Chapter 6
Inverse Inference with Fuzzy Relations Tuning

Diagnosis, i.e. determination of the identity of the observed phenomena, is the most important stage of decision making in different domains of human activity: medicine, engineering, economics, military affairs, and others. In the case of the diagnosis of problems where physical mechanisms are not well known due to high complexity and nonlinearity, a fuzzy relational model may be useful. A fuzzy relational model for simulating cause and effect connections in diagnosing problems has been introduced by Sanchez [1, 2]. A model for diagnosis can be built on the basis of Zadeh's compositional rule of inference [3], in which the fuzzy matrix of "causes-effects" relations serves as the support of the diagnostic information. In this case, the problem of diagnosis amounts to solving fuzzy relational equations.

Inverse problem resolution is of interest to both exact methods and approximate ones. The complete bibliographical notes are presented in [4]. Analytically exact methods for fuzzy relational equations on various lattices and with different kinds of composition laws for fuzzy relations are given in [4 – 8]. There exist tasks in which approximate solutions instead of exact ones are reasonable [9]. Solvability and approximate solvability conditions of fuzzy relational equations are considered in [10 – 14]. In the general case, an optimization environment is a convenient tool for decomposing fuzzy relations. Solving fuzzy relational equations by neural networks is described in [15, 16]. The use of genetic optimization for decomposition of fuzzy relations is proposed in [17].

The necessary condition of diagnostic problem solving is to ascertain the cause-effect relationship. A general methodological scheme envisages structure determination, parameter identification and model validation [18, 19]. An approach of integrated genetic and gradient-based learning in construction of fuzzy relational models is proposed in [20]. An approach of identification of fuzzy relational models by fuzzy neural networks is proposed in [21, 22].

In those cases, when domain experts are involved in developing fuzzy models, construction of the cause-effect connections can be considered as rough tuning of the fuzzy relational model [23]. The observed (output) and diagnosed (input) parameters of a system are considered as linguistic variables [3]. Fuzzy terms, e.g.

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"temperature rise", "pressure drop" etc., associated with causes and effects are used for these variables evaluation. The use of the expert relational matrix cannot guarantee the coincidence of theoretical results of diagnosis and real data. In other words, the "quality" of the model strongly depends on the "quality" of the expert forming the diagnostic matrix. In addition, the problem of solving fuzzy relational equations is still relevant – as of yet there does not exist a satisfactory answer for computing a complete solution set [4].

In chapter 5, a pure expert system using a genetic algorithm [24, 25] as a tool to solve the diagnosis problem was proposed. In this chapter, we propose an approach for building fuzzy systems of diagnosis, which enables solving fuzzy relational equations together with design and tuning of fuzzy relations on the basis of expert and experimental information [26, 27]. The essence of tuning consists of the selection of such membership functions of the fuzzy terms for the input and output variables (causes and effects) and such "causes-effects" fuzzy relations, which provide minimal difference between theoretical and experimental results of diagnosis.

To overcome the *NP*-hardness, chapter 5 used the ideology of genetic optimization [24, 25], which quickly established the domain of global minimum of the discrepancy between the left and right sides of the system of equations followed by a fine adjustment of the solution by search methods available. The genetic algorithm uses all the available experimental information for the optimization, i.e., operates off-line and becomes toilful and inefficient if new experimental data are obtained, i.e., in the on-line mode. The process of diagnosis should be augmented by a hybrid genetic and neuro approach to designing adaptive diagnostic systems [28]. The essence of the approach is in constructing and training a special neuro-fuzzy network isomorphic to the diagnostic equations, which allows on-line correction of decisions.

This chapter is written using original work materials [26 – 28].

6.1 Diagnostic Approximator Based on Fuzzy Relations

The diagnosis object is treated as a black box with n inputs and m outputs (Fig. 6.1). Outputs of the object are associated with the observed effects (symptoms). Inputs correspond to the causes of the observed effects (diagnoses). The problem of diagnosis consists of restoration and identification of the causes (inputs) through the observed effects (outputs). Inputs and outputs can be considered as linguistic variables given on the corresponding universal sets. Fuzzy terms are used for these linguistic variables evaluation.

We shall denote:

$\{x_1, x_2, ..., x_n\}$ is the set of input parameters, $x_i \in [\underline{x}_i, \overline{x}_i]$, $i = \overline{1, n}$;

$\{y_1, y_2, ..., y_m\}$ is the set of output parameters, $y_j \in [\underline{y}_j, \overline{y}_j]$, $j = \overline{1, m}$;

$\{c_{i1}, c_{i2}, ..., c_{ik_i}\}$ is the set of linguistic terms for parameter x_i evaluation, $i = \overline{1, n}$;

$\{e_{j1}, e_{j2}, ..., e_{jq_j}\}$ is the set of linguistic terms for parameter y_j evaluation, $j = \overline{1, m}$.

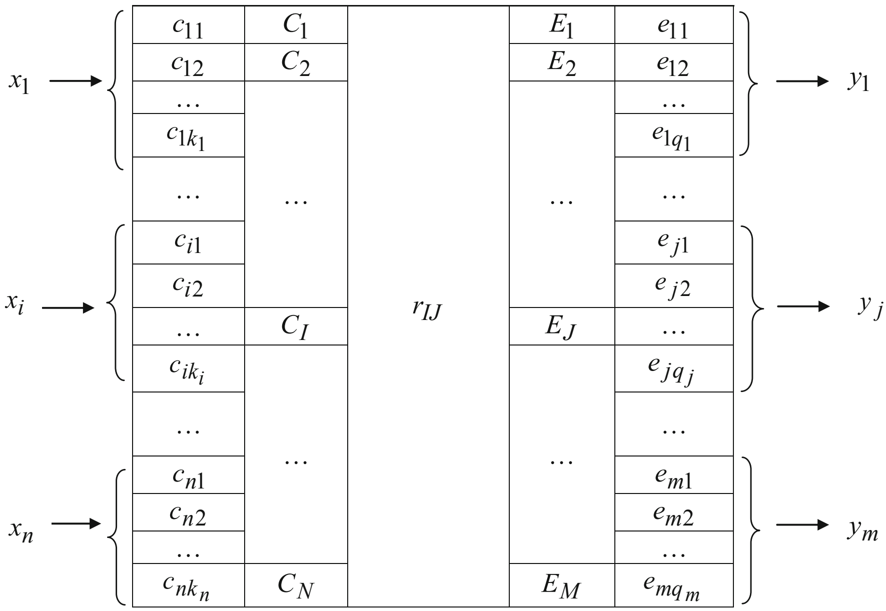


Fig. 6.1. The object of diagnosis

Each term-assessment is described with the help of a fuzzy set:

$$c_{il} = \{(x_i, \mu^{c_{il}}(x_i))\}, \quad i = \overline{1,n}, \quad l = \overline{1,k_i};$$

$$e_{jp} = \{(y_j, \mu^{e_{jp}}(y_j))\}, \quad j = \overline{1,m}, \quad p = \overline{1,q_j}.$$

where $\mu^{c_{il}}(x_i)$ is a membership function of variable x_i to the term-assessment c_{il}, $i = \overline{1,n}$, $l = \overline{1,k_i}$;

$\mu^{e_{jp}}(y_j)$ is a membership function of variable y_j to the term-assessment e_{jp}, $j = \overline{1,m}$, $p = \overline{1,q_j}$.

We shall redenote the set of input and output terms-assessments in the following way:

$\{C_1, C_2, ..., C_N\} = \{c_{11}, c_{12}, ..., c_{1k_1}, ..., c_{n1}, c_{n2}, ..., c_{nk_n}\}$ is the set of terms for input parameters evaluation, where $N = k_1 + k_2 + ... + k_n$;

$\{E_1, E_2, ..., E_M\} = \{e_{11}, e_{12}, ..., e_{1q_1}, ..., e_{m1}, e_{m2}, ..., e_{mq_m}\}$ is the set of terms for output parameters evaluation, where $M = q_1 + q_2 + ... + q_m$.

Set $\{C_I, I = \overline{1,N}\}$ is called fuzzy causes (diagnoses), and set $\{E_J, J = \overline{1,M}\}$ is called fuzzy effects (symptoms).

The diagnostic problem is set in the following way: it is necessary to restore and identify the values of the input parameters $(x_1^*, x_2^*, ..., x_n^*)$ through the values of the observed output parameters $(y_1^*, y_2^*, ..., y_m^*)$.

"Causes-effects" interconnection is given by the matrix of fuzzy relations

$$\mathbf{R} \subseteq C_I \times E_J = [\, r_{IJ}\,,\ I = \overline{1,N}\,,\ J = \overline{1,M}\,].$$

An element of this matrix is a number $r_{IJ} \in [0, 1]$, characterizing the degree to which cause C_I influences upon the rise of effect E_J.

In the presence of matrix **R** the "causes-effects" dependency can be described with the help of Zadeh's compositional rule of inference [3]

$$\boldsymbol{\mu}^E = \boldsymbol{\mu}^C \circ \mathbf{R}, \tag{6.1}$$

where $\boldsymbol{\mu}^C = (\mu^{C_1}, \mu^{C_2}, ..., \mu^{C_N})$ is the fuzzy causes vector with elements $\mu^{C_I} \in [0, 1]$, interpreted as some significance measures of C_I causes;

$\boldsymbol{\mu}^E = (\mu^{E_1}, \mu^{E_2}, ..., \mu^{E_M})$ is the fuzzy effects vector with elements $\mu^{E_J} \in [0, 1]$, interpreted as some significance measures of E_J effects;

$\circ$ is the operation of *max-min* composition [3].

Finding vector $\boldsymbol{\mu}^C$ amounts to the solution of the fuzzy relational equations:

$$\begin{aligned}
\mu^{E_1} &= (\mu^{C_1} \wedge r_{11}) \vee (\mu^{C_2} \wedge r_{21}) ... \vee (\mu^{C_N} \wedge r_{N1}) \\
\mu^{E_2} &= (\mu^{C_1} \wedge r_{12}) \vee (\mu^{C_2} \wedge r_{22}) ... \vee (\mu^{C_N} \wedge r_{N2}) \\
& \cdots \qquad \cdots \qquad \cdots \qquad \cdots \\
\mu^{E_M} &= (\mu^{C_1} \wedge r_{1M}) \vee (\mu^{C_2} \wedge r_{2M}) ... \vee (\mu^{C_N} \wedge r_{NM}),
\end{aligned} \tag{6.2}$$

which is derived from relation (6.1). Taking into account the fact that operations $\vee$ and $\wedge$ are replaced by *max* and *min* in fuzzy set theory [3], system (6.2) is rewritten in the form:

$$\mu^{E_J} = \max_{I=\overline{1,N}} (\min(\mu^{C_I}, r_{IJ})),\ J = \overline{1,M}. \tag{6.3}$$

In order to translate the specific values of the input and output variables into the measures of the causes and effects significances it is necessary to define a membership function of fuzzy terms C_I and E_J, $I = \overline{1,N}$, $J = \overline{1,M}$. We use a bell-shaped membership function model of variable u to arbitrary term T in the form:

$$\mu^T(u) = \frac{1}{1 + \left(\frac{u - \beta}{\sigma} \right)^2}, \tag{6.4}$$

where β is a coordinate of function maximum, $\mu^T(\beta)=1$; σ is a parameter of concentration-extension (Fig. 6.2).

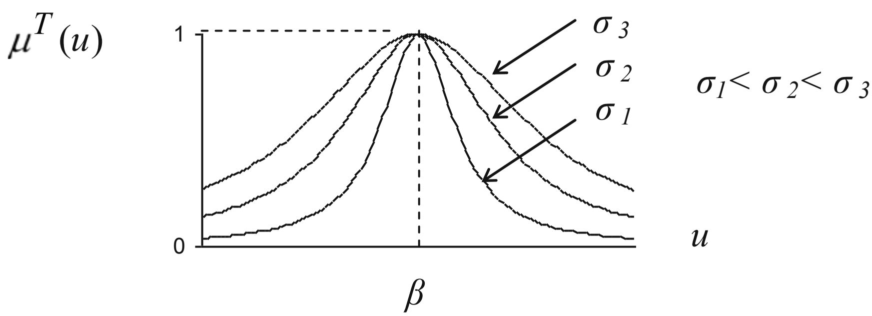


Fig. 6.2. Model of the bell-shaped membership function

This function was determined in [23] and was used for nonlinear dependencies identification by fuzzy IF-THEN rules [29, 30].

Correlations (6.3) and (6.4) define the generalized fuzzy model of diagnosis as follows:

$$\boldsymbol{\mu}^E(\mathbf{Y},\mathbf{B}_E,\boldsymbol{\Omega}_E)=F_R(\mathbf{X},\mathbf{R},\mathbf{B}_C,\boldsymbol{\Omega}_C), \tag{6.5}$$

where $\mathbf{B}_C=(\beta^{C_1},\beta^{C_2},...,\beta^{C_N})$ and $\boldsymbol{\Omega}_C=(\sigma^{C_1},\sigma^{C_2},...,\sigma^{C_N})$ are the vectors of β- and σ- parameters for fuzzy causes C_1, $C_2,\ldots,$ C_N membership functions;

$\mathbf{B}_E=(\beta^{E_1},\beta^{E_2},...,\beta^{E_M})$ and $\boldsymbol{\Omega}_E=(\sigma^{E_1},\sigma^{E_2},...,\sigma^{E_M})$ are the vectors of β- and σ- parameters for fuzzy effects E_1, $E_2,\ldots,E_M$ membership functions;

F_R is the operator of inputs-outputs connection, corresponding to formulae (6.3), (6.4).

6.2 Optimization Problem for Fuzzy Relations Based Inverse Inference

Following the approach proposed in [24, 25], the problem of solving fuzzy relational equations (6.3) is formulated as follows. Fuzzy causes vector $\boldsymbol{\mu}^C=(\mu^{C_1},\mu^{C_2},...,\mu^{C_N})$ should be found which satisfies the constraints $\mu^{C_I}\in[0,1]$, $I=\overline{1,N}$, and also provides the least distance between observed and model measures of effects significances, that is between the left and the right parts of each system equation (6.3):

$$\sum_{J=1}^{M}[\mu^{E_J}-\max_{I=\overline{1,N}}(\min(\mu^{C_I},r_{IJ}))]^2=\min_{\boldsymbol{\mu}^C}. \tag{6.6}$$

Following [4], in the general case, system (6.3) has a solution set $S(\mathbf{R},\boldsymbol{\mu}^E)$, which is completely characterized by the unique greatest solution $\bar{\boldsymbol{\mu}}^C$ and the set of lower solutions $S^*(\mathbf{R},\boldsymbol{\mu}^E)=\{\underline{\boldsymbol{\mu}}_l^C,\ l=\overline{1,T}\}$:

$$S(\mathbf{R},\boldsymbol{\mu}^E) = \bigcup_{\underline{\boldsymbol{\mu}}_l^C \in S^*} \left[\underline{\boldsymbol{\mu}}_l^C, \bar{\boldsymbol{\mu}}^C\right]. \tag{6.7}$$

Here $\bar{\boldsymbol{\mu}}^C = (\bar{\mu}^{C_1}, \bar{\mu}^{C_2}, ..., \bar{\mu}^{C_N})$ and $\underline{\boldsymbol{\mu}}_l^C = (\underline{\mu}_l^{C_1}, \underline{\mu}_l^{C_2}, ..., \underline{\mu}_l^{C_N})$ are the vectors of the upper and lower bounds of causes C_I significance measures, where the union is taken over all $\underline{\boldsymbol{\mu}}_l^C \in S^*(\mathbf{R},\boldsymbol{\mu}^E)$.

Following [24, 25], formation of intervals (6.7) is accomplished by way of solving a multiple optimization problem (6.6) and it begins with the search for its null solution. As the null solution of optimization problem (6.6) we designate $\boldsymbol{\mu}_0^C = (\mu_0^{C_1}, \mu_0^{C_2}, ..., \mu_0^{C_N})$, where $\mu_0^{C_I} \le \bar{\mu}^{C_I}$, $I = \overline{1,N}$. The upper bound ($\bar{\mu}^{C_I}$) is found in the range $\left[\mu_0^{C_I}, 1\right]$. The lower bound ($\underline{\mu}_l^{C_I}$) for $l = 1$ is found in the range $\left[0, \mu_0^{C_I}\right]$, and for $l > 1$ – in the range $\left[0, \bar{\mu}^{C_I}\right]$, where the minimal solutions $\underline{\mu}_k^C$, $k < l$, are excluded from the search space.

Let $\boldsymbol{\mu}^C(t) = (\mu^{C_1}(t), \mu^{C_2}(t), ..., \mu^{C_N}(t))$ be some t-th solution of optimization problem (6.6), that is $F(\boldsymbol{\mu}^C(t)) = F(\boldsymbol{\mu}_0^C)$, since for all $\boldsymbol{\mu}^C \in S(\mathbf{R},\boldsymbol{\mu}^E)$ we have the same value of criterion (6.6). While searching for upper bounds ($\bar{\mu}^{C_I}$) it is suggested that $\mu^{C_I}(t) \ge \mu^{C_I}(t-1)$, and while searching for lower bounds ($\underline{\mu}_l^{C_I}$) it is suggested that $\mu^{C_I}(t) \le \mu^{C_I}(t-1)$ (Fig. 6.3).

The definition of the upper (lower) bounds follows the rule: if $\boldsymbol{\mu}^C(t) \ne \boldsymbol{\mu}^C(t-1)$, then $\bar{\mu}^{C_I}$ ($\underline{\mu}_l^{C_I}$)$=\mu^{C_I}(t)$, $I = \overline{1,N}$. If $\boldsymbol{\mu}^C(t) = \boldsymbol{\mu}^C(t-1)$, then the search for the interval solution $\left[\underline{\boldsymbol{\mu}}_l^C, \bar{\boldsymbol{\mu}}^C\right]$ is stopped. Formation of intervals (6.7) will go on until the condition $\underline{\boldsymbol{\mu}}_l^C \ne \underline{\boldsymbol{\mu}}_k^C$, $k < l$, has been satisfied.

The hybrid genetic and neuro approach is proposed for solving optimization problem (6.6).

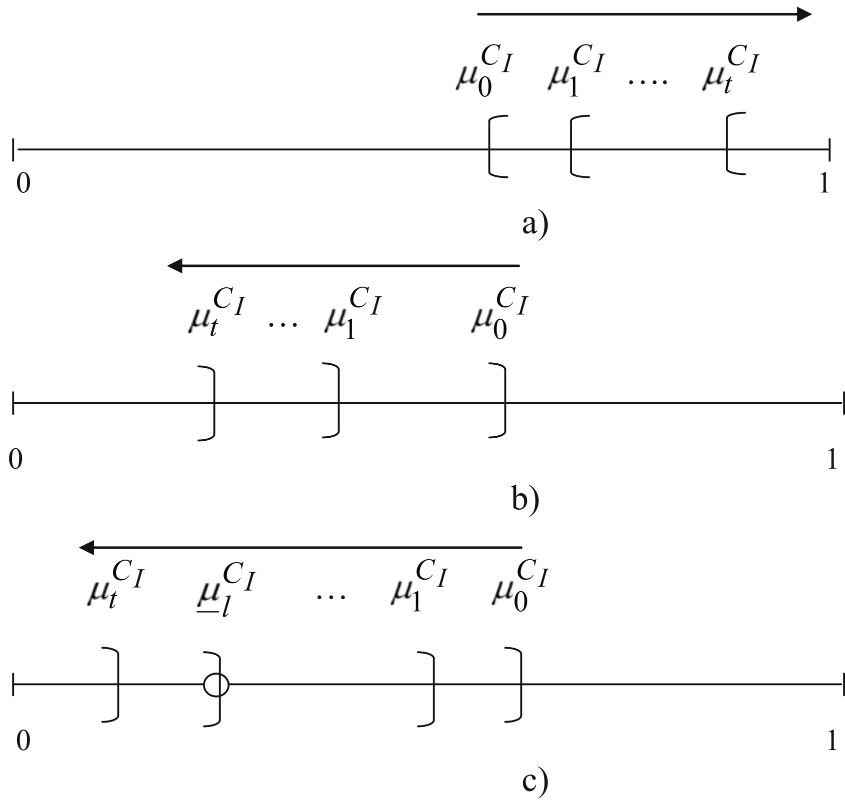


Fig. 6.3. Search for the upper (a) and lower bounds of the intervals for $l = 1$ (b) and $l > 1$ (c)

6.3 Genetic Algorithm for Fuzzy Relations Based Inverse Inference

The chromosome needed in the genetic algorithm for solving the optimization problem (6.6) is defined as the vector-line of binary codes of the lower and upper bounds of the solutions μ^{C_I}, $I = \overline{1,N}$ (Fig. 6.4) [31].

$\underline{\mu}^{C_1}$	$\underline{\mu}^{C_2}$	...	$\underline{\mu}^{C_N}$	$\overline{\mu}^{C_1}$	$\overline{\mu}^{C_2}$	...	$\overline{\mu}^{C_N}$

Fig. 6.4. Structure of the chromosome

The crossover operation is defined in Fig. 6.5, and is carried out by way of exchanging genes inside each solution μ^{C_I}. The points of cross-over shown in dotted lines are selected randomly. Upper symbols (1 and 2) in the vectors of parameters correspond to the first and second chromosomes-parents.

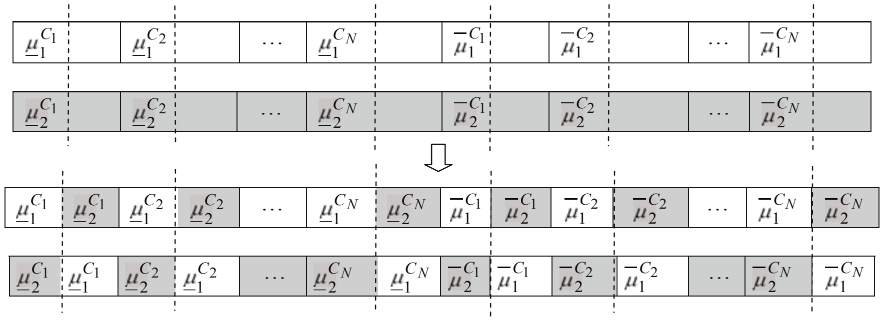


Fig. 6.5. Structure of the crossover operation

A mutation operation implies random change (with some probability) of chromosome elements

$$Mu\,(\mu^{C_l}) = RANDOM\,([\underline{\mu}^{C_l}, \overline{\mu}^{C_l}])\,,$$

where $RANDOM\,([\underline{x}, \overline{x}])$ denotes a random number within the interval $[\underline{x}, \overline{x}]$.

We choose a fitness function as the negative of criterion (6.6).

6.4 Neuro-fuzzy Network for Fuzzy Relations Based Inverse Inference

A neuro-fuzzy network isomorphic to the system of fuzzy logic equations (6.3) is presented in Fig. 6.6. Table 3.1 shows elements of the neuro-fuzzy network [28].

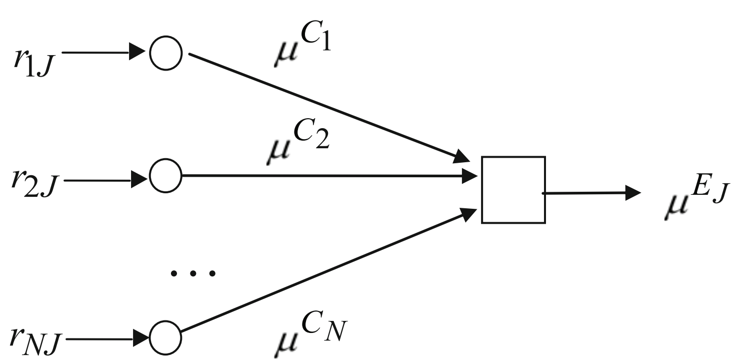


Fig. 6.6. Neuro-fuzzy model of diagnostic equations

The network is designed so that the adjusted weights of arcs are the unknown significance measures of $C_1, C_2, ..., C_N$ causes.

Network inputs are elements of the matrix of fuzzy relations. As follows from the system of fuzzy logic equations (6.3), the fuzzy relation r_{IJ} is the significance measure of the effect μ^{E_J} provided that the significance measure μ^{C_I} is equal to unity, and the significance measures of other causes are equal to zero, i.e. $r_{IJ} = \mu^{E_J}$ (μ^{C_I} =1, μ^{C_K} =0), $K = \overline{1,N}$, $K \neq I$. At the network outputs, actual significance measures of the effects $\max\limits_{I=\overline{1,N}}[\min(\mu^{C_I}, r_{IJ})]$ obtained with allowance for the actual weights of arcs μ^{C_I} are united.

Thus, the problem of solving the system of fuzzy logic equations (6.3) is reduced to the problem of training of a neuro fuzzy network (see Fig. 6.6) with the use of points

$$(r_{1J}, r_{2J}, ..., r_{NJ}, \mu^{E_J}), \quad J = \overline{1,M} .$$

To train the parameters of the neuro-fuzzy network, the recurrent relations:

$$\mu^{C_I}(t+1) = \mu^{C_I}(t) - \eta \frac{\partial \varepsilon_t}{\partial \mu^{C_I}(t)} , \tag{6.8}$$

that minimize the criterion

$$\varepsilon_t = \frac{1}{2}(\hat{\boldsymbol{\mu}}^E(t) - \boldsymbol{\mu}^E(t))^2 , \tag{6.9}$$

applied in the neural network theory, where

$\hat{\boldsymbol{\mu}}^E(t)$ and $\boldsymbol{\mu}^E(t)$ are the experimental and the model fuzzy effects vectors at the t-th step of training;

$\mu^{C_I}(t)$ are the significance measures of causes C_I at the t-th step of training;

η is a parameter of training, which can be selected according to the results from [32].

The partial derivatives appearing in recurrent relations (6.8) characterize the sensitivity of the error (ε_t) to variations in parameters of the neuro-fuzzy network and can be calculated as follows:

$$\frac{\partial \varepsilon_t}{\partial \mu^{C_I}} = \sum_{J=1}^{M} \left[\frac{\partial \varepsilon_t}{\partial \mu^{E_J}} \cdot \frac{\partial \mu^{E_J}}{\partial \mu^{C_I}} \right].$$

Since determining the element "fuzzy output" from Table 3.1 involves the *min* and *max* fuzzy-logic operations, the relations for training are obtained using finite differences.

6.5 Expert Method of Fuzzy Relations Construction

To obtain matrix **R** between causes $C_1, C_2, ..., C_N$ and effects $E_1, E_2, ..., E_M$, included in correlation (6.1), we shall use the method of membership functions construction proposed in [33] on the basis of the 9-mark scale of Saaty's paired comparisons [34].

We consider an effect E_J as a fuzzy set, which is given on the universal set of causes as follows:

$$E_J = \left\{ \frac{r_{1J}}{C_1}, \frac{r_{2J}}{C_2}, ..., \frac{r_{NJ}}{C_N} \right\}, \; J = \overline{1,M} \,, \tag{6.10}$$

where r_{1J}, r_{2J},..., r_{NJ} represent the degrees of membership of causes $C_1, C_2, ..., C_N$ to fuzzy set E_J, and correspond to the J-th column of the fuzzy relational matrix.

Following [33], to obtain membership degrees r_{IJ}, included in (6.10), it is necessary to form the matrix of paired comparisons for each effect E_J, which reflects the influence of causes $C_1, C_2, ..., C_N$ upon the rise of effect E_J, $J = \overline{1,M}$.

For an effect E_J the matrix of paired comparisons looks as follows:

$$\mathbf{A}_J = \begin{array}{c|cccc|} & C_1 & C_2 & \dots & C_N \\ \hline C_1 & a_{11}^J & a_{12}^J & \dots & a_{1N}^J \\ C_2 & a_{21}^J & a_{22}^J & \dots & a_{2N}^J \\ \vdots & \dots & \dots & \dots & \dots \\ C_N & a_{N1}^J & a_{N2}^J & \dots & a_{NN}^J \end{array} \,, \quad J = \overline{1,M} \,, \tag{6.11}$$

where the element a_{IK}^J is evaluated by an expert according to the 9-mark Saaty's scale:

1 — if cause C_K has *no advantage* over cause C_I;

3 — if C_K has *a weak advantage* over C_I;

5 — if C_K has *an essential advantage* over C_I;

7 — if C_K has *an obvious advantage* over C_I;

9— if C_K has *an absolute advantage* over C_I.

Values of 2, 4, 6, 8 correspond to *intermediate* comparative assessments

In accordance with [33], we assume that matrix (6.11) has the following properties:

- elements placed symmetrically relative to the main diagonal are connected by correlation $a_{IK}^J = 1/a_{KI}^J$;
- transitivity, i. e., $a_{IL}^J a_{LK}^J = a_{IK}^J$;
- diagonality, i.e., $a_{II}^J = 1$, $I = \overline{1,N}$, as the consequence from symmetry and transitivity.

These properties allow us to define all elements of matrix (6.11) by using elements of only a single row. If the L-th row is known, i. e. the elements a_{LK}^{J}, $K=\overline{1,N}$, then an arbitrary element a_{IK}^{J} is defined as follows:

$$a_{IK}^{J}=\frac{a_{LK}^{J}}{a_{LI}^{J}}, \qquad I,K,L=\overline{1,N}, \qquad J=\overline{1,M}.$$

After defining matrix (6.11), the degrees of membership needed for constructing fuzzy set (6.10) are calculated by formula [33]:

$$r_{IJ}=\frac{1}{a_{I1}^{J}+a_{I2}^{J}+...+a_{IN}^{J}}, \quad I=\overline{1,N}, \; J=\overline{1,M}. \tag{6.12}$$

Obtained membership degrees (6.12) are to be normalized by way of dividing into the highest degree of membership.

6.6 Problem of Fuzzy Relations Tuning

It is assumed that the training data which is given in the form of L pairs of experimental data is known:

$$\left\langle \hat{\mathbf{X}}_p, \hat{\mathbf{Y}}_p \right\rangle, \; p=\overline{1,L},$$

where $\hat{\mathbf{X}}_p=(\hat{x}_1^p,\hat{x}_2^p,...,\hat{x}_n^p)$ and $\hat{\mathbf{Y}}_p=(\hat{y}_1^p,\hat{y}_2^p,...,\hat{y}_m^p)$ are the vectors of the values of the input and output variables in the experiment number p.

Let $\mathbf{\Lambda}=(\lambda_1,\lambda_2,..,\lambda_M)$ be the vector of concentration parameters for fuzzy sets of effects (6.10), such as:

$$\mathbf{R}=\begin{bmatrix} (r_{11})^{\lambda_1} & (r_{12})^{\lambda_2} & ... & (r_{1M})^{\lambda_M} \\ (r_{21})^{\lambda_1} & (r_{22})^{\lambda_2} & ... & (r_{2M})^{\lambda_M} \\ ... & ... & ... & ... \\ (r_{N1})^{\lambda_1} & (r_{N2})^{\lambda_2} & ... & (r_{NM})^{\lambda_M} \end{bmatrix}.$$

The essence of tuning of the fuzzy model (6.5) consists of finding such null solutions $\boldsymbol{\mu}_0^C(\hat{x}_1^p,\hat{x}_2^p,...,\hat{x}_n^p)$ of the inverse problem, which minimize criterion (6.6) for all the points of the training data:

$$\sum_{p=1}^{L}[F_R(\boldsymbol{\mu}_0^{\mathrm{C}}(\hat{x}_1^p,\hat{x}_2^p,...,\hat{x}_n^p))-\hat{\boldsymbol{\mu}}^E(\hat{y}_1^p,\hat{y}_2^p,...,\hat{y}_m^p)]^2=min.$$

In other words, the essence of tuning of the fuzzy model (6.5) consists of finding such a vector of concentration parameters $\mathbf{\Lambda}$ and such vectors of membership functions parameters $\mathbf{B}_C$, $\mathbf{\Omega}_C$, $\mathbf{B}_E$, $\mathbf{\Omega}_E$, which provide the least distance between model and experimental fuzzy effects vectors:

$$\sum_{p=1}^{L}[F_R(\hat{\mathbf{X}}_p,\mathbf{\Lambda},\mathbf{B}_C,\mathbf{\Omega}_C)-\hat{\boldsymbol{\mu}}^E(\hat{\mathbf{Y}}_p,\mathbf{B}_E,\mathbf{\Omega}_E)]^2=\min_{\mathbf{\Lambda},\mathbf{B}_C,\mathbf{\Omega}_C,\mathbf{B}_E,\mathbf{\Omega}_E} . \quad (6.13)$$

6.7 Genetic Algorithm of Fuzzy Relations Tuning

The chromosome needed in the genetic algorithm for solving the optimization problem (6.13) is defined as the vector-line of binary codes of parameters $\mathbf{\Lambda}$, $\mathbf{B}_C$, $\mathbf{\Omega}_C$, $\mathbf{B}_E$, $\mathbf{\Omega}_E$ (Fig. 6.7) [31].

$\mathbf{\Lambda}$	$\mathbf{B}_C$	$\mathbf{\Omega}_C$	$\mathbf{B}_E$	$\mathbf{\Omega}_E$

Fig. 6.7. Structure of the chromosome

The crossover operation is defined in Fig. 6.8, and is carried out by way of exchanging genes inside the vector of concentration parameters ($\mathbf{\Lambda}$) and each of the vectors of membership functions parameters $\mathbf{B}_C$, $\mathbf{\Omega}_C$, $\mathbf{B}_E$, $\mathbf{\Omega}_E$. The points of cross-over shown in dotted lines are selected randomly. Upper symbols (1 and 2) in the vectors of parameters correspond to the first and second chromosomes-parents.

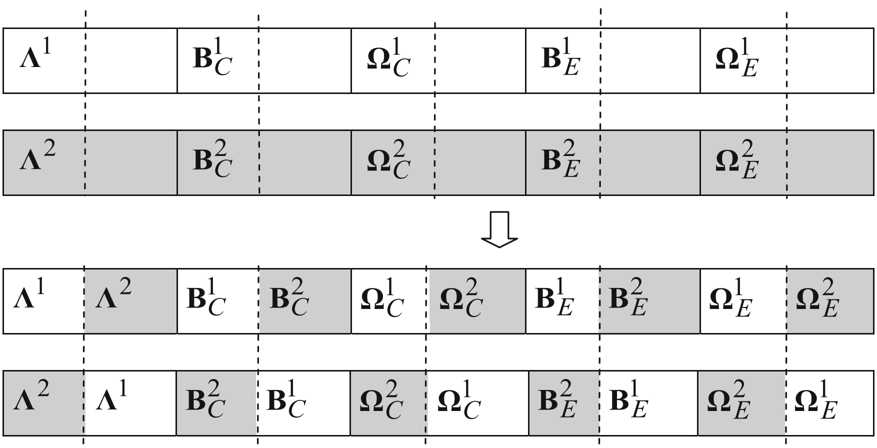


Fig. 6.8. Structure of the crossover operation

A mutation operation implies random change (with some probability) of chromosome elements:

$$Mu\left(\beta^{C_I}\right) = RANDOM\left(\left[\ \underline{\beta}^{C_I}, \overline{\beta}^{C_I}\ \right]\right);\ Mu\left(\sigma^{C_I}\right) = RANDOM\left(\left[\ \underline{\sigma}^{C_I}, \overline{\sigma}^{C_I}\ \right]\right);$$

$$Mu\left(\beta^{E_J}\right) = RANDOM\left(\left[\ \underline{\beta}^{E_J}, \overline{\beta}^{E_J}\ \right]\right);\ Mu\left(\sigma^{E_J}\right) = RANDOM\left(\left[\underline{\sigma}^{E_J}, \overline{\sigma}^{E_J}\ \right]\right);$$

$$Mu\ (\lambda_J) = RANDOM\ ([\underline{\lambda}_J, \overline{\lambda}_J]),$$

where $RANDOM([\underline{x}, \overline{x}])$ denotes a random number within the interval $[\underline{x}, \overline{x}]$.

We choose criterion (6.13) with the negative sign as the fitness function; that is, the higher the degree of adaptability of the chromosome to perform the criterion of optimization the greater is the fitness function.

6.8 Adaptive Tuning of Fuzzy Relations

The neuro-fuzzy model of the object of diagnostics (6.5) is represented in Fig. 6.9, and the nodes are in Table. 3.1. The neuro-fuzzy model is obtained by embedding the matrix of fuzzy relations into the neural network so that the weights of arcs subject to tuning are fuzzy relations and membership functions for causes and effects fuzzy terms [28, 30].

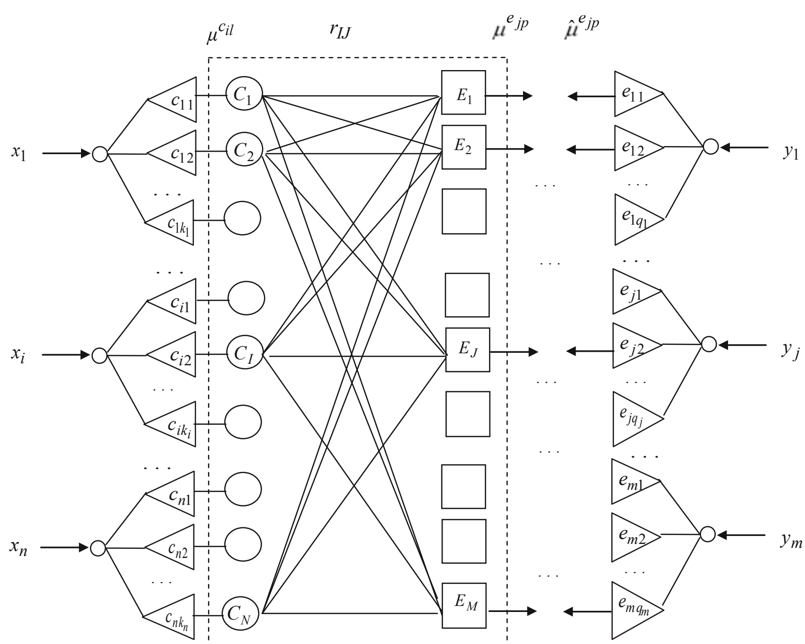


Fig. 6.9. Neuro-fuzzy model of the object of diagnostics

To train the parameters of the neuro-fuzzy network, the recurrent relations:

$$r_{IJ}(t+1) = r_{IJ}(t) - \eta \frac{\partial \varepsilon_t}{\partial r_{IJ}(t)};$$

$$\beta^{c_{il}}(t+1) = \beta^{c_{il}}(t) - \eta \frac{\partial \varepsilon_t}{\partial \beta^{c_{il}}(t)}; \quad \sigma^{c_{il}}(t+1) = \sigma^{c_{il}}(t) - \eta \frac{\partial \varepsilon_t}{\partial \sigma^{c_{il}}(t)};$$

$$\beta^{e_{jp}}(t+1) = \beta^{e_{jp}}(t) - \eta \frac{\partial \varepsilon_t}{\partial \beta^{e_{jp}}(t)}; \quad \sigma^{e_{jp}}(t+1) = \sigma^{e_{jp}}(t) - \eta \frac{\partial \varepsilon_t}{\partial \sigma^{e_{jp}}(t)}, \quad (6.14)$$

minimizing criterion (6.9) are used, where

$r_{IJ}(t)$ are fuzzy relations at the t-th step of training;

$\beta^{c_{il}}(t)$, $\sigma^{c_{il}}(t)$, $\beta^{e_{jp}}(t)$, $\sigma^{e_{jp}}(t)$ are the parameters of the membership functions for causes and effects fuzzy terms at the t-th step of training.

The partial derivatives appearing in recurrent relations (6.14) characterize the sensitivity of the error (ε_t) to variations in parameters of the neuro-fuzzy network and can be calculated as follows:

$$\frac{\partial \varepsilon_t}{\partial r_{IJ}} = \frac{\partial \varepsilon_t}{\partial \mu^{E_J}(X)} \cdot \frac{\partial \mu^{E_J}(X)}{\partial r_{IJ}};$$

$$\frac{\partial \varepsilon_t}{\partial \beta^{c_{il}}} = \sum_{j=1}^{m} \sum_{p=1}^{q_j} \left[\frac{\partial \varepsilon_t}{\partial \mu^{e_{jp}}(x_i)} \cdot \frac{\partial \mu^{e_{jp}}(x_i)}{\partial \mu^{c_{il}}(x_i)} \cdot \frac{\partial \mu^{c_{il}}(x_i)}{\partial \beta^{c_{il}}} \right];$$

$$\frac{\partial \varepsilon_t}{\partial \sigma^{c_{il}}} = \sum_{j=1}^{m} \sum_{p=1}^{q_j} \left[\frac{\partial \varepsilon_t}{\partial \mu^{e_{jp}}(x_i)} \cdot \frac{\partial \mu^{e_{jp}}(x_i)}{\partial \mu^{c_{il}}(x_i)} \cdot \frac{\partial \mu^{c_{il}}(x_i)}{\partial \sigma^{c_{il}}} \right];$$

$$\frac{\partial \varepsilon_t}{\partial \beta^{e_{jp}}} = \frac{\partial \varepsilon_t}{\partial \mu^{e_{jp}}(y_j)} \cdot \frac{\partial \mu^{e_{jp}}(y_j)}{\partial \beta^{e_{jp}}}; \quad \frac{\partial \varepsilon_t}{\partial \sigma^{e_{jp}}} = \frac{\partial \varepsilon_t}{\partial \mu^{e_{jp}}(y_j)} \cdot \frac{\partial \mu^{e_{jp}}(y_j)}{\partial \sigma^{e_{jp}}}.$$

Since determining the element "fuzzy output" (see Table 3.1) involves the *min* and *max* fuzzy-logic operations, the relations for training are obtained using finite differences.

6.9 Computer Simulations

The aim of the experiment consists of checking the performance of the above proposed models and algorithms of diagnosis with the help of the target "input-output" model. The target model was some analytical function $y = f(x)$. This function was approximated by the rule of inference (6.1), and served simultaneously as training and testing data generator. The input values (x) restored for each output (y) were compared with the target values.

The target model is given by the formula:

$$y = \frac{(1.8x+0.8)(5x-1.1)(4x-2.9)(3x-2.1)(9.5x-9.5)(3x-0.05)+20}{80},$$

which is represented in Fig. 6.10 together with the fuzzy terms of causes C_1=*low (L)*, C_2=*lower than average* (*lA*), C_3=*average* (*A*), C_4=*higher than average* (*hA*), C_5=*lower than high*, C_6=*high* (*H*) and effects E_1=*lower than average* (*lA*), E_2=*average* (*A*), E_3=*higher than average* (*hA*), E_4=*high* (*H*).

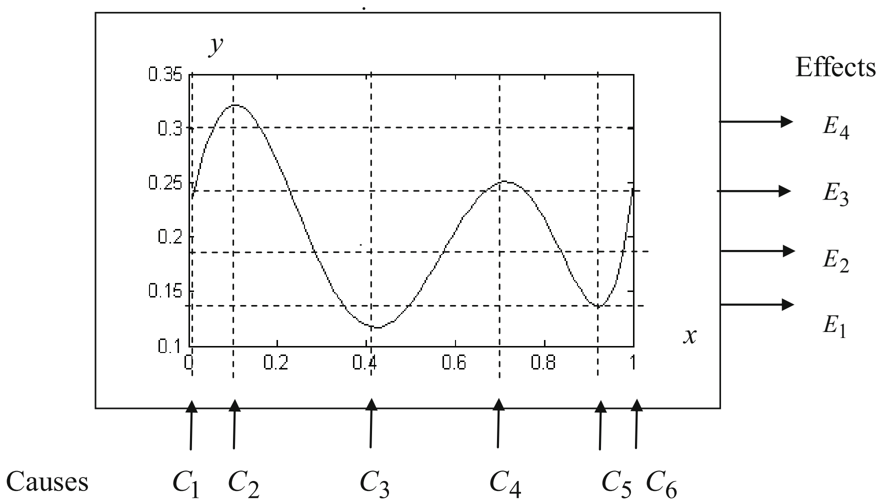


Fig. 6.10. "Input-output" model-generator

A fuzzy relational matrix was formed on the basis of expert assessments. For example, the procedure of fuzzy relations construction for effect E_1 consists of the following. Cause C_2 is the least important for effect E_1, so that the visual difference between the output values $y = E_1$ and $y(x = C_2)$, i.e. $|E_1 - y(x = C_2)|$, is maximal. Therefore, we start forming the matrix of paired comparisons $\mathbf{A}_1$ (6.11) from the 2nd row. This row is formed by an expert and contains the assessments, which define the degree of advantage of the rest causes C_K,

$K = \overline{1,6}$, over C_2. The advantage of cause C_K over cause C_2 is defined by the fact, how much the distance $|E_1 - y(x = C_K)|$ is less than the distance $|E_1 - y(x = C_2)|$. Matrix $\mathbf{A}_1$ (6.11) is defined by the known 2nd row as follows:

$\mathbf{A}_1 =$

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	1	1/3	3	1	8/3	1
C_2	3	1	9	3	8	3
C_3	1/3	1/9	1	1/3	8/9	1/3
C_4	1	1/3	3	1	8/3	1
C_5	3/8	1/8	9/8	3/8	1	3/8
C_6	1	1/3	3	1	8/3	1

Matrix $\mathbf{A}_1$ allows us to construct fuzzy set E_1 (6.10) using formula (6.12). The degrees of membership r_{I1} of causes C_I to fuzzy set E_1 are defined as follows:

$r_{11} = (1+1/3+3+1+8/3+1)^{-1} = 0.11$;

$r_{21} = (3+1+9+3+8+3)^{-1} = 0.04$;

$r_{31} = (1/3+1/9+1+1/3+8/9+1/3)^{-1} = 0.33$;

$r_{41} = (1+1/3+3+1+8/3+1)^{-1} = 0.11$;

$r_{51} = (3/8+1/8+9/8+3/8+1+3/8)^{-1} = 0.30$;

$r_{61} = (1+1/3+3+1+8/3+1)^{-1} = 0.11$.

The obtained membership degrees should be normalized, i.e. r_{11} =0.11/0.33 $\approx$ 0.33; r_{21} =0.04/0.33 $\approx$ 0.12; r_{31} =0.33/0.33=1.00; r_{41} =0.11/0.33 $\approx$ 0.33; r_{51} =0.30/0.33 $\approx$ 0.91; r_{61} =0.11/0.33 $\approx$ 0.33.

Thus, fuzzy set E_1, whose elements correspond to the 1st column of the fuzzy relational matrix, takes the form:

$$E_1 = \left\{ \frac{0.33}{C_1}, \frac{0.12}{C_2}, \frac{1.00}{C_3}, \frac{0.33}{C_4}, \frac{0.91}{C_5}, \frac{0.33}{C_6} \right\}.$$

The resulting expert fuzzy relational matrix takes the form:

R =

	E_1	E_2	E_3	E_4
C_1	0.33	1.00	0.67	0.21
C_2	0.12	0.10	0.33	1.00
C_3	1.00	0.23	0.11	0.11
C_4	0.33	0.33	1.00	0.21
C_5	0.91	0.77	0.22	0.34
C_6	0.33	0.90	0.67	0.21

The results of the fuzzy model tuning are given in Tables 6.1, 6.2.

Table 6.1. Parameters of the membership functions for the causes fuzzy terms before (after) tuning

Fuzzy terms	Parameters (β -, σ -)		
	Before tuning	Genetic algorithm	Neural net
C_1	(0, 0.17)	(0, 0.114)	(0, 0.114)
C_2	(0.1, 0.17)	(0.091, 0.121)	(0.091, 0.121)
C_3	(0.4, 0.17)	(0.430, 0.115)	(0.446, 0.115)
C_4	(0.7, 0.17)	(0.703, 0.100)	(0.711, 0.118)
C_5	(0.9, 0.17)	(0.919, 0.112)	(0.919, 0.112)
C_6	(1.0, 0.08)	(1.0, 0.041)	(1.0, 0.041)

Table 6.2. Parameters of the membership functions for the effects fuzzy terms before (after) tuning

Fuzzy terms	Parameters (β -, σ -)		
	Before tuning	Genetic algorithm	Neural net
E_1	(0.15, 0.05)	(0.171, 0.032)	(0.172, 0.037)
E_2	(0.2, 0.05)	(0.209, 0.040)	(0.209, 0.040)
E_3	(0.25, 0.05)	(0.257, 0.039)	(0.259, 0.041)
E_4	(0.3, 0.05)	(0.350, 0.037)	(0.352, 0.040)

Fuzzy relational equations after tuning take the form:

$$\mu^{E_1}=(\mu^{C_1}\wedge 0.27)\vee(\mu^{C_2}\wedge 0.13)\vee(\mu^{C_3}\wedge 0.97)\vee(\mu^{C_4}\wedge 0.20)\vee(\mu^{C_5}\wedge 0.86)\vee(\mu^{C_6}\wedge 0.21)$$
$$\mu^{E_2}=(\mu^{C_1}\wedge 0.93)\vee(\mu^{C_2}\wedge 0.09)\vee(\mu^{C_3}\wedge 0.28)\vee(\mu^{C_4}\wedge 0.44)\vee(\mu^{C_5}\wedge 0.75)\vee(\mu^{C_6}\wedge 0.82)$$
$$\mu^{E_3}=(\mu^{C_1}\wedge 0.63)\vee(\mu^{C_2}\wedge 0.41)\vee(\mu^{C_3}\wedge 0.15)\vee(\mu^{C_4}\wedge 0.95)\vee(\mu^{C_5}\wedge 0.26)\vee(\mu^{C_6}\wedge 0.67)$$
$$\mu^{E_4}=(\mu^{C_1}\wedge 0.12)\vee(\mu^{C_2}\wedge 0.88)\vee(\mu^{C_3}\wedge 0.07)\vee(\mu^{C_4}\wedge 0.08)\vee(\mu^{C_5}\wedge 0.32)\vee(\mu^{C_6}\wedge 0.12) \quad (6.15)$$

The results of solving the problem of inverse inference before and after tuning are shown in Fig. 6.11 and 6.12. The same figure depicts the membership functions of the fuzzy terms for the causes and effects before and after tuning.

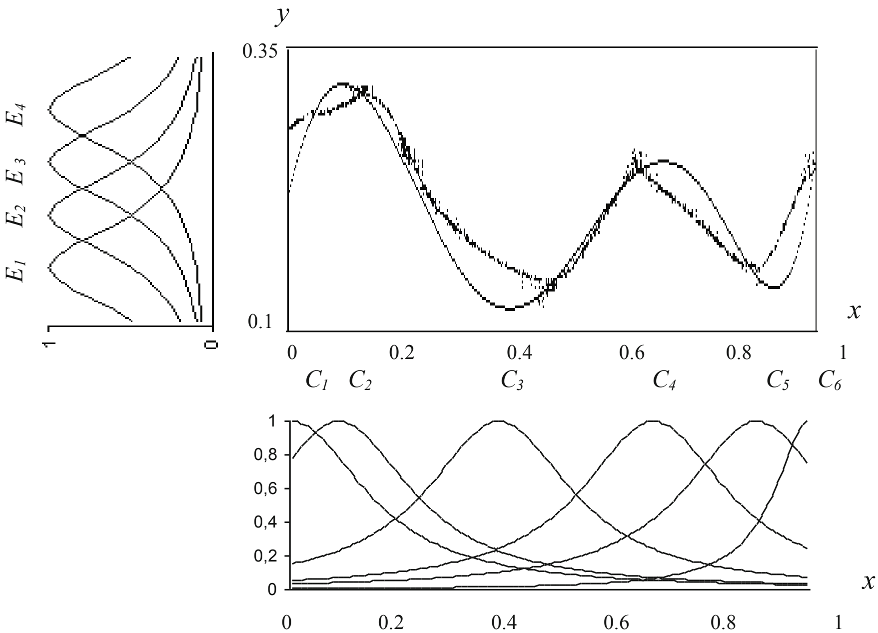
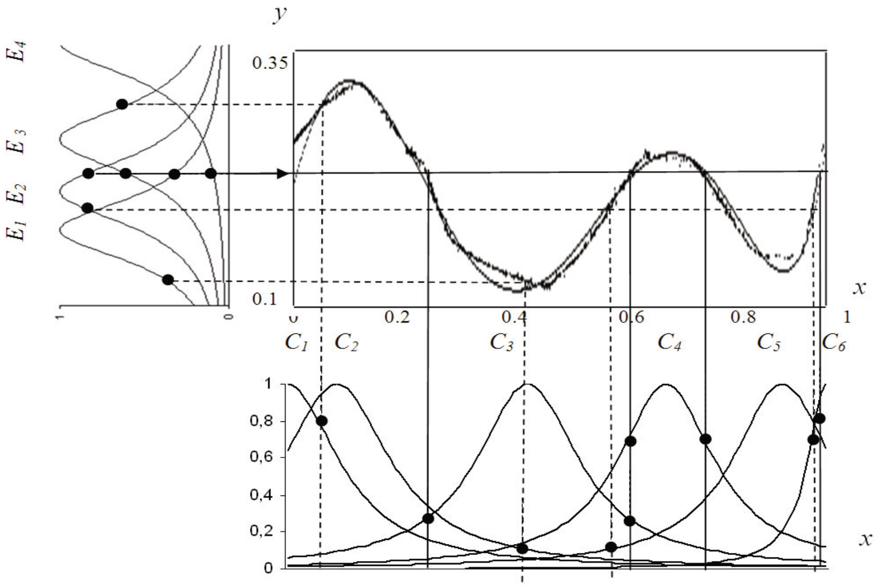
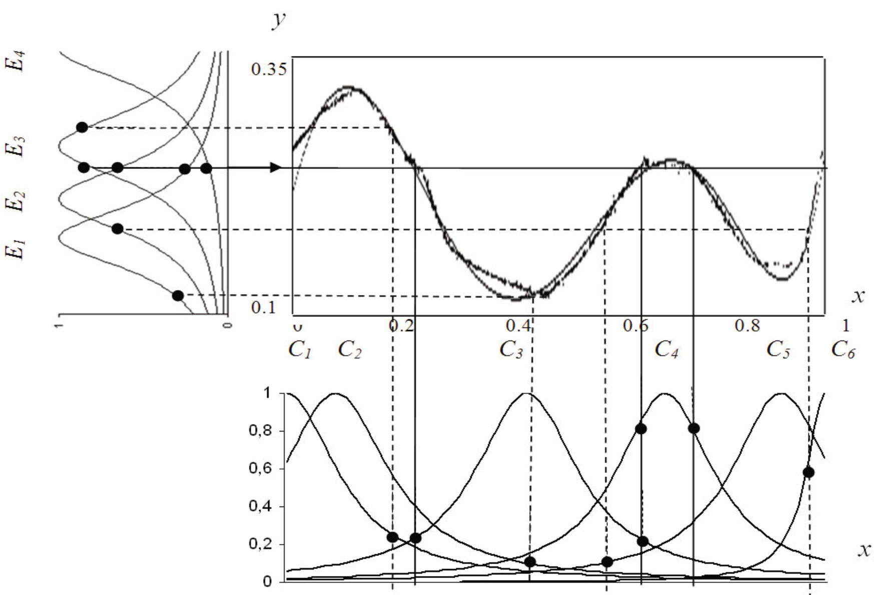


Fig. 6.11. Solution to the problem of inverse fuzzy inference before tuning



a)



b)

Fig. 6.12. Solution to the problem of inverse fuzzy inference after tuning:(a) y^*=0.23; (b) y^*=0.24

Let a specific value of the output variable consists of y^*=0.23. The measures of the effects significances for this value can be defined with the help of the membership functions in Fig. 6.12,a:

$$\boldsymbol{\mu}^E(y^*) = (\mu^{E_1}=0.29;\ \mu^{E_2}=0.78;\ \mu^{E_3}=0.67;\ \mu^{E_4}=0.10).$$

The genetic algorithm yields a null solution

$$\boldsymbol{\mu}_0^C = (\mu_0^{C_1}=0.78, \mu_0^{C_2}=0.10, \mu_0^{C_3}=0.29, \mu_0^{C_4}=0.67, \mu_0^{C_5}=0.07, \mu_0^{C_6}=0.45)\,, \quad (6.16)$$

for which the value of the optimization criterion (6.6) is F=0.0004.

The obtained null solution allows us to arrange for the genetic search for the solution set $S(\mathbf{R},\boldsymbol{\mu}^E)$, which is completely determined by the greatest solution

$$\bar{\boldsymbol{\mu}}^C = (\bar{\mu}^{C_1}=0.78,\ \bar{\mu}^{C_2}=0.12,\ \bar{\mu}^{C_3}=0.29,\ \bar{\mu}^{C_4}=0.67,\ \bar{\mu}^{C_5}=0.12,\ \bar{\mu}^{C_6}=0.78)$$

and the three lower solutions $S^* = \{\underline{\boldsymbol{\mu}}_1^C, \underline{\boldsymbol{\mu}}_2^C, \underline{\boldsymbol{\mu}}_3^C\}$

$$\underline{\boldsymbol{\mu}}_1^C = (\underline{\mu}_1^{C_1}=0.78,\ \underline{\mu}_1^{C_2}=0,\ \underline{\mu}_1^{C_3}=0.29,\ \underline{\mu}_1^{C_4}=0,\ \underline{\mu}_1^{C_5}=0,\ \underline{\mu}_1^{C_6}=0.67);$$
$$\underline{\boldsymbol{\mu}}_2^C = (\underline{\mu}_2^{C_1}=0.78,\ \underline{\mu}_2^{C_2}=0,\ \underline{\mu}_2^{C_3}=0.29,\ \underline{\mu}_2^{C_4}=0.67,\ \underline{\mu}_2^{C_5}=0,\ \underline{\mu}_2^{C_6}=0);$$
$$\underline{\boldsymbol{\mu}}_3^C = (\underline{\mu}_3^{C_1}=0,\ \underline{\mu}_3^{C_2}=0,\ \underline{\mu}_3^{C_3}=0.29,\ \underline{\mu}_3^{C_4}=0,\ \underline{\mu}_3^{C_5}=0,\ \underline{\mu}_3^{C_6}=0.78).$$

Thus, the solution of fuzzy relational equations (6.15) can be represented in the form of intervals:

$$\begin{aligned} S(\mathbf{R},\boldsymbol{\mu}^E) = &\{\mu^{C_1}=0.78; \mu^{C_2}\in[0,0.12]; \mu^{C_3}=0.29; \mu^{C_4}\in[0,0.67]; \mu^{C_5}\in[0,0.12]; \mu^{C_6}\in[0.67,0.78]\} \\ \cup &\{\mu^{C_1}=0.78;\ \mu^{C_2}\in[0,0.12];\ \mu^{C_3}=0.29;\ \mu^{C_4}=0.67;\ \mu^{C_5}\in[0,0.12];\ \mu^{C_6}\in[0,0.78]\} \\ \cup &\{\mu^{C_1}\in[0,0.78];\ \mu^{C_2}\in[0,0.12];\ \mu^{C_3}=0.29;\ \mu^{C_4}\in[0,0.67];\ \mu^{C_5}\in[0,0.12];\ \mu^{C_6}=0.78\}. \end{aligned} \quad (6.17)$$

The intervals of the values of the input variable for each interval in solution (6.17) can be defined with the help of the membership functions in Fig. 6.12,a:

- x^*=0.060 or $x^*\in$ [0.060, 1.0] for C_1;
- $x^*\in$ [0.418, 1.0] for C_2;
- x^*=0.264 or x^*=0.628 for C_3;
- x^*=0.628, $x^*\in$ [0, 0.628], x^*=0.794 or $x^*\in$ [0.794, 1.0] for C_4;
- $x^*\in$ [0, 0.610] for C_5;
- $x^*\in$ [0.971, 0.978], $x^*\in$ [0, 0.978] or x^*=0.978 for C_6.

The restoration of the input set for y^*=0.23, i.e. points (0.264, 0.230), (0.628, 0.230), (0.794, 0.230) and (0.978, 0.230), is shown by the continuous line in Fig. 6.12, a, in which the values of the causes and effects significances measures are marked. The rest of the found input values correspond to other values of the output variable with the same measures of effects significances. The restoration of these points is shown by the dotted line in Fig. 6.12,a.

Assume the value of the output variable has changed from y^*=0.23 to y^*=0.24 (Fig. 6.12,b). For the new value, the fuzzy effects vector is

$$\boldsymbol{\mu}^E(y^*) = (\mu^{E_1}=0.23;\ \mu^{E_2}=0.62;\ \mu^{E_3}=0.82;\ \mu^{E_4}=0.11).$$

A neural adjustment of the null solution (6.16) has yielded a fuzzy causes vector

$$\boldsymbol{\mu}_0^C = (\mu_0^{C_1}=0.17, \mu_0^{C_2}=0.04, \mu_0^{C_3}=0.23, \mu_0^{C_4}=0.82, \mu_0^{C_5}=0.09, \mu_0^{C_6}=0.62)\,\mathbf{,}$$

for which the value of the optimization criterion (6.6) has constituted F=0.0001.

The resulting null solution has allowed adjustment of the bounds in the solution (6.17) and generation of the set of solutions $S(\mathbf{R},\boldsymbol{\mu}^E)$ determined by the greatest solution

$$\overline{\boldsymbol{\mu}}^C = (\overline{\mu}^{C_1}=0.23,\ \overline{\mu}^{C_2}=0.12,\ \overline{\mu}^{C_3}=0.23,\ \overline{\mu}^{C_4}=0.82,\ \overline{\mu}^{C_5}=0.12,\ \overline{\mu}^{C_6}=0.62)$$

and the two lower solutions $S^* = \{\underline{\boldsymbol{\mu}}_1^C, \underline{\boldsymbol{\mu}}_2^C\}$

$$\underline{\boldsymbol{\mu}}_1^C = (\underline{\mu}_1^{C_1}=0.23,\ \underline{\mu}_1^{C_2}=0,\ \underline{\mu}_1^{C_3}=0,\ \underline{\mu}_1^{C_4}=0.82,\ \underline{\mu}_1^{C_5}=0,\ \underline{\mu}_1^{C_6}=0.62);$$
$$\underline{\boldsymbol{\mu}}_2^C = (\underline{\mu}_2^{C_1}=0,\ \underline{\mu}_2^{C_2}=0,\ \underline{\mu}_2^{C_3}=0.23,\ \underline{\mu}_2^{C_4}=0.82,\ \underline{\mu}_2^{C_5}=0,\ \underline{\mu}_2^{C_6}=0.62).$$

Thus, the solution of fuzzy relational equations (6.15) for the new value can be represented in the form of intervals:

$$S(\mathbf{R},\boldsymbol{\mu}^E) = \{\mu^{C_1}=0.23;\ \mu^{C_2}\in[0, 0.12];\ \mu^{C_3}\in[0, 0.23];\ \mu^{C_4}=0.82;\ \mu^{C_5}\in[0, 0.12];\ \mu^{C_6}=0.62\}$$
$$\cup\,\{\mu^{C_1}\in[0, 0.23];\ \mu^{C_2}\in[0, 0.12];\ \mu^{C_3}=0.23;\ \mu^{C_4}=0.82;\ \mu^{C_5}\in[0, 0.12];\ \mu^{C_6}=0.62\}. \quad (6.18)$$

Solution (6.18) differs from (6.17) in the significance measures of the causes C_1, C_3, C_4 and C_6, for which the ranges of the input variable have been determined using the membership functions in Fig. 6.12,b:

- x^*=0.208 or $x^* \in$ [0.208, 1.0] for C_1;
- x^*=0.236, $x^* \in$ [0, 0.236], x^*=0.656 or $x^* \in$ [0.656, 1.0] for C_3;

- x^* =0.656 or x^* =0.766 for C_4;
- x^* =0.968 for C_6.

The restoration of the input set for y^*=0.24, i.e., points (0.236, 0.240), (0.656, 0.240), (0.766, 0.240), is shown in Fig. 6.12,b.

6.10 Example 5: Oil Pump Diagnosis

Let us consider the algorithm's performance having the recourse to the example of the fuel pump faults causes diagnosis.

Input parameters are (variation ranges are indicated in parentheses):

x_1 – engine speed (2600 – 3200 rpm);

x_2 – filter clear area (30 – 45 cm^2/kw);

x_3 – throat ring side clearance (0.1 – 0.3 mm);

x_4 – suction conduit leakage (0.5 – 2.0 cm^3/h);

x_5 – force main resistance (1.2–3.4 kg/cm^2).

The fault causes to be identified (input term-assessments) are: c_{11} – engine speed x_1 drop; c_{21} – decrease of clear area x_2, i.e. filter clogging; c_{31} (c_{32}) – decrease (increase) of side clearance x_3, i.e. assembling defect (throat ring wear-out); c_{41} – increase of leakage x_4, i.e. fuel escape; c_{51} – high resistance of the force main x_5.

Output parameters are (variation ranges are indicated in parentheses):

y_1 – productivity (20–45 m^3/h);

y_2 – force main pressure (3.7–5.5 kg/cm^2);

y_3 – consumed power (15–30 kw);

y_4 – suction conduit pressure (0.5–1.0 kg/cm^2).

The observed effects (output term-assessments) are: e_{11} – productivity y_1 fall; e_{21} (e_{22}) – force main pressure y_2 drop (rise); e_{31} (e_{32}) – consumed power y_3 drop (rise); e_{41} – pressure in suction conduit y_4 rise.

We shall define the set of causes and effects in the following way:

$$\{C_1, C_2, C_3, C_4, C_5, C_6\} = \{c_{11},\ c_{21},\ c_{31},\ c_{32},\ c_{41},\ c_{51}\};$$

$$\{E_1, E_2, E_3, E_4, E_5, E_6\} = \{e_{11},\ e_{21},\ e_{22},\ e_{31},\ e_{32},\ e_{41}\}.$$

"Causes-effects" relations were formed on the basis of expert assessments. For example, the procedure of fuzzy relations construction for effect E_1 consists of the following. Cause C_3 is the least important for effect E_1. Therefore, we start forming the matrix of paired comparisons $\mathbf{A}_1$ (6.11) from the 3rd row. This row is formed by an expert and contains the assessments, which define the degree of advantage of the rest of the causes over C_3. Not a single cause has an absolute advantage over C_3. Therefore, matrix $\mathbf{A}_1$ contains a fictitious cause C_7, where C_7 has *absolute advantage* over C_3. Matrix $\mathbf{A}_1$ (6.11) is defined by the known 3rd row as follows:

$\mathbf{A}_1 =$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	1	7/2	1/2	4	3	1	9/2
C_2	2/7	1	1/7	8/7	6/7	2/7	9/7
C_3	2	7	1	8	6	2	9
C_4	1/4	7/8	1/8	1	3/4	1/4	9/8
C_5	1/3	7/6	1/6	4/3	1	1/3	3/2
C_6	1	7/2	1/2	4	3	1	9/2
C_7	2/9	7/9	1/9	8/9	2/3	2/9	1

Matrix $\mathbf{A}_1$ allows us to construct fuzzy set E_1 (6.10) using formula (6.12). The degrees of membership r_{I1} of causes C_I to fuzzy set E_1 are defined as follows:

$$r_{11} = (1+7/2+1/2+4+3+1+9/2)^{-1} = 0.06;$$

$$r_{21} = (2/7+1+1/7+8/7+6/7+2/7+9/7)^{-1} = 0.20;$$

$$r_{31} = (2+7+1+8+6+2+9)^{-1} = 0.03;$$

$$r_{41} = (1/4+7/8+1/8+1+3/4+1/4+9/8)^{-1} = 0.23;$$

$$r_{51} = (1/3+7/6+1/6+4/3+1+1/3+3/2)^{-1} = 0.17;$$

$$r_{61} = (1+7/2+1/2+4+3+1+9/2)^{-1} = 0.06;$$

$$r_{71} = (2/9+7/9+1/9+8/9+2/3+2/9+1)^{-1} = 0.26.$$

The obtained membership degrees should be normalized, i.e. r_{11} =0.06/0.26 $\approx$ 0.23; r_{21} =0.20/0.26 $\approx$ 0.77; r_{31} =0.03/0.26=0.11; r_{41} =0.23/0.26 $\approx$ 0.88; r_{51} =0.17/0.26 $\approx$ 0.65; r_{61} =0.06/0.26=0.23.

Thus, fuzzy set E_1, whose elements correspond to the 1st column of the fuzzy relational matrix, takes the form:

$$E_1 = \left\{ \frac{0.23}{C_1}, \frac{0.77}{C_2}, \frac{0.11}{C_3}, \frac{0.88}{C_4}, \frac{0.65}{C_5}, \frac{0.23}{C_6} \right\}.$$

The resulting expert fuzzy relational matrix takes the form:

R =

	E_1	E_2	E_3	E_4	E_5	E_6
C_1	0.23	0.90	0.44	0.88	0.11	0.76
C_2	0.77	0.21	0.89	0.23	0.22	0.32
C_3	0.11	0.45	0.22	0.69	0.89	0.24
C_4	0.88	0.21	0.67	0.12	0.11	0.68
C_5	0.65	0.10	0.33	0.12	0.11	0.88
C_6	0.23	0.55	0.11	0.81	0.40	0.12

For the fuzzy model tuning we used the results of diagnosis for 340 pumps. The results of the fuzzy model tuning are given in Tables 6.3, 6.4 and in Fig. 6.13.

Table 6.3. Parameters of the membership functions for the causes and effects fuzzy terms after genetic tuning

Parameter	Fuzzy terms					
	C_1	C_2	C_3	C_4	C_5	C_6
β -	2700	34.75	0.11	0.26	1.84	3.15
σ -	107.12	3.18	0.04	0.05	0.33	0.65
Parameter	Fuzzy terms					
	E_1	E_2	E_3	E_4	E_5	E_6
β -	22.79	3.84	5.32	15.94	28.84	0.89
σ -	5.02	0.92	0.35	3.76	1.85	0.16

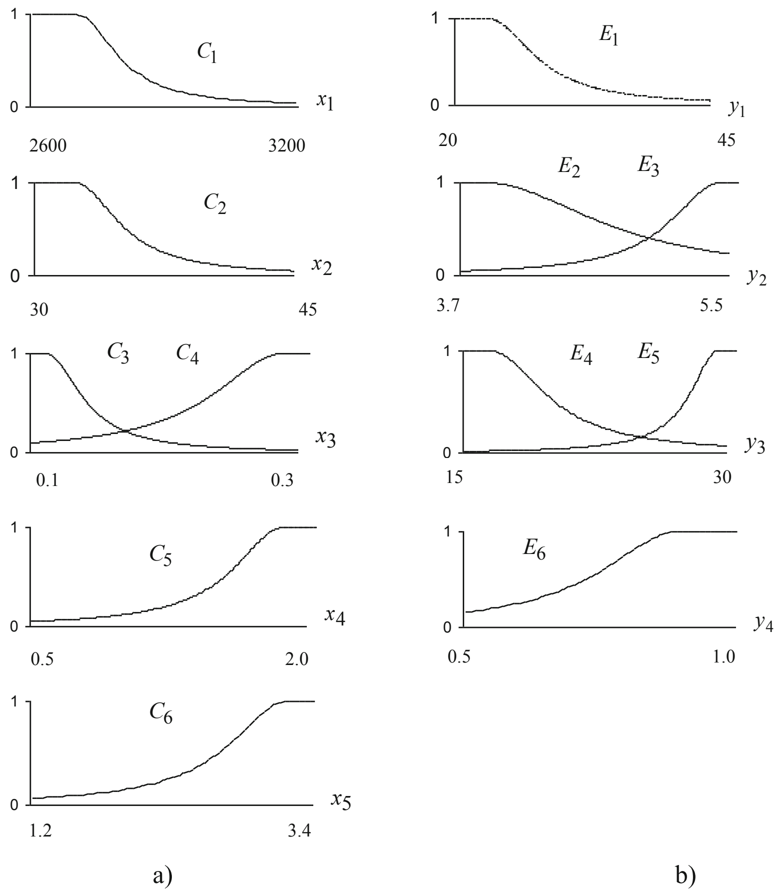


Fig. 6.13. Membership functions of the causes (a) and effects (b) fuzzy terms after tuning

Table 6.4. Parameters of the membership functions for the causes and effects fuzzy terms after neural tuning

Parameter	Fuzzy terms					
	C_1	C_2	C_3	C_4	C_5	C_6
β -	2700	32.27	0.11	0.28	1.82	3.19
σ -	104.57	2.94	0.03	0.06	0.31	0.54
Parameter	Fuzzy terms					
	E_1	E_2	E_3	E_4	E_5	E_6
β -	22.98	3.86	5.37	16.45	28.92	0.89
σ -	4.93	0.87	0.38	3.54	1.82	0.17

Diagnostic equations after tuning take the form:

$$\mu^{E_1} = (\mu^{C_1} \wedge 0.21) \vee (\mu^{C_2} \wedge 0.78) \vee (\mu^{C_3} \wedge 0.15) \vee (\mu^{C_4} \wedge 0.84) \vee (\mu^{C_5} \wedge 0.73) \vee (\mu^{C_6} \wedge 0.18)$$
$$\mu^{E_2} = (\mu^{C_1} \wedge 0.97) \vee (\mu^{C_2} \wedge 0.20) \vee (\mu^{C_3} \wedge 0.43) \vee (\mu^{C_4} \wedge 0.18) \vee (\mu^{C_5} \wedge 0.14) \vee (\mu^{C_6} \wedge 0.58)$$
$$\mu^{E_3} = (\mu^{C_1} \wedge 0.48) \vee (\mu^{C_2} \wedge 0.59) \vee (\mu^{C_3} \wedge 0.85) \vee (\mu^{C_4} \wedge 0.63) \vee (\mu^{C_5} \wedge 0.34) \vee (\mu^{C_6} \wedge 0.12)$$
$$\mu^{E_4} = (\mu^{C_1} \wedge 0.94) \vee (\mu^{C_2} \wedge 0.21) \vee (\mu^{C_3} \wedge 0.64) \vee (\mu^{C_4} \wedge 0.18) \vee (\mu^{C_5} \wedge 0.16) \vee (\mu^{C_6} \wedge 0.74)$$
$$\mu^{E_5} = (\mu^{C_1} \wedge 0.16) \vee (\mu^{C_2} \wedge 0.14) \vee (\mu^{C_3} \wedge 0.92) \vee (\mu^{C_4} \wedge 0.08) \vee (\mu^{C_5} \wedge 0.10) \vee (\mu^{C_6} \wedge 0.41)$$
$$\mu^{E_6} = (\mu^{C_1} \wedge 0.64) \vee (\mu^{C_2} \wedge 0.82) \vee (\mu^{C_3} \wedge 0.21) \vee (\mu^{C_4} \wedge 0.72) \vee (\mu^{C_5} \wedge 0.99) \vee (\mu^{C_6} \wedge 0.09)$$

(6.19)

Let us represent the vector of the observed parameters for a specific pump:

$$\mathbf{Y}^* = (y_1^* = 26.12 \text{ m}^3/\text{h};\ y_2^* = 5.08 \text{ kg/cm}^2;\ y_3^* = 24 \text{ kw};\ y_4^* = 0.781 \text{ kg/cm}^2).$$

The measures of the effects significances for these values can be defined with the help of the membership functions in Fig. 6.13,b:

$$\boldsymbol{\mu}^E(\mathbf{Y}^*) = (\mu^{E_1} = 0.71;\ \mu^{E_2} = 0.34;\ \mu^{E_3} = 0.63;\ \mu^{E_4} = 0.18;\ \mu^{E_5} = 0.12;\ \mu^{E_6} = 0.71).$$

The genetic algorithm yields a null solution

$$\boldsymbol{\mu}_0^C = (\mu_0^{C_1} = 0.26, \mu_0^{C_2} = 0.54, \mu_0^{C_3} = 0.14, \mu_0^{C_4} = 0.69, \mu_0^{C_5} = 0.71, \mu_0^{C_6} = 0.08)\,, \quad (6.20)$$

for which the value of the optimization criterion (6.6) is F=0.0144.

The obtained null solution allows us to arrange for the genetic search for the solution set $S(\mathbf{R}, \boldsymbol{\mu}^E)$, which is completely determined by the greatest solution

$$\bar{\boldsymbol{\mu}}^C = (\bar{\mu}^{C_1} = 0.26,\ \bar{\mu}^{C_2} = 0.71,\ \bar{\mu}^{C_3} = 0.16,\ \bar{\mu}^{C_4} = 0.71,\ \bar{\mu}^{C_5} = 0.71,\ \bar{\mu}^{C_6} = 0.16)$$

and the three lower solutions $S^* = \{\underline{\boldsymbol{\mu}}_1^C, \underline{\boldsymbol{\mu}}_2^C, \underline{\boldsymbol{\mu}}_3^C\}$

$$\underline{\boldsymbol{\mu}}_1^C = (\underline{\mu}_1^{C_1} = 0.26,\ \underline{\mu}_1^{C_2} = 0.71,\ \underline{\mu}_1^{C_3} = 0,\ \underline{\mu}_1^{C_4} = 0.63,\ \underline{\mu}_1^{C_5} = 0,\ \underline{\mu}_1^{C_6} = 0);$$
$$\underline{\boldsymbol{\mu}}_2^C = (\underline{\mu}_2^{C_1} = 0.26,\ \underline{\mu}_2^{C_2} = 0,\ \underline{\mu}_2^{C_3} = 0,\ \underline{\mu}_2^{C_4} = 0.71,\ \underline{\mu}_2^{C_5} = 0,\ \underline{\mu}_2^{C_6} = 0);$$
$$\underline{\boldsymbol{\mu}}_3^C = (\underline{\mu}_3^{C_1} = 0.26,\ \underline{\mu}_3^{C_2} = 0,\ \underline{\mu}_3^{C_3} = 0,\ \underline{\mu}_3^{C_4} = 0.63,\ \underline{\mu}_3^{C_5} = 0.71,\ \underline{\mu}_3^{C_6} = 0).$$

Thus, the solution of fuzzy relational equations (6.19) can be represented in the form of intervals:

$$S(\mathbf{R},\boldsymbol{\mu}^E)=\{\mu^{C_1}=0.26;\mu^{C_2}=0.71;\mu^{C_3}\in[0,0.16];\mu^{C_4}\in[0.63,0.71];\mu^{C_5}\in[0,0.71];\mu^{C_6}\in[0,0.16]\}$$
$$\bigcup\{\mu^{C_1}=0.26;\ \mu^{C_2}\in[0,0.71];\ \mu^{C_3}\in[0,0.16];\ \mu^{C_4}=0.71;\ \mu^{C_5}\in[0,0.71];\ \mu^{C_6}\in[0,0.16]\}$$
$$\bigcup\{\mu^{C_1}=0.26;\ \mu^{C_2}\in[0,0.71];\ \mu^{C_3}\in[0,0.16];\ \mu^{C_4}\in[0.63,0.71];\ \mu^{C_5}=0.71;\ \mu^{C_6}\in[0,0.16]\}. \tag{6.21}$$

The intervals of the values of the input variables for each interval in solution (6.21) can be defined with the help of the membership functions in Fig. 6.13,b:

- x_1^*=2877 rpm for C_1;
- x_2^*=34.15 or $x_2^*\in$ [34.15, 45] cm^2/kw for C_2;
- $x_3^*\in$ [0.178, 0.300] mm for C_3;
- x_3^*=0.242 or $x_3^*\in$ [0.234, 0.242] mm for C_4;
- x_4^*=1.62 or $x_4^*\in$ [0.5, 1.62] cm^3/h for C_5;
- $x_5^*\in$ [1.2, 1.95] kg/cm^2 for C_6.

The obtained solution allows the analyst to make the preliminary conclusions. The cause of the observed pump state should be located and identified as the filter clogging, the throat ring wear-out or fuel escape in the suction conduit (clear area decreased up to 34.15-45 cm^2/kw, side clearance increased up to 0.234-0.242 mm, and leakage increased up to 0.5-1.62 cm^3/h), since the significance measures of the causes C_2, C_4 and C_5 are sufficiently high. An assembly defect of the throat ring for the side clearance within 0.178-0.300 mm should be excluded since the significance measure of the cause C_3 is small. The engine speed reduced to 2877 rpm can also tell on the pump's proper functioning, the significance measure of which is indicative of the cause C_1. Resistance of the force main increased up to 1.2-1.95 kg/cm^2 practically has no influence on the pump fault, so that the significance measure of cause C_6 is small.

Assume a repeated measurement has revealed a decrease in the pump delivery up to y_1^*=24.97 m^3/h and an increase in the suction pressure up to y_4^*=0.792 kg/cm^2, the values of μ^{E_1} increasing up to 0.86, μ^{E_6} up to 0.75, and the values of other parameters remaining unchanged.

A neural adjustment of the null solution (6.20) has yielded a fuzzy causes vector

$$\boldsymbol{\mu}_0^C=(\mu_0^{C_1}=0.26,\mu_0^{C_2}=0.17,\mu_0^{C_3}=0.10,\mu_0^{C_4}=0.93,\mu_0^{C_5}=0.75,\mu_0^{C_6}=0.05)\,,$$

for which the value of the optimization criterion (6.6) has constituted F=0.0148.

The resulting null solution has allowed adjustment of the bounds in the solution (6.21) and generation of the set of solutions $S(\mathbf{R},\boldsymbol{\mu}^E)$ determined by the greatest solution

$$\bar{\boldsymbol{\mu}}^C=(\bar{\mu}^{C_1}=0.26,\ \bar{\mu}^{C_2}=0.75,\ \bar{\mu}^{C_3}=0.16,\ \bar{\mu}^{C_4}=1.00,\ \bar{\mu}^{C_5}=0.75,\ \bar{\mu}^{C_6}=0.16)$$

and the two lower solutions $S^*=\{\underline{\boldsymbol{\mu}}_1^C,\underline{\boldsymbol{\mu}}_2^C\}$

$$\underline{\boldsymbol{\mu}}_1^C=(\underline{\mu}_1^{C_1}=0.26,\ \underline{\mu}_1^{C_2}=0.75,\ \underline{\mu}_1^{C_3}=0,\ \underline{\mu}_1^{C_4}=0.84,\ \underline{\mu}_1^{C_5}=0,\ \underline{\mu}_1^{C_6}=0);$$
$$\underline{\boldsymbol{\mu}}_2^C=(\underline{\mu}_2^{C_1}=0.26,\ \underline{\mu}_2^{C_2}=0,\ \underline{\mu}_2^{C_3}=0,\ \underline{\mu}_2^{C_4}=0.84,\ \underline{\mu}_2^{C_5}=0.75,\ \underline{\mu}_2^{C_6}=0).$$

Thus, the solution of fuzzy relational equations (6.19) can be represented in the form of intervals:

$$S(\mathbf{R},\boldsymbol{\mu}^E)=\{\mu^{C_1}=0.26;\mu^{C_2}=0.75;\mu^{C_3}\in[0,0.16];\mu^{C_4}\in[0.84,1.00];\mu^{C_5}\in[0,0.75];\mu^{C_6}\in[0,0.16]\}$$
$$\cup\{\mu^{C_1}=0.26;\ \mu^{C_2}\in[0,0.75];\ \mu^{C_3}\in[0,0.16];\ \mu^{C_4}\in[0.84,1.00];\ \mu^{C_5}=0.75;\ \mu^{C_6}\in[0,0.16]\}. \tag{6.22}$$

Solution (6.22) differs from (6.21) in the significance measures of the causes C_2, C_4 and C_5. The ranges of input variables have been determined for these causes using the membership functions in Fig. 6.13,a:

- $x_2^*=33.97$ or $x_2^*\in[33.97, 45]$ cm^2/kw for C_2;
- $x_3^*\in[0.254, 0.300]$ mm for C_4;
- $x_4^*=1.64$ or $x_4^*\in[0.5, 1.64]$ cm^3/h for C_5.

The solution obtained allows for the final conclusions. The state of the pump being observed is due to the throat ring wear-out (the side clearance increased to 0.254-0.300 mm), since the significance measure of the cause C_4 is maximal. The causes of the pump failure are still the filter clogging and fuel escape in the suction conduit (the flow area decreased to 33.97-45 cm^2/kw and the leakage increased to 0.5-1.64 cm^3/h), since the significance measures of the causes C_2 and C_5 are reasonably high. The values of other parameters have not changed.

To test the fuzzy model we used the results of diagnosis for 250 pumps with different kinds of faults. The tuning algorithm efficiency characteristics for the testing data are given in Table 6.5. Attaining an average accuracy rate of 95% required 30 min of the operation of a genetic algorithm and 4 min of the operation of a neural network (Intel Core 2 Duo P7350 2.0 GHz).

Table 6.5. Tuning algorithm efficiency characteristics

Causes (diagnoses)	Number of cases in the data sample	Probability of the correct diagnosis		
		Before tuning	After tuning	
			Null solution (genetic algorithm)	Refined diagnoses (neural network)
C_1	105	83 / 105 = 0.79	99 / 105 = 0.94	103 / 105 = 0.98
C_2	203	164 / 203 = 0.81	186 / 203 = 0.92	197 / 203 = 0.97
C_3	59	52 / 59 = 0.88	54 / 59 = 0.91	57 / 59 = 0.97
C_4	187	154 / 187 = 0.82	174 / 187 = 0.93	178 / 187 = 0.95
C_5	94	85 / 94 = 0.90	90 / 94 = 0.96	93 / 94 = 0.99
C_6	75	64 / 75 = 0.85	69 / 75 = 0.92	73 / 75 = 0.97

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