Chapter 7
Inverse Inference Based on Fuzzy Rules

The wide class of the problems, arising from engineering, medicine, economics and other domains, belongs to the class of inverse problems [1]. The typical representative of the inverse problem is the problem of medical and technical diagnosis, which amounts to the restoration and the identification of the unknown causes of the disease or the failure through the observed effects, i.e. the symptoms or the external signs of the failure. The diagnosis problem, which is based on a cause and effect analysis and abductive reasoning can be formally described by neural networks [2] or Bayesian networks [3, 4]. In the cases, when domain experts are involved in developing cause-effect connections, the dependency between unobserved and observed parameters can be modelled using the means of fuzzy sets theory [5, 6]: fuzzy relations and fuzzy IF-THEN rules. Fuzzy relational calculus plays the central role as a uniform platform for inverse problem resolution on various fuzzy approximation operators [7, 8]. In the case of a multiple variable linguistic model, the cause-effect dependency is extended to the multidimensional fuzzy relational structure [6], and the problem of inputs restoration and identification amounts to solving a system of multidimensional fuzzy relational equations [9, 10]. Fuzzy IF-THEN rules enable us to consider complex combinations in cause-effect connections as being simpler and more natural, which are difficult to model with fuzzy relations. In rule-based models, an inputs-outputs connection is described by a hierarchical system of simplified fuzzy relational equations with max-min and dual min-max laws of composition [11 – 13].

In works [14 – 16], an expert system using a genetic algorithm [17] as a tool to solve the diagnosis problem was proposed. The diagnosis problem based on a cause and effect analysis was formally described by the single input single output fuzzy relation approximator [18 – 20]. This chapter proposes an approach for inverse problem solution based on the description of the interconnection between unobserved and observed parameters of an object (causes and effects) with the help of fuzzy IF-THEN rules. The problem consists of not only solving a system of fuzzy logical equations, which correspond to IF-THEN rules, but also in selection of such forms of the fuzzy terms membership functions and such weights of the fuzzy IF–THEN rules, which provide maximal proximity between model and real results of diagnosis [21].

The essence of the proposed approach consists of formulating and solving the optimization problems, which, on the one hand, find the roots of fuzzy logical equations, corresponding to IF-THEN rules, and, on the other hand, tune the fuzzy

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model using the readily available experimental data. The hybrid genetic and neuro approach is proposed for solving the formulated optimization problems.

This chapter is written on the basis of [11 – 13].

7.1 Diagnostic Approximator Based on Fuzzy Rules

The diagnosis object is treated as a black box with n inputs and m outputs (Fig. 7.1). Outputs of the object are associated with the observed effects (symptoms). Inputs correspond to the causes of the observed effects (diagnoses). The problem of diagnosis consists of restoration and identification of the causes (inputs) through the observed effects (outputs). Inputs and outputs can be considered as linguistic variables given on the corresponding universal sets. Fuzzy terms are used for these linguistic variables evaluation.

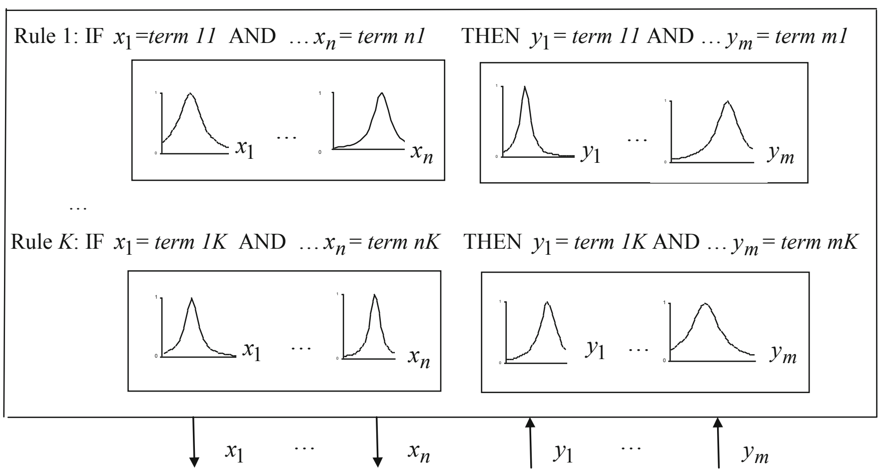

Fig. 7.1. Object of diagnosis

We shall denote the following:

$\{x_1, x_2, ..., x_n\}$ is the set of input parameters, $x_i \in [\underline{x}_i, \overline{x}_i]$, $i = \overline{1, n}$;

$\{y_1, y_2, ..., y_m\}$ is the set of output parameters, $y_j \in [\underline{y}_j, \overline{y}_j]$, $j = \overline{1, m}$;

$\{c_{i1}, c_{i2}, ..., c_{ik_i}\}$ is the set of linguistic terms for parameter x_i evaluation, $i = \overline{1, n}$;

$\{e_{j1}, e_{j2}, ..., e_{jq_j}\}$ is the set of linguistic terms for parameter y_j evaluation, $j = \overline{1, m}$.

Each term-assessment is described with the help of a fuzzy set:

$$c_{il} = \{(x_i, \mu^{c_{il}}(x_i))\}, \quad i = \overline{1,n}, \; l = \overline{1,k_i};$$

$$e_{jp} = \{(y_j, \mu^{e_{jp}}(y_j))\}, \quad j = \overline{1,m}, \; p = \overline{1,q_j}.$$

where $\mu^{c_{il}}(x_i)$ is a membership function of variable x_i to the term-assessment c_{il}, $i = \overline{1,n}$, $l = \overline{1,k_i}$;

$\mu^{e_{jp}}(y_j)$ is a membership function of variable y_j to the term-assessment e_{jp}, $j = \overline{1,m}$, $p = \overline{1,q_j}$.

We shall redenote the set of input and output terms-assessments in the following way:

$\{C_1, C_2, ..., C_N\} = \{c_{11}, c_{12}, ..., c_{1k_1}, ..., c_{n1}, c_{n2}, ..., c_{nk_n}\}$ is the set of terms for input parameters evaluation, where $N = k_1 + k_2 + ... + k_n$;

$\{E_1, E_2, ..., E_M\} = \{e_{11}, e_{12}, ..., e_{1q_1}, ..., e_{m1}, e_{m2}, ..., e_{mq_m}\}$ is the set of terms for output parameters evaluation, where $M = q_1 + q_2 + ... + q_m$.

Set $\{C_I, I = \overline{1,N}\}$ is called fuzzy causes (diagnoses), and set $\{E_J, J = \overline{1,M}\}$ is called fuzzy effects (symptoms).

Causes - effects interconnection can be represented using the expert matrix of knowledge (Table 7.1).

Table 7.1. Fuzzy knowledge base

№ rule	Inputs				Outputs						
	x_1	x_2	…	x_n	y_1	Weight	y_2	Weight		y_m	Weight
1	a_{11}	a_{21}	…	a_{n1}	b_{11}	w_{11}	b_{21}	w_{21}	…	b_{m1}	w_{m1}
2	a_{12}	a_{22}	…	a_{n2}	b_{12}	w_{12}	b_{22}	w_{22}	…	b_{m2}	w_{m2}
…	…	…	…	…	…	…	…	…	…	…	…
K	a_{1K}	a_{2K}	…	a_{nK}	b_{1K}	w_{1K}	b_{2K}	w_{2K}	…	b_{mK}	w_{mK}

The fuzzy knowledge base below corresponds to this matrix:

Rule l: IF $x_1 = a_{1l}$ AND $x_2 = a_{2l}$ … AND $x_n = a_{nl}$

THEN $y_1 = b_{1l}$ (with weight w_{1l})

AND $y_2 = b_{2l}$ (with weight w_{2l}) …

AND $y_m = b_{ml}$ (with weight w_{ml}), $l = \overline{1,K}$; (7.1) where a_{il} is a fuzzy term for variable x_i evaluation in the rule with number l;

b_{jl} is a fuzzy term for variable y_j evaluation in the rule with number l;

w_{jl} is a rule weight, i.e. a number in the range [0, 1], characterizing the measure of confidence of an expert relative to the statement with number jl;

K is the number of fuzzy rules.

The problem of diagnosis is set in the following way: it is necessary to restore and identify the values of the input parameters $(x_1^*, x_2^*, ..., x_n^*)$ through the values of the observed output parameters $(y_1^*, y_2^*, ..., y_m^*)$.

7.2 Interconnection of Fuzzy Rules and Relations

This fuzzy rules base is modelled by the fuzzy relational matrices presented in Table 7.2. These relational matrices can be translated as a set of fuzzy IF-THEN rules

IF $\mathbf{X} = A_L$

(i.e., $x_1 = C_1$ (with weight v_{1L})

AND … $x_i = C_I$ (with weight v_{IL}) …

AND $x_n = C_N$ (with weight v_{NL}))

THEN $y_j = E_J$ (with weight r_{LJ}) (7.2)

Here A_L is the combination of input terms in rule L, $L = \overline{1, K}$.

Table 7.2. Fuzzy relational matrices

	IF inputs											THEN outputs										
	x_1			…	x_i			…	x_n			y_1			…	y_j			…	y_m		
	c_{11}	…	c_{1k_1}	…	c_{i1}	…	c_{ik_i}	…	c_{n1}	…	c_{nk_n}	e_{11}	…	e_{1q_1}	…	e_{j1}	…	e_{jq_j}	…	e_{m1}	…	e_{mq_m}
	C_1	…			C_I	…					C_N	E_1	…			E_J	…					E_M
A_1	v_{11}	…			v_{I1}	…					v_{N1}	r_{11}	…			r_{1J}	…					r_{1M}
…	…	…			…	…					…	…	…			…	…					…
A_L	v_{1L}	…			v_{IL}	…					v_{NL}	r_{L1}	…			r_{LJ}	…					r_{LM}
…	…	…			…	…					…	…	…			…	…					…
A_K	v_{1K}	…			v_{IK}	…					v_{NK}	r_{K1}	…			r_{KJ}	…					r_{KM}

"Causes – rules – effects" interconnection is given by the hierarchical system of relational matrices $\mathbf{V} \subseteq C_I \times A_L = [v_{IL}, I = \overline{1,N}, L = \overline{1,K}]$ and $\mathbf{R} \subseteq A_L \times E_J = [r_{LJ}, L = \overline{1,K}, J = \overline{1,M}]$. An element of binary matrix **V** is the weight of term $v_{IL} \in \{0,1\}$, where $v_{IL} = 1(0)$ if term C_I is present (absent) in the causes combination A_L. An element of fuzzy relational matrix **R** is the weight of rule $r_{LJ} \in [0, 1]$, characterizing the degree to which causes combination A_L influences upon the rise of effect E_J.

Given the matrices **R** and **V**, the "causes-effects" dependency can be described with the help of Zadeh's compositional rule of inference [5]

$$\boldsymbol{\mu}^E = \boldsymbol{\mu}^A \circ \mathbf{R}, \tag{7.3}$$

where

$$\boldsymbol{\mu}^A = \boldsymbol{\mu}^C \bullet \overline{\mathbf{V}}. \tag{7.4}$$

Here $\overline{\mathbf{V}}$ is the complement of the matrix of terms weights $\mathbf{V}$;

$\boldsymbol{\mu}^C = (\mu^{C_1}, \mu^{C_2}, ..., \mu^{C_N})$ is the fuzzy causes vector with elements $\mu^{C_I} \in [0, 1]$, interpreted as some significance measures of C_I causes;

$\boldsymbol{\mu}^E = (\mu^{E_1}, \mu^{E_2}, ..., \mu^{E_M})$ is the fuzzy effects vector with elements $\mu^{E_J} \in [0, 1]$, interpreted as some significance measures of E_J effects;

$\boldsymbol{\mu}^A = (\mu^{A_1}, \mu^{A_2}, ..., \mu^{A_K})$ is the fuzzy causes combinations vector with elements $\mu^{A_L} \in [0, 1]$, interpreted as some significance measures of A_L causes combinations;

- ($\circ$) is the operation of *min-max* (*max-min*) composition [5].

Finding vector $\boldsymbol{\mu}^C$ amounts to the solution of the hierarchical system of simplified fuzzy relational equations with *max-min* and dual *min-max* laws of composition

$$\mu^{E_1} = (\mu^{A_1} \wedge r_{11}) \vee (\mu^{A_2} \wedge r_{21}) \vee ... \vee (\mu^{A_K} \wedge r_{K1})$$

$$\mu^{E_2} = (\mu^{A_1} \wedge r_{12}) \vee (\mu^{A_2} \wedge r_{22}) \vee ... \vee (\mu^{A_K} \wedge r_{K2})$$

$$\dots$$

$$\mu^{E_M} = (\mu^{A_1} \wedge r_{1M}) \vee (\mu^{A_2} \wedge r_{2M}) \vee ... \vee (\mu^{A_K} \wedge r_{KM}), \tag{7.5}$$

where

$$\mu^{A_1} = (\mu^{C_1} \vee \bar{v}_{11}) \wedge (\mu^{C_2} \vee \bar{v}_{21}) \wedge ... \wedge (\mu^{C_N} \vee \bar{v}_{N1})$$

$$\mu^{A_2} = (\mu^{C_1} \vee \bar{v}_{12}) \wedge (\mu^{C_2} \vee \bar{v}_{22}) \wedge ... \wedge (\mu^{C_N} \vee \bar{v}_{N2})$$

$$\dots$$

$$\mu^{A_K} = (\mu^{C_1} \vee \bar{v}_{1K}) \wedge (\mu^{C_2} \vee \bar{v}_{2K}) \wedge ... \wedge (\mu^{C_N} \vee \bar{v}_{NK}), \tag{7.6}$$

which is derived from relations (7.3) and (7.4).

Since the operations $\vee$ and $\wedge$ are replaced by *max* and *min* in fuzzy set theory [5], systems (7.5) and (7.6) can be rearranged as:

$$\mu^{E_J} = \max_{L=\overline{1,K}}(\min(\mu^{A_L}, r_{LJ})),\ J = \overline{1,M}\ , \tag{7.7}$$

where

$$\mu^{A_L} = \min_{I=\overline{1,N}}(\max(\mu^{C_I}, \bar{v}_{IL})),\ L = \overline{1,K} \tag{7.8}$$

or

$$\mu^{E_J} = \max_{L=\overline{1,K}}\left(\min\left(\min_{I=\overline{1,N}}(\max(\mu^{C_I}, \bar{v}_{IL})), r_{LJ} \right)\right),\ J = \overline{1,M}\ . \tag{7.9}$$

To translate the specific values of the input and output variables into the measures of the causes and effects significances it is necessary to define a membership function of linguistic terms C_I and E_J, $I = \overline{1,N}$, $J = \overline{1,M}$, used in the fuzzy rules (7.1). We use a bell-shaped membership function model of variable u to arbitrary term T in the form:

$$\mu^T(u) = \frac{1}{1+\left(\frac{u-\beta}{\sigma}\right)^2}, \tag{7.10}$$

where β is a coordinate of function maximum, $\mu^T(\beta)=1$; σ is a parameter of concentration-extension.

Correlations (7.9), (7.10) define the generalized fuzzy model of diagnosis as follows:

$$\mathbf{\mu}^E(\mathbf{Y}, \mathbf{B}_E, \mathbf{\Omega}_E) = F_Y(\mathbf{X}, \mathbf{R}, \mathbf{B}_C, \mathbf{\Omega}_C), \tag{7.11}$$

where $\mathbf{R} = (r_{11}, r_{12}, ..., r_{1K}, ..., r_{m1}, r_{m2}, ..., r_{mK})$ is the vector of rules weights;

$\mathbf{B}_C = (\beta^{C_1}, \beta^{C_2}, ..., \beta^{C_N})$ and $\mathbf{\Omega}_C = (\sigma^{C_1}, \sigma^{C_2}, ..., \sigma^{C_N})$ are the vectors of β - and σ - parameters for fuzzy causes C_1, C_2,..., C_N membership functions;

$\mathbf{B}_E = (\beta^{E_1}, \beta^{E_2}, ..., \beta^{E_M})$ and $\mathbf{\Omega}_E = (\sigma^{E_1}, \sigma^{E_2}, ..., \sigma^{E_M})$ are the vectors of β - and σ - parameters for fuzzy effects E_1, E_2,..., E_M membership functions;

F_Y is the operator of inputs-outputs connection, corresponding to formulae (7.9), (7.10).

7.3 Optimization Problem for Fuzzy Rules Based Inverse Inference

Following the approach, proposed in [14 – 16], the problem of solving fuzzy logic equations (7.9) is formulated as follows. Fuzzy causes vector $\mathbf{\mu}^C = (\mu^{C_1}, \mu^{C_2}, ..., \mu^{C_N})$

should be found which satisfies the constraints $\mu^{C_I} \in [0,1]$, $I = \overline{1,N}$, and also provides the least distance between observed and model fuzzy effects vectors:

$$F = \sum_{J=1}^{M} \left[\mu^{E_J} - \max_{L=1,K} \left(\min \left(\min_{I=1,N} (\max(\mu^{C_I}, \bar{v}_{IL})), r_{LJ} \right) \right) \right]^2 = \min_{\mathbf{\mu}^C} . \tag{7.12}$$

Solving hierarchical system of fuzzy relational equations (7.9) is accomplished by way of consequent solving system (7.7) with *max-min* law of composition and system (7.8) with *min-max* law of composition.

The problem of solving fuzzy logic equations (7.7) is formulated as follows. Fuzzy causes combinations vector $\mathbf{\mu}^A = (\mu^{A_1}, \mu^{A_2}, ..., \mu^{A_K})$ should be found which satisfies the constraints $\mu^{A_L} \in [0,1]$, $L = \overline{1,K}$, and also provides the least distance between observed and model fuzzy effects vectors:

$$F_1 = \sum_{J=1}^{M} \left[\mu^{E_J} - \max_{L=1,K} (\min(\mu^{A_L}, r_{LJ})) \right]^2 = \min_{\mathbf{\mu}^A} \tag{7.13}$$

The problem of solving fuzzy logic equations (7.8) is formulated as follows. Fuzzy causes vector $\mathbf{\mu}^C = (\mu^{C_1}, \mu^{C_2}, ..., \mu^{C_N})$, should be found which satisfies the constraints $\mu^{C_I} \in [0,1]$, $I = \overline{1,N}$, and also provides the least distance between observed and model fuzzy causes combinations vectors:

$$F_2 = \sum_{L=1}^{K} \left[\mu^{A_L} - \min_{I=1,N} (\max(\mu^{C_I}, \bar{v}_{IL})) \right]^2 = \min_{\mathbf{\mu}^C} \tag{7.14}$$

Following [8], in the general case, system (7.7) has a solution set $S(\mathbf{R}, \mathbf{\mu}^E)$, which is completely characterized by the unique greatest solution $\overline{\mathbf{\mu}}^A$ and the set of lower solutions $S^*(\mathbf{R}, \mathbf{\mu}^E) = \{ \underline{\mathbf{\mu}}_k^A,\ k = \overline{1,T} \}$:

$$S(\mathbf{R}, \mathbf{\mu}^E) = \bigcup_{\underline{\mathbf{\mu}}_k^A \in S^*} \left[\underline{\mathbf{\mu}}_k^A, \overline{\mathbf{\mu}}^A \right]. \tag{7.15}$$

Here $\overline{\mathbf{\mu}}^A = (\bar{\mu}^{A_1}, \bar{\mu}^{A_2}, ..., \bar{\mu}^{A_K})$ and $\underline{\mathbf{\mu}}_k^A = (\underline{\mu}_k^{A_1}, \underline{\mu}_k^{A_2}, ..., \underline{\mu}_k^{A_K})$ are the vectors of the upper and lower bounds of causes combinations A_L significance measures, where the union is taken over all $\underline{\mathbf{\mu}}_k^A \in S^*(\mathbf{R}, \mathbf{\mu}^E)$.

For the greatest solution $\overline{\boldsymbol{\mu}}^A$, system (7.8) has a solution set $\overline{D}(\overline{\boldsymbol{\mu}}^A)$, which is completely characterized by the unique least solution $\underline{\boldsymbol{\mu}}^C$ and the set of upper solutions $\overline{D}^*(\overline{\boldsymbol{\mu}}^A) = \{\overline{\boldsymbol{\mu}}_l^C, l = \overline{1,H}\}$:

$$\overline{D}\left(\overline{\boldsymbol{\mu}}^A\right) = \bigcup_{\overline{\boldsymbol{\mu}}_l^C \in \overline{D}^*} \left[\underline{\boldsymbol{\mu}}^C, \overline{\boldsymbol{\mu}}_l^C\right]. \tag{7.16}$$

Here $\underline{\boldsymbol{\mu}}^C = (\underline{\mu}^{C_1}, \underline{\mu}^{C_2}, ..., \underline{\mu}^{C_N})$ and $\overline{\boldsymbol{\mu}}_l^C = (\overline{\mu}_l^{C_1}, \overline{\mu}_l^{C_2}, ..., \overline{\mu}_l^{C_N})$ are the vectors of the lower and upper bounds of causes C_I significance measures, where the union is taken over all $\overline{\boldsymbol{\mu}}_l^C \in \overline{D}^*\left(\overline{\boldsymbol{\mu}}^A\right)$.

For each lower solution $\underline{\boldsymbol{\mu}}_k^A$, $k = \overline{1,T}$, system (7.8) has a solution set $\underline{D}_k(\underline{\boldsymbol{\mu}}_k^A)$, which is completely characterized by the unique least solution $\underline{\boldsymbol{\mu}}_k^C$ and the set of upper solutions $\underline{D}_k^*(\underline{\boldsymbol{\mu}}_k^A) = \left\{\overline{\boldsymbol{\mu}}_{kl}^C, l = \overline{1, H_k}\right\}$:

$$\underline{D}_k(\underline{\boldsymbol{\mu}}_k^A) = \bigcup_{\overline{\boldsymbol{\mu}}_{kl}^C \in \underline{D}_k^*} \left[\underline{\boldsymbol{\mu}}_k^C, \overline{\boldsymbol{\mu}}_{kl}^C\right]. \tag{7.17}$$

Here $\underline{\boldsymbol{\mu}}_k^C = (\underline{\mu}_k^{C_1}, \underline{\mu}_k^{C_2}, ..., \underline{\mu}_k^{C_N})$ and $\overline{\boldsymbol{\mu}}_{kl}^C = (\overline{\mu}_{kl}^{C_1}, \overline{\mu}_{kl}^{C_2}, ..., \overline{\mu}_{kl}^{C_N})$ are the vectors of the lower and upper bounds of causes C_I significance measures, where the union is taken over all $\overline{\boldsymbol{\mu}}_{kl}^C \in \underline{D}_k^*(\underline{\boldsymbol{\mu}}_k^A)$.

Following [14 – 16], formation of diagnostic results begins with the search for the null solution $\boldsymbol{\mu}_0^C = (\mu_0^{C_1}, \mu_0^{C_2}, ..., \mu_0^{C_N})$ of optimization problem (7.12). Formation of intervals (7.15) begins with the search for the null vector of the causes combinations significances measures $\boldsymbol{\mu}_0^A(\boldsymbol{\mu}_0^C) = (\mu_0^{A_1}, \mu_0^{A_2}, ..., \mu_0^{A_K})$, which corresponds to the obtained null solution $\boldsymbol{\mu}_0^C$. The upper bound ($\overline{\mu}^{A_L}$) is found in the range $[\mu_0^{A_L}, 1]$. The lower bound ($\underline{\mu}_k^{A_L}$) for $k = 1$ is found in the range $[0, \mu_0^{A_L}]$, and for $k > 1$ – in the range $[0, \overline{\mu}^{A_L}]$, where the minimal solutions $\underline{\boldsymbol{\mu}}_p^A$, $p < k$, are excluded from the search space.

Let $\boldsymbol{\mu}^A(t) = (\mu^{A_1}(t), \mu^{A_2}(t), ..., \mu^{A_K}(t))$ be some t-th solution of optimization problem (7.13). While searching for upper bounds ($\overline{\mu}^{A_L}$) it is suggested that

$\mu^{A_L}(t) \geq \mu^{A_L}(t-1)$, and while searching for lower bounds ($\underline{\mu}_k^{A_L}$) it is suggested that $\mu^{A_L}(t) \leq \mu^{A_L}(t-1)$ (Fig. 7.2a). The definition of the upper (lower) bounds follows the rule: if $\boldsymbol{\mu}^A(t) \neq \boldsymbol{\mu}^A(t-1)$, then $\overline{\mu}^{A_L}$ ($\underline{\mu}_k^{A_L}$)= $\mu^{A_L}(t)$, $L = \overline{1,K}$. If $\boldsymbol{\mu}^A(t) = \boldsymbol{\mu}^A(t-1)$, then the search for the interval solution $[\underline{\boldsymbol{\mu}}_k^A, \overline{\boldsymbol{\mu}}^A]$ is stopped. Formation of intervals (7.15) will go on until the condition $\underline{\boldsymbol{\mu}}_k^A \neq \underline{\boldsymbol{\mu}}_p^A$, $p < k$, has been satisfied.

Formation of intervals (7.16) begins with the search for the null solution $\overline{\boldsymbol{\mu}}_0^C = (\overline{\mu}_0^{C_1}, \overline{\mu}_0^{C_2}, ..., \overline{\mu}_0^{C_N})$ for the greatest solution $\overline{\boldsymbol{\mu}}^A$. The lower bound ($\underline{\mu}^{C_I}$) of solution set (7.16) is found in the range $[0, \overline{\mu}_0^{C_I}]$. The upper bound ($\overline{\mu}_l^{C_I}$) for $l = 1$ is found in the range $[\overline{\mu}_0^{C_I}, 1]$, and for $l > 1$ – in the range $[\underline{\mu}^{C_I}, 1]$, where the maximal solutions $\overline{\mu}_p^{C_I}$, $p < l$, are excluded from the search space.

Formation of intervals (7.17) begins with the search for the null solutions $\underline{\boldsymbol{\mu}}_{0k}^C = (\underline{\mu}_{0k}^{C_1}, \underline{\mu}_{0k}^{C_2}, ..., \underline{\mu}_{0k}^{C_N})$ for each of the lower solutions $\underline{\boldsymbol{\mu}}_k^A$, $k = \overline{1,T}$. The lower bound ($\underline{\mu}_k^{C_I}$) of solution set (7.17) is found in the range $[0, \underline{\mu}_{0k}^{C_I}]$. The upper bound ($\overline{\mu}_{kl}^{C_I}$) for $l = 1$ is found in the range $[\underline{\mu}_{0k}^{C_I}, 1]$, and for $l > 1$ – in the range $[\underline{\mu}_k^{C_I}, 1]$, where the maximal solutions $\overline{\mu}_{kp}^{C_I}$, $p < l$, are excluded from the search space.

Let $\boldsymbol{\mu}^C(t) = (\mu^{C_1}(t), \mu^{C_2}(t), ..., \mu^{C_N}(t))$ be some t-th solution of optimization problem (7.14). While searching for upper bounds ($\overline{\mu}_l^{C_I}$ or $\overline{\mu}_{kl}^{C_I}$) it is suggested that $\mu^{C_I}(t) \geq \mu^{C_I}(t-1)$, and while searching for lower bounds ($\underline{\mu}^{C_I}$ or $\underline{\mu}_k^{C_I}$) it is suggested that $\mu^{C_I}(t) \leq \mu^{C_I}(t-1)$ (Fig. 7.2b,c). The definition of the upper (lower) bounds follows the rule: if $\boldsymbol{\mu}^C(t) \neq \boldsymbol{\mu}^C(t-1)$, then $\overline{\mu}_l^{C_I}$ ($\underline{\mu}^{C_I}$)= $\mu^{C_I}(t)$ or $\overline{\mu}_{kl}^{C_I}$ ($\underline{\mu}_k^{C_I}$)= $\mu^{C_I}(t)$, $I = \overline{1,N}$. If $\boldsymbol{\mu}^C(t) = \boldsymbol{\mu}^C(t-1)$, then the search for the interval solution $[\underline{\boldsymbol{\mu}}_l^C, \overline{\boldsymbol{\mu}}^C]$ or $[\underline{\boldsymbol{\mu}}_{kl}^C, \overline{\boldsymbol{\mu}}_k^C]$ is stopped. Formation of intervals (7.16) and (7.17) will go on until the conditions $\overline{\boldsymbol{\mu}}_l^C \neq \overline{\boldsymbol{\mu}}_p^C$ and $\overline{\boldsymbol{\mu}}_{kl}^C \neq \overline{\boldsymbol{\mu}}_{kp}^C$, $p < l$, have been satisfied.

The hybrid genetic and neuro approach is proposed for solving optimization problems (7.12) – (7.14).

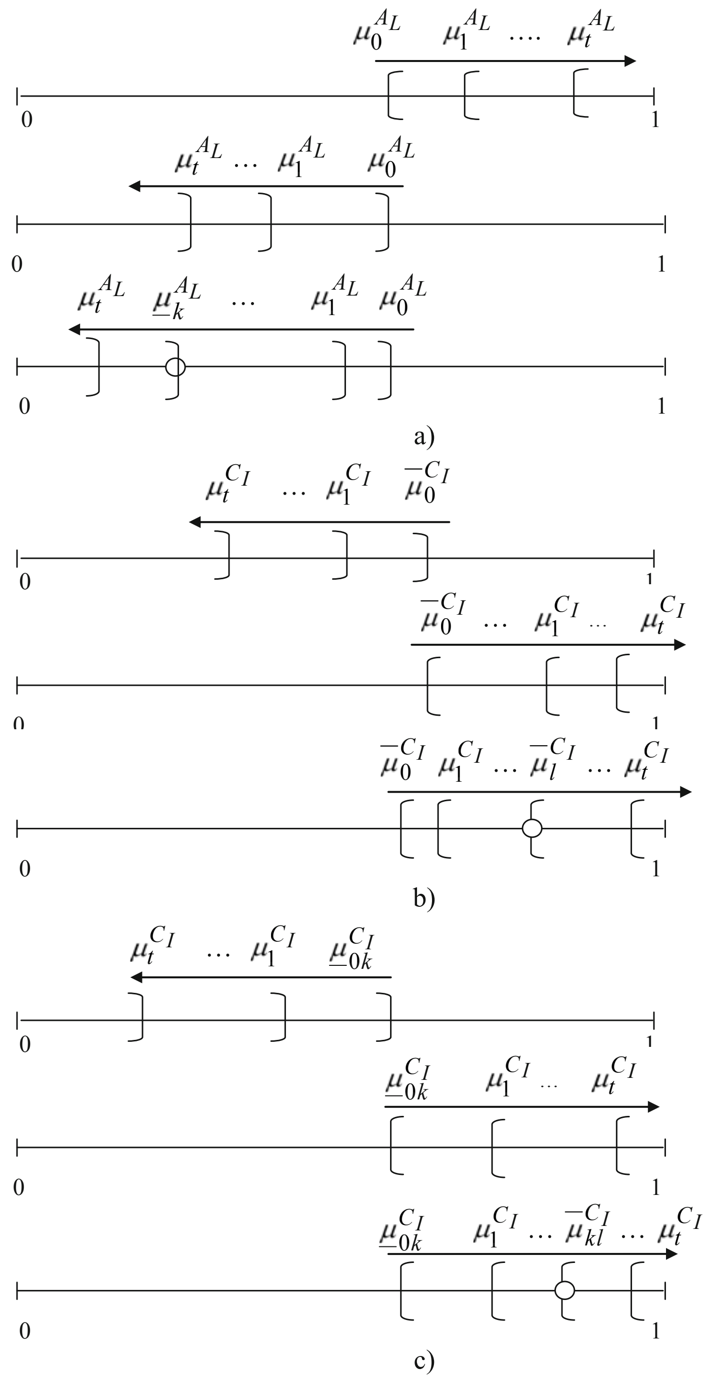

Fig. 7.2. Search for the solution sets (7.15) (a), (7.16) (b), (7.17) (c)

7.4 Genetic Algorithm for Fuzzy Rules Based Inverse Inference

The chromosome needed in the genetic algorithm [14 – 16] for solving optimization problems (7.12) – (7.14) includes the binary codes of parameters μ^{C_I}, $I = \overline{1,N}$, and μ^{A_L}, $L = \overline{1,K}$ (Fig. 7.3).

μ^{C_1}	μ^{C_2}	…	μ^{C_N}	μ^{A_1}	μ^{A_2}	...	μ^{A_K}

Fig. 7.3. Structure of the chromosome

The crossover operation is defined in Fig. 7.4, and is carried out by way of exchanging genes inside each of the solutions μ^{C_I} and μ^{A_L}. The points of crossover shown in dotted lines are selected randomly. Upper symbols (1 and 2) in the vectors of parameters correspond to the first and second chromosomes-parents.

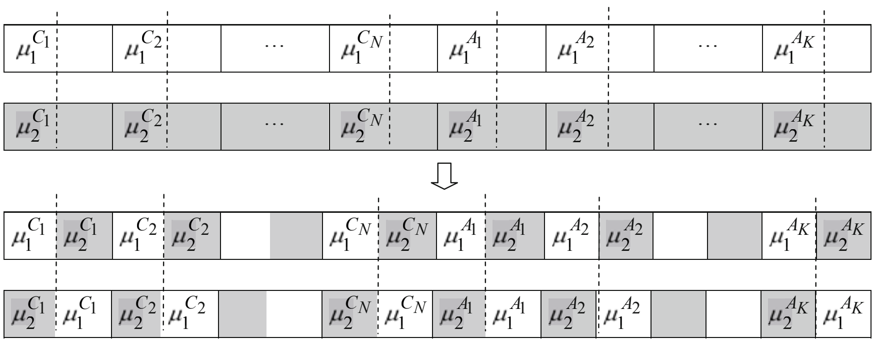

Fig. 7.4. Structure of the crossover operation

A mutation operation implies random change (with some probability) of chromosome elements

$$Mu\ (\mu^{C_I}) = RANDOM\left(\left[\ \underline{\mu}^{C_I}, \overline{\mu}^{C_I}\ \right]\right);$$

$$Mu\ (\mu^{A_L}) = RANDOM\left(\left[\ \underline{\mu}^{A_L}, \overline{\mu}^{A_L}\ \right]\right).$$

where $RANDOM([\underline{x}, \overline{x}])$ denotes a random number within the interval $[\underline{x}, \overline{x}]$.

Fitness function is built on the basis of criteria (7.12) – (7.14).

7.5 Neuro-fuzzy Network for Fuzzy Rules Based Inverse Inference

The neuro-fuzzy networks isomorphic to the systems of fuzzy logical equations (7.7) – (7.9), are presented in Fig. 7.5, a-c, respectively, and the elements of the neuro-fuzzy networks are shown in Table 3.1 [16].

The network in Fig. 7.5,a is designed so that the adjusted weights of arcs are the unknown significance measures of causes combinations μ^{A_L}, $L = \overline{1,K}$. The network in Fig. 7.5b is designed so that the adjusted weights of arcs are the unknown significance measures of causes μ^{C_I}, $I = \overline{1,N}$.

Network inputs in Fig. 7.5a are elements of the matrix of rules weights. As follows from the system of fuzzy relational equations (7.7), the rule weight r_{LJ} is the significance measure of the effect μ^{E_J} provided that the significance measure of the causes combination μ^{A_L} is equal to unity, and the significance measures of other combinations are equal to zero, i. e., $r_{LJ} = \mu^{E_J}$ (μ^{A_L} =1, μ^{A_P} =0), $P = \overline{1,K}$, $P \neq L$. At the network outputs, actual significance measures of the effects $\max_{L=\overline{1,K}}[\min(\mu^{A_L}, r_{LJ})]$ obtained for the actual weights of arcs μ^{A_L} are united.

Network inputs in Fig. 7.5,b are elements of the matrix of terms weights. As follows from the system of fuzzy relational equations (7.8), the term weight v_{IL} is the maximal possible significance measure of the cause μ^{C_I} in the combination μ^{A_L}. At the network outputs, actual significance measures of the causes $\min(\mu^{C_I}, v_{IL})$ obtained for the actual weights of arcs μ^{C_I} are united.

The neuro-fuzzy model in Fig. 7.5c is obtained by embedding the matrix of fuzzy relations into the neural network so that the adjusted weights of arcs are the unknown significance measures of the causes μ^{C_I}, $I = \overline{1,N}$. Network inputs in Fig. 7.5,c are elements of the matrix of terms weights. At the network outputs, actual significance measures of the effects $\max_{L=\overline{1,K}}\left(\min\left(\min_{I=\overline{1,N}}(\mu^{C_I}, v_{IL}), r_{LJ}\right)\right)$ obtained for the actual weights of arcs μ^{C_I} and r_{LJ} are united.

Thus, the problem of solving the system of fuzzy logic equations (7.9) is reduced to the problem of training of a neuro fuzzy network (see Fig. 7.5c) with the use of points

$$(v_{1L}, v_{2L}, ..., v_{NL}, \mu^{E_J}),\ L = \overline{1,K},\ J = \overline{1,M}.$$

The problem of solving the system of fuzzy logic equations (7.7) is reduced to the problem of training of a neuro fuzzy network (see Fig. 7.5a) with the use of points

$$(r_{1J}, r_{2J}, ..., r_{KJ}, \mu^{E_J}),\ J = \overline{1,M}.$$

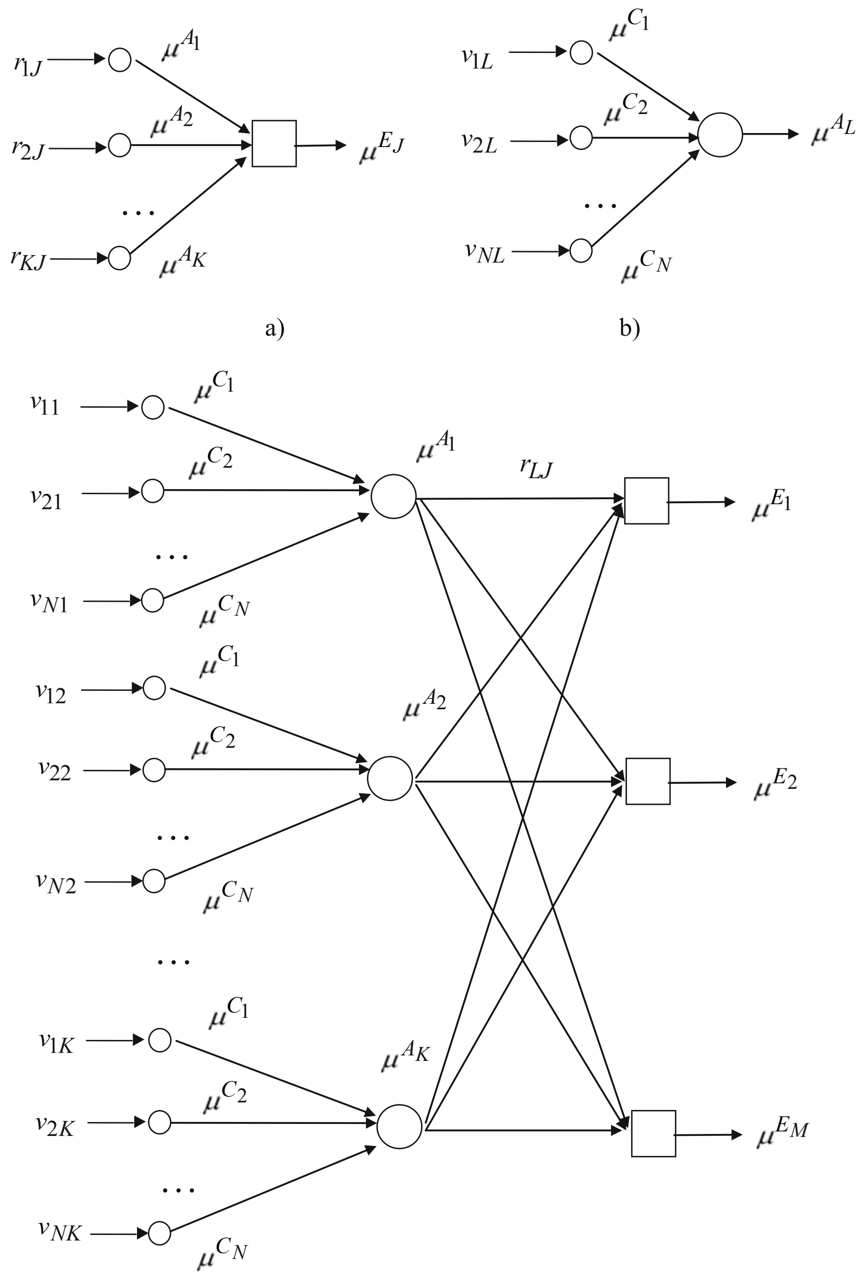

Fig. 7.5. Neuro-fuzzy models of diagnostic equations

The problem of solving the system of fuzzy logic equations (7.8) is reduced to the problem of training of a neuro fuzzy network (see Fig. 7.5b) with the use of points

$$\left(v_{1L}, v_{2L}, ..., v_{NL}, \mu^{A_L}\right),\ L = \overline{1,K} .$$

The adjustment of parameters of the neuro-fuzzy networks employs the recurrent relations

$$\mu^{C_I}(t+1) = \mu^{C_I}(t) - \eta \frac{\partial \varepsilon_t^E}{\partial \mu^{C_I}(t)} ,$$

$$\mu^{A_L}(t+1) = \mu^{A_L}(t) - \eta \frac{\partial \varepsilon_t^E}{\partial \mu^{A_L}(t)} , \tag{7.18}$$

$$\mu^{C_I}(t+1) = \mu^{C_I}(t) - \eta \frac{\partial \varepsilon_t^A}{\partial \mu^{C_I}(t)} ,$$

that minimize the criteria

$$\varepsilon_t^E = \frac{1}{2}(\hat{\boldsymbol{\mu}}^E(t) - \boldsymbol{\mu}^E(t))^2 , \tag{7.19}$$

$$\varepsilon_t^A = \frac{1}{2}(\hat{\boldsymbol{\mu}}^A(t) - \boldsymbol{\mu}^A(t))^2 , \tag{7.20}$$

where $\hat{\boldsymbol{\mu}}^E(t)$ and $\boldsymbol{\mu}^E(t)$ are the experimental and the model fuzzy effects vectors at the t-th step of training;

$\hat{\boldsymbol{\mu}}^A(t)$ and $\boldsymbol{\mu}^A(t)$ are the experimental and the model fuzzy causes combinations vectors at the t-th step of training;

$\mu^{C_I}(t)$ and $\mu^{A_L}(t)$ are the significance measures of causes C_I and causes combinations A_L at the t-th step of training;

η is a parameter of training, which can be selected according to the results from [22].

The partial derivatives appearing in recurrent relations (7.18) characterize the sensitivity of the error (ε_t^E or ε_t^A) to variations in parameters of the neuro-fuzzy network and can be calculated as follows:

$$\frac{\partial \varepsilon_t^E}{\partial \mu^{C_I}} = \sum_{J=1}^{M} \left[\frac{\partial \varepsilon_t^E}{\partial \mu^{E_J}} \cdot \sum_{L=1}^{K} \left[\frac{\partial \mu^{E_J}}{\partial \mu^{A_L}} \cdot \frac{\partial \mu^{A_L}}{\partial \mu^{C_I}} \right] \right];$$

$$\frac{\partial \varepsilon_t^E}{\partial \mu^{A_L}} = \sum_{J=1}^{M} \left[\frac{\partial \varepsilon_t^E}{\partial \mu^{E_J}} \cdot \frac{\partial \mu^{E_J}}{\partial \mu^{A_L}} \right];\quad \frac{\partial \varepsilon_t^A}{\partial \mu^{C_I}} = \sum_{L=1}^{K} \left[\frac{\partial \varepsilon_t^A}{\partial \mu^{A_L}} \cdot \frac{\partial \mu^{A_L}}{\partial \mu^{C_I}} \right].$$

Since determining the element “fuzzy output” from Table 3.1 involves the *min* and *max* fuzzy-logic operations, the relations for training are obtained using finite differences.

7.6 Problem of Fuzzy Rules Tuning

It is assumed that the training data which is given in the form of L pairs of experimental data is known: $\langle \hat{\mathbf{X}}_p, \hat{\mathbf{Y}}_p \rangle$, $p = \overline{1, L}$, where $\hat{\mathbf{X}}_p = (\hat{x}_1^p, \hat{x}_2^p, ..., \hat{x}_n^p)$ and $\hat{\mathbf{Y}}_p = (\hat{y}_1^p, \hat{y}_2^p, ..., \hat{y}_m^p)$ are the vectors of the values of the input and output variables in the experiment number p.

The essence of tuning of the fuzzy model (7.11) consists of finding such null solutions $\boldsymbol{\mu}_0^C(\hat{x}_1^p, \hat{x}_2^p, ..., \hat{x}_n^p)$ of the inverse problem, which minimize criterion (7.12) for all the points of the training data:

$$\sum_{p=1}^{L} [F_Y(\boldsymbol{\mu}_0^{\mathrm{C}}(\hat{x}_1^p, \hat{x}_2^p, ..., \hat{x}_n^p)) - \hat{\boldsymbol{\mu}}^E(\hat{y}_1^p, \hat{y}_2^p, ..., \hat{y}_m^p)]^2 = \min.$$

In other words, the essence of tuning of the fuzzy model (7.11) consists of finding such a vector of rules weights $\mathbf{R}$ and such vectors of membership functions parameters $\mathbf{B}_C$, $\boldsymbol{\Omega}_C$, $\mathbf{B}_E$, $\boldsymbol{\Omega}_E$, which provide the least distance between model and experimental fuzzy effects vectors:

$$\sum_{p=1}^{L} [F_Y(\hat{\mathbf{X}}_p, \mathbf{R}, \mathbf{B}_{\mathrm{C}}, \boldsymbol{\Omega}_{\mathrm{C}}) - \hat{\boldsymbol{\mu}}^E(\hat{\mathbf{Y}}_p, \mathbf{B}_{\mathrm{E}}, \boldsymbol{\Omega}_{\mathrm{E}})]^2 = \min_{\mathbf{R}, \mathbf{B}_{\mathrm{C}}, \boldsymbol{\Omega}_{\mathrm{C}}, \mathbf{B}_{\mathrm{E}}, \boldsymbol{\Omega}_{\mathrm{E}}}. \quad (7.21)$$

7.7 Genetic Algorithm for Fuzzy Rules Tuning

The chromosome needed in the genetic algorithm [23, 24] for solving the optimization problem (7.21) is defined as the vector-line of binary codes of parameters $\mathbf{R}$, $\mathbf{B}_C$, $\boldsymbol{\Omega}_C$, $\mathbf{B}_E$, $\boldsymbol{\Omega}_E$ (Fig. 7.6).

$\mathbf{R}$	$\mathbf{B}_C$	$\boldsymbol{\Omega}_C$	$\mathbf{B}_E$	$\boldsymbol{\Omega}_E$

Fig. 7.6. Structure of the chromosome

The crossover operation is defined in Fig. 7.7, and is carried out by way of exchanging genes inside the vector of rules weights ($\mathbf{R}$) and each of the vectors of membership functions parameters $\mathbf{B}_C$, $\boldsymbol{\Omega}_C$, $\mathbf{B}_E$, $\boldsymbol{\Omega}_E$. The points of cross-over shown in dotted lines are selected randomly. Upper symbols (1 and 2) in the vectors of parameters correspond to the first and second chromosomes-parents.

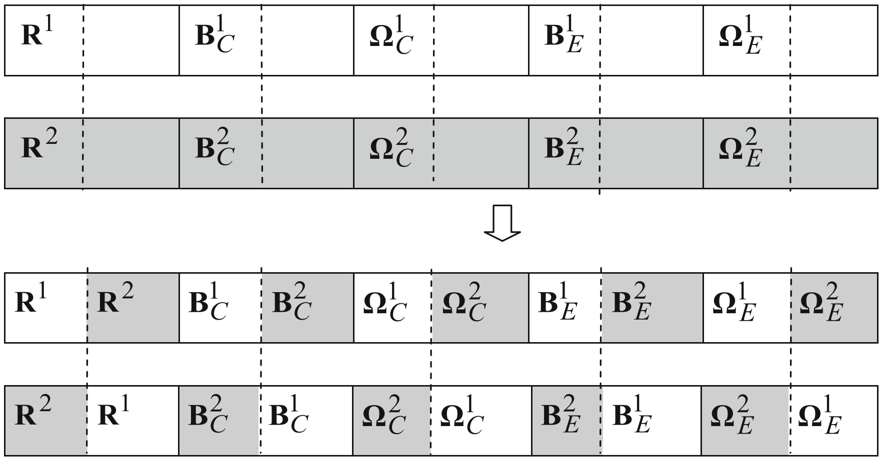

Fig. 7.7. Structure of the crossover operation

A mutation operation implies random change (with some probability) of chromosome elements:

$$Mu\,(\beta^{C_I}) = RANDOM\left(\left[\,\underline{\beta}^{C_I}, \overline{\beta}^{C_I}\,\right]\right);\ Mu\,(\sigma^{C_I}) = RANDOM\left(\left[\,\underline{\sigma}^{C_I}, \overline{\sigma}^{C_I}\,\right]\right);$$

$$Mu\,(\beta^{E_J}) = RANDOM\left(\left[\,\underline{\beta}^{E_j}, \overline{\beta}^{E_J}\,\right]\right);\ Mu\,(\sigma^{E_J}) = RANDOM\left(\left[\,\underline{\sigma}^{E_J}, \overline{\sigma}^{E_J}\,\right]\right);$$

$$Mu\,(r_{LJ}) = RANDOM\,([\underline{r}_{LJ}, \overline{r}_{LJ}])\,,$$

where $RANDOM\,([\underline{x}, \overline{x}])$ denotes a random number within the interval $[\underline{x}, \overline{x}]$.

The fitness function is built on the basis of criterion (7.21).

7.8 Adaptive Tuning of Fuzzy Rules

The neuro-fuzzy model of the object of diagnostics is shown in Fig. 7.8, and the nodes are represented in Table 3.1. The neuro-fuzzy model in Fig. 7.8 is obtained by embedding the matrices of fuzzy relations into the neural network so that the weights of arcs subject to tuning are rules weights (fuzzy relations) and the membership functions for causes and effects fuzzy terms [16, 25].

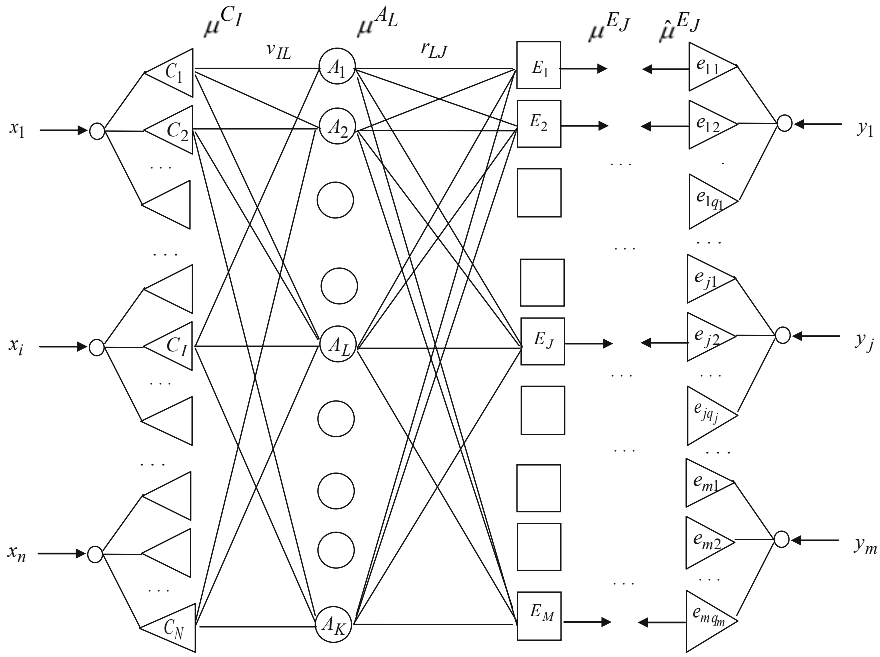

Fig. 7.8. Neuro-fuzzy model of the object of diagnostics

To train the parameters of the neuro-fuzzy network, the recurrent relations:

$$r_{LJ}(t+1) = r_{LJ}(t) - \eta \frac{\partial \varepsilon_t}{\partial r_{LJ}(t)};$$

$$\beta^{C_I}(t+1) = \beta^{C_I}(t) - \eta \frac{\partial \varepsilon_t}{\partial \beta^{C_I}(t)}; \;\; \sigma^{C_I}(t+1) = \sigma^{C_I}(t) - \eta \frac{\partial \varepsilon_t}{\partial \sigma^{C_I}(t)};$$

$$\beta^{E_J}(t+1) = \beta^{E_J}(t) - \eta \frac{\partial \varepsilon_t}{\partial \beta^{E_J}(t)}; \;\; \sigma^{E_J}(t+1) = \sigma^{E_J}(t) - \eta \frac{\partial \varepsilon_t}{\partial \sigma^{E_J}(t)}, \qquad (7.22)$$

minimizing criterion (7.19) are used, where

$r_{LJ}(t)$ are fuzzy relations (rules weights) at the *t*-th step of training;

$\beta^{C_I}(t)$, $\sigma^{C_I}(t)$, $\beta^{E_J}(t)$, $\sigma^{E_J}(t)$ are parameters of the membership functions for causes and effects fuzzy terms at the *t*-th step of training.

The partial derivatives appearing in recurrent relations (7.22) characterize the sensitivity of the error (ε_t) to variations in parameters of the neuro-fuzzy network and can be calculated as follows:

$$\frac{\partial \varepsilon_t}{\partial r_{LJ}} = \frac{\partial \varepsilon_t}{\partial \mu^{E_J}(X)} \cdot \frac{\partial \mu^{E_J}(X)}{\partial r_{LJ}};$$

$$\frac{\partial \varepsilon_t}{\partial \beta^{C_I}} = \sum_{J=1}^{M} \left[\frac{\partial \varepsilon_t}{\partial \mu^{E_J}(x_i)} \cdot \sum_{L=1}^{K} \left[\frac{\partial \mu^{E_J}(x_i)}{\partial \mu^{A_L}(x_i)} \cdot \frac{\partial \mu^{A_L}(x_i)}{\partial \mu^{C_I}(x_i)} \cdot \frac{\partial \mu^{C_I}(x_i)}{\partial \beta^{C_I}} \right] \right];$$

$$\frac{\partial \varepsilon_t}{\partial \sigma^{C_I}} = \sum_{J=1}^{M} \left[\frac{\partial \varepsilon_t}{\partial \mu^{E_J}(x_i)} \cdot \sum_{L=1}^{K} \left[\frac{\partial \mu^{E_J}(x_i)}{\partial \mu^{A_L}(x_i)} \cdot \frac{\partial \mu^{A_L}(x_i)}{\partial \mu^{C_I}(x_i)} \cdot \frac{\partial \mu^{C_I}(x_i)}{\partial \sigma^{C_I}} \right] \right];$$

$$\frac{\partial \varepsilon_t}{\partial \beta^{E_J}} = \frac{\partial \varepsilon_t}{\partial \mu^{E_J}(y_j)} \cdot \frac{\partial \mu^{E_J}(y_j)}{\partial \beta^{E_J}}; \qquad \frac{\partial \varepsilon_t}{\partial \sigma^{E_J}} = \frac{\partial \varepsilon_t}{\partial \mu^{E_J}(y_j)} \cdot \frac{\partial \mu^{E_J}(y_j)}{\partial \sigma^{E_J}}.$$

Since determining the element “fuzzy output” (see Table 3.1) involves the *min* and *max* fuzzy-logic operations, the relations for training are obtained using finite differences.

7.9 Computer Simulations

The aim of the experiment consists of checking the performance of the above proposed models and algorithms with the help of the target “two inputs (x_1, x_2) – two outputs (y_1, y_2)” model. Some analytical functions $y_1 = f_1(x_1, x_2)$ and $y_2 = f_2(x_1, x_2)$ were approximated by the combined fuzzy knowledge base, and served simultaneously as training and testing data generator. The input values (x_1, x_2) restored for each output combination (y_1, y_2) were compared with the target level lines.

The target model is given by the formulae:

$$y_1 = f_1(x_1, x_2) = \frac{1}{10}(2z - 0.9)\ (7z - 1)\ (17z - 19)\ (15z - 2), \tag{7.23}$$

$$y_2 = f_2(x_1, x_2) = -\frac{1}{2} y_1 + 1,$$

where $z = \dfrac{(x_1 - 3.0)^2 + (x_2 - 2.5)^2}{40}$.

The target model is represented in Fig. 7.9.

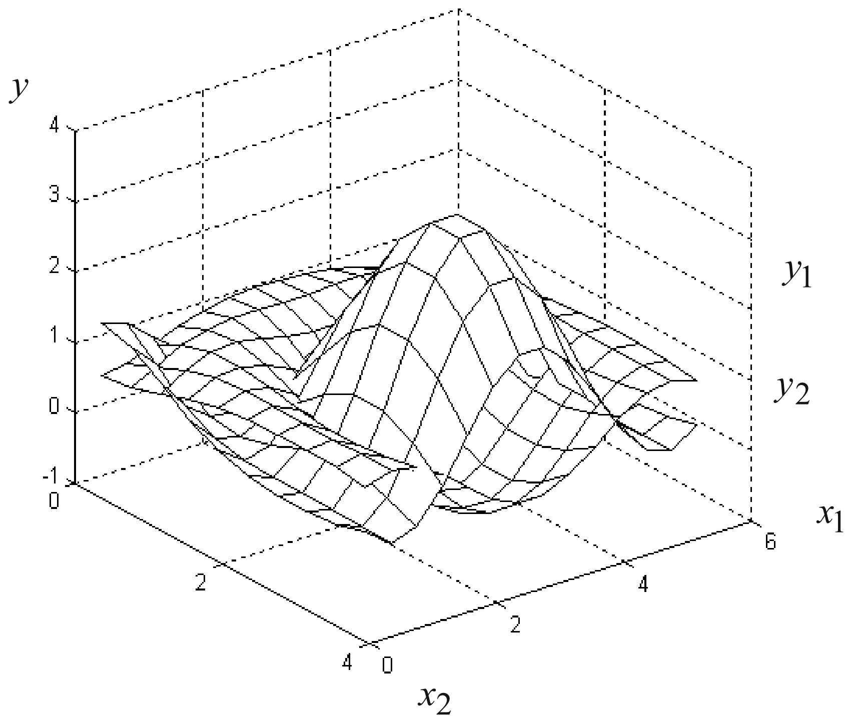

Fig. 7.9. "Inputs-outputs" model-generator

The fuzzy IF-THEN rules correspond to this model:

Rule 1: IF x_1 =*L* AND x_2 =*L* THEN y_1 =*hA* AND y_2 =*lA*;
Rule 2: IF x_1 =*A* AND x_2 =*L* THEN y_1 =*hL* AND y_2 =*A*;
Rule 3: IF x_1 =*H* AND x_2 =*L* THEN y_1 =*hA* AND y_2 =*lA*;
Rule 4: IF x_1 =*L* AND x_2 =*H* THEN y_1 =*hL* AND y_2 =*A*;
Rule 5: IF x_1 =*A* AND x_2 =*H* THEN y_1 =*H* AND y_2 =*L*;
Rule 6: IF x_1 =*H* AND x_2 =*H* THEN y_1 =*hL* AND y_2 =*A*.

where the total number of the input and output terms-assessments consists of: c_{11} *Low (L)*, c_{12} *Average (A)*, c_{13} *High (H)* for x_1, c_{21} *(Low)*, c_{22} *(High)* for x_2; e_{11} *higher than Low* (*hL*), e_{12} *higher than Average (hA),* e_{13} *High (H)* for y_1; e_{21} *Low (L)*, e_{22} *lower than Average (lA)*, e_{23} *Average (A)* for y_2.

We shall define the set of causes and effects in the following way:

$$\{C_1, C_2, \ldots, C_5\} = \{c_{11}, c_{12}, c_{13}, c_{21}, c_{22}\};$$

$$\{E_1, E_2, \ldots, E_6\} = \{e_{11},\ e_{12},\ e_{13},\ e_{21},\ e_{22},\ e_{23}\}.$$

This fuzzy rule base is modelled by the fuzzy relational matrix presented in Table 7.3.

Table 7.3. Fuzzy knowledge matrix

IF inputs			THEN outputs					
	x_1	x_2	y_1			y_2		
			hL	*hA*	*H*	*L*	*lA*	*A*
A_1	*L*	*L*	0	1	0	0	1	0
A_2	*A*	*L*	1	0	0	0	0	1
A_3	*H*	*L*	0	1	0	0	1	0
A_4	*L*	*H*	1	0	0	0	0	1
A_5	*A*	*H*	0	0	1	1	0	0
A_6	*H*	*H*	1	0	0	0	0	1

The results of the fuzzy model tuning are given in Tables 7.4, 7.5.

Table 7.4. Parameters of the membership functions for the causes fuzzy terms before (after) tuning

Parameter	Fuzzy terms				
	C_1	C_2	C_3	C_4	C_5
β -	0 (0.03)	3.0 (3.03)	6.0 (5.98)	0 (0.02)	3.0 (3.05)
σ -	1.0 (0.71)	2.0 (0.62)	1.0 (0.69)	1.0 (0.73)	2.0 (0.60)

Table 7.5. Parameters of the membership functions for the effects fuzzy terms before (after) tuning

Parameter	Fuzzy terms					
	E_1	E_2	E_3	E_4	E_5	E_6
β -	0 (0.02)	1.0 (1.10)	3.5 (3.36)	-0.7 (-0.67)	0.5 (0.44)	0.8 (0.89)
σ -	0.5 (0.27)	0.5 (0.29)	2.0 (1.91)	2.0 (1.70)	0.5 (0.31)	0.5 (0.25)

Fuzzy logic equations after tuning take the form:

$$\mu^{E_1} = (\mu^{A_2} \wedge 0.75) \vee (\mu^{A_4} \wedge 0.78) \vee (\mu^{A_6} \wedge 0.86)$$

$$\mu^{E_2} = (\mu^{A_1} \wedge 0.80) \vee (\mu^{A_3} \wedge 0.92)$$

$$\mu^{E_3} = (\mu^{A_5} \wedge 0.97)$$

$$\mu^{E_4} = (\mu^{A_1} \wedge 0.50) \vee (\mu^{A_3} \wedge 0.48) \vee (\mu^{A_5} \wedge 0.77)$$

$$\mu^{E_5} = (\mu^{A_1} \wedge 0.76) \vee (\mu^{A_3} \wedge 0.72)$$

$$\mu^{E_6} = (\mu^{A_2} \wedge 0.96) \vee (\mu^{A_4} \wedge 0.82) \vee (\mu^{A_6} \wedge 0.87) \ , \qquad (7.24)$$

where

$$\mu^{A_1} = \mu^{C_1} \wedge \mu^{C_4}$$

$$\mu^{A_2} = \mu^{C_2} \wedge \mu^{C_4}$$

$$\mu^{A_3} = \mu^{C_3} \wedge \mu^{C_4}$$

$$\mu^{A_4} = \mu^{C_1} \wedge \mu^{C_5}$$

$$\mu^{A_5} = \mu^{C_2} \wedge \mu^{C_5}$$

$$\mu^{A_6} = \mu^{C_3} \wedge \mu^{C_5} \ . \qquad (7.25)$$

The results of solving the problem of inverse inference before and after tuning are shown in Fig. 7.10, 7.11. The same figure depicts the membership functions of the fuzzy terms for the causes and effects before and after tuning.

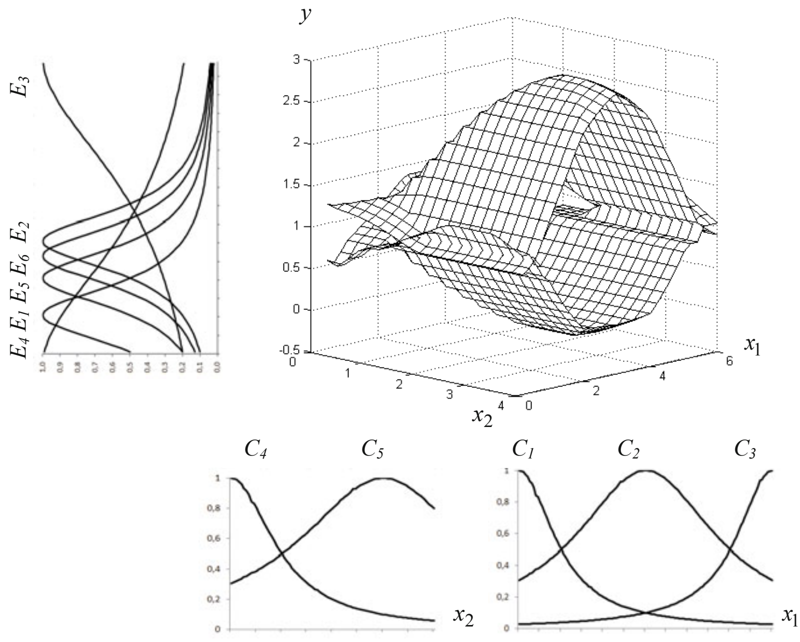

Fig. 7.10. Solution to the problem of inverse fuzzy inference before tuning

Let the specific values of the output variables consist of y_1^*=0.20 and y_2^*=0.80. The fuzzy effects vector for these values can be defined with the help of the membership functions in Fig. 7.11:

$$\boldsymbol{\mu}^E = (\mu^{E_1}(y_1^*) = 0.69;\ \mu^{E_2}(y_1^*) = 0.09;\ \mu^{E_3}(y_1^*) = 0.27;$$
$$\mu^{E_4}(y_2^*) = 0.57;\ \mu^{E_5}(y_2^*) = 0.43;\ \mu^{E_6}(y_2^*) = 0.89).$$

The genetic algorithm yields a null solution of the optimization problem (7.12)

$$\boldsymbol{\mu}_0^C = (\mu_0^{C_1} = 0.26, \mu_0^{C_2} = 0.93, \mu_0^{C_3} = 0.20, \mu_0^{C_4} = 0.89, \mu_0^{C_5} = 0.42)\ , \qquad (7.26)$$

for which the value of the optimization criterion (7.12) is F=0.1064.

The null vector of the causes combinations significances measures

$$\boldsymbol{\mu}_0^A = (\mu_0^{A_1} = 0.26, \mu_0^{A_2} = 0.89, \mu_0^{A_3} = 0.20, \mu_0^{A_4} = 0.26, \mu_0^{A_5} = 0.42, \mu_0^{A_6} = 0.20)$$

corresponds to the obtained null solution.

The obtained null solution allows us to arrange for the genetic search for the solution set $S(\mathbf{R}, \boldsymbol{\mu}^E)$, which is completely determined by the greatest solution

$$\overline{\boldsymbol{\mu}}^A = (\ \overline{\mu}^{A_1} = 0.26,\ \overline{\mu}^{A_2} = 0.89,\ \overline{\mu}^{A_3} = 0.26,\ \overline{\mu}^{A_4} = 0.75,\ \overline{\mu}^{A_5} = 0.42,\ \overline{\mu}^{A_6} = 0.75)$$

and the two lower solutions $S^* = \{\underline{\boldsymbol{\mu}}_1^A, \underline{\boldsymbol{\mu}}_2^A\}$

$$\underline{\boldsymbol{\mu}}_1^A = (\underline{\mu}_1^{A_1} = 0.26,\ \underline{\mu}_1^{A_2} = 0.89,\ \underline{\mu}_1^{A_3} = 0,\ \underline{\mu}_1^{A_4} = 0,\ \underline{\mu}_1^{A_5} = 0.42,\ \underline{\mu}_1^{A_6} = 0);$$

$$\underline{\boldsymbol{\mu}}_2^A = (\underline{\mu}_2^{A_1} = 0,\ \underline{\mu}_2^{A_2} = 0.89,\ \underline{\mu}_2^{A_3} = 0.26,\ \underline{\mu}_2^{A_4} = 0,\ \underline{\mu}_2^{A_5} = 0.42,\ \underline{\mu}_2^{A_6} = 0).$$

Thus, the solution of fuzzy relational equations (7.24) can be represented in the form of intervals:

$$S(\mathbf{R}, \boldsymbol{\mu}^E) = \{\mu^{A_1} = 0.26,\ \mu^{A_2} = 0.89,\ \mu^{A_3} \in [0, 0.26],\ \mu^{A_4} \in [0, 0.75],\ \mu^{A_5} = 0.42,\ \mu^{A_6} \in [0, 0.75]\}$$
$$\cup \{\mu^{A_1} \in [0, 0.26],\ \mu^{A_2} = 0.89,\ \mu^{A_3} = 0.26,\ \mu^{A_4} \in [0, 0.75],\ \mu^{A_5} = 0.42,\ \mu^{A_6} \in [0, 0.75]\}. \tag{7.27}$$

We next apply the genetic algorithm for solving the optimization problem (7.14) for the greatest solution $\overline{\boldsymbol{\mu}}^A$ and the two lower solutions $\underline{\boldsymbol{\mu}}_1^A$ and $\underline{\boldsymbol{\mu}}_2^A$.

For the greatest solution $\overline{\boldsymbol{\mu}}^A$, the genetic algorithm yields a null solution of the optimization problem (7.14)

$$\overline{\boldsymbol{\mu}}_0^C = (\overline{\mu}_0^{C_1} = 0.49, \overline{\mu}_0^{C_2} = 0.96, \overline{\mu}_0^{C_3} = 0.49, \overline{\mu}_0^{C_4} = 0.90, \overline{\mu}_0^{C_5} = 0.49)\,, \tag{7.28}$$

for which the value of the optimization criterion (7.14) is F=0.2459.

The obtained null solution allows us to arrange for the genetic search for the solution set $\overline{D}$ ($\overline{\boldsymbol{\mu}}^A$), which is completely determined by the least solution

$$\underline{\boldsymbol{\mu}}^C = (\underline{\mu}^{C_1} = 0.49,\ \underline{\mu}^{C_2} = 0.89,\ \underline{\mu}^{C_3} = 0.49,\ \underline{\mu}^{C_4} = 0.89,\ \underline{\mu}^{C_5} = 0.49)$$

and the two upper solutions $\overline{D}^* = \{\overline{\boldsymbol{\mu}}_1^C, \overline{\boldsymbol{\mu}}_2^C\}$

$$\overline{\boldsymbol{\mu}}_1^C = (\overline{\mu}_1^{C_1} = 0.49,\ \overline{\mu}_1^{C_2} = 0.89,\ \overline{\mu}_1^{C_3} = 0.49,\ \overline{\mu}_1^{C_4} = 1.0,\ \overline{\mu}_1^{C_5} = 0.49);$$

$$\overline{\boldsymbol{\mu}}_2^C = (\overline{\mu}_2^{C_1} = 0.49,\ \overline{\mu}_2^{C_2} = 1.0,\ \overline{\mu}_2^{C_3} = 0.49,\ \overline{\mu}_2^{C_4} = 0.89,\ \overline{\mu}_2^{C_5} = 0.49).$$

Thus, the solution of fuzzy relational equations (7.25) for the greatest solution $\overline{\boldsymbol{\mu}}^A$ can be represented in the form of intervals:

$$\overline{D}(\overline{\boldsymbol{\mu}}^A) = \{\mu^{C_1} = 0.49,\ \mu^{C_2} = 0.89,\ \mu^{C_3} = 0.49,\ \mu^{C_4} \in [0.89, 1.0],\ \mu^{C_5} = 0.49\}$$
$$\cup \{\mu^{C_1} = 0.49,\ \mu^{C_2} \in [0.89, 1.0],\ \mu^{C_3} = 0.49,\ \mu^{C_4} = 0.89,\ \mu^{C_5} = 0.49\}. \tag{7.29}$$

For the first lower solution $\underline{\boldsymbol{\mu}}_1^A$, the genetic algorithm yields a null solution of the optimization problem (7.14)

$$\underline{\boldsymbol{\mu}}_{01}^C = (\underline{\mu}_{01}^{C_1} = 0.13, \underline{\mu}_{01}^{C_2} = 0.89, \underline{\mu}_{01}^{C_3} = 0, \underline{\mu}_{01}^{C_4} = 0.94, \underline{\mu}_{01}^{C_5} = 0.42)\,, \tag{7.30}$$

for which the value of the optimization criterion (7.14) is F=0.0338.

The obtained null solution allows us to arrange for the genetic search for the solution set $\underline{D}_1(\underline{\mathbf{\mu}}_1^A)$, which is completely determined by the least solution

$$\underline{\mathbf{\mu}}^C = (\underline{\mu}^{C_1} = 0.13,\ \underline{\mu}^{C_2} = 0.89,\ \underline{\mu}^{C_3} = 0,\ \underline{\mu}^{C_4} = 0.89,\ \underline{\mu}^{C_5} = 0.42)$$

and the two upper solutions $\underline{D}_1^* = \{\overline{\mathbf{\mu}}_1^C, \overline{\mathbf{\mu}}_2^C\}$

$$\overline{\mathbf{\mu}}_1^C = (\overline{\mu}_1^{C_1} = 0.13,\ \overline{\mu}_1^{C_2} = 0.89,\ \overline{\mu}_1^{C_3} = 0,\ \overline{\mu}_1^{C_4} = 1.0,\ \overline{\mu}_1^{C_5} = 0.42);$$

$$\overline{\mathbf{\mu}}_2^C = (\overline{\mu}_2^{C_1} = 0.13,\ \overline{\mu}_2^{C_2} = 1.0,\ \overline{\mu}_2^{C_3} = 0,\ \overline{\mu}_2^{C_4} = 0.89,\ \overline{\mu}_2^{C_5} = 0.42).$$

Thus, the solution of fuzzy relational equations (7.25) for the first lower solution $\underline{\mathbf{\mu}}_1^A$ can be represented in the form of intervals:

$$\underline{D}_1(\underline{\mathbf{\mu}}_1^A) = \{\mu^{C_1} = 0.13,\ \mu^{C_2} = 0.89,\ \mu^{C_3} = 0,\ \mu^{C_4} \in [0.89, 1.0],\ \mu^{C_5} = 0.42\}$$
$$\cup \{\mu^{C_1} = 0.13,\ \mu^{C_2} \in [0.89, 1.0],\ \mu^{C_3} = 0,\ \mu^{C_4} = 0.89,\ \mu^{C_5} = 0.42\}. \quad (7.31)$$

For the second lower solution $\underline{\mathbf{\mu}}_2^A$, the genetic algorithm yields a null solution of the optimization problem (7.14)

$$\underline{\mathbf{\mu}}_{02}^C = (\underline{\mu}_{02}^{C_1} = 0, \underline{\mu}_{02}^{C_2} = 0.97, \underline{\mu}_{02}^{C_3} = 0.13, \underline{\mu}_{02}^{C_4} = 0.89, \underline{\mu}_{02}^{C_5} = 0.42), \quad (7.32)$$

for which the value of the optimization criterion (7.14) is F=0.0338.

The obtained null solution allows us to arrange for the genetic search for the solution set $\underline{D}_2(\underline{\mathbf{\mu}}_2^A)$, which is completely determined by the least solution

$$\underline{\mathbf{\mu}}^C = (\underline{\mu}^{C_1} = 0,\ \underline{\mu}^{C_2} = 0.89,\ \underline{\mu}^{C_3} = 0.13,\ \underline{\mu}^{C_4} = 0.89,\ \underline{\mu}^{C_5} = 0.42)$$

and the two upper solutions $\underline{D}_2^* = \{\overline{\mathbf{\mu}}_1^C, \overline{\mathbf{\mu}}_2^C\}$

$$\overline{\mathbf{\mu}}_1^C = (\overline{\mu}_1^{C_1} = 0,\ \overline{\mu}_1^{C_2} = 0.89,\ \overline{\mu}_1^{C_3} = 0.13,\ \overline{\mu}_1^{C_4} = 1.0,\ \overline{\mu}_1^{C_5} = 0.42);$$

$$\overline{\mathbf{\mu}}_2^C = (\overline{\mu}_2^{C_1} = 0,\ \overline{\mu}_2^{C_2} = 1.0,\ \overline{\mu}_2^{C_3} = 0.13,\ \overline{\mu}_2^{C_4} = 0.89,\ \overline{\mu}_2^{C_5} = 0.42).$$

Thus, the solution of fuzzy relational equations (7.25) for the second lower solution $\underline{\mathbf{\mu}}_2^A$ can be represented in the form of intervals:

$$\underline{D}_2(\underline{\mathbf{\mu}}_2^A) = \{\mu^{C_1} = 0,\ \mu^{C_2} = 0.89,\ \mu^{C_3} = 0.13,\ \mu^{C_4} \in [0.89, 1.0],\ \mu^{C_5} = 0.42\}$$
$$\cup \{\mu^{C_1} = 0,\ \mu^{C_2} \in [0.89, 1.0],\ \mu^{C_3} = 0.13,\ \mu^{C_4} = 0.89,\ \mu^{C_5} = 0.42\}. \quad (7.33)$$

The intervals of the values of the input variable for each interval in solutions (7.29), (7.31), (7.33) can be defined with the help of the membership functions in Fig. 7.11:

- x_1^* =0.75 or x_1^* =1.85 or x_1^* =6.00 for C_1;
- $x_1^* \in$ [2.81, 3.25] for C_2;
- x_1^* =5.27 or x_1^* =4.20 or x_1^* =0 for C_3;
- $x_2^* \in$ [0, 0.27] for C_4;
- x_2^* =2.44 and x_2^* =3.66 or x_2^* =2.35 and x_2^* =3.75 for C_5.

The restoration of the input set for y_1^* =0.20 and y_2^* =0.80 is shown in Fig. 7.11, in which the values of the causes $C_1 \div C_5$ and effects $E_1 \div E_6$ significances measures are marked. The comparison of the target and restored level lines for y_1^* =0.20 and y_2^* =0.80 is shown in Fig. 7.12.

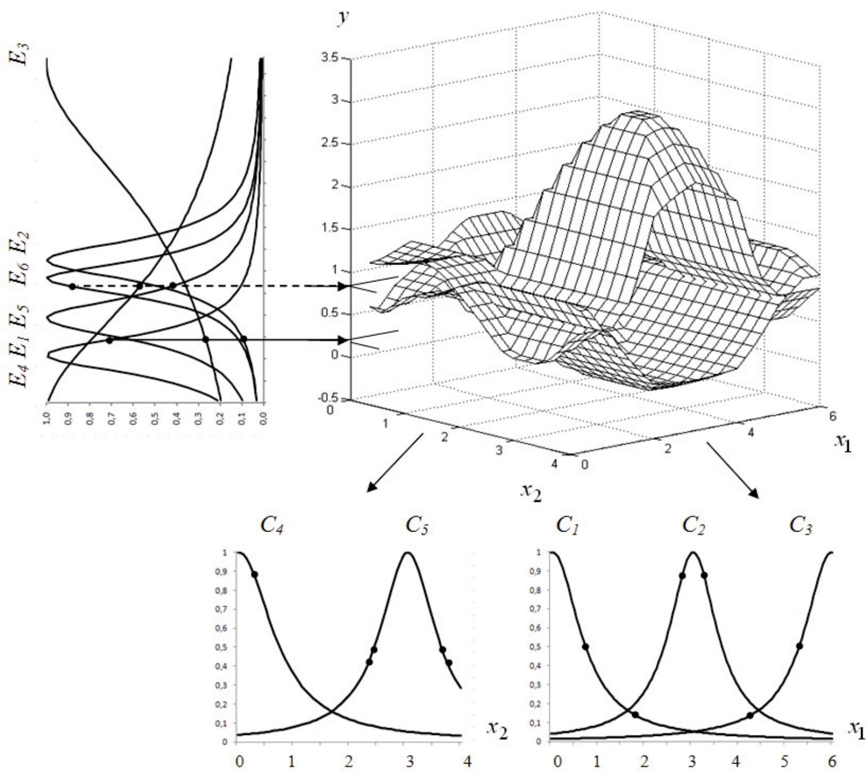

Fig. 7.11. Solution to the problem of inverse fuzzy inference for y_1^* =0.20 and y_2^* =0.80

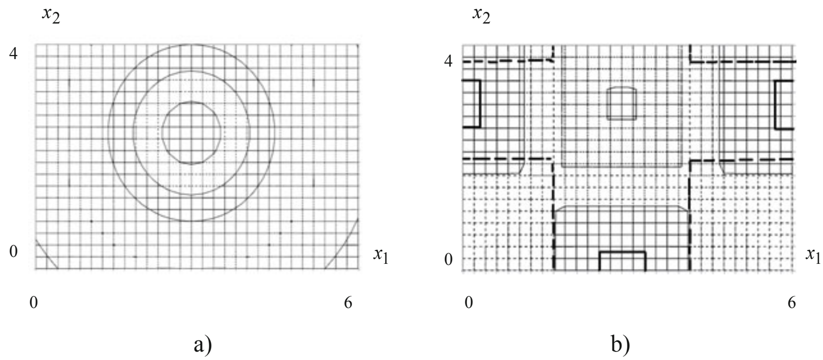

Fig. 7.12. Comparison of the target (a) and restored (b) level linesfor y_1^*=0.20 and y_2^*=0.80

Let the values of the output variables have changed with y_1^*=0.20 and y_2^*=0.80 to y_1^*=1.00 and y_2^*=0.60 (Fig. 7.13). For the new values, the fuzzy effects vector is

$$\mathbf{\mu}^E = (\mu^{E_1}(y_1^*) = 0.07;\ \mu^{E_2}(y_1^*) = 0.89;\ \mu^{E_3}(y_1^*) = 0.40;$$
$$\mu^{E_4}(y_2^*) = 0.64;\ \mu^{E_5}(y_2^*) = 0.79;\ \mu^{E_6}(y_2^*) = 0.43).$$

A neural adjustment of the null solution (7.26) of the optimization problem (7.12) has yielded a fuzzy causes vector

$$\mathbf{\mu}_0^C = (\mu_0^{C_1} = 0.84, \mu_0^{C_2} = 0.32, \mu_0^{C_3} = 0.89, \mu_0^{C_4} = 0.95, \mu_0^{C_5} = 0.32)\ ,$$

for which the value of the optimization criterion (7.12) has constituted *F*=0.1015.

The null vector of the causes combinations significances measures

$$\mathbf{\mu}_0^A = (\mu_0^{A_1} = 0.84, \mu_0^{A_2} = 0.32, \mu_0^{A_3} = 0.89, \mu_0^{A_4} = 0.32, \mu_0^{A_5} = 0.32, \mu_0^{A_6} = 0.32)\ ,$$

corresponds to the obtained null solution.

The resultant null solution has allowed adjusting the bounds in the solution (7.27) and generating the set of solutions $S(\mathbf{R}, \mathbf{\mu}^E)$ determined by the greatest solution

$$\bar{\mathbf{\mu}}^A = (\bar{\mu}^{A_1} = 1.0,\ \bar{\mu}^{A_2} = 0.32,\ \bar{\mu}^{A_3} = 0.89,\ \bar{\mu}^{A_4} = 0.32,\ \bar{\mu}^{A_5} = 0.32,\ \bar{\mu}^{A_6} = 0.32)$$

and the three lower solutions $S^* = \{\underline{\mathbf{\mu}}_1^A, \underline{\mathbf{\mu}}_2^A, \underline{\mathbf{\mu}}_3^A\}$

$$\underline{\mathbf{\mu}}_1^A = (\underline{\mu}_1^{A_1} = 0.76,\ \underline{\mu}_1^{A_2} = 0.32,\ \underline{\mu}_1^{A_3} = 0.89,\ \underline{\mu}_1^{A_4} = 0,\ \underline{\mu}_1^{A_5} = 0.32,\ \underline{\mu}_1^{A_6} = 0);$$
$$\underline{\mathbf{\mu}}_2^A = (\underline{\mu}_2^{A_1} = 0.76,\ \underline{\mu}_2^{A_2} = 0,\ \underline{\mu}_2^{A_3} = 0.89,\ \underline{\mu}_2^{A_4} = 0.32,\ \underline{\mu}_2^{A_5} = 0.32,\ \underline{\mu}_2^{A_6} = 0);$$
$$\underline{\mathbf{\mu}}_3^A = (\underline{\mu}_3^{A_1} = 0.76,\ \underline{\mu}_3^{A_2} = 0,\ \underline{\mu}_3^{A_3} = 0.89,\ \underline{\mu}_3^{A_4} = 0,\ \underline{\mu}_3^{A_5} = 0.32,\ \underline{\mu}_3^{A_6} = 0.32).$$

Thus, the solution of fuzzy relational equations (7.24) for the new values can be represented in the form of intervals:

$$S(\mathbf{R},\boldsymbol{\mu}^E)=\{\mu^{A_1}\in[0.76,1.0],\ \mu^{A_2}=0.32,\ \mu^{A_3}=0.89,\ \mu^{A_4}\in[0,0.32],\ \mu^{A_5}=0.32,\ \mu^{A_6}\in[0,0.32]\}$$
$$\cup\{\mu^{A_1}\in[0.76,1.0],\ \mu^{A_2}\in[0,0.32],\ \mu^{A_3}=0.89,\ \mu^{A_4}=0.32,\ \mu^{A_5}=0.32,\ \mu^{A_6}\in[0,0.32]\}$$
$$\cup\{\mu^{A_1}\in[0.76,1.0],\ \mu^{A_2}\in[0,0.32],\ \mu^{A_3}=0.89,\ \mu^{A_4}\in[0,0.32],\ \mu^{A_5}=0.32,\ \mu^{A_6}=0.32]\}.$$

For the greatest solution $\overline{\boldsymbol{\mu}}^A$, a neural adjustment of the null solution (7.28) has yielded a fuzzy causes vector

$$\overline{\boldsymbol{\mu}}_0^C=(\overline{\mu}_0^{C_1}=1.0,\ \overline{\mu}_0^{C_2}=0.32,\ \overline{\mu}_0^{C_3}=0.89,\ \overline{\mu}_0^{C_4}=1.0,\ \overline{\mu}_0^{C_5}=0.32),$$

for which the value of the optimization criterion (7.14) has constituted F=0.0.

The resultant null solution has allowed adjusting the bounds in the solution (7.29) and generating the set of solutions $\overline{D}(\overline{\boldsymbol{\mu}}^A)$ determined by the unique (null) solution

$$\overline{D}(\overline{\boldsymbol{\mu}}^A)=\{\mu^{C_1}=1.0,\ \mu^{C_2}=0.32,\ \mu^{C_3}=0.89,\ \mu^{C_4}=1.0,\ \mu^{C_5}=0.32\}. \quad (7.34)$$

For the first lower solution $\underline{\boldsymbol{\mu}}_1^A$, a neural adjustment of the null solution (7.30) has yielded a fuzzy causes vector

$$\underline{\boldsymbol{\mu}}_{01}^C=(\underline{\mu}_{01}^{C_1}=0.76,\ \underline{\mu}_{01}^{C_2}=0.32,\ \underline{\mu}_{01}^{C_3}=0.89,\ \underline{\mu}_{01}^{C_4}=0.92,\ \underline{\mu}_{01}^{C_5}=0.11),$$

for which the value of the optimization criterion (7.14) has constituted F=0.0683.

The resulting null solution has allowed adjusting the bounds in the solution (7.31) and generating the set of solutions $\underline{D}_1(\underline{\boldsymbol{\mu}}_1^A)$, which is completely determined by the least solution

$$\underline{\boldsymbol{\mu}}^C=(\underline{\mu}^{C_1}=0.76,\ \underline{\mu}^{C_2}=0.32,\ \underline{\mu}^{C_3}=0.89,\ \underline{\mu}^{C_4}=0.89,\ \underline{\mu}^{C_5}=0.11)$$

and the two upper solutions $\underline{D}_1^*=\{\overline{\boldsymbol{\mu}}_1^C,\overline{\boldsymbol{\mu}}_2^C\}$

$$\overline{\boldsymbol{\mu}}_1^C=(\overline{\mu}_1^{C_1}=0.76,\ \overline{\mu}_1^{C_2}=0.32,\ \overline{\mu}_1^{C_3}=0.89,\ \overline{\mu}_1^{C_4}=1.0,\ \overline{\mu}_1^{C_5}=0.11);$$
$$\overline{\boldsymbol{\mu}}_2^C=(\overline{\mu}_2^{C_1}=0.76,\ \overline{\mu}_2^{C_2}=0.32,\ \overline{\mu}_2^{C_3}=1.0,\ \overline{\mu}_2^{C_4}=0.89,\ \overline{\mu}_2^{C_5}=0.11).$$

Thus, the solution of fuzzy relational equations (7.24) for the first lower solution $\underline{\boldsymbol{\mu}}_1^A$ can be represented in the form of intervals:

$$\underline{D}_1(\underline{\boldsymbol{\mu}}_1^A)=\{\mu^{C_1}=0.76,\ \mu^{C_2}=0.32,\ \mu^{C_3}=0.89,\ \mu^{C_4}\in[0.89,1.0],\ \mu^{C_5}=0.11\}$$
$$\cup\{\mu^{C_1}=0.76,\ \mu^{C_2}=0.32,\ \mu^{C_3}\in[0.89,1.0],\ \mu^{C_4}=0.89,\ \mu^{C_5}=0.11\}. \quad (7.35)$$

For the second lower solution $\underline{\mathbf{\mu}}_2^A$, a neural adjustment of the null solution (7.32) has yielded a fuzzy causes vector

$$\underline{\mathbf{\mu}}_{02}^C = (\underline{\mu}_{02}^{C_1} = 0.76, \underline{\mu}_{02}^{C_2} = 0.16, \underline{\mu}_{02}^{C_3} = 0.89, \underline{\mu}_{02}^{C_4} = 1.0, \underline{\mu}_{02}^{C_5} = 0.16) \text{ ,}$$

and for the third lower solution $\underline{\mathbf{\mu}}_3^A$, a neural adjustment of the null solution (7.32) has yielded a fuzzy causes vector

$$\underline{\mathbf{\mu}}_{03}^C = (\underline{\mu}_{03}^{C_1} = 0.76, \underline{\mu}_{03}^{C_2} = 0.16, \underline{\mu}_{03}^{C_3} = 0.96, \underline{\mu}_{03}^{C_4} = 0.89, \underline{\mu}_{03}^{C_5} = 0.16) \text{ ,}$$

for which the value of the optimization criterion (7.14) has constituted F=0.1024.

The resulting null solutions have allowed adjusting the bounds in the solution (7.33) and generating the sets of solutions $\underline{D}_2(\underline{\mathbf{\mu}}_2^A)$ and $\underline{D}_3(\underline{\mathbf{\mu}}_3^A)$, which are completely determined by the least solution

$$\underline{\mathbf{\mu}}^C = (\underline{\mu}^{C_1} = 0.76,\ \underline{\mu}^{C_2} = 0.16,\ \underline{\mu}^{C_3} = 0.89,\ \underline{\mu}^{C_4} = 0.89,\ \underline{\mu}^{C_5} = 0.16)$$

and the two upper solutions $\underline{D}_2^* = \underline{D}_3^* = \{\overline{\mathbf{\mu}}_1^C, \overline{\mathbf{\mu}}_2^C\}$

$$\overline{\mathbf{\mu}}_1^C = (\overline{\mu}_1^{C_1} = 0.76,\ \overline{\mu}_1^{C_2} = 0.16,\ \overline{\mu}_1^{C_3} = 0.89,\ \overline{\mu}_1^{C_4} = 1.0,\ \overline{\mu}_1^{C_5} = 0.16);$$
$$\overline{\mathbf{\mu}}_2^C = (\overline{\mu}_2^{C_1} = 0.76,\ \overline{\mu}_2^{C_2} = 0.16,\ \overline{\mu}_2^{C_3} = 1.0,\ \overline{\mu}_2^{C_4} = 0.89,\ \overline{\mu}_2^{C_5} = 0.16).$$

Thus, the solution of fuzzy relational equations (7.24) for the second and third lower solutions $\underline{\mathbf{\mu}}_2^A$ and $\underline{\mathbf{\mu}}_3^A$ can be represented in the form of intervals:

$$\underline{D}_2(\underline{\mathbf{\mu}}_2^A) = \underline{D}_3(\underline{\mathbf{\mu}}_3^A) = \{\ \mu^{C_1} = 0.76,\ \mu^{C_2} = 0.16,\ \mu^{C_3} = 0.89,\ \mu^{C_4} \in [0.89, 1.0],\ \mu^{C_5} = 0.16\}$$
$$\bigcup \{\ \mu^{C_1} = 0.76,\ \mu^{C_2} = 0.16,\ \mu^{C_3} \in [0.89, 1.0],\ \mu^{C_4} = 0.89,\ \mu^{C_5} = 0.16\}. \qquad (7.36)$$

The intervals of the values of the input variable for each interval in solutions (7.34), (7.35), (7.36) can be defined with the help of the membership functions in Fig. 7.13:

- x_1^*=0.43 or x_1^*=0 for C_1;
- x_1^*=2.12 and x_1^*=3.93 or x_1^*=1.60 and x_1^*=4.45 for C_2;
- $x_1^* \in$ [5.74, 6.0] for C_3;
- $x_2^* \in$ [0, 0.27] for C_4;
- x_2^*=2.17 and x_2^*=3.92 or x_2^*=1.68 or x_2^*=1.35 for C_5.

The restoration of the input set for y_1^*=1.00 and y_2^*=0.60 is shown in Fig. 7.13, in which the values of the causes $C_1 \div C_5$ and effects $E_1 \div E_6$ significances measures are marked. The comparison of the target and restored level lines for y_1^*=1.00 and y_2^*=0.60 is shown in Fig. 7.14.

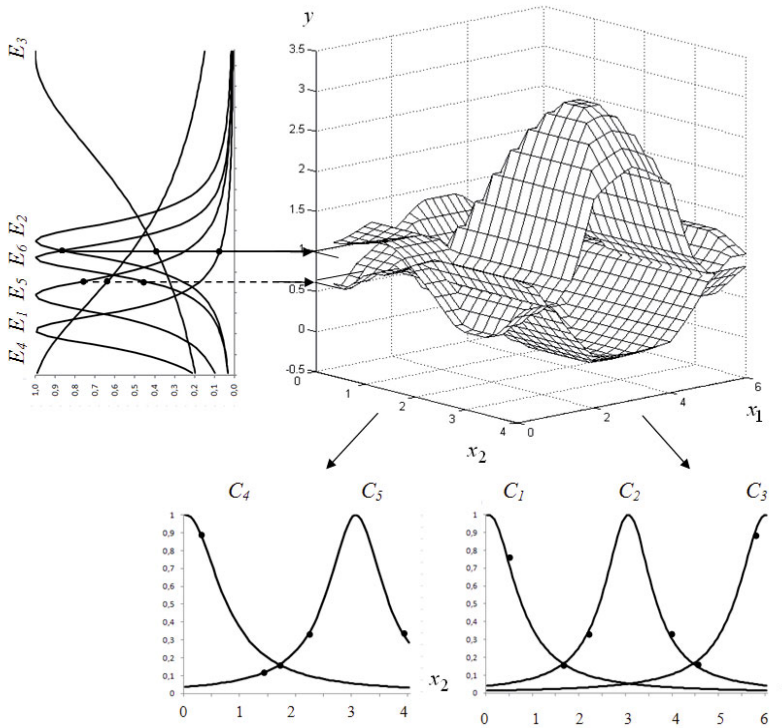

Fig. 7.13. Solution to the problem of inverse fuzzy inference for y_1^*=1.00 and y_2^*=0.60

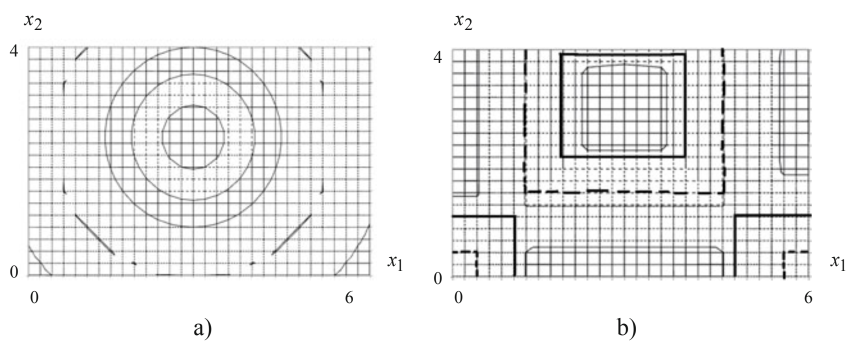

Fig. 7.14. Comparison of the target (a) and restored (b) level lines for y_1^*=1.00 (____) and y_2^*=0.60 (_ _ _)

7.10 Example 6: Hydro Elevator Diagnosis

Let us consider the algorithm's performance having the recourse to the example of the hydraulic elevator faults causes diagnosis. Input parameters of the hydro elevator are (variation ranges are indicated in parentheses):

x_1 – engine speed (30 – 50 r.p.s);

x_2 – inlet pressure (0.02 – 0.15 kg/cm^2);

x_3 – feed change gear clearance (0.1 – 0.3 mm).

Output parameters of the elevator are:

y_1 – productivity (13 – 24 l/min);

y_2 – consumed power (2.1 – 3.0 kw);

y_3 – suction conduit pressure (0.5 – 1 kg/cm^2).

"Causes-effects" interconnection is described with the help of the following system of fuzzy IF-THEN rules:

Rule 1: IF $x_1 = I$ AND $x_2 = I$ AND $x_3 = I$ THEN $y_1 = D$ AND $y_2 = I$ AND $y_3 = D$;

Rule 2: IF $x_1 = D$ AND $x_2 = D$ AND $x_3 = D$ THEN $y_1 = D$ AND $y_2 = D$ AND $y_3 = I$;

Rule 3: IF $x_1 = I$ AND $x_2 = I$ AND $x_3 = D$ THEN $y_1 = D$ AND $y_2 = D$ AND $y_3 = D$;

Rule 4: IF $x_1 = I$ AND $x_2 = D$ AND $x_3 = D$ THEN $y_1 = I$ AND $y_2 = D$ AND $y_3 = D$;

Rule 5: IF $x_1 = D$ AND $x_2 = I$ AND $x_3 = D$ THEN $y_1 = I$ AND $y_2 = D$ AND $y_3 = I$.

where the total number of the causes and effects consists of: c_{11} *Decrease* (*D*) and c_{12} *Increase* (*I*) for x_1; c_{21} (*D*) and c_{21} (*I*) for x_2; c_{31} (*D*) and c_{32} (*I*) for x_3; e_{11} (*D*) and e_{12} (*I*) for y_1; e_{21} (*D*) and e_{22} (*I*) for y_2; e_{31} (*D*) and e_{32} (*I*) for y_3.

We shall define the set of causes and effects in the following way:

$$\{ C_1, C_2, \ldots, C_6 \} = \{ c_{11},\ c_{12},\ c_{21},\ c_{22},\ c_{31},\ c_{32} \};$$

$$\{ E_1, E_2, \ldots, E_6 \} = \{ e_{11},\ e_{12},\ e_{21},\ e_{22},\ e_{31},\ e_{32} \}.$$

This fuzzy rule base is modelled by the fuzzy relational matrix presented in Table 7.6.

Table 7.6. Fuzzy knowledge matrix

IF inputs				THEN outputs					
	x_1	x_2	x_3	y_1		y_2		y_3	
				D	*I*	*D*	*I*	*D*	*I*
A_1	*I*	*I*	*I*	1	0	0	1	1	0
A_2	*D*	*D*	*D*	1	0	1	0	0	1
A_3	*I*	*I*	*D*	1	0	1	0	1	0
A_4	*I*	*D*	*D*	0	1	1	0	1	0
A_5	*D*	*I*	*D*	0	1	1	0	0	1

For the fuzzy model tuning we used the results of diagnosis for 200 hydraulic elevators. The results of the fuzzy model tuning are given in Tables 7.7, 7.8 and in Fig. 7.15.

Table 7.7. Parameters of the membership functions for the causes fuzzy terms after tuning

Parameter	Fuzzy terms					
	C_1	C_2	C_3	C_4	C_5	C_6
β -	32.15	48.65	0.021	0.144	0.11	0.27
σ -	7.75	6.27	0.054	0.048	0.06	0.08

Table 7.8. Parameters of the membership functions for the effects fuzzy terms after tuning

Parameter	Fuzzy terms					
	E_1	E_2	E_3	E_4	E_5	E_6
β -	13.58	21.43	2.24	2.85	0.53	0.98
σ -	4.76	4.58	0.35	0.17	0.31	0.22

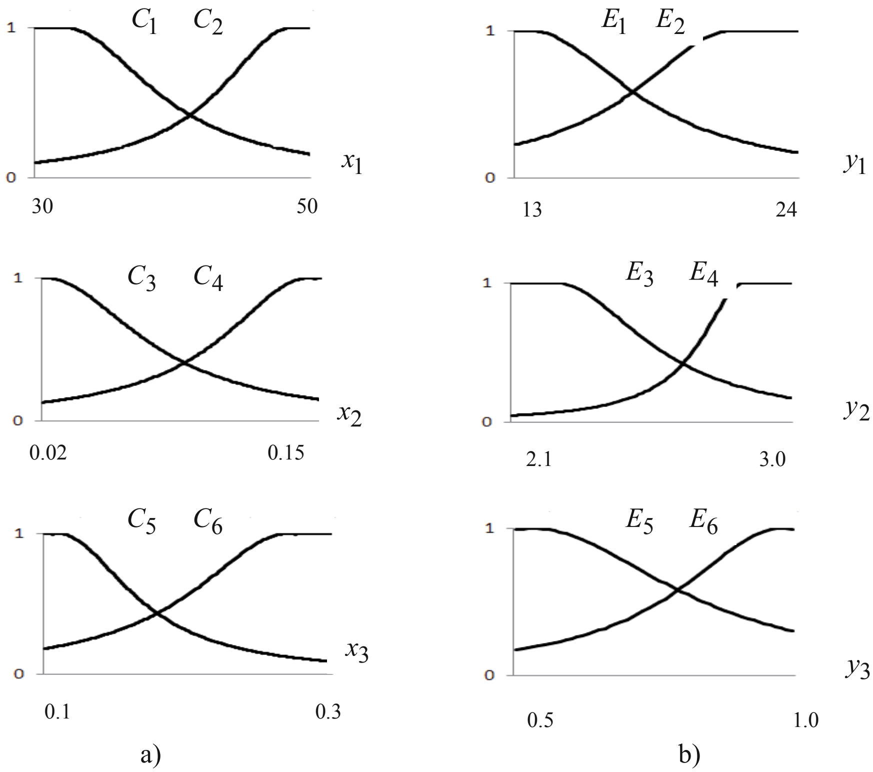

Fig. 7.15. Membership functions of the causes (a) and effects (b) fuzzy terms after tuning

Diagnostic equations after tuning take the form:

$$\mu^{E_1} = (\mu^{A_1} \wedge 0.97) \vee (\mu^{A_2} \wedge 0.65) \vee (\mu^{A_3} \wedge 0.77)$$

$$\mu^{E_2} = (\mu^{A_4} \wedge 1.00) \vee (\mu^{A_5} \wedge 0.46)$$

$$\mu^{E_3} = (\mu^{A_2} \wedge 0.99) \vee (\mu^{A_3} \wedge 0.80) \vee (\mu^{A_4} \wedge 0.69) \vee (\mu^{A_5} \wedge 0.93)$$

$$\mu^{E_4} = (\mu^{A_1} \wedge 0.96)$$

$$\mu^{E_5} = (\mu^{A_1} \wedge 0.72) \vee (\mu^{A_3} \wedge 0.47) \vee (\mu^{A_4} \wedge 0.76)$$

$$\mu^{E_6} = (\mu^{A_2} \wedge 0.92) \vee (\mu^{A_5} \wedge 0.87), \qquad (7.37)$$

where

$$\mu^{A_1} = \mu^{C_2} \wedge \mu^{C_4} \wedge \mu^{C_6}$$
$$\mu^{A_2} = \mu^{C_1} \wedge \mu^{C_3} \wedge \mu^{C_5}$$
$$\mu^{A_3} = \mu^{C_2} \wedge \mu^{C_4} \wedge \mu^{C_5}$$
$$\mu^{A_4} = \mu^{C_2} \wedge \mu^{C_3} \wedge \mu^{C_5}$$
$$\mu^{A_5} = \mu^{C_1} \wedge \mu^{C_4} \wedge \mu^{C_5} . \tag{7.38}$$

Let us represent the vector of the observed parameters for a specific elevator:

$$\mathbf{Y}^* = (y_1^* = 17.10 \text{ l/min};\ y_2^* = 2.45 \text{ kw};\ y_3^* = 0.87 \text{ kg/cm}^2).$$

The measures of the effects significances for these values can be defined with the help of the membership functions in Fig. 7.15,b:

$$\boldsymbol{\mu}^E = (\mu^{E_1}(y_1^*) = 0.65;\ \mu^{E_2}(y_1^*) = 0.53;$$
$$\mu^{E_3}(y_2^*) = 0.74;\ \mu^{E_4}(y_2^*) = 0.15;$$
$$\mu^{E_5}(y_3^*) = 0.45;\ \mu^{E_6}(y_3^*) = 0.80).$$

The genetic algorithm yields a null solution of the optimization problem (7.12)

$$\boldsymbol{\mu}_0^C = (\mu_0^{C_1} = 0.77, \mu_0^{C_2} = 0.49, \mu_0^{C_3} = 0.77, \mu_0^{C_4} = 0.62, \mu_0^{C_5} = 0.77, \mu_0^{C_6} = 0.15) , \tag{7.39}$$

for which the value of the optimization criterion (7.12) takes the value of F=0.0050.

The null vector of the causes combinations significances measures

$$\boldsymbol{\mu}_0^A = (\mu_0^{A_1} = 0.15, \mu_0^{A_2} = 0.77, \mu_0^{A_3} = 0.49, \mu_0^{A_4} = 0.49, \mu_0^{A_5} = 0.62)$$

corresponds to the obtained null solution.

The obtained null solution allows us to arrange for the genetic search for the solution set $S(\mathbf{R}, \boldsymbol{\mu}^E)$, which is completely determined by the greatest solution

$$\overline{\boldsymbol{\mu}}^A = (\overline{\mu}^{A_1} = 0.15,\ \overline{\mu}^{A_2} = 0.77,\ \overline{\mu}^{A_3} = 0.65,\ \overline{\mu}^{A_4} = 0.49,\ \overline{\mu}^{A_5} = 0.77)$$

and the two lower solutions $S^* = \{\underline{\boldsymbol{\mu}}_1^A, \underline{\boldsymbol{\mu}}_2^A\}$

$$\underline{\boldsymbol{\mu}}_1^A = (\underline{\mu}_1^{A_1} = 0.15,\ \underline{\mu}_1^{A_2} = 0.77,\ \underline{\mu}_1^{A_3} = 0,\ \underline{\mu}_1^{A_4} = 0.49,\ \underline{\mu}_1^{A_5} = 0);$$
$$\underline{\boldsymbol{\mu}}_2^A = (\underline{\mu}_2^{A_1} = 0.15,\ \underline{\mu}_2^{A_2} = 0,\ \underline{\mu}_2^{A_3} = 0.65,\ \underline{\mu}_2^{A_4} = 0.49,\ \underline{\mu}_2^{A_5} = 0.77).$$

Thus, the solution of fuzzy relational equations (7.37) can be represented in the form of intervals:

$$S(\mathbf{R},\boldsymbol{\mu}^E)=\{\mu^{A_1}=0.15,\ \mu^{A_2}=0.77,\ \mu^{A_3}\in[0, 0.65],\ \mu^{A_4}=0.49,\ \mu^{A_5}\in[0, 0.77]\}$$
$$\bigcup\{\mu^{A_1}=0.15,\ \mu^{A_2}\in[0, 0.77],\ \mu^{A_3}=0.65,\ \mu^{A_4}=0.49,\ \mu^{A_5}=0.77\}. \quad (7.40)$$

We next apply the genetic algorithm for solving optimization problem (7.14) for the greatest solution $\bar{\boldsymbol{\mu}}^A$ and the two lower solutions $\underline{\boldsymbol{\mu}}_1^A$ and $\underline{\boldsymbol{\mu}}_2^A$.

For the greatest solution $\bar{\boldsymbol{\mu}}^A$, the genetic algorithm yields a null solution of the optimization problem (7.14)

$$\bar{\boldsymbol{\mu}}_0^C=(\bar{\mu}_0^{C_1}=0.77,\bar{\mu}_0^{C_2}=0.57,\bar{\mu}_0^{C_3}=0.94,\bar{\mu}_0^{C_4}=0.80,\bar{\mu}_0^{C_5}=0.79,\bar{\mu}_0^{C_6}=0.15), \quad (7.41)$$

for which the value of the optimization criterion (7.14) is F=0.0128.

The obtained null solution allows us to arrange for the genetic search for the solution set $\bar{D}(\bar{\boldsymbol{\mu}}^A)$, which is completely determined by the least solution

$$\underline{\boldsymbol{\mu}}^C=(\underline{\mu}^{C_1}=0.77,\ \underline{\mu}^{C_2}=0.57,\ \underline{\mu}^{C_3}=0.77,\ \underline{\mu}^{C_4}=0.77,\ \underline{\mu}^{C_5}=0.77,\ \underline{\mu}^{C_6}=0.15)$$

and the four upper solutions $\bar{D}^*=\{\bar{\boldsymbol{\mu}}_1^C,\bar{\boldsymbol{\mu}}_2^C,\bar{\boldsymbol{\mu}}_3^C,\bar{\boldsymbol{\mu}}_4^C\}$

$$\bar{\boldsymbol{\mu}}_1^C=(\bar{\mu}_1^{C_1}=0.77,\ \bar{\mu}_1^{C_2}=0.57,\ \bar{\mu}_1^{C_3}=1.0,\ \bar{\mu}_1^{C_4}=1.0,\ \bar{\mu}_1^{C_5}=1.0,\ \bar{\mu}_1^{C_6}=0.15);$$
$$\bar{\boldsymbol{\mu}}_2^C=(\bar{\mu}_2^{C_1}=1.0,\ \bar{\mu}_2^{C_2}=0.57,\ \bar{\mu}_2^{C_3}=0.77,\ \bar{\mu}_2^{C_4}=1.0,\ \bar{\mu}_2^{C_5}=1.0,\ \bar{\mu}_2^{C_6}=0.15);$$
$$\bar{\boldsymbol{\mu}}_3^C=(\bar{\mu}_3^{C_1}=1.0,\ \bar{\mu}_3^{C_2}=0.57,\ \bar{\mu}_3^{C_3}=1.0,\ \bar{\mu}_3^{C_4}=0.77,\ \bar{\mu}_3^{C_5}=1.0,\ \bar{\mu}_3^{C_6}=0.15);$$
$$\bar{\boldsymbol{\mu}}_4^C=(\bar{\mu}_4^{C_1}=1.0,\ \bar{\mu}_4^{C_2}=0.57,\ \bar{\mu}_4^{C_3}=1.0,\ \bar{\mu}_4^{C_4}=1.0,\ \bar{\mu}_4^{C_5}=0.77,\ \bar{\mu}_4^{C_6}=0.15).$$

Thus, the solution of fuzzy relational equations (7.38) for the greatest solution $\bar{\boldsymbol{\mu}}^A$ can be represented in the form of intervals:

$$\bar{D}(\bar{\boldsymbol{\mu}}^A)=\{\mu^{C_1}=0.77,\mu^{C_2}=0.57,\mu^{C_3}\in[0.77,1.0],\mu^{C_4}\in[0.77,1.0],\mu^{C_5}\in[0.77,1.0],\mu^{C_6}=0.15\}$$
$$\bigcup\{\mu^{C_1}\in[0.77,1.0],\ \mu^{C_2}=0.57,\ \mu^{C_3}=0.77,\ \mu^{C_4}\in[0.77,1.0],\ \mu^{C_5}\in[0.77,1.0],\ \mu^{C_6}=0.15\}$$
$$\bigcup\{\mu^{C_1}\in[0.77,1.0],\ \mu^{C_2}=0.57,\ \mu^{C_3}\in[0.77,1.0],\ \mu^{C_4}=0.77,\ \mu^{C_5}\in[0.77,1.0],\ \mu^{C_6}=0.15\}$$
$$\bigcup\{\mu^{C_1}\in[0.77,1.0],\ \mu^{C_2}=0.57,\ \mu^{C_3}\in[0.77,1.0],\ \mu^{C_4}\in[0.77,1.0],\ \mu^{C_5}=0.77,\ \mu^{C_6}=0.15\}. \quad (7.42)$$

For the first lower solution $\underline{\boldsymbol{\mu}}_1^A$, the genetic algorithm yields a null solution of the optimization problem (7.14)

$$\underline{\boldsymbol{\mu}}_{01}^C=(\underline{\mu}_{01}^{C_1}=0.77,\underline{\mu}_{01}^{C_2}=0.49,\underline{\mu}_{01}^{C_3}=0.84,\underline{\mu}_{01}^{C_4}=0,\underline{\mu}_{01}^{C_5}=0.92,\underline{\mu}_{01}^{C_6}=0), \quad (7.43)$$

for which the value of the optimization criterion (7.14) is F=0.0225.

The obtained null solution allows us to arrange for the genetic search for the solution set $\underline{D}_1(\underline{\boldsymbol{\mu}}_1^A)$, which is completely determined by the least solution

$$\underline{\boldsymbol{\mu}}^C = (\underline{\mu}^{C_1}=0.77,\ \underline{\mu}^{C_2}=0.49,\ \underline{\mu}^{C_3}=0.77,\ \underline{\mu}^{C_4}=0,\ \underline{\mu}^{C_5}=0.77,\ \underline{\mu}^{C_6}=0)$$

and the three upper solutions $\underline{D}_1^* = \{\overline{\boldsymbol{\mu}}_1^C, \overline{\boldsymbol{\mu}}_2^C, \overline{\boldsymbol{\mu}}_3^C\}$

$$\overline{\boldsymbol{\mu}}_1^C = (\overline{\mu}_1^{C_1}=0.77,\ \overline{\mu}_1^{C_2}=0.49,\ \overline{\mu}_1^{C_3}=1.0,\ \overline{\mu}_1^{C_4}=0,\ \overline{\mu}_1^{C_5}=1.0,\ \overline{\mu}_1^{C_6}=0);$$
$$\overline{\boldsymbol{\mu}}_2^C = (\overline{\mu}_2^{C_1}=1.0,\ \overline{\mu}_2^{C_2}=0.49,\ \overline{\mu}_2^{C_3}=0.77,\ \overline{\mu}_2^{C_4}=0,\ \overline{\mu}_2^{C_5}=1.0,\ \overline{\mu}_2^{C_6}=0);$$
$$\overline{\boldsymbol{\mu}}_3^C = (\overline{\mu}_3^{C_1}=1.0,\ \overline{\mu}_3^{C_2}=0.49,\ \overline{\mu}_3^{C_3}=1.0,\ \overline{\mu}_3^{C_4}=0,\ \overline{\mu}_3^{C_5}=0.77,\ \overline{\mu}_3^{C_6}=0).$$

Thus, the solution of fuzzy relational equations (7.38) for the first lower solution $\underline{\boldsymbol{\mu}}_1^A$ can be represented in the form of intervals:

$$\underline{D}_1(\underline{\boldsymbol{\mu}}_1^A) = \{\mu^{C_1}=0.77,\ \mu^{C_2}=0.49,\ \mu^{C_3}\in[0.77, 1.0],\ \mu^{C_4}=0,\ \mu^{C_5}\in[0.77, 1.0],\ \mu^{C_6}=0\}$$
$$\bigcup \{\mu^{C_1}\in[0.77, 1.0],\ \mu^{C_2}=0.49,\ \mu^{C_3}=0.77,\ \mu^{C_4}=0,\ \mu^{C_5}\in[0.77, 1.0],\ \mu^{C_6}=0\}$$
$$\bigcup \{\mu^{C_1}\in[0.77, 1.0],\ \mu^{C_2}=0.49,\ \mu^{C_3}\in[0.77, 1.0],\ \mu^{C_4}=0,\ \mu^{C_5}=0.77,\ \mu^{C_6}=0\}. \quad (7.44)$$

For the second lower solution $\underline{\boldsymbol{\mu}}_2^A$, the genetic algorithm yields a null solution of the optimization problem (7.14)

$$\underline{\boldsymbol{\mu}}_{02}^C = (\underline{\mu}_{02}^{C_1}=0.77, \underline{\mu}_{02}^{C_2}=0.65, \underline{\mu}_{02}^{C_3}=0.25, \underline{\mu}_{02}^{C_4}=0.97, \underline{\mu}_{02}^{C_5}=0.85, \underline{\mu}_{02}^{C_6}=0.15), \quad (7.45)$$

for which the value of the optimization criterion (7.14) is F=0.1201.

The obtained null solution allows us to arrange for the genetic search for the solution set $\underline{D}_2(\underline{\boldsymbol{\mu}}_2^A)$, which is completely determined by the least solution

$$\underline{\boldsymbol{\mu}}^C = (\underline{\mu}^{C_1}=0.77,\ \underline{\mu}^{C_2}=0.65,\ \underline{\mu}^{C_3}=0.25,\ \underline{\mu}^{C_4}=0.77,\ \underline{\mu}^{C_5}=0.77,\ \underline{\mu}^{C_6}=0.15)$$

and the three upper solutions $\underline{D}_2^* = \{\overline{\boldsymbol{\mu}}_1^C, \overline{\boldsymbol{\mu}}_2^C, \overline{\boldsymbol{\mu}}_3^C\}$

$$\overline{\boldsymbol{\mu}}_1^C = (\overline{\mu}_1^{C_1}=0.77,\ \overline{\mu}_1^{C_2}=0.65,\ \overline{\mu}_1^{C_3}=0.25,\ \overline{\mu}_1^{C_4}=1.0,\ \overline{\mu}_1^{C_5}=1.0,\ \overline{\mu}_1^{C_6}=0.15);$$
$$\overline{\boldsymbol{\mu}}_2^C = (\overline{\mu}_2^{C_1}=1.0,\ \overline{\mu}_2^{C_2}=0.65,\ \overline{\mu}_2^{C_3}=0.25,\ \overline{\mu}_2^{C_4}=0.77,\ \overline{\mu}_2^{C_5}=1.0,\ \overline{\mu}_2^{C_6}=0.15);$$
$$\overline{\boldsymbol{\mu}}_3^C = (\overline{\mu}_3^{C_1}=1.0,\ \overline{\mu}_3^{C_2}=0.65,\ \overline{\mu}_3^{C_3}=0.25,\ \overline{\mu}_3^{C_4}=1.0,\ \overline{\mu}_3^{C_5}=0.77,\ \overline{\mu}_3^{C_6}=0.15).$$

Thus, the solution of fuzzy relational equations (7.38) for the second lower solution $\underline{\boldsymbol{\mu}}_2^A$ can be represented in the form of intervals:

$$\underline{D}_2(\underline{\boldsymbol{\mu}}_2^A)=\{\mu^{C_1}=0.77,\ \mu^{C_2}=0.65,\ \mu^{C_3}=0.25,\ \mu^{C_4}\in[0.77,1.0],\ \mu^{C_5}\in[0.77,1.0],\ \mu^{C_6}=0.15\}$$

$$\bigcup\{\mu^{C_1}\in[0.77,1.0],\ \mu^{C_2}=0.65,\ \mu^{C_3}=0.25,\ \mu^{C_4}=0.77,\ \mu^{C_5}\in[0.77,1.0],\ \mu^{C_6}=0.15\}$$

$$\bigcup\{\mu^{C_1}\in[0.77,1.0],\ \mu^{C_2}=0.65,\ \mu^{C_3}=0.25,\ \mu^{C_4}\in[0.77,1.0],\ \mu^{C_5}=0.77,\ \mu^{C_6}=0.15\}. \quad (7.46)$$

Following the solutions (7.42), (7.44), (7.46), the causes C_1, C_3, C_4 and C_5 are the causes of the observed elevator state, so that $\mu^{C_1}>\mu^{C_2}$, $\mu^{C_3}=\mu^{C_4}$, $\mu^{C_5}>\mu^{C_6}$. The intervals of the values of the input variables for these causes can be defined with the help of the membership functions in Fig. 7.15,a:

- $x_1^*\in[30.0, 36.4]$ r.p.s for C_1;
- $x_2^*\in[0.020, 0.050]$ kg/cm^2 for C_3 and $x_2^*\in[0.118, 0.150]$ kg/cm^2 for C_4;
- $x_3^*\in[0.100, 0.143]$ mm for C_5.

The obtained solution allows the analyst to make the preliminary conclusions. The elevator failure may be because of the engine speed reduced to 30 – 36 r.p.s, the inlet pressure decreased to 0.020 – 0.050 kg/cm^2 or increased to 0.118 – 0.150 kg/cm^2, and the feed change gear clearance decreased to 100 – 143 mkm.

Assume a repeated measurement has revealed an increase in the elevator productivity up to y_1^*=18.80 l/min, an increase of the consumed power up to y_2^*=2.51 kw, and a decrease in the suction pressure up to y_3^*=0.75 kg/cm^2.

For the new values, the fuzzy effects vector is

$$\boldsymbol{\mu}^E=(\mu^{E_1}(y_1^*)=0.45;\ \mu^{E_2}(y_1^*)=0.75;$$
$$\mu^{E_3}(y_2^*)=0.63;\ \mu^{E_4}(y_2^*)=0.20;$$
$$\mu^{E_5}(y_3^*)=0.67;\ \mu^{E_6}(y_3^*)=0.48).$$

A neural adjustment of the null solution (7.39) of the optimization problem (7.12) has yielded a fuzzy causes vector

$$\boldsymbol{\mu}_0^C=(\mu_0^{C_1}=0.46,\ \mu_0^{C_2}=0.69,\ \mu_0^{C_3}=0.75,\ \mu_0^{C_4}=0.25,\ \mu_0^{C_5}=0.92,\ \mu_0^{C_6}=0.20),$$

for which the value of the optimization criterion (7.12) has constituted F=0.0094.

The null vector of the causes combinations significances measures

$$\boldsymbol{\mu}_0^A=(\mu_0^{A_1}=0.20,\ \mu_0^{A_2}=0.46,\ \mu_0^{A_3}=0.25,\ \mu_0^{A_4}=0.69,\ \mu_0^{A_5}=0.25),$$

corresponds to the obtained null solution.

The resulting null solution has allowed adjusting the bounds in the solution (7.40) and generating the set of solutions $S(\mathbf{R},\boldsymbol{\mu}^E)$ determined by the greatest solution

$$\overline{\boldsymbol{\mu}}^A = (\overline{\mu}^{A_1}=0.20,\ \overline{\mu}^{A_2}=0.46,\ \overline{\mu}^{A_3}=0.46,\ \overline{\mu}^{A_4}=0.69,\ \overline{\mu}^{A_5}=0.46)$$

and the two lower solutions $S^* = \{\underline{\boldsymbol{\mu}}_1^A, \underline{\boldsymbol{\mu}}_2^A\}$

$$\underline{\boldsymbol{\mu}}_1^A = (\underline{\mu}_1^{A_1}=0.20,\ \underline{\mu}_1^{A_2}=0.46,\ \underline{\mu}_1^{A_3}=0,\ \underline{\mu}_1^{A_4}=0.69,\ \underline{\mu}_1^{A_5}=0);$$

$$\underline{\boldsymbol{\mu}}_2^A = (\underline{\mu}_2^{A_1}=0.20,\ \underline{\mu}_2^{A_2}=0,\ \underline{\mu}_2^{A_3}=0.46,\ \underline{\mu}_2^{A_4}=0.69,\ \underline{\mu}_2^{A_5}=0.46).$$

Thus, the solution of fuzzy relational equations (7.37) for the new values can be represented in the form of intervals:

$$S(\mathbf{R},\boldsymbol{\mu}^E) = \{\mu^{A_1}=0.20,\ \mu^{A_2}=0.46,\ \mu^{A_3}\in[0,\,0.46],\ \mu^{A_4}=0.69,\ \mu^{A_5}\in[0,\,0.46]\}$$
$$\cup\{\mu^{A_1}=0.20,\ \mu^{A_2}\in[0,\,0.46],\ \mu^{A_3}=0.46,\ \mu^{A_4}=0.69,\ \mu^{A_5}=0.46\}.$$

For the greatest solution $\overline{\boldsymbol{\mu}}^A$, a neural adjustment of the null solution (7.41) has yielded a fuzzy causes vector

$$\overline{\boldsymbol{\mu}}_0^C = (\overline{\mu}_0^{C_1}=0.46,\ \overline{\mu}_0^{C_2}=0.78,\ \overline{\mu}_0^{C_3}=0.69,\ \overline{\mu}_0^{C_4}=0.46,\ \overline{\mu}_0^{C_5}=0.91,\ \overline{\mu}_0^{C_6}=0.20),$$

for which the value of the optimization criterion (7.14) has constituted F=0.

The resultant null solution has allowed adjusting the bounds in the solution (7.42) and generating the set of solutions $\overline{D}(\overline{\boldsymbol{\mu}}^A)$, which is completely determined by the least solution

$$\underline{\boldsymbol{\mu}}^C = (\underline{\mu}^{C_1}=0.46,\ \underline{\mu}^{C_2}=0.69,\ \underline{\mu}^{C_3}=0.69,\ \underline{\mu}^{C_4}=0.46,\ \underline{\mu}^{C_5}=0.69,\ \underline{\mu}^{C_6}=0.20)$$

and the three upper solutions $\overline{D}^* = \{\overline{\boldsymbol{\mu}}_1^C, \overline{\boldsymbol{\mu}}_2^C, \overline{\boldsymbol{\mu}}_3^C\}$

$$\overline{\boldsymbol{\mu}}_1^C = (\overline{\mu}_1^{C_1}=0.46,\ \overline{\mu}_1^{C_2}=0.69,\ \overline{\mu}_1^{C_3}=1.0,\ \overline{\mu}_1^{C_4}=0.46,\ \overline{\mu}_1^{C_5}=1.0,\ \overline{\mu}_1^{C_6}=0.20);$$

$$\overline{\boldsymbol{\mu}}_2^C = (\overline{\mu}_2^{C_1}=0.46,\ \overline{\mu}_2^{C_2}=1.0,\ \overline{\mu}_2^{C_3}=0.69,\ \overline{\mu}_2^{C_4}=0.46,\ \overline{\mu}_2^{C_5}=1.0,\ \overline{\mu}_2^{C_6}=0.20);$$

$$\overline{\boldsymbol{\mu}}_3^C = (\overline{\mu}_3^{C_1}=0.46,\ \overline{\mu}_3^{C_2}=1.0,\ \overline{\mu}_3^{C_3}=1.0,\ \overline{\mu}_3^{C_4}=0.46,\ \overline{\mu}_3^{C_5}=0.69,\ \overline{\mu}_3^{C_6}=0.20).$$

Thus, the solution of fuzzy relational equations (7.38) for the greatest solution $\bar{\boldsymbol{\mu}}^A$ can be represented in the form of intervals:

$$\overline{D}(\bar{\boldsymbol{\mu}}^A)=\{\mu^{C_1}=0.46,\ \mu^{C_2}=0.69,\ \mu^{C_3}\in[0.69,1.0],\ \mu^{C_4}=0.46,\ \mu^{C_5}\in[0.69,1.0],\ \mu^{C_6}=0.20\}$$
$$\cup\{\mu^{C_1}=0.46,\ \mu^{C_2}\in[0.69,1.0],\ \mu^{C_3}=0.69,\ \mu^{C_4}=0.46,\ \mu^{C_5}\in[0.69,1.0],\ \mu^{C_6}=0.20\}$$
$$\cup\{\mu^{C_1}=0.46,\ \mu^{C_2}\in[0.69,1.0],\ \mu^{C_3}\in[0.69,1.0],\ \mu^{C_4}=0.46,\ \mu^{C_5}=0.69,\ \mu^{C_6}=0.20\} \quad (7.47)$$

For the first lower solution $\underline{\boldsymbol{\mu}}_1^A$, a neural adjustment of the null solution (7.43) has yielded a fuzzy causes vector

$$\underline{\boldsymbol{\mu}}_{01}^C=(\underline{\mu}_{01}^{C_1}=0.46,\ \underline{\mu}_{01}^{C_2}=0.92,\ \underline{\mu}_{01}^{C_3}=0.86,\ \underline{\mu}_{01}^{C_4}=0.10,\ \underline{\mu}_{01}^{C_5}=0.69,\ \underline{\mu}_{01}^{C_6}=0.10),$$

for which the value of the optimization criterion (7.14) has constituted F=0.0300.

The resulting null solution has allowed adjusting the bounds in the solution (7.44) and generating the set of solutions $\underline{D}_1(\underline{\boldsymbol{\mu}}_1^A)$, which is completely determined by the least solution

$$\underline{\boldsymbol{\mu}}^C=(\underline{\mu}^{C_1}=0.46,\ \underline{\mu}^{C_2}=0.69,\ \underline{\mu}^{C_3}=0.69,\ \underline{\mu}^{C_4}=0.10,\ \underline{\mu}^{C_5}=0.69,\ \underline{\mu}^{C_6}=0.10)$$

and the three upper solutions $\underline{D}_1^*=\{\bar{\boldsymbol{\mu}}_1^C,\bar{\boldsymbol{\mu}}_2^C,\bar{\boldsymbol{\mu}}_3^C\}$

$$\bar{\boldsymbol{\mu}}_1^C=(\bar{\mu}_1^{C_1}=0.46,\ \bar{\mu}_1^{C_2}=0.69,\ \bar{\mu}_1^{C_3}=1.0,\ \bar{\mu}_1^{C_4}=0.10,\ \bar{\mu}_1^{C_5}=1.0,\ \bar{\mu}_1^{C_6}=0.10);$$
$$\bar{\boldsymbol{\mu}}_2^C=(\bar{\mu}_2^{C_1}=0.46,\ \bar{\mu}_2^{C_2}=1.0,\ \bar{\mu}_2^{C_3}=0.69,\ \bar{\mu}_2^{C_4}=0.10,\ \bar{\mu}_2^{C_5}=1.0,\ \bar{\mu}_2^{C_6}=0.10);$$
$$\bar{\boldsymbol{\mu}}_3^C=(\bar{\mu}_3^{C_1}=0.46,\ \bar{\mu}_3^{C_2}=1.0,\ \bar{\mu}_3^{C_3}=1.0,\ \bar{\mu}_3^{C_4}=0.10,\ \bar{\mu}_3^{C_5}=0.69,\ \bar{\mu}_3^{C_6}=0.10).$$

Thus, the solution of fuzzy relational equations (7.38) for the first lower solution $\underline{\boldsymbol{\mu}}_1^A$ can be represented in the form of intervals:

$$\underline{D}_1(\underline{\boldsymbol{\mu}}_1^A)=\{\mu^{C_1}=0.46,\ \mu^{C_2}=0.69,\ \mu^{C_3}\in[0.69,1.0],\ \mu^{C_4}=0.10,\ \mu^{C_5}\in[0.69,1.0],\ \mu^{C_6}=0.10\}$$
$$\cup\{\mu^{C_1}=0.46,\ \mu^{C_2}\in[0.69,1.0],\ \mu^{C_3}=0.69,\ \mu^{C_4}=0.10,\ \mu^{C_5}\in[0.69,1.0],\ \mu^{C_6}=0.10\}$$
$$\cup\{\mu^{C_1}=0.46,\ \mu^{C_2}\in[0.69,1.0],\ \mu^{C_3}\in[0.69,1.0],\ \mu^{C_4}=0.10,\ \mu^{C_5}=0.69,\ \mu^{C_6}=0.10\}. \quad (7.48)$$

For the second lower solution $\underline{\boldsymbol{\mu}}_2^A$, a neural adjustment of the null solution (7.45) has yielded a fuzzy causes vector

$$\underline{\boldsymbol{\mu}}_{02}^C = (\underline{\mu}_{02}^{C_1} = 0.23, \underline{\mu}_{02}^{C_2} = 0.69, \underline{\mu}_{02}^{C_3} = 0.83, \underline{\mu}_{02}^{C_4} = 0.46, \underline{\mu}_{02}^{C_5} = 0.97, \underline{\mu}_{02}^{C_6} = 0.20),$$

for which the value of the optimization criterion (7.14) has constituted F=0.1058.

The resulting null solution has allowed adjusting the bounds in the solution (7.46) and generating the sets of solutions $\underline{D}_2(\underline{\boldsymbol{\mu}}_2^A)$, which is completely determined by the least solution

$$\underline{\boldsymbol{\mu}}^C = (\underline{\mu}^{C_1} = 0.23,\ \underline{\mu}^{C_2} = 0.69,\ \underline{\mu}^{C_3} = 0.69,\ \underline{\mu}^{C_4} = 0.46,\ \underline{\mu}^{C_5} = 0.69,\ \underline{\mu}^{C_6} = 0.20)$$

and the three upper solutions $\underline{D}_2^* = \{\overline{\boldsymbol{\mu}}_1^C, \overline{\boldsymbol{\mu}}_2^C, \overline{\boldsymbol{\mu}}_3^C\}$

$$\overline{\boldsymbol{\mu}}_1^C = (\overline{\mu}_1^{C_1} = 0.23,\ \overline{\mu}_1^{C_2} = 0.69,\ \overline{\mu}_1^{C_3} = 1.0,\ \overline{\mu}_1^{C_4} = 0.46,\ \overline{\mu}_1^{C_5} = 1.0,\ \overline{\mu}_1^{C_6} = 0.20);$$
$$\overline{\boldsymbol{\mu}}_2^C = (\overline{\mu}_2^{C_1} = 0.23,\ \overline{\mu}_2^{C_2} = 1.0,\ \overline{\mu}_2^{C_3} = 0.69,\ \overline{\mu}_2^{C_4} = 0.46,\ \overline{\mu}_2^{C_5} = 1.0,\ \overline{\mu}_2^{C_6} = 0.20);$$
$$\overline{\boldsymbol{\mu}}_3^C = (\overline{\mu}_3^{C_1} = 0.23,\ \overline{\mu}_3^{C_2} = 1.0,\ \overline{\mu}_3^{C_3} = 1.0,\ \overline{\mu}_3^{C_4} = 0.46,\ \overline{\mu}_3^{C_5} = 0.69,\ \overline{\mu}_3^{C_6} = 0.20).$$

Thus, the solution of fuzzy relational equations (7.38) for the second lower solution $\underline{\boldsymbol{\mu}}_2^A$ can be represented in the form of intervals:

$$\begin{aligned}\underline{D}_2(\underline{\boldsymbol{\mu}}_2^A) = \{&\mu^{C_1} = 0.23,\ \mu^{C_2} = 0.69,\ \mu^{C_3} \in [0.69, 1.0],\ \mu^{C_4} = 0.46,\ \mu^{C_5} \in [0.69, 1.0],\ \mu^{C_6} = 0.20\} \\ \cup\{&\mu^{C_1} = 0.23,\ \mu^{C_2} \in [0.69, 1.0],\ \mu^{C_3} = 0.69,\ \mu^{C_4} = 0.46,\ \mu^{C_5} \in [0.69, 1.0],\ \mu^{C_6} = 0.20\} \\ \cup\{&\mu^{C_1} = 0.23,\ \mu^{C_2} \in [0.69, 1.0],\ \mu^{C_3} \in [0.69, 1.0],\ \mu^{C_4} = 0.46,\ \mu^{C_5} = 0.69,\ \mu^{C_6} = 0.20\}.\end{aligned} \quad (7.49)$$

Following the resultant solutions (7.47), (7.48), (7.49), the causes C_2, C_3 and C_5 are the causes of the observed elevator state, since $\mu^{C_2} > \mu^{C_1}$, $\mu^{C_3} > \mu^{C_4}$, $\mu^{C_5} > \mu^{C_6}$. The intervals of the values of the input variables for these causes can be defined with the help of the membership functions in Fig. 7.15,a:

- $x_1^* \in [44.4, 50.0]$ r.p.s for C_2;
- $x_2^* \in [0.020, 0.057]$ kg/cm^2 for C_3;
- $x_3^* \in [0.100, 0.150]$ mm for C_5.

The solution obtained allows the final conclusions. Thus, the causes of the observed elevator state should be located and identified as the increase of the engine speed to 45-50 r.p.s, the decrease of the inlet pressure to 0.020 – 0.057 kg/cm^2, and the decrease of the feed change gear clearance to 100-150 mk.

To test the fuzzy model we used the results of diagnosis for 192 elevators with different kinds of faults. The tuning algorithm efficiency characteristics for the testing data are given in Table 7.9. Attaining a 96% correctness of the diagnostics required 30 min of operation of the genetic algorithm and 7 min of operation of the neural network (Intel Core 2 Duo P7350 2.0 GHz).

Table 7.9. Tuning algorithm efficiency characteristics

Causes (diagnoses)	Number of cases in the data sample	Probability of the correct diagnosis		
		Before tuning	After tuning	
			Null solution (genetic algorithm)	Refined diagnoses (neural network)
C_1	104	86 / 104=0.82	96 / 104=0.92	101 / 104=0.97
C_2	88	67 / 88=0.76	80 / 88=0.91	84 / 88=0.95
C_3	92	74 / 92=0.80	82 / 92=0.89	88 / 92=0.95
C_4	100	70 / 100=0.70	93 / 100=0.93	97 / 100=0.97
C_5	122	103 / 122=0.84	109 / 122=0.89	117 / 122=0.96
C_6	70	51 / 70=0.73	61 / 70=0.87	68 / 70=0.97

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