitations imposed on knowledge base volume and provides the least distance between model and experimental outputs of the object:

$$\sum_{s=1}^{L} [F_R(\hat{\mathbf{X}}_s, \mathbf{R}, \mathbf{B}, \mathbf{\Omega}) - \hat{\mathbf{Y}}_s]^2 = \min_{\mathbf{R},\mathbf{B},\mathbf{\Omega}} . \tag{8.10}$$

If R_0 is a solution of the optimization problem (8.10), then R_0 is the exact solution of the composite system of fuzzy relational equations:

$$\hat{\boldsymbol{\mu}}^A(\hat{\mathbf{X}}_s) \circ \mathbf{R} = \hat{\boldsymbol{\mu}}^B(\hat{\mathbf{X}}_s), \tag{8.11}$$

where the experimental input and output matrices

$$\hat{\boldsymbol{\mu}}^A = \begin{bmatrix} \hat{\mu}^{C_1}(\hat{\mathbf{X}}_1) & \dots & \hat{\mu}^{C_N}(\hat{\mathbf{X}}_1) \\ \dots & \dots & \dots \\ \hat{\mu}^{C_1}(\hat{\mathbf{X}}_L) & \dots & \hat{\mu}^{C_N}(\hat{\mathbf{X}}_L) \end{bmatrix}, \ \hat{\boldsymbol{\mu}}^B = \begin{bmatrix} \hat{\mu}^{E_1}(\hat{\mathbf{X}}_1) & \dots & \hat{\mu}^{E_M}(\hat{\mathbf{X}}_1) \\ \dots & \dots & \dots \\ \hat{\mu}^{E_1}(\hat{\mathbf{X}}_L) & \dots & \hat{\mu}^{E_M}(\hat{\mathbf{X}}_L) \end{bmatrix}$$

are obtained for the given training data.

Following [2], the system (8.11) has a solution set $S(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$, which is determined by the unique maximal solution $\overline{\mathbf{R}}$ and the set of minimal solutions $S^*(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B) = \{\underline{\mathbf{R}}_I, \ I = \overline{1,T}\}$:

$$S^*(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B) = \bigcup_{\underline{\mathbf{R}}_I \in S^*} [\underline{\mathbf{R}}_I, \overline{\mathbf{R}}] \ . \tag{8.12}$$

Here $\overline{\mathbf{R}} = [\overline{r}_{lk}]$ and $\underline{\mathbf{R}}_I = [\underline{r}^I_{lk}]$ are the matrices of the upper and lower bounds of the fuzzy relations r_{lk}, where the union is taken over all $\underline{\mathbf{R}}_I \in S^*(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$.

The problem of solving fuzzy relational equations (8.11) is formulated as follows [19 – 21]. Fuzzy relation matrix $\mathbf{R} = [r_{lk}]$, $l = \overline{1,N}$, $k = \overline{1,M}$, should be found which satisfies the constraints $r_{lk} \in [0,1]$ and also provides the least distance between model and experimental outputs of the object; that is, the minimum value of the criterion (8.10).

Following [19 – 21], formation of the intervals (8.12) is accomplished by way of solving a multiple optimization problem (8.10) and it begins with the search for its null solution $\mathbf{R}_0 = [r^0_{lk}]$, where $r^0_{lk} \le \overline{r}_{lk}$, $l = \overline{1,N}$, $k = \overline{1,M}$. The upper bound ($\overline{r}_{lk}$) is found in the range $[r^0_{lk}, 1]$. The lower bound ($\underline{r}^I_{lk}$) for $I = 1$ is found in the range $[0, r^0_{lk}]$, and for $I > 1$ in the range $[0, \overline{r}_{lk}]$, where the minimal solutions $\underline{\mathbf{R}}_J$, $J < I$, are excluded from the search space.

Let $\mathbf{R}(t) = [r_{lk}(t)]$ be some t-th solution of optimization problem (8.10), that is $F(\mathbf{R}(t)) = F(\mathbf{R}_0)$, since for all $\mathbf{R} \in S(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$ we have the same value of criterion (8.10). While searching for upper bounds $\overline{r}_{lk}$ it is suggested that $r_{lk}(t) \ge r_{lk}(t-1)$, and while searching for lower bounds $\underline{r}^I_{lk}$ it is suggested that $r_{lk}(t) \le r_{lk}(t-1)$ (Fig. 8.1).

The definition of the upper (lower) bounds follows the rule: if $\mathbf{R}(t) \ne \mathbf{R}(t-1)$, then $\overline{r}_{lk}$ ($\underline{r}^I_{lk}$)$= r_{lk}(t)$. If $\mathbf{R}(t) = \mathbf{R}(t-1)$, then the search for the interval solution $[\underline{\mathbf{R}}_I, \overline{\mathbf{R}}]$ is stopped. Formation of intervals (8.12) will go on until the condition $\underline{\mathbf{R}}_I \ne \underline{\mathbf{R}}_J$, $J < I$, has been satisfied.

The hybrid genetic and neuro approach is proposed for solving optimization problem (8.10).

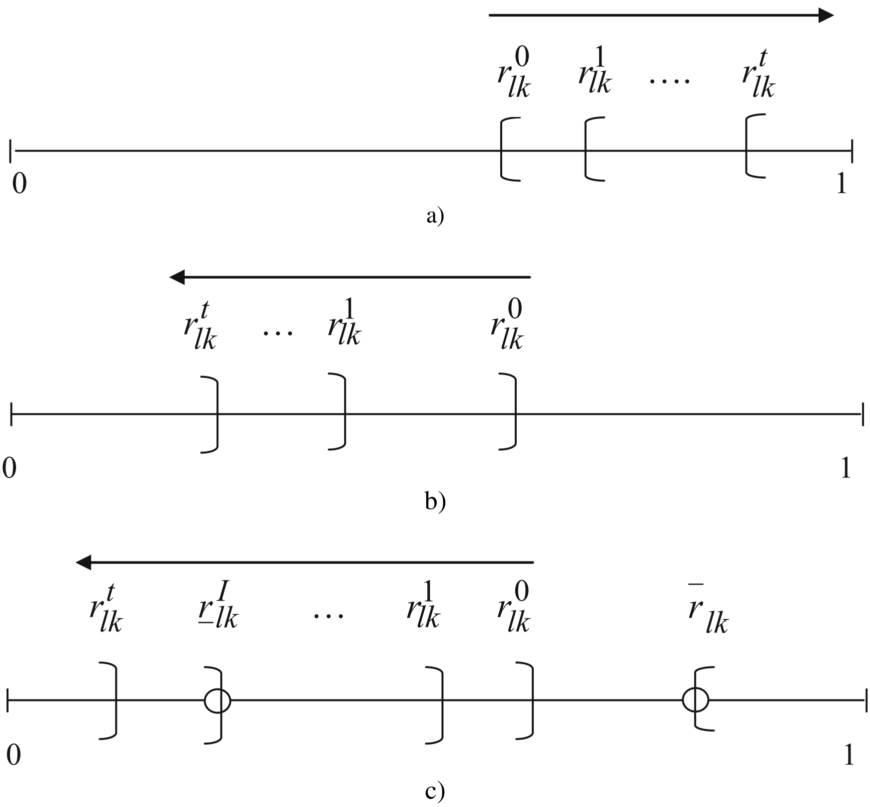

Fig. 8.1. Search for the upper (a) and lower bounds of the intervals for $I = 1$ (b) and $I > 1$ (c)

8.4 Genetic Algorithm for Fuzzy Relations Extraction

To describe the chromosome for the parameters matrix (Table 8.2), we use the string shown in Fig. 8.2, where C_l is the code of IF-THEN rule with number l, $l = \overline{1,N}$. The chromosome needed in the genetic algorithm for solving fuzzy relational equations (8.11) includes only the codes of parameters r_{lk}, $l = \overline{1,N}$, $k = \overline{1,M}$. Parameters of membership functions are defined simultaneously with the null solution.

The crossover operation is defined in Fig. 8.3, and is carried out by way of exchanging chromosomes parts inside each rule C_l ($l = \overline{1,N}$) and inside matrix of rules weights $\mathbf{R}$. The total number of exchange points is equal to $\overline{N} + 1$.

A mutation operation (Mu) implies random change (with some probability) of chromosome elements:

$$Mu(r_{lk}) = RANDOM([0,1]) \ ,$$

$$Mu(\beta^{a_{il}}) = RANDOM([\underline{x}_i, \overline{x}_i]) \ , \ Mu(\sigma^{a_{il}}) = RANDOM([\underline{\sigma}^{a_{il}}, \overline{\sigma}^{a_{il}}]) \ ,$$

where $RANDOM([\underline{x}, \overline{x}])$ denotes a random number within the interval $[\underline{x}, \overline{x}]$.

The fitness function is evaluated on the basis of criterion (8.10).

If $P(t)$ are chromosomes-parents and $C(t)$ are chromosomes-offsprings on a t-th iteration, then the genetic procedure of optimization will be carried out according to the following algorithm [24, 25]:

begin
 t:=0;
 To set the initial population $P(t)$;
 To evaluate the $P(t)$ for the null solution using criterion (8.10);
 while (no condition of null solution formation) **do**
  To generate the $C(t)$ by operation of cross-over with $P(t)$;
  To evaluate the $C(t)$ for the null solution using criterion (8.10);
  To select the population $P(t+1)$ from $P(t)$ and $C(t)$;
  t:=t+1;
 end
 while (no condition of interval set formation) **do**
  To generate the $C(t)$ by operation of cross-over with $P(t)$;
  To evaluate the $C(t)$ for the bounds of intervals (8.12) using criterion (8.10);
  To select the population $P(t+1)$ from $P(t)$ and $C(t)$;
  t:=t+1;
 end
end

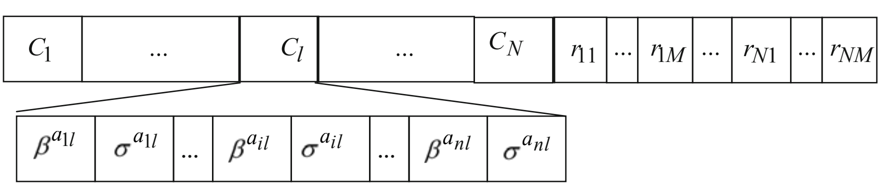

Fig. 8.2. Coding of parameters matrix

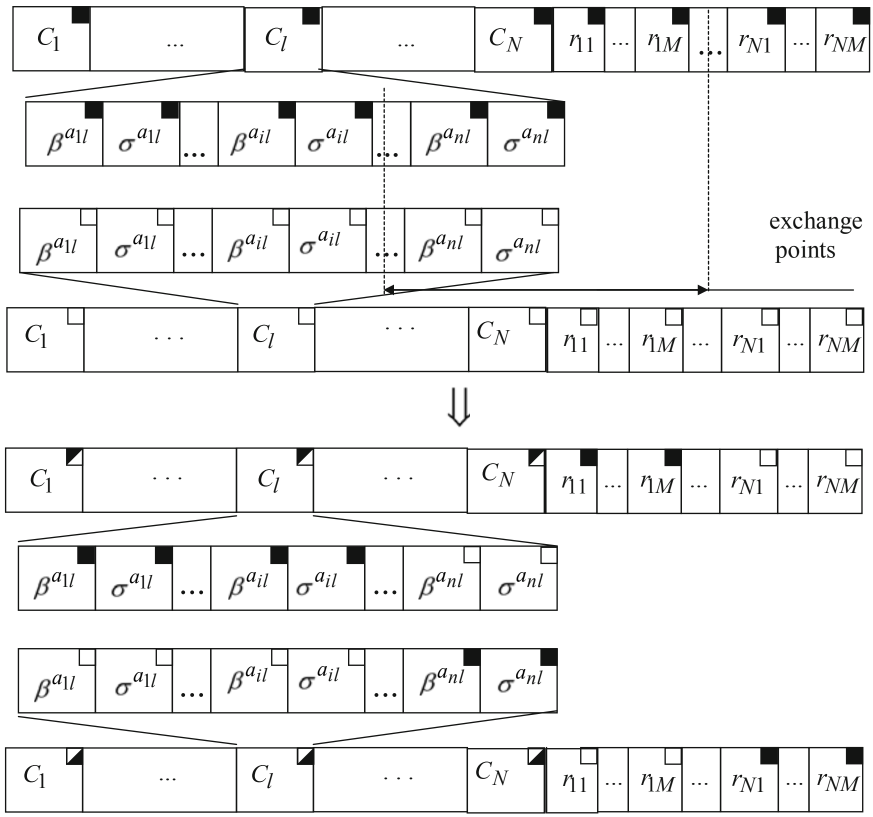

Fig. 8.3. Crossover operation (■, □ - chromosomes-parents, ◪, ◩ - chromosomes-offsprings)

8.5 Neuro-fuzzy Network for Fuzzy Relations Extraction

Let us impose limitations on the knowledge base (8.2) volume in the following form:

$$k_1 \le \overline{k_1}\ ,\ k_2 \le \overline{k_2}\ , ...,\ k_n \le \overline{k_n}\ ,$$

where $\overline{k}_i$ is the maximum permissible total number of fuzzy terms describing a variable x_i , $i=\overline{1,n}$.

This allows embedding system (8.2) into the special neuro-fuzzy network, which is able to extract knowledge [16, 21]. The neuro-fuzzy network for knowledge extraction is shown in Fig. 8.4, and the nodes are presented in Table 3.1.

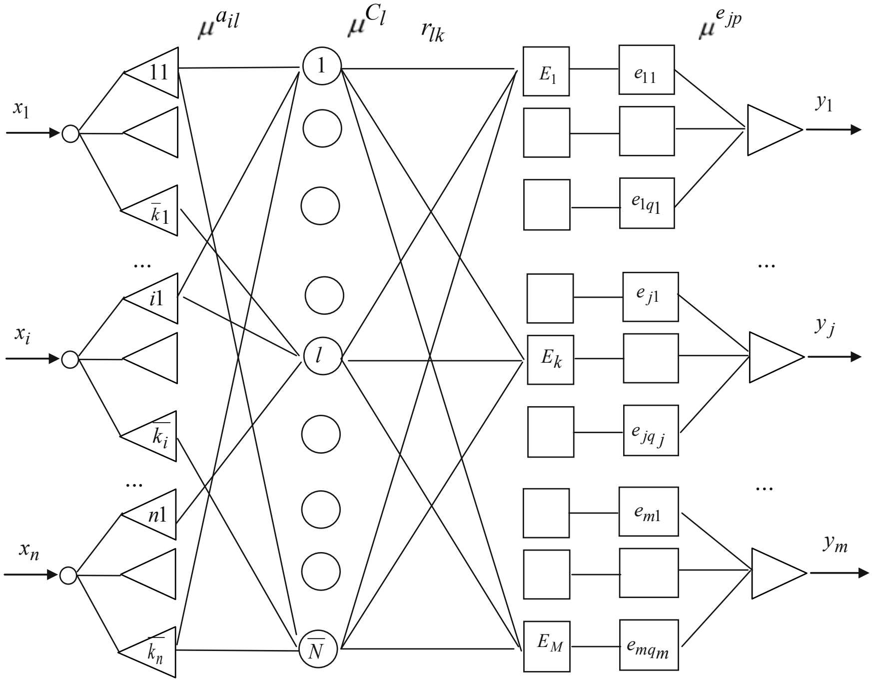

Fig. 8.4. Neuro-fuzzy network for knowledge extraction

As is seen from Fig. 8.4 the neuro-fuzzy network has the following structure:

layer 1 for object identification inputs (the number of nodes is equal to n),

layer 2 for fuzzy terms used in the knowledge base (the number of nodes is equal to $\bar{k}_1 + \bar{k}_2 + ... + \bar{k}_n$),

layer 3 for strings-conjunctions (the number of nodes is equal to $\bar{k}_1 \cdot \bar{k}_2 \cdot ... \cdot \bar{k}_n$),

layer 4 for fuzzy rules making classes (the layer is fully connected, the number of nodes is equal to the number of output classes M),

layer 5 for a defuzzification operation for each output.

To train the parameters of the neuro-fuzzy network, the recurrent relations

$$r_{lk}(t+1) = r_{lk}(t) - \eta \frac{\partial \varepsilon_t}{\partial r_{lk}(t)} ;$$

$$\beta^{a_{il}}(t+1) = \beta^{a_{il}}(t) - \eta \frac{\partial \varepsilon_t}{\partial \beta^{a_{il}}(t)} ; \quad \sigma^{a_{il}}(t+1) = \sigma^{a_{il}}(t) - \eta \frac{\partial \varepsilon_t}{\partial \sigma^{a_{il}}(t)} , \quad (8.13)$$

are used which minimize the criterion

$$\varepsilon_t = \frac{1}{2}(\hat{y}_t - y_t)^2,$$

where $\hat{y}_t$ and y_t are the experimental and the model outputs of the object at the t-th step of training;

$r_{lk}(t)$ are fuzzy relations at the t-th step of training;

$\beta^{a_{il}}(t)$, $\sigma^{a_{il}}(t)$ are parameters for the fuzzy terms membership functions at the t-th step of training.

η is a parameter of training [26].

The partial derivatives appearing in recurrent relations (8.13) can be obtained according to the results from Section 7.8.

8.6 Computer Simulations

Experiment 1

The aim of the experiment is to generate the system of IF-THEN rules for the target "input (x) – output (y)" model presented in Fig. 8.5.

$$y = \frac{(1.8x+0.8)(5x-1.1)(4x-2.9)(3x-2.1)(9.5x-9.5)(3x-0.05)+20}{80}. \quad (8.14)$$

The training data in the form of the interval values of input and output variable is presented in Table 8.3.

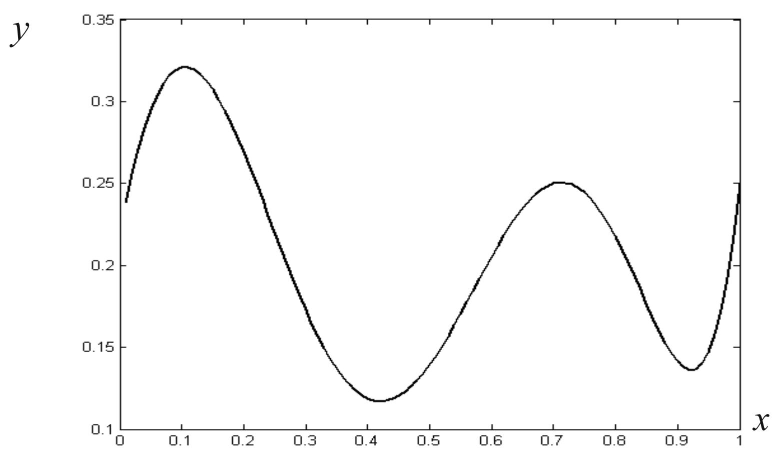

Fig. 8.5. Input-output model-generator

Table 8.3. Training data ($\hat{x}_s$, $\hat{y}_s$)

s	Input	Output
	x_1	y_1
1	[0, 0.1]	[0.22, 0.32]
2	[0.1, 0.2]	[0.32, 0.27]
3	[0.2, 0.3]	[0.27, 0.17]
4	[0.3, 0.4]	[0.17, 0.12]
5	[0.4, 0.5]	[0.12, 0.14]
6	[0.5, 0.6]	[0.14, 0.21]
7	[0.6, 0.7]	[0.21, 0.25]
8	[0.7, 0.8]	[0.25, 0.22]
9	[0.8, 0.9]	[0.22, 0.14]
10	[0.9, 1.0]	[0.14, 0.25]

The total number of fuzzy terms for the input variable is limited to six. The total number of classes for the output variable is limited to four.

The classes for output variable evaluation are formed as follows:

$$[\underline{y}, \overline{y}] = [\underbrace{0.10, 0.15}_{e_{11}})\cup[\underbrace{0.15, 0.20)}_{e_{12}}]\cup[\underbrace{0.20, 0.25}_{e_{13}}]\cup[\underbrace{0.25, 0.35}_{e_{14}}],$$

The null solution $\mathbf{R}_0$ presented in Table 8.4 together with the parameters of the knowledge matrix is obtained using the genetic algorithm.

Table 8.4. Fuzzy relational matrix (null solution)

IF input x		THEN output y			
		e_{11}	e_{12}	e_{13}	e_{14}
C_1	(0, 0.14)	0.3	0.9	0.7	0.1
C_2	(0.09, 0.14)	0.2	0.2	0.4	0.9
C_3	(0.40, 0.12)	0.8	0.3	0.3	0.1
C_4	(0.72, 0.12)	0.1	0.3	0.9	0.2
C_5	(0.92, 0.11)	0.9	0.6	0.2	0.3
C_6	(1.0, 0.07)	0.3	0.9	0.6	0.1

The obtained null solution allows us to arrange for the genetic search for the solution set of the system (8.11), where the matrices $\hat{\boldsymbol{\mu}}^A(\hat{x}_s)$ and $\hat{\boldsymbol{\mu}}^B(\hat{x}_s)$ for the training data take the following form:

$\hat{\boldsymbol{\mu}}^A =$

	$\hat{\mu}^{C_1}$	$\hat{\mu}^{C_2}$	$\hat{\mu}^{C_3}$	$\hat{\mu}^{C_4}$	$\hat{\mu}^{C_5}$	$\hat{\mu}^{C_6}$
$\hat{x}_1$	[0.67, 1.0]	[0.75, 1.0]	[0.09, 0.14]	[0.03, 0.04]	[0.01, 0.02]	[0, 0.01]
$\hat{x}_2$	[0.33, 0.67]	[0.62, 0.98]	[0.14, 0.26]	[0.04, 0.05]	0.02	0.01
$\hat{x}_3$	[0.18, 0.33]	[0.31, 0.62]	[0.26, 0.59]	[0.05, 0.08]	[0.02, 0.03]	0.01
$\hat{x}_4$	[0.11, 0.18]	[0.17, 0.31]	[0.59, 1.0]	[0.08, 0.17]	[0.03, 0.04]	0.01
$\hat{x}_5$	[0.07, 0.11]	[0.10, 0.17]	[0.59, 1.0]	[0.17, 0.30]	[0.04, 0.06]	[0.01, 0.02]
$\hat{x}_6$	[0.05, 0.07]	[0.07, 0.10]	[0.25, 0.59]	[0.30, 0.50]	[0.06, 0.11]	[0.02, 0.03]
$\hat{x}_7$	[0.04, 0.05]	[0.05, 0.07]	[0.17, 0.26]	[0.50, 0.97]	[0.11, 0.20]	[0.03, 0.05]
$\hat{x}_8$	[0.03, 0.04]	[0.04, 0.05]	[0.08, 0.17]	[0.69, 1.0]	[0.20, 0.46]	[0.05, 0.11]
$\hat{x}_9$	[0.02, 0.03]	[0.03, 0.04]	[0.05, 0.08]	[0.33, 0.69]	[0.46, 0.97]	[0.11, 0.33]
$\hat{x}_{10}$	0.02	[0.02, 0.03]	[0.04, 0.05]	[0.16, 0.33]	[0.70, 1.0]	[0.33, 1.0]

$\hat{\boldsymbol{\mu}}^B =$

	$\hat{\mu}^{E_1}$	$\hat{\mu}^{E_2}$	$\hat{\mu}^{E_3}$	$\hat{\mu}^{E_4}$
$\hat{x}_1$	0.30	[0.67, 0.90]	[0.67, 0.70]	[0.75, 0.90]
$\hat{x}_2$	0.30	[0.33, 0.67]	[0.40, 0.67]	[0.62, 0.90]
$\hat{x}_3$	[0.30, 0.59]	[0.30, 0.33]	[0.31, 0.40]	[0.31, 0.62]
$\hat{x}_4$	[0.59, 0.80]	0.30	[0.30, 0.31]	[0.17, 0.31]
$\hat{x}_5$	[0.59, 0.80]	0.30	0.30	[0.17, 0.20]
$\hat{x}_6$	[0.26, 0.59]	0.30	[0.30, 0.50]	0.20
$\hat{x}_7$	[0.17, 0.26]	0.30	[0.50, 0.90]	0.20
$\hat{x}_8$	[0.20, 0.46]	[0.30, 0.46]	[0.69, 0.90]	[0.20, 0.30]
$\hat{x}_9$	[0.46, 0.90]	[0.46, 0.60]	[0.33, 0.69]	0.30
$\hat{x}_{10}$	[0.70, 0.90]	[0.60, 0.90]	[0.33, 0.60]	0.30

The complete solution set for the fuzzy relation matrix is presented in Table 8.5, where input x is described by fuzzy terms *Low (L)*, *lower than Average (lA), Average (A)*, *higher than Average* (*hA*), *lower than High (lH), High (H)*; output y is described by fuzzy terms *higher than Low* (*hL*), *lower than Average (lA), Average (A)*, *High (H).*

Table 8.5. Fuzzy relational matrix (complete solution set)

IF input x		THEN output y			
		hL	lA	A	H
C_1	L	0.30	0.90	0.70	[0, 0.75]
C_2	lA	0.30	0.30	0.40	0.90
C_3	A	0.80	0.30	0.30	[0, 0.20]
C_4	hA	[0, 0.26]	0.30	[0.69, 0.90]	0.20
C_5	lH	0.90	0.60	$[0.33, 0.60] \cup [0, 0.60]$	0.30
C_6	H	[0, 0.70]	[0.60, 0.90]	$[0, 0.60] \cup [0.33, 0.60]$	[0, 0.30]

The obtained solution provides the approximation of the object shown in Fig. 8.6.

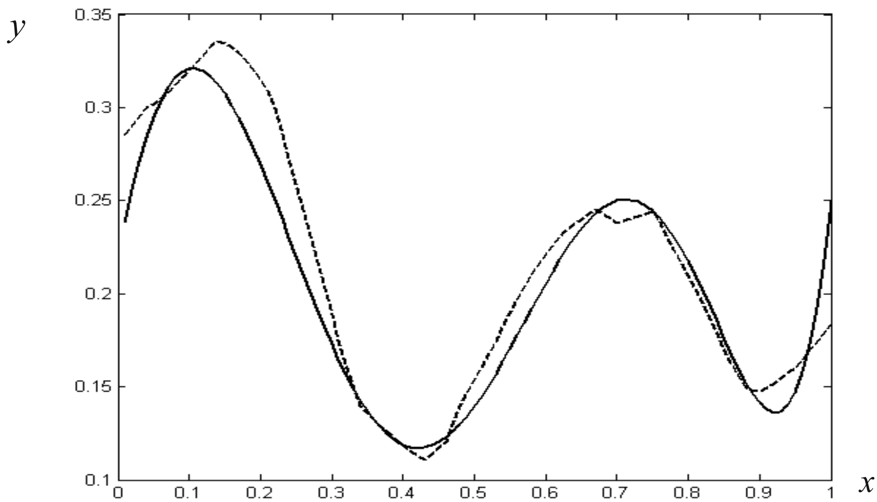

Fig. 8.6. Input-output model extracted from data

The resulting solution can be linguistically interpreted as the set of the two possible rules bases (See Table 8.6), which differ in the fuzzy terms describing output y in rule 6 with overlapping weights.

Table 8.6. System of IF-THEN rules

Rule	IF x	THEN y
1	L	lA
2	lA	H
3	A	hL
4	hA	A
5	lH	hL
6	H	hL or lA

Experiment 2

The aim of the experiment is to generate the system of IF-THEN rules for the target "two inputs (x_1, x_2) – two outputs (y_1, y_2)" model presented in Fig. 8.7:

$$y_1 = f_1(x_1, x_2) = \frac{1}{10}(2z - 0.9)\ (7z - 1)\ (17z - 19)\ (15z - 2)\,, \tag{8.15}$$

$$y_2 = f_2(x_1, x_2) = -\frac{1}{2} y_1 + 1\,,$$

where $z = \dfrac{(x_1 - 3.0)^2 + (x_2 - 2.5)^2}{40}$.

The training data in the form of the interval values of input and output variables is presented in Table 8.7.

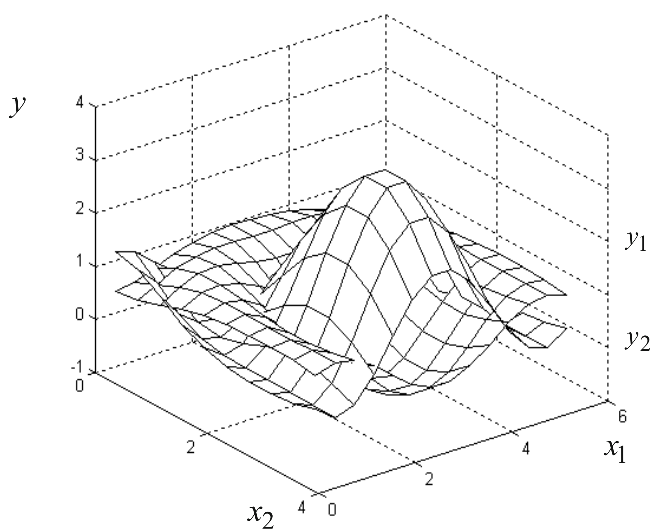

Fig. 8.7. Inputs-outputs model-generator

Table 8.7. Training data ($\hat{\mathbf{X}}_s$, $\hat{\mathbf{Y}}_s$)

s	Inputs		Outputs	
	x_1	x_2	y_1	y_2
1	[0.2, 1.2]	[0.3, 1.6]	[0, 1.0]	[0.5, 1.0]
2	[0.2, 1.2]	[1.3, 4.0]	[0, 0.8]	[0.6, 1.0]
3	[0.7, 3.0]	[0.3, 1.6]	[0, 2.3]	[-0.15, 1.0]
4	[0.7, 3.0]	[1.3, 4.0]	[0, 3.4]	[-0.7, 1.0]
5	[3.0, 5.3]	[0.3, 1.6]	[0, 2.3]	[-0.15, 1.0]
6	[3.0, 5.3]	[1.3, 4.0]	[0, 3.4]	[-0.7, 1.0]
7	[4.8, 5.8]	[0.3, 1.6]	[0, 1.0]	[0.5, 1.0]
8	[4.8, 5.8]	[1.3, 4.0]	[0, 0.8]	[0.6, 1.0]

The total number of fuzzy terms for input variables is limited to three. The total number of combinations of input terms is limited to six.

The classes for output variables evaluation are formed as follows:

$$[\underline{y}_1, \overline{y}_1] = [\underbrace{0, 0.2}_{e_{11}}) \cup [\underbrace{0.2, 1.2)}_{e_{12}}] \cup [\underbrace{1.2, 3.4}_{e_{13}}],$$

$$[\underline{y}_2, \overline{y}_2] = [\underbrace{-0.7, 0}_{e_{21}}) \cup [\underbrace{0, 1.2}_{e_{22}}].$$

The null solution $\mathbf{R}_0$ presented in Table 8.8 together with the parameters of the knowledge matrix is obtained using the genetic algorithm.

Table 8.8. Fuzzy relational matrix (null solution)

IF inputs			THEN outputs				
			y_1			y_2	
	x_1	x_2	e_{11}	e_{12}	e_{13}	e_{21}	e_{22}
C_1	(0.03, 0.72)	(0.01, 1.10)	0.15	0.78	0.24	0.52	0.48
C_2	(3.00, 1.77)	(0.02, 1.14)	0.85	0.16	0.02	0.76	0.15
C_3	(5.96, 0.71)	(0.04, 0.99)	0.10	0.92	0.27	0.50	0.43
C_4	(0.00, 0.75)	(2.99, 2.07)	0.86	0.04	0.30	0.80	0.30
C_5	(3.02, 1.80)	(2.97, 2.11)	0.21	0.11	0.10	0.15	0.97
C_6	(5.99, 0.74)	(3.02, 2.10)	0.94	0.08	0.30	0.75	0.30

The obtained null solution allows us to arrange for the genetic search for the solution set of the system (8.11), where the matrices $\hat{\boldsymbol{\mu}}^A(\hat{\mathbf{X}}_s)$ and $\hat{\boldsymbol{\mu}}^B(\hat{\mathbf{X}}_s)$ for the training data take the following form:

$\hat{\boldsymbol{\mu}}^A$ =

	$\hat{\mu}^{C_1}$	$\hat{\mu}^{C_2}$	$\hat{\mu}^{C_3}$	$\hat{\mu}^{C_4}$	$\hat{\mu}^{C_5}$	$\hat{\mu}^{C_6}$
$\hat{\mathbf{X}}_1$	[0.16, 0.74]	[0.16, 0.52]	0	[0.33, 0.61]	[0.28, 0.52]	0
$\hat{\mathbf{X}}_2$	[0.21, 0.46]	[0.21, 0.46]	0	[0.35, 0.90]	[0.28, 0.52]	0
$\hat{\mathbf{X}}_3$	[0, 0.50]	[0.16, 0.74]	0	[0, 0.50]	[0.33, 0.61]	0
$\hat{\mathbf{X}}_4$	[0, 0.46]	[0.21, 0.46]	0	[0, 0.50]	[0.37, 0.95]	0
$\hat{\mathbf{X}}_5$	0	[0.16, 0.74]	[0, 0.50]	0	[0.33, 0.61]	[0, 0.50]
$\hat{\mathbf{X}}_6$	0	[0.21, 0.46]	[0, 0.46]	0	[0.34, 0.95]	[0, 0.50]
$\hat{\mathbf{X}}_7$	0	[0.16, 0.52]	[0.16, 0.74]	0	[0.28, 0.52]	[0.33, 0.61]
$\hat{\mathbf{X}}_8$	0	[0.21, 0.46]	[0.21, 0.46]	0	[0.28, 0.52]	[0.35, 0.90]

$\hat{\boldsymbol{\mu}}^B$ =

	$\hat{\mu}^{E_1}$	$\hat{\mu}^{E_2}$	$\hat{\mu}^{E_3}$	$\hat{\mu}^{E_4}$	$\hat{\mu}^{E_5}$
$\hat{\mathbf{X}}_1$	[0.33, 0.61]	[0.16, 0.74]	[0.30, 0.52]	[0.33, 0.61]	[0.30, 0.52]
$\hat{\mathbf{X}}_2$	[0.35, 0.86]	[0.21, 0.46]	[0.30, 0.52]	[0.35, 0.80]	[0.30, 0.52]
$\hat{\mathbf{X}}_3$	[0.21, 0.74]	[0.16, 0.50]	[0.33, 0.61]	[0.16, 0.74]	[0.33, 0.61]
$\hat{\mathbf{X}}_4$	[0.21, 0.46]	[0.16, 0.46]	[0.37, 0.95]	[0.21, 0.50]	[0.37, 0.95]
$\hat{\mathbf{X}}_5$	[0.21, 0.74]	[0.16, 0.50]	[0.33, 0.61]	[0.16, 0.74]	[0.33, 0.61]
$\hat{\mathbf{X}}_6$	[0.21, 0.50]	[0.16, 0.46]	[0.34, 0.95]	[0.21, 0.50]	[0.34, 0.95]
$\hat{\mathbf{X}}_7$	[0.33, 0.61]	[0.16, 0.74]	[0.30, 0.52]	[0.33, 0.61]	[0.30, 0.52]
$\hat{\mathbf{X}}_8$	[0.35, 0.90]	[0.21, 0.46]	[0.30, 0.52]	[0.35, 0.75]	[0.30, 0.52]

The complete solution set for the fuzzy relation matrix is presented in Table 8.9, where input x_1 is described by fuzzy terms *Low (L)*, *Average (A)*, *High (H)*; input

x_2 is described by fuzzy terms *Low (L)*, *High (H)*; output y_1 is described by fuzzy terms *higher than Low* (*hL*), *lower than Average (lA)*, *High (H)*; output y_2 is described by fuzzy terms *Low (L)*, *lower than Average (lA).*

Table 8.9. Fuzzy relational matrix (complete solution set)

IF inputs			THEN outputs				
			y_1			y_2	
	x_1	x_2	*hL*	*lA*	*H*	*lA*	*L*
C_1	*L*	*L*	[0, 0.21]	[0.74, 1.0]	[0, 0.30]	[0.33, 0.61]	[0, 0.52]
C_2	*A*	*L*	[0.74, 1.0]	[0, 0.16] ⋃0.16	[0, 0.30]	[0.74, 1.0]	[0, 0.30]
C_3	*H*	*L*	[0, 0.21]	[0.74, 1.0]	[0, 0.30]	[0.33, 0.61]	[0, 0.52]
C_4	*L*	*H*	0.86	[0, 0.16]	0.30	0.80	0.30
C_5	*A*	*H*	0.21	0.16⋃ [0, 0.16]	[0.95, 1.0]	[0, 0.16]	[0.97, 1.0]
C_6	*H*	*H*	[0.90, 1.0]	[0, 0.16]	0.30	0.75	0.30

The obtained solution provides the approximation of the object shown in Fig. 8.8.

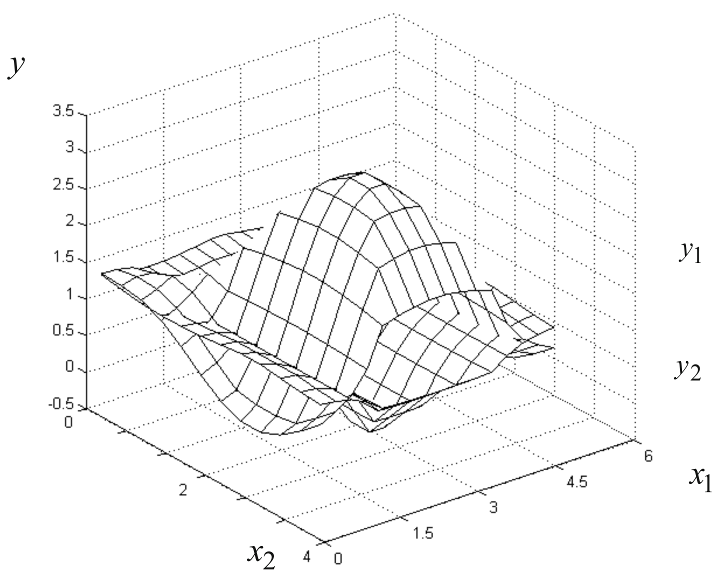

Fig. 8.8. Inputs-outputs model extracted from data

The resulting solution can be linguistically interpreted as the set of the four possible rules bases (See Table 8.10), which differ in the fuzzy terms describing output y_2 in rule 1 and rule 3 with overlapping weights.

Table 8.10. System of IF-THEN rules

Rule	IF inputs		THEN outputs	
	x_1	x_2	y_1	y_2
1	*L*	*L*	*lA*	*lA* or *L*
2	*A*	*L*	*hL*	*lA*
3	*H*	*L*	*lA*	*lA* or *L*
4	*L*	*H*	*hL*	*lA*
5	*A*	*H*	*H*	*L*
6	*H*	*H*	*hL*	*lA*

8.7 Example 7: Fuzzy Relations Extraction for Heart Diseases Diagnosis

The aim is to generate the system of IF-THEN rules for diagnosis of heart diseases. Input parameters are (variation ranges are indicated in parentheses):

x_1 – aortic valve size (0.75 – 2.5 cm^2);
x_2 – mitral valve size (1 – 2 cm^2);
x_3 – tricuspid valve size (0.5 – 2.7 cm^2);
x_4 – lung artery pressure (65 – 100 mm Hg).

Output parameters are:

y_1 – left ventricle size (11–14 mm);
y_2 – left auricle size (40–70 mm);
y_3 – right ventricle size (36–41 mm);
y_4 – right auricle size (38–45 mm).

The training data obtained in the Vinnitsa Clinic of Cardiology is represented in Table 8.11 [27].

In current clinical practice, the number of combined heart diseases (aortic-mitral, mitral-tricuspid, mitral with lung hypertension etc.) is limited to six ($\overline{N} = 6$).

The classes for output variables evaluation are formed as follows:

$$[\underline{y}_1, \overline{y}_1] = [\underbrace{11, 12}_{e_{11}}) \cup [\underbrace{13, 14}_{e_{12}}], \quad [\underline{y}_2, \overline{y}_2] = [\underbrace{41, 50}_{e_{21}}) \cup [\underbrace{50, 70}_{e_{22}}],$$

$$[\underline{y}_3, \overline{y}_3] = [\underbrace{36, 38}_{e_{31}}) \cup [\underbrace{38, 41}_{e_{32}}], \quad [\underline{y}_4, \overline{y}_4] = [\underbrace{38, 40}_{e_{41}}) \cup [\underbrace{40, 45}_{e_{42}}].$$

Table 8.11. Training data

s	Input parameters				Output parameters			
	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4
1	0.75-2	2	2	65-69	12-14	41-44	36	38
2	2.0-2.5	2	2	65-69	11-13	40-41	36	38
3	2.0-2.5	1-2	2	71-80	11	40	38-40	40-45
4	2.0-2.5	2	2	71-80	11	50-70	37-38	38-40
5	2.0-2.5	2	0.5-2	72-90	11-12	60-70	40-41	40-45
6	2.0-2.5	1-2	2-2.7	80-90	11-12	40	40-41	38
7	2.0-2.5	2	2	80-100	11	50-60	36	38
8	2.0-2.5	1-2	2-2.7	80-100	11	40	40-41	38-40

In clinical practice these classes correspond to the types of diagnoses e_{j1} *low inflation* and e_{j2} *dilation* (*hypertrophy*) of heart sections $y_1 \div y_4$. The aim of the diagnosis is to translate a set of specific parameters $x_1 \div x_4$ into decision e_{jp} for each output $y_1 \div y_4$.

The null solution $\mathbf{R}_0$ presented in Table 8.12 together with the parameters of the knowledge matrix is obtained using the genetic algorithm.

Table 8.12. Fuzzy relational matrix (null solution)

IF inputs				THEN outputs							
x_1	x_2	x_3	x_4	y_1		y_2		y_3		y_4	
				e_{11}	e_{12}	e_{21}	e_{22}	e_{31}	e_{32}	e_{41}	e_{42}
(0.75, 1.30)	(2.00, 0.63)	(2.35, 0.92)	(65.54, 8.81)	0.21	0.95	0.76	0.16	0.95	0.10	0.90	0.10
(2.50, 0.95)	(2.00, 0.65)	(2.44, 1.15)	(64.90, 9.57)	0.40	0.63	0.93	0.15	0.90	0.12	0.85	0.06
(2.52, 1.04)	(1.00, 0.82)	(2.32, 0.88)	(69.32, 10.23)	0.92	0.20	0.86	0.08	0.31	0.75	0.14	0.82
(2.55, 0.98)	(2.00, 0.72)	(2.36, 0.90)	(95.07, 21.94)	0.90	0.15	0.24	0.59	0.55	0.02	0.64	0.26
(2.51, 1.10)	(1.92, 0.75)	(0.50, 0.90)	(100.48, 26.14)	0.85	0.18	0.12	0.95	0.10	0.90	0.21	0.93
(2.55, 0.96)	(1.00, 0.94)	(2.30, 1.20)	(95.24, 22.46)	0.80	0.37	0.76	0.31	0.22	0.88	0.75	0.14

The obtained null solution allows us to arrange for the genetic search for the solution set of the system (8.11), where the matrices $\hat{\boldsymbol{\mu}}^A(\hat{\mathbf{X}}_s)$ and $\hat{\boldsymbol{\mu}}^B(\hat{\mathbf{X}}_s)$ for the training data take the following form:

$\hat{\boldsymbol{\mu}}^A =$

	$\hat{\mu}^{C_1}$	$\hat{\mu}^{C_2}$	$\hat{\mu}^{C_3}$	$\hat{\mu}^{C_4}$	$\hat{\mu}^{C_5}$	$\hat{\mu}^{C_6}$
$\hat{\mathbf{X}}_1$	[0.62, 0.94]	[0.32, 0.74]	[0.30, 0.40]	[0.09, 0.31]	[0.07, 0.35]	[0.08, 0.29]
$\hat{\mathbf{X}}_2$	[0.35, 0.62]	[0.74, 0.90]	0.40	[0.09, 0.31]	[0.07, 0.35]	[0.08, 0.29]
$\hat{\mathbf{X}}_3$	[0.21, 0.54]	[0.2, 0.52]	[0.22, 0.56]	[0.31, 0.72]	0.35	[0.29, 0.77]
$\hat{\mathbf{X}}_4$	[0.21, 0.54]	[0.2, 0.52]	[0.22, 0.40]	[0.31, 0.72]	0.35	[0.29, 0.41]
$\hat{\mathbf{X}}_5$	[0.1, 0.54]	[0.08, 0.52]	[0.07, 0.56]	[0.31, 0.86]	[0.35, 0.89]	[0.29, 0.41]
$\hat{\mathbf{X}}_6$	[0.1, 0.21]	[0.08, 0.21]	[0.07, 0.22]	[0.72, 0.86]	[0, 0.35]	[0.41, 0.85]
$\hat{\mathbf{X}}_7$	[0, 0.21]	[0, 0.21]	[0, 0.22]	[0.72, 0.90]	0.35	0.41
$\hat{\mathbf{X}}_8$	[0, 0.21]	[0, 0.21]	[0, 0.22]	[0.72, 0.90]	[0, 0.35]	[0.41, 1.0]

$\hat{\boldsymbol{\mu}}^B =$

	$\hat{\mu}^{E_1}$	$\hat{\mu}^{E_2}$	$\hat{\mu}^{E_3}$	$\hat{\mu}^{E_4}$	$\hat{\mu}^{E_5}$	$\hat{\mu}^{E_6}$	$\hat{\mu}^{E_7}$	$\hat{\mu}^{E_8}$
$\hat{\mathbf{X}}_1$	[0.32, 0.40]	[0.62, 0.94]	[0.62, 0.76]	[0.16, 0.35]	[0.62, 0.94]	[0.30, 0.40]	[0.62, 0.90]	[0.30, 0.40]
$\hat{\mathbf{X}}_2$	0.40	0.63	[0.74, 0.90]	[0.16, 0.35]	[0.74, 0.90]	0.40	[0.74, 0.85]	0.40
$\hat{\mathbf{X}}_3$	[0.35, 0.77]	[0.21, 0.54]	[0.29, 0.76]	[0.35, 0.59]	[0.31, 0.55]	[0.35, 0.77]	[0.31, 0.75]	[0.35, 0.56]
$\hat{\mathbf{X}}_4$	[0.35, 0.72]	[0.21, 0.54]	[0.29, 0.54]	[0.35, 0.59]	[0.31, 0.55]	[0.35, 0.41]	[0.31, 0.64]	[0.35, 0.4]
$\hat{\mathbf{X}}_5$	[0.35, 0.89]	[0.10, 0.54]	[0.29, 0.56]	[0.35, 0.89]	[0.31, 0.55]	[0.35, 0.89]	[0.31, 0.64]	[0.35, 0.89]
$\hat{\mathbf{X}}_6$	[0.72, 0.86]	0.37	[0.41, 0.76]	0.59	0.55	[0.41, 0.85]	[0.64, 0.75]	[0.26, 0.35]
$\hat{\mathbf{X}}_7$	[0.72, 0.90]	0.37	0.41	0.59	0.55	0.41	0.64	0.35
$\hat{\mathbf{X}}_8$	[0.72, 0.90]	0.37	[0.41, 0.76]	0.59	0.55	[0.41, 0.88]	[0.64, 0.75]	[0.26, 0.35]

The complete solution set for the fuzzy relation matrix is presented in Table 8.13, where, according to current clinical practice, the valve sizes $x_1 \div x_3$ are described by fuzzy terms *stenosis* (*S*) and *insufficiency* (*I*); pressure x_4 is described by fuzzy terms *normal* (*N*) and *lung hypertension* (*H*).

The obtained solution provides the results of diagnosis presented in Table 8.14 for 57 patients. Heart diseases diagnosis obtained an average accuracy rate of 90% after 10000 iterations of the genetic algorithm (100 min on Intel Core 2 Duo P7350 2.0 GHz).

The resulting solution can be linguistically interpreted as the set of the four possible rules bases (See Table 8.15), which differ in the fuzzy terms describing outputs y_1 and y_3 in rule 3 with overlapping weights.

Table 8.13. Fuzzy relational matrix (complete solution set)

IF inputs				THEN outputs							
x_1	x_2	x_3	x_4	y_1		y_2		y_3		y_4	
				L	*D*	*L*	*D*	*L*	*D*	*L*	*D*
S	*I*	*I*	*N*	[0, 0.40]	[0.94, 1.0]	0.76	0.16	[0.94, 1.0]	[0, 0.30]	0.90	[0, 0.30]
I	*I*	*I*	*N*	0.40	0.63	[0.90, 1.0]	[0, 0.35]	[0.90, 1.0]	[0, 0.30]	0.85	[0, 0.30]
I	*S*	*I*	*N*	[0.40, 1.0]	[0, 0.54]	[0.56, 1.0]	[0, 0.35]	[0, 0.55]	[0.40, 1.0]	[0, 0.31]	[0.56, 1.0]
I	*I*	*I*	*H*	[0.90, 1.0]	[0, 0.37] ∪0.37	[0, 0.41]	0.59	0.55	[0, 0.41]	0.64	0.26 ∪ [0, 0.26]
I	*I*	*S*	*H*	[0.89, 1.0]	[0, 0.54]	[0, 0.56]	[0.89, 1.0]	[0, 0.55]	[0.89, 1.0]	[0, 0.31]	[0.89, 1.0]
I	*S*	*I*	*H*	[0.77, 0.90]	0.37 ∪ [0, 0.37]	0.76	[0, 0.59]	[0, 0.55]	[0.85, 1.0]	0.75	[0, 0.26] ∪0.26

Table 8.14. Genetic algorithm efficiency characteristics

Output parameter	Type of diagnosis	Number of cases	Probability of the correct diagnosis
y_1	e_{11}	20	17 / 20 = 0.85
	e_{12}	37	34 / 37 = 0.92
y_2	e_{21}	26	23 / 26 = 0.88
	e_{22}	31	28 / 31 = 0.90
y_3	e_{31}	28	25 / 28 = 0.89
	e_{32}	29	27 / 29 = 0.93
y_4	e_{41}	40	37 / 40 = 0.92
	e_{42}	17	15 / 17 = 0.88

Table 8.15. System of IF-THEN rules

Rule	IF inputs				THEN outputs			
	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4
1	*S*	*I*	*I*	*N*	*D*	*L*	*L*	*L*
2	*I*	*I*	*I*	*N*	*D*	*L*	*L*	*L*
3	*I*	*S*	*I*	*N*	*L* or *D*	*L*	*L or D*	*D*
4	*I*	*I*	*I*	*H*	*L*	*D*	*L*	*L*
5	*I*	*I*	*S*	*H*	*L*	*D*	*D*	*D*
6	*I*	*S*	*I*	*H*	*L*	*L*	*D*	*L*

References

1. Di Nola, A., Sessa, S., Pedrycz, W., Sancez, E.: Fuzzy Relational Equations and Their Applications to Knowledge Engineering, p. 430. Kluwer, Dordrecht (1989)
2. Peeva, K., Kyosev, Y.: Fuzzy Relational Calculus Theory, Applications and Software. World Scientific Publishing Company, Singapore (2004) CD-ROM, `http://mathworks.net`
3. Higashi, M., Klir, G.J.: Identification of fuzzy relation systems. IEEE Trans. Syst., Man, and Cybernetics 14, 349–355 (1984)
4. Pedrycz, W.: An identification algorithm in fuzzy relational systems. Fuzzy Sets Syst. 13, 153–167 (1984)
5. Pedrycz, W.: Fuzzy models and relational equations. Mathematical Modeling 9(6), 427–434 (1987)
6. Zadeh, L.: The Concept of Linguistic Variable and It's Application to Approximate Decision Making. Mir, Moscow (1976)
7. Yager, R.R., Filev, D.P.: Essentials of Fuzzy Modeling and Control. John Willey & Sons, New York (1994)
8. Pedrycz, W., Gomide, F.: An introduction to fuzzy sets / Analysis and Design. – A Bradford Book, p. 465. The MIT Press (1998)
9. Bourke, M.M., Grant Fisher, D.: Calculation and application of a minimum fuzzy relational matrix. Fuzzy Sets and Syst. 74(2), 225–236 (1995)
10. Bourke, M.M., Grant Fisher, D.: Identification algorithms for fuzzy relational matrices, Part 1: Non-optimizing algorithms. Fuzzy Sets and Systems 109(3), 305–320 (2000)
11. Yi, S.Y., Chung, M.J.: Identification of fuzzy relational model and its application to control. Fuzzy Sets and Systems 59(1), 25–33 (1993)
12. Branco, P.J., Dente, J.A.: A fuzzy relational identification algorithm and its application to predict the behaviour of a motor drive system. Fuzzy Sets and Systems 109(3), 343–354 (2000)
13. Wong, C.H., Shah, S.L., Fisher, D.G.: Fuzzy relational predictive identification. Fuzzy Sets and Systems 113(3), 417–426 (2000)
14. Bourke, M.M., Fisher, D.G.: Identification algorithms for fuzzy relational matrices. Part 2: Optimizing algorithms. Fuzzy Sets and Systems 109(3), 321–341 (2000)
15. Blanco, A., Delgado, M., Requena, I.: Identification of fuzzy relational equations by fuzzy neural networks. Fuzzy Sets and Systems 71(2), 215–226 (1995)
16. Chakraborty, D., Pal, N.R.: Integrated Feature Analysis and Fuzzy Rule-Based System Identification in a Neuro-Fuzzy Paradigm. IEEE Transactions on Systems, Man and Cybernetics – Part B: Cybernetics 31(3), 391–400 (2001)
17. Ciaramella, A., Tagliaferri, R., Pedrycz, W., Di Nola, A.: Fuzzy relational neural network. International Journal of Approximate Reasoning 41(2), 146–163 (2006)
18. Rotshtein, A., Posner, M., Rakytyanska, H.: Cause and Effect Analysis by Fuzzy Relational Equations and a Genetic Algorithm. Reliability Engineering & System Safety 91(9), 1095–1101 (2006)
19. Rotshtein, A., Rakytyanska, H.: Fuzzy Relation-based Diagnosis. Automation and Remote Control 68(12), 2198–2213 (2007)
20. Rotshtein, A., Rakytyanska, H.: Diagnosis Problem Solving using Fuzzy Relations. IEEE Transactions on Fuzzy Systems 16(3), 664–675 (2008)
21. Rotshtein, A., Rakytyanska, H.: Adaptive Diagnostic System based on Fuzzy Relations. Cybernetics and Systems Analysis 45(4), 623–637 (2009)

22. Rotshtein, A., Mityushkin, Y.: Extraction of Fuzzy Knowledge Bases from Experimental Data by Genetic Algorithms. Cybernetics and Systems Analysis 37(4), 501–508 (2001)
23. Rotshtein, A.P., Rakytyanska, H.B.: Fuzzy Genetic Object Identification: Multiple Inputs/Multiple Outputs Case. In: Hippe, Z.S., Kulikowski, J.L., Mroczek, T. (eds.) Human - Computer Systems Interaction. AISC, vol. 99, pp. 375–394. Springer, Heidelberg (2012)
24. Gen, M., Cheng, R.: Genetic Algorithms and Engineering Design, p. 352. John Wiley & Sons, New York (1997)
25. Cordon, O., Herrera, F., Hoffmann, F., Magdalena, L.: Genetic Fuzzy Systems. Evolutionary Tuning and Learning of Fuzzy Knowledge Bases. World Scientific, New York (2001)
26. Tsypkin, Y.Z.: Information Theory of Identification, p. 320. Nauka, Moscow (1984) (in Russian)
27. Frid, A., Grines, C.: Cardiology. Practice, Moscow (1996) (in Russian)