Chapter 4
Fuzzy Rules Extraction from Experimental Data

The necessary condition for nonlinear object identification on the basis of fuzzy logic is the availability of IF-THEN rules interconnecting linguistic estimations of input and output variables. Earlier we assumed that IF-THEN rules are generated by an expert who knows the object very well. What is to be done when there is no expert? In this case the generation of IF-THEN rules becomes of interest because it means the generation of fuzzy knowledge base from accessible experimental data [1].

Transformation of experimental information into fuzzy knowledge bases may turn out to be a useful method of data processing in medicine, banking, management and in other fields where persons making decisions instead of strict quantitative relations give preference to the use of transparent easily interpreted verbal rules [2, 3]. In this case proximity of linguistic approximation results and corresponding experimental data is the criterion for the quality of extracted regularities.

Fuzzy-neural networks and genetic algorithms are traditionally used for knowledge extraction from experimental data [4]. Fuzzy-neural network is an excellent approach for automatic rules formation and adjustment due to the mechanisms of pruning redundant membership functions and rules [5 – 7]. However, convergence of the training depends on the initial structure of the fuzzy model. On the other hand, genetic algorithms grow the appropriate structure of fuzzy inference automatically [8, 9]. In this case, the restriction of the total number of fuzzy terms and fuzzy rules prevents the construction of more compact structure of the fuzzy model. Combinations of both paradigms stipulated for the development of a new hybrid approach, which consists of automatic generation of fuzzy-neural network based on the genetic algorithm [10 – 13].

The extraction of fuzzy IF-THEN rules has two phases. In the first phase we define the fuzzy model structure by using the generalized fuzzy approximator proposed in [3, 14]. The second phase consists of finding optimal parameters of rules which provide the least distance between the model and experimental outputs of the object. For solving the optimization problem we use a combination of genetic algorithm and neural network. The genetic algorithm provides a rough finding of the appropriate structure of the fuzzy inference [15, 16]. We use the neural network for fine adjustment and adaptive correction of approximating rules by pruning the redundant membership functions and rules [17].

This chapter is written using original work materials [15 – 17].

A.P. Rotshtein et al.: Fuzzy Evidence in Identif., Forecast. and Diagn., STUDFUZZ 275, pp. 119–148.
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4.1 Fuzzy Rules for "Multiple Inputs – Single Output" Object

Let us consider an object of this form

$$y = f(x_1, x_2, ..., x_n) \tag{4.1}$$

with n inputs and one output for which the following is known:

- intervals of inputs and output change:

$$x_i \in [\underline{x}_i, \overline{x}_i]\ ,\ i = \overline{1,n}\ ,\ y \in [\underline{y}, \overline{y}]\ ,$$

- classes of decisions d_j $(j = \overline{1,m})$ in case of discrete output:

$$[\underline{y}, \overline{y}] = [\underbrace{\underline{y}, y_1}_{d_1}) \cup \ldots \cup [\underbrace{y_{j-1}, y_j}_{d_j}) \cup \ldots \cup [\underbrace{y_{m-1}, \overline{y}}_{d_m}]\ .$$

- training sample in the form of M pairs of experimental data "inputs-output":

$\{\mathbf{X}_p, y_p\}$ - for objects with continuous output,

$\{\mathbf{X}_p, d_p\}$ - for objects with discrete output,

where $\mathbf{X}_p = \{x_1^p, x_2^p, ..., x_n^p\}$ - input vector in p-th pair, $p = \overline{1,M}$.

It is required: to synthesize knowledge about object (4.1) in the form of fuzzy logical expressions system:

$$\begin{aligned}
&\text{IF}\ \left[\ (x_1 = a_1^{j1})\ \text{AND}\ (x_2 = a_2^{j1})\ \text{AND} ...\ (x_n = a_n^{j1})\ \right] \text{(with weight } w_{j1}\text{)} \\
&\text{OR}\ \left[\ (x_1 = a_1^{j2})\ \text{AND}\ (x_2 = a_2^{j2})\ \text{AND} ...\ (x_n = a_n^{j2})\ \right] \text{(with weight } w_{j2}\text{)} ... \\
&....\text{OR}\ \left[\ \left(x_1 = a_1^{jk_j}\right)\ \text{AND}\ \left(x_2 = a_2^{jk_j}\right)\ \text{AND} ...\ \left(x_n = a_n^{jk_j}\right)\ \right] \text{(with weight } w_{jk_j}\text{)}, \\
&\quad \text{THEN}\quad y \in d_j = [y_{j-1}, y_j]\ \text{, for all}\ \ j = \overline{1,m},
\end{aligned} \tag{4.2}$$

where a_i^{jp} is the linguistic term for variable x_i evaluation in the row with number $p = \overline{1,k_j}$,

k_j is the number of conjunction rows corresponding to the class d_j, $j = \overline{1,m}$,

w_{jp} is a number in the range $[0, 1]$, which characterizes the weight of the expression with number jp .

4.2 Rules Extraction as Optimization Problem

It was shown that object (4.1) model in the form of the following calculation relations corresponds to knowledge base (4.2):

$$y = \frac{\underline{y}\mu^{d_1}(y) + y_1\mu^{d_2}(y) + ... + y_{m-1}\mu^{d_m}(y)}{\mu^{d_1}(y) + \mu^{d_2}(y) + ... + \mu^{d_m}(y)} \quad , \tag{4.3}$$

$$\mu^{d_j}(y) = \max_{p=\overline{1,k_j}} \left\{ w_{jp} \min_{i=\overline{1,n}} [\mu^{jp}(x_i)] \right\} \quad , \tag{4.4}$$

$$\mu^{jp}(x_i) = \frac{1}{1 + \left(\frac{x_i - b_i^{jp}}{c_i^{jp}} \right)} \quad , \quad i = \overline{1,n} \ , \ p = \overline{1,k_j} \ , \ j = \overline{1,m} \ , \tag{4.5}$$

where $\mu^{d_j}(y)$ is the membership function of the output y to the class d_j,

$\mu^{jp}(x_i)$ is the membership function of the input x_i to the term a_i^{jp},

b_i^{jp} and c_i^{jp} are the tuning parameters for the input variables x_i membership functions.

Relations (4.3) - (4.5) define the model of the object (4.1) which is written down in this form:

$y = F(\mathbf{X}, \mathbf{W}, \mathbf{B}, \mathbf{C})$ - for continuous output,

$\mu^{d_j}(y) = \mu^{d_j}(\mathbf{X}, \mathbf{W}, \mathbf{B}, \mathbf{C})$ - for discrete output,

where $\mathbf{X} = (x_1, x_2, ..., x_n)$ is the input vector, $\mathbf{W} = (w_1, w_2, ..., w_N)$ is the vector of rules-rows in the fuzzy knowledge base (4.2), $\mathbf{B} = (b_1, b_2, ..., b_q)$ and $\mathbf{C} = (c_1, c_2, ..., c_q)$ are the vectors of fuzzy terms membership functions tuning parameters in (4.5), N is the total number of rules-rows, q is the total number of terms, F is the operator of inputs-output connection corresponding to relations (4.3) - (4.5).

Let us impose limitations on the knowledge base (4.2) volume in one of the following forms:

a) $N = k_1 + k_2 + ... + k_m \leq \overline{N}$,

b) $k_1 \leq \overline{k_1}$, $k_2 \leq \overline{k_2}$, ..., $k_m \leq \overline{k_m}$,

where $\overline{N}$ is the maximum permissible total number of conjunction rows in (4.2),

$\overline{k_j}$ is the maximum permissible number of conjunction rows in rules of j-th decision class, $j = \overline{1,m}$.

So as the content and number of linguistic terms a_i^{jp} ($i = \overline{1,n}$, $p = \overline{1,k_j}$, $j = \overline{1,m}$), used in the knowledge base (4.2), are not known beforehand then it is suggested to interpret them on the basis of membership functions (4.5) parameter values (b_i^{jp}, c_i^{jp}). Therefore, knowledge base (4.2) synthesis is reduced to obtaining the parameter matrix shown in Table 4.1.

Table 4.1. Knowledge base parameters matrix

Rule	IF			Weight	THEN
	x_1	… x_i …	x_n		y
11	(b_1^{11}, c_1^{11})	(b_i^{11}, c_i^{11})	(b_n^{11}, c_n^{11})	w_{11}	
12	(b_1^{12}, c_1^{12})	(b_i^{12}, c_i^{12})	(b_n^{12}, c_n^{12})	w_{12}	d_1
…	…	…	…	…	
$1\,k_1$	$(b_1^{1k_1}, c_1^{1k_1})$	$(b_i^{1k_1}, c_i^{1k_1})$	$(b_n^{1k_1}, c_n^{1k_1})$	w_{1k_1}	
…	…	…	…	…	
$j\,1$	(b_1^{j1}, c_1^{j1})	(b_i^{j1}, c_i^{j1})	(b_n^{j1}, c_n^{j1})	w_{j1}	
$j\,2$	(b_1^{j2}, c_1^{12})	(b_i^{j2}, c_i^{j2})	(b_n^{j2}, c_n^{j2})	w_{j2}	
…	…	…	…	…	d_j
$j\,k_j$	$\left(b_1^{jk_j}, c_1^{jk_j}\right)$	$\left(b_i^{jk_j}, c_i^{jk_j}\right)$	$\left(b_n^{jk_j}, c_n^{jk_j}\right)$	w_{jk_j}	
…	…	…	…	…	
$m\,1$	(b_1^{m1}, c_1^{m1})	(b_i^{m1}, c_i^{m1})	(b_n^{m1}, c_n^{m1})	w_{m1}	
$m\,2$	(b_1^{m2}, c_1^{m2})	(b_i^{m2}, c_i^{m2})	(b_n^{m2}, c_n^{m2})	w_{m2}	
…	…	…	…	…	d_m
$m\,k_m$	$\left(b_1^{mk_m}, c_1^{mk_m}\right)$	$\left(b_i^{mk_m}, c_i^{mk_m}\right)$	$\left(b_n^{mk_m}, c_n^{mk_m}\right)$	w_{mk_m}	

In terms of mathematical programming this problem can be formulated in the following way. It is required to find such matrix (Table 4.1) which satisfying limitations imposed on parameters $(\mathbf{W}, \mathbf{B}, \mathbf{C})$ change ranges and number of rows provides for:

$$\sum_{p=1}^{M} [F(\mathbf{X}_p, \mathbf{W}, \mathbf{B}, \mathbf{C}) - y_p]^2 = \min_{\mathbf{W},\mathbf{B},\mathbf{C}}, \tag{4.6}$$

for the object with continuous output,

$$\sum_{p=1}^{M} \left\{ \sum_{j=1}^{m} \left[\mu^{d_j}(\mathbf{X}_p, \mathbf{W}, \mathbf{B}, \mathbf{C}) - \mu_p^{d_j}(y) \right]^2 \right\} = \min_{\mathbf{W},\mathbf{B},\mathbf{C}}, \tag{4.7}$$

for the object with discrete output, where

$$\mu_p^{d_j} = \begin{cases} 1, & if \quad d_j = d_p \\ 0, & if \quad d_j \neq d_p \end{cases}.$$

To solve these optimization problems it is appropriate to use a hybrid genetic and neuro approach.

4.3 Genetic Algorithm for Rules Extraction

The chromosome describing desired parameter matrix (Table 4.1), we define by the row shown in Fig. 4.1, where r_{jp} is the code of IF-THEN rule with number jp, $p=\overline{1,k_j}$, $j=\overline{1,m}$.

The operation of chromosomes crossover is defined in Fig. 4.2. It consists of exchanging parts of chromosomes in each rule r_{jp} ($j=\overline{1,m}$) and rules weights vector. The total number of exchange points makes $\overline{k_1}+\overline{k_2}+...+\overline{k_m}+1$: one for each rule and one for rules weights vector.

The operation of mutation (Mu) consists in random change (with some probability) of chromosome elements:

$$Mu(w_{jp}) = RANDOM([0,1]) \ ,$$

$$Mu\ (b_i^{jp}) = RANDOM([\underline{x}_i, \overline{x}_i]) \ ,$$

$$Mu\ (c_i^{jp}) = RANDOM([\underline{c}_i^{jp}, \overline{c}_i^{jp}]) \ ,$$

where $RANDOM([\underline{x},\overline{x}])$ is the operation of finding random number which is uniformly distributed on the interval $[\underline{x},\overline{x}]$.

If rules weights can take values 1 (rule available) or 0 (rule not available), then weights mutation can take place by way of random choice of 1 or 0.

Fitness function of chromosomes - solutions is calculated on the basis of (4.6) and (4.7) criteria.

If $P(t)$ - parent chromosomes, and $C(t)$ - offspring chromosomes on t-th iteration then genetic procedure of optimization will be carried out according to the following algorithm [18, 19]:

begin
t:=0 ;
assign initial value $P(t)$;
estimate $P(t)$ using criteria (4.6) and (4.7);
while (**not** condition for completion) **do**
Crossover $P(t)$ to obtain $C(t)$;
Estimate *C(t)* using criteria (4.6) and (4.7);
Choose $P(t+1)$ from $P(t)$ and $C(t)$;
t:=t+1 ;
end
end

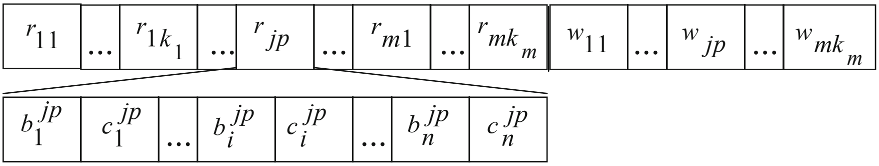

Fig. 4.1. Coding of parameter matrix

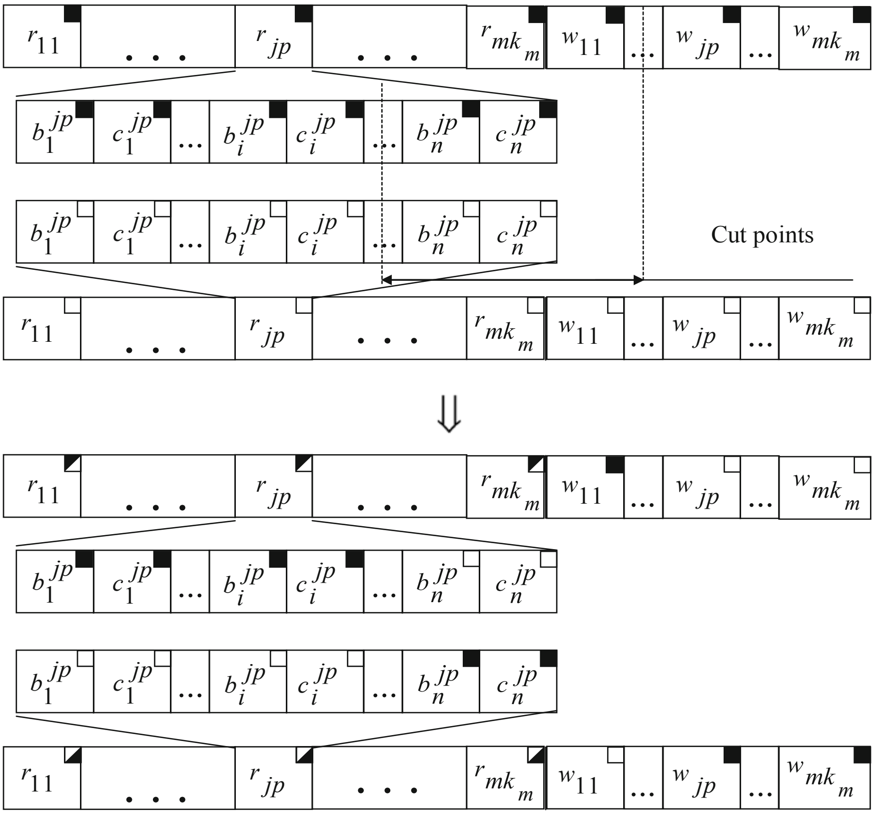

Fig. 4.2. Crossover operation (■, □ - parents symbols, ◢, ◩ - offspring symbols)

4.4 Neuro-fuzzy Network for Rules Extraction from Data

Let us impose limitations on the knowledge base (4.2) volume in the following form:

$$q_1 \leq \overline{q_1}\,,\; q_2 \leq \overline{q_2}\,, \ldots,\; q_n \leq \overline{q_n}\,,$$

where $\bar{q}_i$ is the maximum permissible total number of fuzzy terms describing a variable x_i, $i = \overline{1,n}$;

This allows embedding system (4.2) into the special neuro-fuzzy network, which is able to extract knowledge [7, 17]. The neuro-fuzzy network for knowledge extraction is shown in Fig. 4.3, and the nodes are presented in Table 3.1.

As is seen from Fig. 4.3 the neuro-fuzzy network has the following structure:

layer 1 for object identification inputs (the number of nodes is equal to *n*),

layer 2 for fuzzy terms used in knowledge base (the number of nodes is equal to $\bar{q}_1 + \bar{q}_2 + ... + \bar{q}_n$),

layer 3 for strings-conjunctions (the number of nodes is equal to $\bar{q}_1 \cdot \bar{q}_2 \cdot ... \cdot \bar{q}_n$),

layer 4 for fuzzy rules making classes (the layer is fully connected, the number of nodes is equal to the number of output classes *m*),

layer 5 for a defuzzification operation.

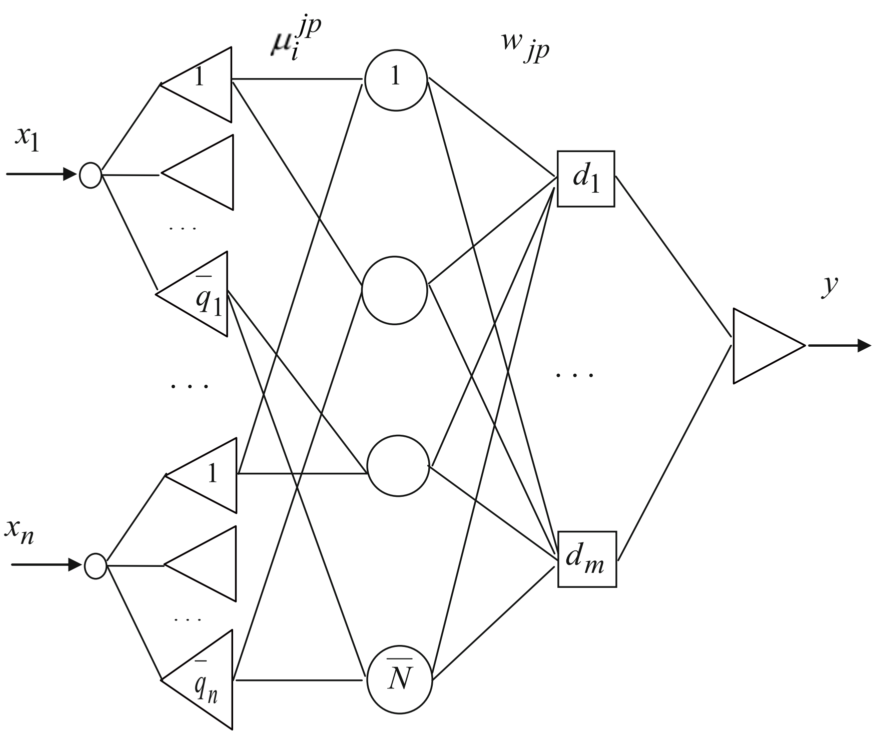

Fig. 4.3. Neuro-fuzzy network for knowledge extraction

To train the parameters of the neuro-fuzzy network, the recurrent relations

$$w_{jp}(t+1) = w_{jp}(t) - \eta \frac{\partial E_t}{\partial w_{jp}(t)} ,$$

$$c_i^{jp}(t+1) = c_i^{jp}(t) - \eta \frac{\partial E_t}{\partial c_i^{jp}(t)} , \quad b_i^{jp}(t+1) = b_i^{jp}(t) - \eta \frac{\partial E_t}{\partial b_i^{jp}(t)}$$

are used which minimize the criterion

$$E_t = \frac{1}{2}(\hat{y}_t - y_t)^2 ,$$

applied in the neural network theory, where $\hat{y}_t$ (y_t) are experimental and model outputs of the object at the t-th step of training;

$w_{jp}(t)$, $c_i^{jp}(t)$, $b_i^{jp}(t)$ are rules weights and parameters for the fuzzy terms membership functions at the t-th step of training;

η is a parameter of training [20].

The partial derivatives appearing in recurrent relations can be obtained according to the results from Section 3.3.

4.5 Computer Simulations

Example 1

Experimental data about the object was generated using the model "one input – one output"

$$y = f(x) = e^{\frac{-x}{4}} \cdot \sin(\tfrac{\pi}{2}x) , \ x \in [0, 10] , \ y \in [-0.47, 0.79] , \tag{4.8}$$

which is represented in Fig. 4.4.

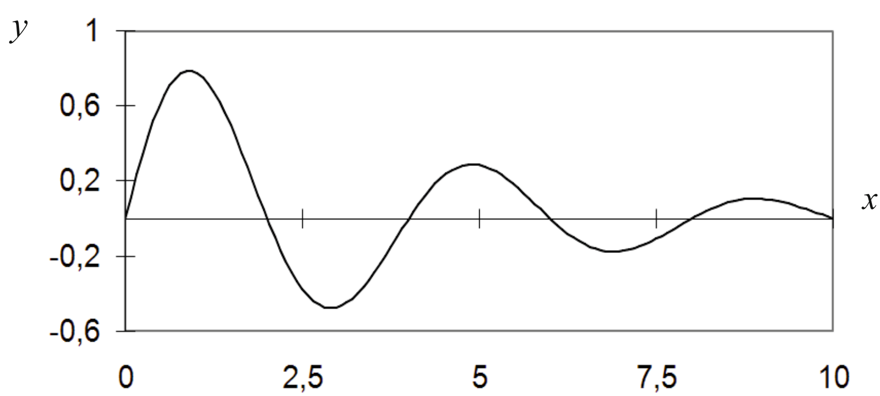

Fig. 4.4. "One input – one output" object behavior

The object output was divided into seven classes:

$$y \in \underbrace{[-0.47,-0.30)}_{d_1} \cup \underbrace{[-0.30,-0.05)}_{d_2} \cup \underbrace{[-0.05,0.15)}_{d_3} \cup \underbrace{[0.15,0.30)}_{d_4} \cup$$
$$\cup \underbrace{[0.30,0.45)}_{d_5} \cup \underbrace{[0.45,0.65)}_{d_6} \cup \underbrace{[0.65,0.78]}_{d_7}$$

The goal was to synthesize 5 rules for every class describing the object (4.8).

Rules weights were accepted as equal to 0 and 1. As the result of using the genetic and neuro algorithm of optimization we obtained the parameters matrix represented in Table 4.2.

Table 4.2. Rules parameters matrix

IF x				THEN y
Genetic algorithm		Neuro-fuzzy network		
Term parameters (b, c)	Weight	Term parameters (b, c)	Weight	
(2.85, 0.96)	1	(2.81, 1.12)	1	d_1
(2.77, 1.05)	1	(2.72, 0.70)	1	
(2.90, 0.88)	1	(2.93, 0.85)	1	
(0.25, 0.85)	0	(0.13, 0.64)	0	
(2.88, 1.24)	1	(2.81, 1.17)	1	
(6.85, 1.94)	1	(6.11, 1.13)	1	d_2
(8.74, 1.26)	1	(3.71, 0.25)	0	
(8.91, 2.17)	1	(6.91, 2.05)	1	
(6.92, 1.83)	1	(6.83, 0.72)	1	
(0.93, 1.21)	0	(1.13, 0.92)	0	
(0.06, 0.74)	1	(0.13, 0.87)	1	d_3
(8.91, 2.53)	0	(9.10, 1.25)	0	
(9.72, 2.12)	1	(8.62, 2.20)	1	
(9.90, 1.30)	1	(9.92, 1.12)	1	
(8.25, 1.15)	0	(8.7, 1.33)	1	
(4.85, 0.11)	1	(4.91, 0.21)	1	d_4
(5.33, 1.72)	1	(5.20, 1.50)	1	
(5.10, 1.08)	1	(5.01, 0.90)	1	
(6.54, 0.70)	0	(5.12, 0.83)	0	
(9.48, 2.31)	0	(9.17, 1.19)	0	
(2.00, 0.94)	0	(2.13, 0.72)	0	d_7
(0.64, 2.46)	0	(0.70, 1.25)	0	
(0.88, 0.76)	1	(0.92, 0.70)	1	
(1.25, 0.67)	0	(0.93, 1.12)	0	
(0.97, 2.18)	1	(1.01, 1.90)	1	

After linguistic interpretation the genetically generated rules look like this:

IF $x =$ *about* 2.8 THEN $y \in d_1$

IF $x =$ *about* 6.9 OR $x =$ *about* 8.8 THEN $y \in d_2$

IF $x =$ *about* 0 OR $x =$ *about* 10 THEN $y \in d_3$

IF $x =$ *about* 5 THEN $y \in d_4$

IF $x =$ *about* 0.9 THEN $y \in d_7$

Rules specified using neural adjustment after linguistic interpretation look like this:

IF $x =$ *about* 2.8 THEN $y \in d_1$

IF $x =$ *about* 6.9 THEN $y \in d_2$

IF $x =$ *about* 0 OR $x =$ *about* 8.8 OR $x =$ *about* 10 THEN $y \in d_3$

IF $x =$ *about* 5 THEN $y \in d_4$

IF $x =$ *about* 0.9 THEN $y \in d_7$

The model derived according to synthesized rules in comparison with the target one is shown in Fig. 4.5, 4.6.

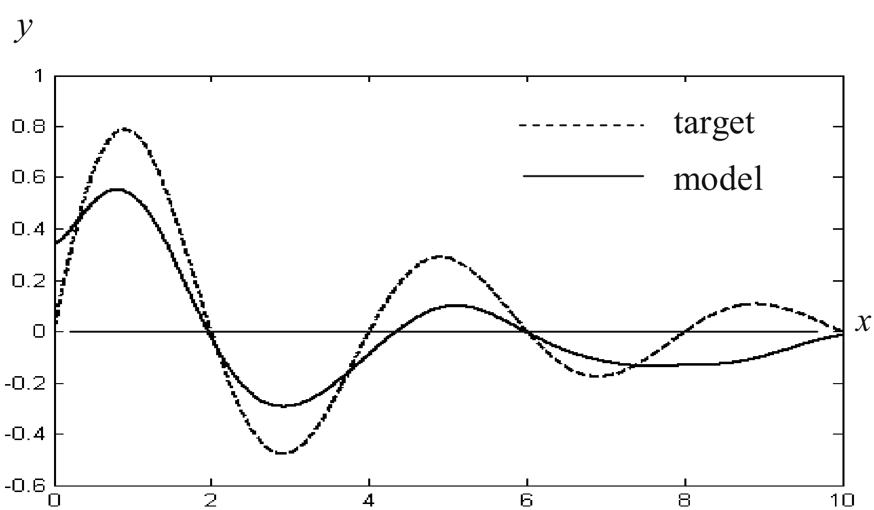

Fig. 4.5. Comparison of the genetically synthesized linguistic model with the standard

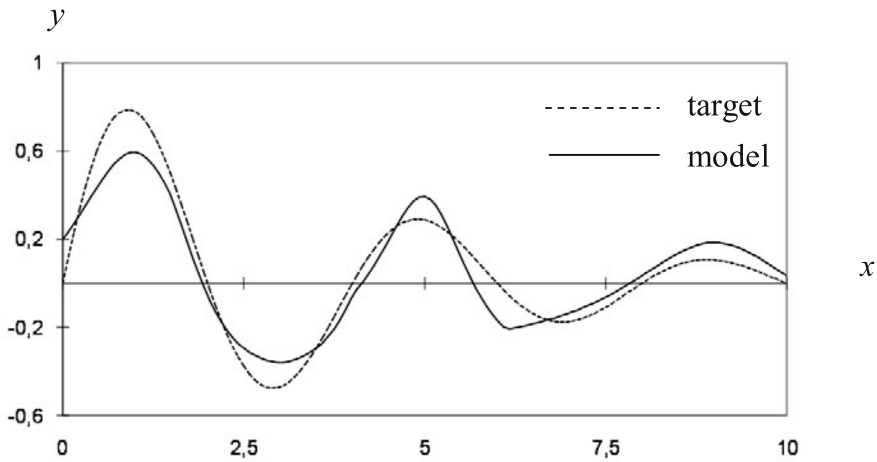

Fig. 4.6. Comparison of the linguistic model specified using neural adjustment with the standard

Further increase of linguistic model precision is possible on the account of its fine tuning.

Example 2

Experimental data about the object was generated using the model "two inputs – one output":

$$y = f(x_1, x_2) = \frac{1}{10}(2z-0.9)\ (7z-1)\ (17z-19)\ (15z-2)\,, \tag{4.9}$$

where $z = \dfrac{(x_1-3.0)^2+(x_2-3.0)^2}{40}$,

which is represented in Fig. 4.7.

The object output was divided into five classes:

$$y \in \underbrace{[-5.08,-4.50)}_{d_1} \cup \underbrace{[-4.50,-3.0)}_{d_2} \cup \underbrace{[-3.0,-0.5)}_{d_3} \cup \underbrace{[-0.5,0)}_{d_4} \cup \underbrace{[0,0.855)}_{d_5}\,.$$

The goal was to synthesize 20 rules for every class describing the object (4.9). Rules weights were accepted as equal to 0 and 1. As the result of using the genetic and neuro algorithm of optimization we obtained the parameters matrix represented in Table 4.3.

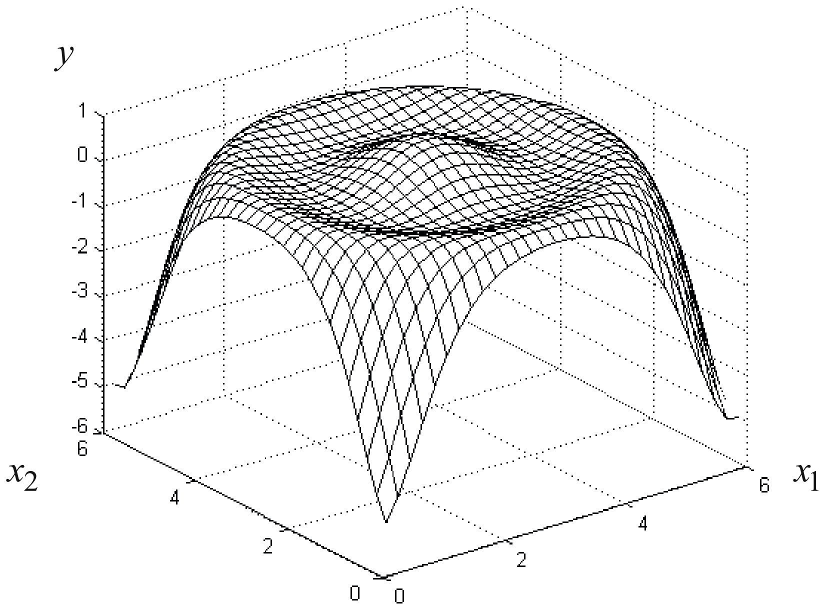

Fig. 4.7. "Two inputs – one output" object behaviour

Table 4.3. Rules parameters (*b* , *c*) matrix

Genetic algorithm			Neuro-fuzzy network			d
x_1	x_2	weight	x_1	x_2	weight	
(0.05, 0.12)	(1.10, 0.99)	1	(0.15, 0.08)	(1.16, 0.83)	1	d_1
(0.39, 0.98)	(0.02, 0.17)	1	(0.32, 0.75)	(0.09, 0.06)	1	
(4.83, 0.86)	(0.20, 0.11)	1	(4.72, 1.14)	(0.18, 0.09)	1	
(5.99, 0.15)	(1.33, 0.84)	1	(5.97, 0.12)	(1.48, 1.17)	1	
(0.20, 0.15)	(5.08, 0.92)	1	(0.17, 0.09)	(5.62, 0.79)	1	
(0.77, 0.96)	(5.92, 0.14)	1	(0.92, 0.81)	(5.99, 0.06)	1	
(5.95, 0.17)	(4.91, 0.83)	1	(5.85, 0.10)	(4.69, 0.72)	1	
(4.93, 1.36)	(5.90, 0.17)	1	(5.24, 1.17)	(5.99, 0.07)	1	
(0.08, 0.12)	(0.16, 0.08)	1	(0.04, 0.06)	(0.05, 0.11)	1	d_2
(5.99, 0.20)	(0.19, 0.18)	1	(5.98, 0.11)	(0.17, 0.04)	1	
(0.13, 0.17)	(5.92, 0.12)	1	(0.10, 0.09)	(5.97, 0.08)	1	
(5.97, 0.11)	(5.90, 0.20)	1	(5.87, 0.09)	(6.00, 0.10)	1	
(0.44, 0.96)	(0.87, 0.91)	1	(0.56, 1.17)	(1.28, 0.99)	0	
(4.06, 0.52)	(0.03, 0.08)	1	(5.88, 0.14)	(0.12, 0.14)	1	
(0.58, 1.07)	(5.71, 1.20)	1	(0.82, 1.34)	(5.86, 0.92)	0	
(4.91, 0.78)	(1.48, 0.77)	1	(5.32, 0.89)	(1.54, 0.65)	0	

Table 4.3. *(continued)*

(0.09, 0.15)	(2.04, 0.56)	1	(0.10, 0.12)	(2.17, 0.45)	1	d_3
(3.65, 0.74)	(1.52, 0.73)	1	(0.44, 0.96)	(0.87, 0.91)	1	
(5.91, 0.08)	(3.71, 0.67)	1	(1.86, 0.37)	(0.16, 0.09)	1	
(0.16, 0.07)	(3.94, 0.64)	1	(4.06, 0.52)	(0.03, 0.08)	1	
(0.04, 0.20)	(3.05, 0.86)	1	(4.91, 0.78)	(1.48, 0.77)	1	
(4.88, 0.84)	(5.32, 0.98)	1	(5.94, 0.09)	(2.11, 0.56)	1	
(3.02, 0.77)	(5.94, 0.13)	1	(0.06, 0.15)	(3.67, 0.39)	1	
(5.91, 0.34)	(0.12, 0.19)	0	(0.58, 1.07)	(5.71, 1.20)	1	
(5.34, 0.76)	(4.18, 0.56)	0	(5.96, 0.04)	(3.94, 0.65)	1	
(0.16, 0.25)	(3.44, 0.95)	0	(5.17, 0.88)	(4.98, 0.70)	1	
(4.97, 0.56)	(5.11, 0.93)	0	(2.02, 0.60)	(5.99, 0.06)	1	
(3.22, 0.91)	(5.99, 0.32)	0	(3.74, 0.49)	(5.87, 0.09)	1	
(0.22, 1.17)	(3.07, 0.85)	1	(0.16, 0.09)	(2.86, 0.59)	1	d_4
(1.25, 0.93)	(1.96, 0.53)	1	(1.07, 1.15)	(2.25, 0.35)	1	
(2.17, 0.75)	(0.74, 0.72)	1	(1.96, 0.54)	(0.37, 0.88)	1	
(3.00, 0.92)	(0.04, 0.26)	1	(3.04, 0.79)	(0.09, 0.16)	1	
(1.08, 0.54)	(3.45, 0.65)	1	(1.06, 0.94)	(3.75, 0.49)	1	
(5.93, 0.18)	(2.16, 0.78)	1	(4.07, 0.52)	(0.42, 0.30)	1	
(1.85, 0.46)	(0.06, 0.15)	1	(1.92, 0.33)	(1.96, 0.51)	1	
(3.03, 0.88)	(2.03, 0.47)	1	(2.96, 0.81)	(2.40, 0.38)	1	
(5.92, 0.20)	(2.34, 0.67)	1	(3.61, 0.42)	(2.08, 0.44)	1	
(2.03, 0.68)	(3.00, 0.91)	1	(4.75, 0.79)	(1.96, 0.50)	1	
(5.99, 0.08)	(2.92, 0.79)	1	(2.17, 0.38)	(3.08, 0.72)	1	
(1.98, 0.93)	(5.74, 1.17)	1	(3.81, 0.54)	(2.99, 0.85)	1	
(3.81, 0.69)	(3.66, 0.61)	1	(5.96, 0.11)	(3.06, 0.69)	1	
(4.82, 1.45)	(3.52, 0.93)	1	(1.77, 0.42)	(3.68, 0.47)	1	
(2.26, 0.74)	(4.65, 1.14)	1	(3.07, 0.68)	(4.05, 0.32)	1	
(3.67, 0.81)	(5.86, 0.26)	1	(3.91, 0.53)	(3.89, 0.37)	1	
(4.55, 1.34)	(3.22, 0.96)	0	(4.78, 1.15)	(3.61, 0.45)	1	
(1.87, 0.72)	(5.08, 0.33)	0	(2.18, 0.39)	(5.67, 0.95)	1	
(3.77, 0.21)	(4.26, 1.91)	0	(3.65, 0.47)	(4.86, 0.71)	1	
(3.08, 0.83)	(5.07, 2.36)	0	(2.97, 0.75)	(5.96, 0.11)	1	
(3.68, 1.31)	(4.78, 1.56)	1	(0.26, 0.81)	(3.02, 0.75)	1	d_5
(2.97, 0.93)	(0.52, 0.09)	1	(3.02, 0.70)	(0.56, 0.15)	1	
(2.92, 0.55)	(3.02, 0.98)	1	(2.96, 0.64)	(3.09, 0.66)	1	
(5.64, 0.97)	(3.00, 1.17)	1	(5.41, 0.79)	(3.03, 0.82)	1	
(3.02, 1.26)	(5.44, 0.97)	1	(3.06, 0.67)	(5.56, 1.13)	1	
(2.33, 0.85)	(2.07, 0.46)	1	(2.17, 1.68)	(1.74, 0.61)	0	
(3.92, 1.45)	(1.89, 0.92)	1	(3.12, 2.65)	(1.28, 1.12)	0	
(3.90, 1.58)	(3.02, 0.77)	1	(3.18, 0.54)	(3.00, 0.38)	0	
(1.82, 0.23)	(3.48, 0.82)	1	(1.89, 0.74)	(3.91, 0.60)	0	
(3.06, 1.72)	(4.01, 2.12)	1	(3.00, 2.16)	(4.871, 0.53)	0	

The generated rules after linguistic interpretation are presented in Table 4.4, where the parameters of fuzzy terms for variables x_1 and x_2 evaluation are interpreted as follows: *about* 0 – *Low* (*L*), *about* 0.5 – *higher than Low* (*hL*), *about* 1.5 – *lower than Average* (*lA*), *about* 3 – *Average* (*A*), *about* 4.5 – *higher than Average* (*hA*), *about* 5.5 – *lower than High* (*lH*), *about* 6 – *High* (*H*).

Table 4.4. Fuzzy knowledge base

Genetic algorithm		Neuro-fuzzy network		d
x_1	x_2	x_1	x_2	
L	*hL*	*L*	*hL*	d_1
hL	*L*	*hL*	*L*	
lH	*L*	*lH*	*L*	
H	*hL*	*H*	*hL*	
L	*lH*	*L*	*lH*	
hL	*H*	*hL*	*H*	
H	*lH*	*H*	*lH*	
lH	*H*	*lH*	*H*	
L	*L*	*L*	*L*	d_2
H	*L*	*H*	*L*	
L	*H*	*L*	*H*	
H	*H*	*H*	*H*	
hL	*hL*			
hA	*L*			
hL	*lH*			
lH	*hL*			
L	*lA*	*L*	*lA*	d_3
hA	*hL*	*hL*	*hL*	
H	*hA*	*lA*	*L*	
L	*hA*	*hA*	*L*	
L	*A*	*lH*	*hL*	
lH	*lH*	*H*	*lA*	
A	*H*	*L*	*hA*	
		hL	*lH*	
		H	*hA*	
		lH	*lH*	
		lA	*H*	
		hA	*H*	

Genetic algorithm		Neuro-fuzzy network		d
x_1	x_2	x_1	x_2	
hL	*A*	*L*	*A*	d_4
hL	*lA*	*hL*	*lA*	
lA	*hL*	*lA*	*hL*	
A	*L*	*A*	*L*	
hL	*hA*	*hL*	*hA*	
H	*lA*	*hA*	*hL*	
lA	*L*	*lA*	*lA*	
A	*lA*	*A*	*lA*	
lH	*lA*	*hA*	*lA*	
lA	*A*	*lH*	*lA*	
H	*A*	*lA*	*A*	
lA	*H*	*hA*	*A*	
hA	*hA*	*H*	*A*	
lH	*hA*	*lA*	*hA*	
lA	*lH*	*A*	*hA*	
hA	*H*	*hA*	*hA*	
		lH	*hA*	
		lA	*lH*	
		hA	*lH*	
		A	*H*	
hA	*lH*	*hL*	*A*	d_5
A	*hL*	*A*	*hL*	
A	*A*	*A*	*A*	
lH	*A*	*lH*	*A*	
A	*lH*	*A*	*lH*	
lA	*lA*			
hA	*lA*			
hA	*A*			
lA	*hA*			
A	*hA*			

The model of the object derived according to synthesized rules is shown in Fig. 4.8, 4.9.

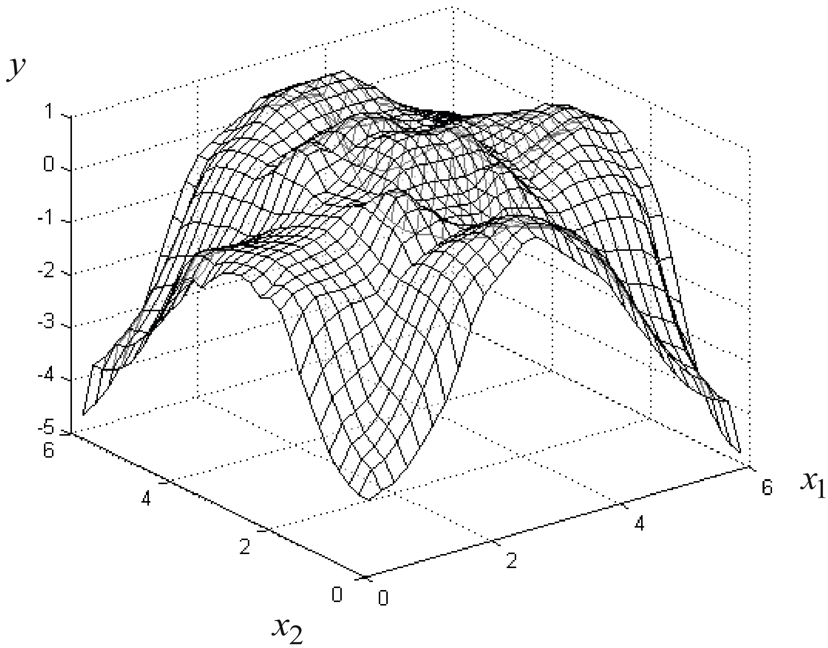

Fig. 4.8. Linguistic model synthesized using the genetic algorithm

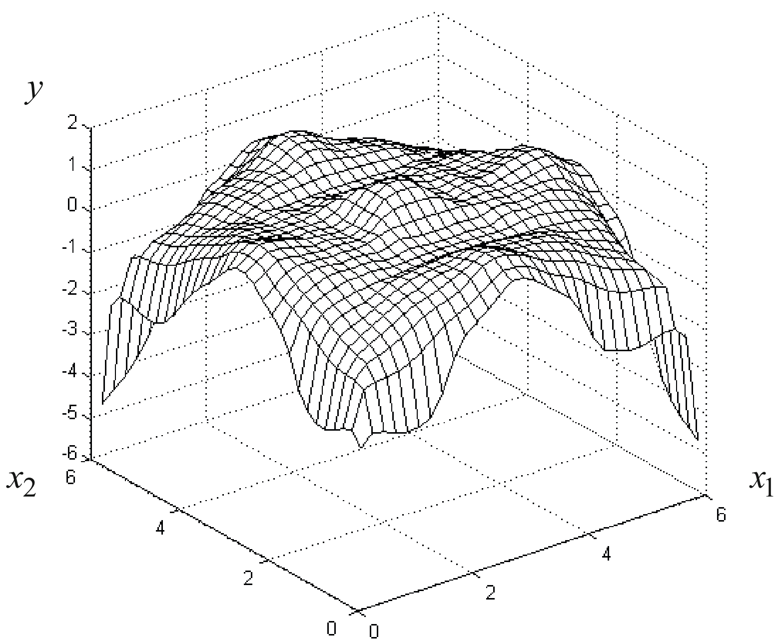

Fig. 4.9. Linguistic model specified using neural adjustment

Further increase of linguistic model precision is possible on the account of its fine tuning.

4.6 Example 3: Rules Extraction for Differential Diagnosis of Heart Disease

In a lot of areas of medicine there are huge experimental data collections and it is necessary to convert these data into the form convenient for decision making. Several well-known methods like mathematical statistics, regression analyses etc. are usually used for data processing [21]. Decision makers in medicine, however, are typically not statisticians or mathematicians. It is therefore important to present the results of data processing in an easily understandable form for decision makers without special mathematical backgrounds.

Fuzzy information granulation in the form of fuzzy IF-THEN rules [1] allows making the results of data analysis easily understandable and well interpretable. But during the development of fuzzy expert systems it is supposed that an initial knowledge base is generated by an expert from the given area of medicine [2, 3]. That is why the quality of these systems depends on the skill of a medical expert.

The aim of this section is (1) to propose the formal procedure of fuzzy IF-THEN rules extraction from histories of diseases and (2) to compare the results of medical diagnosis using extracted IF-THEN rules and the similar rules proposed by an expert [3].

A specific feature of fuzzy rules bases for medical diagnosis consists of their hierarchical character. In this section we propose the formal procedure for extraction of a hierarchical system of fuzzy rules for medical diagnosis from real histories of diseases. The suggested procedure is based on the optimal solution growing from a set of primary IF-THEN rules variants using the genetic cross-over, mutation and selection operations [18, 19]. The neural approach is used for adaptive correction of the diagnostic rules by pruning redundant membership functions and rules.

The efficiency of proposed genetic and neuro algorithms is illustrated by an example of ischemia heart disease (IHD) diagnosis [3].

4.6.1 Hierarchical System of IF-THEN Rules

Let us consider the object (3.30) - (3.32) for which the following is known:

- intervals of inputs (parameters of the patient state) change $x_i \in [\underline{x_i}, \overline{x_i}]$, $i = \overline{1,n}$,

- classes of decisions d_j $(j = \overline{1,m})$ (types of diagnoses),

- training data (histories of diseases) in the form of M pairs of experimental data “parameters of patient state - type of diagnose” $\{\mathbf{X}_p, d_p\}$, where $\mathbf{X}_p = \{x_1^p, x_2^p, ..., x_n^p\}$ - input vector in p-th pair, $p = \overline{1,M}$.

It is necessary to transfer the available training data into the following systems of the fuzzy IF-THEN rules:

1) for the instrumental danger y depending on parameters $\{x_2, x_3, x_4, x_5, x_{10}, x_{11}\}$:

$$\text{IF}\left[\ (x_2 = a_2^{j1}) \text{ AND } (x_3 = a_3^{j1}) \text{ AND} \ldots\ (x_{11} = a_{11}^{j1})\ \right] \text{(with weight } w_{j1}^y\text{)}$$

...

$$\text{OR}\left[\ \left(x_2 = a_2^{jk_j}\right) \text{ AND } \left(x_3 = a_3^{jk_j}\right) \text{ AND} \ldots\ \left(x_{11} = a_{11}^{jk_j}\right)\ \right] \text{(with weight } w_{jk_j}^y\text{),}$$

$$\text{THEN} \quad y \in y_j \text{, for all } j = \overline{1,5}\,; \tag{4.10}$$

2)) for the biochemical danger z depending on parameters $\{x_6, x_7, x_8, x_9, x_{12}\}$:

$$\text{IF}\left[\ (x_6 = a_6^{j1}) \text{ AND } (x_7 = a_7^{j1}) \text{ AND} \ldots\ (x_{12} = a_{12}^{j1})\ \right] \text{(with weight } w_{j1}^z\text{)}$$

...

$$\text{OR}\left[\ \left(x_6 = a_6^{jk_j}\right) \text{ AND } \left(x_7 = a_7^{jk_j}\right) \text{ AND} \ldots\ \left(x_{12} = a_{12}^{jk_j}\right)\ \right] \text{(with weight } w_{jk_j}^z\text{),}$$

$$\text{THEN } z \in z_j \text{, for all } j = \overline{1,5}\,; \tag{4.11}$$

3) *for the danger of IHD d depending on parameters* $\{x_1, y, z\}$ *:*

$$\text{IF}\left[\ (x_1 = a_1^{j1}) \text{ AND } (y = a_y^{j1}) \text{ AND } (z = a_z^{j1})\ \right] \text{(with weight } w_{j1}\text{)}$$

...

$$\text{OR}\left[\ \left(x_1 = a_1^{jk_j}\right) \text{ AND } \left(y = a_y^{jk_j}\right) \text{ AND } \left(z = a_z^{jk_j}\right)\ \right] \text{(with weight } w_{jk_j}\text{),}$$

$$\text{THEN } d \in d_j \text{, for all } j = \overline{1,m}, \tag{4.12}$$

where a_i^{jp} is the linguistic term for the estimation of variable x_i in the row with number $p = \overline{1,k_j}$,

a_y^{jp} (a_z^{jp}) is the linguistic term for the estimation of variable *y (z)* in the row with number $p = \overline{1,k_j}$, and it is supposed that term a_y^{jp} (a_z^{jp}) should be chosen from estimates y_j (z_j), $j = \overline{1,5}$;

k_j is the number of conjunction rows corresponding to the classes d_j, y_j, z_j;

w^y_{jp}, w^z_{jp}, w_{jp} the weights of the expressions with number jp in (4.10) - (4.12).

4.6.2 Hierarchical System of Parameter Matrices

The problem of fuzzy IF-THEN rules (4.10) - (4.12) extraction can be considered as finding three matrices presented in Tables 4.5 - 4.7. Each element (b_i^{jp}, c_i^{jp}) of these matrices corresponds to the membership function parameters and can be interpreted as a fuzzy term (low, average, high, etc.). Each element a_y^{jp} (a_z^{jp}) in Table 4.7 is chosen from the decision classes y_j (z_j) in Table 4.5, 4.6.

Table 4.5. Matrix of IF-THEN rules parameters for model (3.31)

Rule	IF			Weight	THEN
№	x_2	…	x_{11}		y
11 … $1\,k_1$	(b_2^{11}, c_2^{11}) … $(b_2^{1k_1}, c_2^{1k_1})$		$(b_{11}^{11}, c_{11}^{11})$ … $(b_{11}^{1k_1}, c_{11}^{1k_1})$	w_{11}^y … $w_{1k_1}^y$	y_1
…	…		…	…	…
51 … $5\,k_5$	(b_2^{51}, c_2^{51}) … $(b_2^{5k_5}, c_2^{5k_5})$		$(b_{11}^{51}, c_{11}^{51})$ … $(b_{11}^{5k_5}, c_{11}^{5k_5})$	w_{51}^y … $w_{5k_5}^y$	y_5

Table 4.6. Matrix of IF-THEN rules parameters for model (3.32)

Rule	IF			Weight	THEN
№	x_6	…	x_{12}		z
11 … $1\,k_1$	(b_6^{11}, c_6^{11}) … $(b_6^{1k_1}, c_6^{1k_1})$		$(b_{12}^{11}, c_{12}^{11})$ … $(b_{12}^{1k_1}, c_{12}^{1k_1})$	w_{11}^z … $w_{1k_1}^z$	z_1
…	…		…	…	…
51 … $5\,k_5$	(b_6^{51}, c_6^{51}) … $\left(b_6^{5k_5}, c_6^{5k_5}\right)$		$(b_{12}^{51}, c_{12}^{51})$ … $\left(b_{12}^{5k_5}, c_{12}^{5k_5}\right)$	w_{51}^z … $w_{5k_5}^z$	z_5

Table 4.7. Matrix of IF-THEN rules parameters for model (3.30)

Rule	IF			Weight	THEN
№	x_1	y	z		d
11	(b_1^{11}, c_1^{11})	a_y^{11}	a_z^{11}	w_{11}	
…	…	…	…	…	d_1
$1\,k_1$	$(b_1^{1k_1}, c_1^{1k_1})$	$a_y^{1k_1}$	$a_z^{1k_1}$	w_{1k_1}	
…	…	…	…	…	…
$m\,1$	(b_1^{m1}, c_1^{m1})	a_y^{m1}	a_z^{m1}	w_{m1}	
…	…	…	…	…	d_m
$m\ k_m$	$\left(b_1^{mk_m}, c_1^{mk_m}\right)$	$a_y^{mk_m}$	$a_z^{mk_m}$	w_{mk_m}	

4.6.3 Computer Experiment

The total number of patients with IHD in our study was 65. The aim of computer experiment was to generate three rules for each class of decision (*y-, z-, d-*) according to the models (3.30) - (3.32). The results of this optimization problem solving using genetic and neuro algorithm are presented in Tables 4.8 - 4.13. According to these tables it is easy to make interpretation of each pairs of parameters using fuzzy terms: *L – Low*, *lA – lower than Average*, *A – Average*, *hA – higher than Average*, *H – High.* For example, the pairs (176.5, 87.8), (256.1, 25.1), (368.3, 49.8) correspond to the membership functions shown in Fig. 4.10, which can be interpreted as *lower than Average* (*lA*), *Average* (*A*)*, High* (*H*).

After linguistic interpretation we can describe the optimal solutions (Tables 4.8 - 4.13) in the form of fuzzy IF-THEN rules matrices (Tables 4.14 - 4.16), where

GA – genetic algorithm;
NN – neuro-fuzzy network.

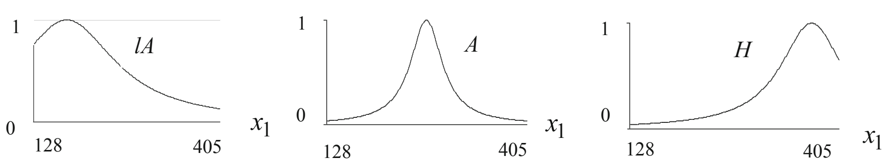

Fig. 4.10. Example of linguistic interpretation

Table 4.8. Parameters of rules for model (3.31) synthesized using the genetic algorithm

x_2	x_3	x_4	x_5	x_{10}	x_{11}	y
(366.22, 83.44)	(941.93, 251.67)	(3.22, 5.24)	(0.43, 0.08)	(34.28, 11.42)	(275.50, 535.50)	
(176.48, 206.91)	(667.20, 120.90)	(1.84, 5.63)	(0.25, 0.02)	(17.79, 41.88)	(298.45, 135.26)	L
(145.31, 50.27)	(109.43, 1350.49)	(0.81, 0.41)	(0.09, 0.11)	(24.23, 3.10)	(65.13, 21.18)	
(368.30, 102.18)	(955.80, 842.19)	(1.31, 2.48)	(0.17, 0.15)	(11.42, 12.05)	(251.02, 7.03)	
(256.11, 90.71)	(128.85, 408.26)	(2.14, 0.46)	(0.32, 0.59)	(40.57, 25.13)	(179.88, 160.36)	lA
(128.00, 48.30)	(92.78, 180.36)	(0.60, 0.58)	(0.10, 0.05)	(7.40, 3.86)	(199.77, 52.74)	
(184.79, 350.26)	(914.18, 1942.50)	(2.41, 5.78)	(0.23, 0.18)	(26.33, 18.37)	(227.31, 229.50)	
(130.77, 80.12)	(808.73, 224.63)	(0.62, 2.60)	(0.40, 0.23)	(8.91, 10.84)	(140.10, 200.05)	A
(162.63, 45.64)	(306.45, 1406.27)	(0.66, 0.39)	(0.12, 0.35)	(8.41, 4.69)	(290.80, 150.46)	
(315.67, 50.92)	(123.30, 917.02)	(0.88, 5.78)	(0.28, 0.27)	(33.53, 7.18)	(191.35, 688.50)	
(188.94, 346.25)	(142.73, 268.38)	(1.89, 2.05)	(0.36, 0.07)	(8.91, 9.04)	(325.23, 116.83)	hA
(128.00, 74.17)	(645.00, 138.73)	(0.76, 0.49)	(0.10, 0.03)	(8.49, 16.79)	(208.95, 10.25)	
(202.79, 120.62)	(597.83, 340.36)	(1.47, 1.42)	(0.11, 0.35)	(16.53, 8.17)	(185.23, 137.25)	
(290.74, 80.56)	(434.10, 380.95)	(1.06, 7.02)	(0.46, 0.21)	(39.90, 18.37)	(277.80, 155.48)	H
(128.00, 60.04)	(114.98, 570.30)	(0.61, 0.78)	(0.09, 0.08)	(7.74, 5.28)	(46.00, 40.34)	

Table 4.9. Parameters of rules for model (3.31) specified using the neuro-fuzzy network

x_2	x_3	x_4	x_5	x_{10}	x_{11}	w	y
(330.21, 207.75)	(539.55, 260.85)	(1.76, 5.78)	(0.26, 0.03)	(18.12, 6.20)	(75.84, 688.50)	0.98	
(314.28, 42.24)	(114.98, 238.65)	(0.77, 5.78)	(0.20, 0.35)	(33.20, 25.13)	(59.77, 535.50)	0.51	L
(205.56, 623.25)	(711.60, 185.93)	(3.64, 5.78)	(0.35, 0.06)	(26.83, 25.46)	(114.09, 688.50)	0.99	
(179.25, 484.75)	(575.63, 2497.50)	(2.68, 0.82)	(0.46, 0.11)	(38.30, 4.69)	(216.60, 688.50)	0.54	
(206.95, 346.25)	(950.25, 1387.50)	(3.49, 0.83)	(0.19, 0.35)	(40.73, 8.29)	(176.82, 229.50)	0.59	lA
(397.38, 623.25)	(1197.23, 1942.50)	(1.37, 7.43)	(0.20, 1.06)	(37.22, 7.04)	(205.12, 688.50)	0.88	
(140.47, 58.86)	(797.63, 1942.50)	(3.19, 0.59)	(0.49, 0.59)	(23.40, 8.38)	(296.16, 69.62)	0.51	
(215.95, 195.29)	(794.85, 8.33)	(3.26, 7.43)	(0.15, 0.59)	(19.21, 41.88)	(259.44, 231.03)	0.70	A
(299.74, 346.25)	(1086.23, 1387.50)	(1.81, 5.78)	(0.22, 0.04)	(24.65, 41.88)	(301.51, 95.63)	0.95	
(226.34, 484.75)	(395.25, 832.50)	(1.59, 2.48)	(0.49, 0.10)	(38.14, 8.12)	(155.40, 74.21)	0.50	
(200.71, 346.25)	(425.78, 2497.50)	(2.14, 2.48)	(0.27, 0.12)	(38.89, 7.79)	(62.07, 65.79)	0.53	hA
(202.10, 74.79)	(1039.05, 563.33)	(0.90, 0.54)	(0.26, 0.20)	(14.10, 25.13)	(332.88, 387.09)	0.97	
(321.21, 623.25)	(148.28, 122.10)	(0.81, 0.43)	(0.33, 0.59)	(36.88, 5.53)	(262.50, 229.50)	0.50	
(146.70, 46.40)	(1061.25 230.33)	(1.44, 4.13)	(0.17, 0.09)	(11.59, 3.10)	(86.55, 45.14)	0.50	H
(232.57, 346.25)	(237.08, 740.93)	(2.49, 4.21)	(0.53, 0.06)	(20.72, 58.63)	(152.34, 382.50)	1.00	

Table 4.10. Parameters of rules for model (3.32) synthesized using the genetic algorithm

x_6	x_7	x_8	x_9	x_{12}	z
(50.32, 26.25) (49.71, 9.16) (35.09, 8.75)	(20.56, 11.28) (22.53, 4.17) (22.84, 2.75)	(13.41, 4.45) (15.47, 41.13) (4.42, 0.57)	(4.50, 5.06) (3.82, 0.72) (1.01, 3.58)	(21.92, 30.85) (16.59, 28.14) (3.90, 10.52)	L
(62.31, 7.80) (61.70, 15.03) (35.01, 8.75)	(26.91, 20.48) (20.87, 3.37) (11.90, 8.51)	(15.88, 25.16) (24.69, 12.04) (3.66, 5.28)	(2.33, 7.95) (2.75, 5.98) (1.01, 1.05)	(23.56, 41.17) (24.74, 27.44) (4.29, 7.20)	lA
(49.10, 6.11) (65.38, 12.34) (56.45, 9.72)	(28.09, 39.38) (27.74, 21.88) (15.71, 4.76)	(16.94, 27.06) (7.30, 12.65) (3.66, 5.88)	(5.32, 1.41) (3.80, 5.00) (2.48, 0.88)	(21.85, 15.22) (20.60, 5.17) (4.10, 3.24)	A
(58.64, 43.75) (47.35, 20.85) (34.66, 78.75)	(16.84, 8.90) (22.36, 5.03) (11.90, 4.55)	(4.60, 4.18) (5.95, 1.03) (5.07, 13.18)	(4.71, 6.27) (3.77, 8.16) (1.00, 0.93)	(24.94, 15.88) (7.91, 11.07) (3.97, 5.43)	hA
(58.72, 26.25) (34.57, 8.75) (34.57, 6.28)	(28.83, 30.63) (15.27, 20.15) (11.90, 4.74)	(24.40, 9.47) (9.24, 22.94) (3.84, 16.32)	(5.32, 10.29) (4.88, 9.84) (1.01, 4.40)	(16.79, 6.29) (6.67, 30.15) (18.76, 3.65)	H

Table 4.11. Parameters of rules for model (3.32) specified using the neuro-fuzzy network

x_6	x_7	x_8	x_9	x_{12}	w	z
(52.79, 43.75) (55.68, 78.75) (63.90, 78.75)	(27.96, 4.38) (24.02, 2.71) (12.82, 30.63)	(9.59, 17.63) (8.12, 52.88) (19.29, 17.63)	(4.77, 3.40) (1.72, 3.53) (2.75, 0.88)	(29.48, 3.48) (10.61, 33.53) (16.46, 5.65)	0.57 0.98 0.69	L
(47.54, 78.75) (39.84, 8.75) (35.90, 44.63)	(20.08, 21.88) (17.24, 30.63) (24.11, 21.88)	(25.28, 29.38) (11.77, 52.88) (23.05, 41.13)	(2.30, 5.88) (2.42, 1.18) (4.30, 3.53)	(21.85, 32.88) (25.86, 46.03) (17.64, 32.88)	0.97 1.00 0.99	lA
(56.73, 43.75) (40.98, 78.75) (56.29, 6.74)	(15.49, 4.33) (28.00, 13.13) (28.35, 9.23)	(17.41, 52.88) (26.63, 5.76) (8.12, 41.13)	(4.45, 10.58) (1.95, 5.88) (4.63, 1.32)	(7.38, 5.33) (20.86, 6.58) (25.33, 19.73)	0.61 0.78 0.93	A
(68.19, 26.25) (37.48, 8.75) (61.10, 78.75)	(13.08, 4.38) (14.96, 30.63) (27.13, 3.98)	(19.87, 29.38) (5.72, 5.88) (6.07, 52.88)	(4.78, 8.23) (2.54, 5.88) (2.36, 3.53)	(26.91, 6.58) (20.27, 32.88) (26.52, 6.05)	0.60 0.70 0.50	hA
(66.18, 43.75) (44.91, 8.75) (49.73, 61.25)	(20.30, 30.63) (26.12, 39.38) (16.41, 39.38)	(21.87, 29.38) (3.78, 5.88) (13.59, 52.88)	(2.35, 3.53) (4.81, 10.58) (1.99, 10.58)	(19.94, 59.18) (18.43, 32.88) (27.37, 32.88)	0.80 1.00 1.00	H

Table 4.12. Parameters of rules for model (3.30) synthesized using the genetic algorithm

x_1	y	z	d
(38.56, 25.19)	H	L	d_1
(54.83, 40.26)	A	H	
(31.07, 10.04)	H	H	
(55.30, 6.74)	hA	A	d_2
(51.25, 10.57)	lA	H	
(31.00, 4.36)	A	lA	
(55.91, 12.11)	lA	A	d_3
(49.83, 4.67)	lA	lA	
(34.38, 5.12)	lA	lA	
(56.04, 12.20)	L	A	d_4
(31.14, 37.21)	lA	hA	
(32.01, 4.23)	L	L	
(42.34, 11.45)	L	lA	d_5
(46.80, 5.17)	hA	hA	
(32.96, 4.82)	L	hA	
(33.30, 6.31)	A	hA	d_6
(45.78, 16.70)	hA	hA	
(31.07, 4.48)	L	lA	

Table 4.13. Parameters of rules for model (3.30) specified using the neuro-fuzzy network

x_1	y	z	w	d
(38.90, 60.75)	hA	L	0.93	d_1
(31.47, 33.75)	L	H	0.70	
(51.05, 33.75)	hA	H	0.70	
(57.46, 19.85)	H	A	0.50	d_2
(45.92, 9.79)	A	H	0.99	
(50.04, 33.75)	L	hA	0.50	
(51.52, 60.75)	A	A	1.00	d_3
(48.15, 33.75)	A	hA	0.70	
(52.40, 33.75)	A	hA	0.50	
(52.06, 6.62)	lA	A	0.50	d_4
(40.38, 47.25)	A	lA	0.83	
(42.00, 20.25)	lA	L	0.50	
(57.53, 47.25)	lA	hA	0.72	d_5
(34.85, 60.75)	H	lA	0.50	
(44.16, 33.75)	lA	lA	0.97	
(36.54, 60.75)	L	hA	1.00	d_6
(44.84, 20.25)	H	lA	0.60	
(31.47, 35.91)	lA	hA	1.00	

Table 4.14. Fuzzy knowledge base for the instrumental danger y

x_2	x_3	x_4	x_5	x_{10}	x_{11}	y
GA / NN	GA / NN	GA / NN	GA / NN	GA / NN	GA / NN	
hA lA / hA L / lA	hA / A A / lA L / A	hA / lA A / L lA / H	hA / lA lA L / A	hA / lA lA / hA A	hA / L hA / L lA	L
hA / lA A / lA L / H	hA / A L / hA L / H	lA / hA A / H L / lA	lA / hA A / lA L / lA	lA / H H L / H	hA / A A A	lA
lA / L L / lA lA / A	hA hA lA / H	A / hA L / hA L / lA	lA / hA hA / lA lA	A L / lA L / A	A / hA lA / hA hA	A
hA / lA lA L lA	L / lA L / lA A / hA	lA A L	A / hA A L / lA	hA / H L / H L / lA	A / lA H / L A / H	hA
lA / hA A / L L / A	A / L lA / hA L / lA	lA / L lA L / A	L / A hA / lA L / H	lA / hA H / L L / lA	A / hA hA / L L / lA	H

Table 4.15. Fuzzy knowledge base for the biochemical danger z

x_6	x_7	x_8	x_9	x_{12}	z
GA / NN	GA / NN	GA / NN	GA / NN	GA / NN	
A A L / hA	A / H A / hA hA / L	A / lA A / lA L / hA	hA A / lA L / lA	hA / H A / lA L / A	L
hA / lA hA / lA L	hA / A A / lA L / hA	A / H H / lA L / hA	lA lA L / hA	hA hA L / A	lA
A H / lA hA / A	H / lA H lA / H	A lA / H L / lA	H / hA A / lA lA / hA	hA / lA hA L / hA	A
hA lA L hA	lA / L A / lA L / hA	L / A lA / L lA / L	hA A / lA L / lA	hA lA / A L / hA	hA
hA / H L / lA L / A	H / A lA / hA L / lA	H / hA lA / L L / A	H / lA hA L / lA	A lA / A A / H	H

Table 4.16. Fuzzy knowledge base for IHD danger d

x_1		y		z		d
GA /	NN	GA /	NN	GA /	NN	
lA		H /	hA	L		d_1
H /	L	A /	L	H		
L /	hA	H /	hA	H		
H		hA /	H	A		d_2
hA /	A	lA /	A	H		
L /	hA	A /	L	lA /	hA	
H /	hA	lA /	A	A		d_3
hA /	A	lA /	A	lA /	hA	
lA /	hA	lA /	A	lA /	hA	
H /	hA	L /	lA	A		d_4
L /	lA	lA /	A	hA /	lA	
L /	A	L /	lA	L		
A /	H	L /	lA	lA /	hA	d_5
A /	lA	hA /	H	hA /	lA	
lA /	A	L /	lA	hA /	lA	
lA		A /	L	hA		d_6
A		hA /	H	hA /	lA	
L		L /	lA	lA /	hA	

4.6.4 Comparison of the Expert and Extracted from Histories of Diseases IF-THEN Rules

Comparison of the expert [3] and extracted from the real histories of diseases IF-THEN rules is presented in Tables 4.17 – 4.19. As can be seen

- fuzzy terms marked by (!) fully coincide;
- instead of terms marked by (+) the adjacent terms were extracted;
- instead of terms marked by (-), the terms which are too far from the expert ones were extracted.

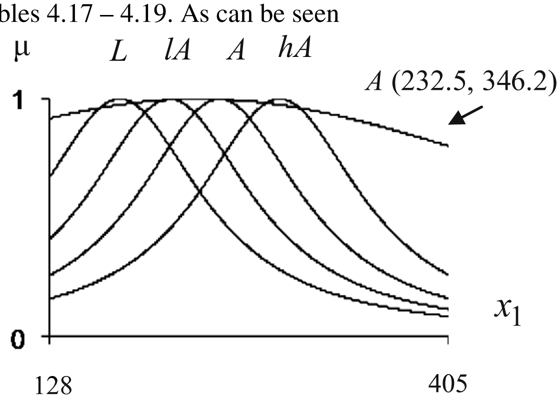

Fig. 4.11. Comparison of fuzzy terms

No coincidences of the terms are due to the parameters c- of membership functions compression-extension. For example, the pair (232.5, 346.2) in the first column of Table 4.9, to which term *Average* (A) corresponds in Fig. 4.11, can be presented by a term set: *L – Low*, *lA – lower than Average*, *A – Average, hA – higher than Average.* If some expert rule contains the term from this set, then this rule is not at variance with the rule extracted from data.

Table 4.17. Comparison of the extracted and expert rules for instrumental danger y

Number of the extracted rule in Table 4.14	Expert rules						y
	x_2	x_3	x_4	x_5	x_{10}	x_{11}	
Rule 3	H (-)	H (-)	H (!)	L (-)	H (-)	H (-)	
Rule 1	H (+)	hA (+)	H (-)	lA (!)	H (-)	H (-)	L
Rule 2	hA (!)	H (-)	hA (-)	L (+)	H (+)	H (-)	
Rule 3	hA (+)	hA (+)	H (-)	lA (!)	H (!)	hA (+)	
Rule 2	H (-)	H (+)	hA (+)	A (+)	H (!)	H (-)	lA
Rule 1	hA (-)	hA (+)	H (+)	lA (-)	hA (+)	hA (+)	
Rule 3	A (!)	A (-)	A (+)	A (+)	A (!)	A (+)	
Rule 2	hA (-)	hA (!)	A (+)	lA (!)	hA (-)	A (+)	A
Rule 1	A (-)	hA (!)	hA (!)	A (+)	hA (+)	hA (!)	
Rule 1	lA (!)	A (+)	lA (!)	hA (!)	lA (-)	lA (!)	
Rule 2	lA (!)	lA (!)	A (!)	A (!)	L (-)	lA (+)	hA
Rule 3	A (+)	lA (-)	lA (+)	hA (-)	lA (!)	A (-)	
Rule 1	L (-)	L (!)	L (!)	hA (+)	L (-)	L (-)	
Rule 3	lA (+)	L (+)	lA (+)	H (!)	L (+)	lA (!)	H
Rule 2	L (!)	lA (-)	lA (!)	hA (-)	L (!)	L (!)	

Table 4.18. Comparison of the extracted and expert rules for biochemical danger z

Number of the extracted rule in Table 4.15	Expert rules					z
	x_6	x_7	x_8	x_9	x_{12}	
Rule 1	H (-)	H (!)	H (-)	H (+)	H (!)	
Rule 2	hA (+)	H (+)	hA (-)	hA (-)	hA (-)	L
Rule 3	H (+)	hA (-)	H (+)	A (+)	hA (+)	
Rule 1	hA (-)	hA (+)	A (-)	A (+)	hA (!)	
Rule 2	A (+)	hA (-)	A (+)	hA (-)	H (+)	lA
Rule 3	A (-)	H (+)	hA (!)	hA (!)	hA (+)	

Table 4.18. *(continued)*

Rule 1 Rule 3 Rule 2	A (!) hA (+) hA (-)	A (+) hA (+) A (-)	A (!) A (+) hA (+)	hA (!) A (+) hA (-)	hA (-) A (+) A (+)	A
Rule 2 Rule 1 Rule 3	lA (!) hA (!) L (-)	A (+) lA (+) A (+)	lA (+) A (!) A (-)	A (+) lA (-) lA (!)	A (!) lA (-) A (+)	hA
Rule 1 Rule 2 Rule 3	L (-) lA (!) L (-)	L (-) L (-) lA (!)	L (-) lA (+) lA (+)	L (+) L (-) L (+)	lA (+) L (-) lA (-)	H

Table 4.19. Comparison of the extracted and expert rules for IHD danger d

Number of the extracted rule in Table 4.16	Expert rules			d
	x_1	y	z	
Rule 1 Rule 2 Rule 3	L (+) lA (+) lA (-)	L (-) L (!) lA (-)	L (!) lA (-) H (!)	d_1
Rule 3 Rule 2 Rule 1	lA (-) lA (+) lA (-)	lA (+) A (!) lA (-)	lA (-) lA (-) A (!)	d_2
Rule 2 Rule 3 Rule 1	lA (+) hA (!) A (+)	A (!) hA (+) hA (+)	A (+) lA (-) A (!)	d_3
Rule 3 Rule 2 Rule 1	A (!) hA (-) hA (!)	hA (-) A (!) lA (!)	hA (-) hA (-) hA (+)	d_4
Rule 1 Rule 3 Rule 2	H (!) hA (+) hA (-)	A (+) hA (-) H (!)	A (+) H (-) hA (-)	d_5
Rule 2 Rule 3 Rule 1	H (-) H (-) H (-)	H (!) hA (-) A (-)	H (-) hA (!) hA (!)	d_6

4.6.5 Comparison of the Results of Medical Diagnosis

The separate aim of our study was to compare the results of medical diagnosis obtained by formally extracted IF-THEN rules (using a genetic and neuro algorithm) and the same rules proposed by a medical expert in the field of ischemia heart disease [3]. The fragment of data sample is presented in Table 4.20.

Comparison of diagnoses for 65 patients shows the following (See Table 4.21). As a result of the genetic algorithm operation, there are full coincidences of all types of diagnoses for 54 patients. In 9 cases we can observe decisions on a boundary between classes of diagnoses (these cases are marked by *). In 2 cases the results of computer decision were too far from the real medical doctor diagnosis (these cases are marked by **). After neural correction of diagnostic rules there are full coincidences of all types of diagnoses for 57 patients. In 8 cases we can observe decisions on a boundary between classes of diagnoses (these cases are marked by *).

These results (obtained by extracted IF-THEN rules) are close enough to similar results obtained by the fuzzy expert system described in [3]. Future quality improvement of extracted fuzzy IF-THEN rules can be reached by increasing the number of tuning parameters.

The number of unknown parameters in our computer experiment was 486, and for the optimization problem solving we spent about 3 hours (Intel Core 2 Duo P7350 2.0 GHz).

Table 4.20. Comparison of the diagnosis results

	IF-THEN rules		
	Expert	Extracted from histories of diseases	
		Genetic algorithm	Neuro-fuzzy network
Full coincidences of all types of diagnoses	56	54	57
Decisions on a boundary between classes of diagnoses (*)	8	9	8
Computer decision is too far from the real medical doctor diagnosis (**)	1	2	0

Table 4.21. Fragment of the data sample and diagnosis results

№	Patient state parameters												Diagnosis			
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	$\hat{d}$	d_e	d_G	d_N
1	324	980	2.8	0.12	34.2	266	50.07	22.76	8.05	3.7	19.3	31	d1	d1	d1	d1
2	330	900	2.9	0.14	29.7	242	56.52	24.33	9.02	4.1	21.0	36	d1	d1	d1	d1
3	260	800	2.3	0.18	28.5	194	51.73	25.62	8.53	4.2	23.8	39	d2	d2	d2	d2
4	272	867	2.5	0.28	28.7	198	59.31	28.44	8.53	4.0	19.4	42	d2	d2	d2	d3*
5	287	491	2.2	0.24	25.3	156	52.77	21.61	8.53	3.5	20.5	48	d3	d3	d3	d3
6	175	507	2.4	0.25	22.4	172	60.70	26.14	10.40	3.9	26.1	53	d3	d3	d3	d3

Table 4.21. *(continued)*

7	247	728	2.0	0.34	26.5	144	62.06	26.14	5.55	2.3	22.9	45	d4	d4	d4	d4
8	231	768	1.5	0.36	20.0	158	62.77	23.01	6.83	2.5	23.8	52	d4	d4	d5*	d4
9	151	610	1.3	0.42	19.8	104	54.49	23.91	5.55	2.4	25.7	32	d5	d5	d5	d5
10	177	542	1.6	0.48	21.7	120	62.06	26.14	5.55	2.3	28.1	45	d5	d6*	d6*	d6*
11	128	349	1.4	0.48	13.9	92	67.03	24.46	5.20	1.9	30.2	38	d6	d6	d6	d6
12	145	304	1.2	0.56	14.4	74	64.15	25.62	7.11	2.6	25.5	38	d6	d6	d6	d6
13	327	930	2.2	0.24	35.4	347	59.31	25.62	7.56	3.3	18.9	40	d1	d2*	d1	d1
14	348	952	1.8	0.20	34.2	352	34.48	20.79	9.56	5.7	21.6	38	d1	d1	d1	d1
15	307	800	1.9	0.21	30.1	304	57.90	25.08	6.83	2.9	19.3	34	d2	d4**	d2	d1*
16	284	738	2.0	0.26	29.7	339	62.06	25.08	8.53	3.4	20.4	48	d2	d2	d2	d2
17	174	600	1.7	0.32	27.2	312	55.18	24.46	8.56	3.8	22.0	35	d3	d3	d1**	d3
18	229	515	2.1	0.30	22.4	300	61.34	22.20	6.83	2.4	23.4	49	d3	d4*	d3	d4*
19	265	421	2.0	0.26	17.7	258	60.07	22.76	4.08	1.8	23.8	58	d4	d4	d4	d4
20	330	650	1.5	0.25	20.3	244	69.49	25.08	6.83	2.5	22.0	49	d4	d4	d4	d4
21	187	475	1.4	0.34	21.4	204	60.39	23.31	5.55	2.1	22.7	48	d5	d5	d5	d5
22	224	400	1.5	0.39	20.4	215	55.18	21.05	7.11	2.7	22.5	42	d5	d5	d5	d5
23	195	100	1.2	0.48	22.6	191	60.70	21.61	7.52	2.7	25.9	32	d6	d6	d5*	d6
24	192	292	1.3	0.45	19.2	188	62.77	23.70	5.55	1.6	24.4	51	d6	d6	d6	d6
25	347	952	2.9	0.10	35.7	298	62.40	23.70	12.50	4.3	19.6	36	d1	d1	d1	d1
26	314	902	3.2	0.14	33.5	287	59.40	24.20	10.50	4.2	18.8	48	d1	d1	d1	d1
27	352	875	3.2	0.16	38.2	322	52.30	22.70	9.50	3.9	19.0	42	d1	d1	d1	d1
28	323	1040	2.7	0.20	30.4	290	59.60	25.20	8.80	3.2	18.2	40	d1	d2*	d1	d2*
29	377	988	2.9	0.09	32.5	275	60.40	24.30	10.20	3.4	17.7	41	d1	d1	d1	d1
30	309	932	3.2	0.15	31.5	312	60.80	25.40	9.40	4.4	18.5	34	d1	d1	d1	d1
31	279	1056	2.7	0.09	33.4	334	59.90	21.30	8.80	3.7	18.7	52	d1	d1	d1	d1
32	376	895	2.7	0.18	30.4	312	61.50	23.60	9.50	3.6	20.1	44	d2	d2	d2	d2
33	304	929	2.6	0.22	32.5	346	58.20	25.10	10.70	3.8	19.2	46	d2	d2	d2	d2
34	292	904	2.2	0.24	29.3	290	56.00	27.90	10.10	4.0	18.5	46	d2	d2	d1*	d2
35	276	885	2.4	0.25	27.8	226	61.40	29.40	11.20	3.6	20.8	42	d2	d2	d2	d2
36	311	930	2.7	0.19	25.6	249	62.50	23.80	9.80	2.9	21.0	31	d2	d1*	d2	d2
37	335	992	2.4	0.22	24.6	255	61.60	24.70	9.90	3.3	20.3	44	d2	d2	d2	d2
38	346	873	2.3	0.18	28.7	267	57.70	22.50	10.60	3.7	18.8	47	d2	d2	d1*	d2
39	288	804	2.4	0.27	20.9	275	60.00	22.20	11.50	3.5	19.5	48	d3	d3	d1**	d3
40	316	875	2.1	0.31	22.5	302	61.40	24.00	9.30	2.8	21.2	50	d3	d4*	d3	d4*
41	292	774	2.0	0.28	26.7	277	62.50	25.90	8.80	3.0	22.5	51	d3	d4*	d3	d3
42	315	766	2.2	0.22	21.4	265	53.70	26.20	8.70	2.7	20.5	54	d3	d4*	d3	d2*
43	300	865	2.1	0.25	21.9	303	59.40	25.80	9.30	3.5	21.4	40	d3	d3	d3	d2*
44	270	777	2.1	0.28	22.3	316	61.00	26.10	9.70	4.1	21.3	36	d3	d3	d3	d3
45	275	859	2.3	0.30	24.0	295	62.50	27.00	9.60	4.2	22.5	34	d3	d3	d3	d3
46	261	776	1.7	0.36	20.4	204	65.00	22.50	8.40	2.7	23.8	52	d4	d4	d5*	d4
47	258	785	1.5	0.36	19.8	225	62.70	23.80	7.60	2.5	24.0	41	d4	d4	d5*	d4
48	290	845	1.8	0.39	18.7	268	57.10	24.00	7.20	2.5	22.5	53	d4	d4	d4	d4
49	203	723	2.0	0.40	17.1	209	58.50	23.70	6.20	2.8	24.7	39	d4	d4	d4	d4
50	244	802	1.7	0.35	18.5	212	62.00	25.30	6.30	3.0	24.9	45	d4	d4	d5*	d4
51	233	795	1.9	0.39	17.4	251	57.90	24.90	5.20	2.4	23.5	46	d4	d4	d4	d4
52	262	805	1.8	0.38	19.2	244	57.90	24.50	7.70	2.2	22.1	54	d4	d4	d4	d4
53	245	595	1.3	0.44	16.5	204	64.20	26.40	5.60	2.1	24.7	51	d5	d5	d5	d5
54	209	772	1.5	0.45	14.7	195	60.20	27.80	5.90	2.4	25.0	40	d5	d5	d5	d5
55	198	621	1.4	0.42	12.2	225	58.80	25.20	6.10	2.6	24.5	42	d5	d5	d5	d5
56	245	523	1.5	0.39	14.1	207	57.50	23.30	6.50	2.2	26.9	44	d5	d5	d5	d5

Table 4.21. *(continued*

57	237	652	1.6	0.45	11.9	262	63.70	24.70	6.40	2.1	24.2	50	d5	d5	d5	d5
58	202	744	1.3	0.45	12.3	226	61.80	25.70	5.70	2.4	22.6	56	d5	d5	d5	d5
59	247	723	1.2	0.38	10.4	230	62.50	26.90	5.60	2.3	25.8	51	d5	d5	d6*	d5
60	192	516	1.1	0.52	9.9	200	60.10	22.70	5.50	2.0	22.9	48	d6	d6	d6	d6
61	188	446	1.2	0.48	9.5	212	59.00	23.50	5.20	2.4	26.7	39	d6	d6	d6	d6
62	212	406	0.9	0.56	8.2	225	61.70	26.00	5.30	1.9	29.4	49	d6	d6	d6	d6
63	247	527	0.7	0.51	7.4	197	62.60	27.40	5.10	2.0	28.5	45	d6	d6	d6	d6
64	206	448	0.8	0.55	7.4	188	57.40	22.10	6.30	2.1	30.1	44	d6	d6	d6	d6
65	228	512	1.0	0.52	7.8	204	53.90	25.60	5.40	2.3	29.5	42	d6	d6	d6	d6

$\hat{d}$ - diagnosis obtained by medical doctor.

d_e - computer diagnosis obtained by the expert IF-THEN rules.

d_G - computer diagnosis obtained by the genetically grown rules.

d_N - computer diagnosis specified using the neural network.

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