Chapter 2
Direct Inference Based on Fuzzy Rules

This chapter is devoted to the methodology aspects of identification and decision making on the basis of intellectual technologies. The essence of intellectuality consists of representation of the structure of the object in the form of linguistic IF-THEN rules, reflecting human reasoning on the common sense and practical knowledge level. The linguistic approach to designing complex systems based on linguistically described models was originally initiated by Zadeh [1] and developed further by Tong [2], Gupta [3], Pedrych [4 – 6], Sugeno [7], Yager [8], Zimmermann [9], Kacprzyk [10], Kandel [11]. The main principles of fuzzy modeling were formulated by Yager [8]. The linguistic model is a knowledge-based system. The set of fuzzy IF-THEN rules takes the place of the usual set of equations used to characterize a system [12 – 14]. The fuzzy sets associated with input and output variables are the parameters of the linguistic model [15]; the number of the rules determines its structure. Different interpretations of the knowledge contained in these rules, which are due to different reasoning mechanisms, result in different types of models.

This monograph can be regarded as one of the possible approaches to modeling intellectual activity on the basis of knowledge engineering. The herein proposed intellectual technique of identification, which supports the human-system approach to the solution of the simulation tasks [16], represents some general framework for design of fuzzy expert systems. The aim of this chapter is to introduce the main formalisms necessary for the definition of fuzzy knowledge bases being the medium of expert information. All intellectual tasks discussed above can be considered to be the tasks of identification having the following common properties [17]:

1) the output variable is associated with the object of identification, that is with the type of the decision made,
2) the input variables are associated with the parameters of the identification object state,
3) output and input variables can have quantitative and qualitative estimations,
4) the structure of the interconnection between output and input variables is described by IF <inputs> THEN <outputs> rules using qualitative estimations of variables and representing fuzzy knowledge bases.

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A fuzzy knowledge base represents some combination of IF <inputs>, THEN <output> rules, which reflect expert experience and the understanding of cause-effect connections in the decision making task considered (control, diagnosis, prediction and other ones). Peculiarity of the similar expressions consists in the fact that their adequacy doesn't change with the insignificant deviations of experiment conditions. Therefore, formation of the fuzzy knowledge base can be treated as an analog of the structural identification [12 – 14] stage, which involves simulation of the rough object model. In this case, the results of fuzzy evidence depend on the forms of fuzzy terms membership functions, which are used to estimate object inputs and outputs. In addition, the combination of IF-THEN rules can be considered as a set of expert points in input-output space. Application of the fuzzy logic evidence apparatus allows us to restore and identify the multidimensional surface according to these points, which allows us to receive output values with various combinations of input variables values available.

Work [17] is the basis of this chapter.

2.1 Formalization of Source Information

2.1.1 *Inputs and Outputs of an Object*

Here we consider an object with one output and n inputs of the form:

$$y = f_y(x_1, x_2, ..., x_n) \quad , \tag{2.1}$$

where y is the output variable; $x_1, x_2, ..., x_n$ are the input variables.

Variables $x_1, x_2, ..., x_n$ and y can be quantitative and qualitative. The examples of quantitative variables are: VEHICLE SPEED = [0, 160] km/h, PATIENT TEMPERATURE = [36, 41] °C, REACTOR LOAD DOZE = [6, 20]%, and other variables, easily measured using accepted for them quantitative scales.

The example of a variable for which there is no natural scale is the LEVEL OF OPERATOR STRESS, which can be estimated by qualitative terms (low, average, high) or measured by artificial scales, for example, using 5-, 10- or 100- points systems.

For quantitative variables some known intervals of change are suggested:

$$U_i = [\, \underline{x}_i, \overline{x}_i \,] \ , \ i = \overline{1,n} \quad , \tag{2.2}$$

$$Y = [\, \underline{y}, \overline{y} \,] \quad , \tag{2.3}$$

where $\underline{x}_i$ $(\overline{x}_i)$ is the lower (upper) value of input variable x_i , $i = \overline{1,n}$;

$\underline{y}$ ($\overline{y}$) is the lower (upper) value of output variable y.

It is suggested that the sets of all possible values for qualitative variables $x_1 \div x_n$ and y are known:

$$U_i = \{v_i^1, v_i^2, ..., v_i^{q_i}\},\ i = \overline{1,n}, \tag{2.4}$$

$$Y = \{y^1, y^2, ..., y^{q_m}\}\ , \tag{2.5}$$

where v_i^1 $\left(v_i^{q_i}\right)$ is the point estimation corresponding to the smallest (largest) value of input variable x_i;

y^1 (y^{q_m}) is the point estimation corresponding to the smallest (largest) value of output variable y;

q_i, $i = \overline{1,n}$ and q_m are the cardinalities of sets (2.4) and (2.5), where in the general case $q_1 \neq q_2 \neq \ ...\ \neq q_n \neq q_m$.

2.1.2 Linguistic Variables

Let $\mathbf{X}^* = \left\langle x_1^*, x_2^*, ..., x_n^* \right\rangle$ be some vector of the input variables fixed values of the considered object, where $x_i^* \in U_i$, $i = \overline{1,n}$. The task of decision making consists of defining the output $y^* \in Y$ on the basis of the information about the vector of inputs $\mathbf{X}^*$. The necessary condition for a formal solution of this task is the availability of dependence (2.1). To define this dependence we consider input variables x_i, $i = \overline{1,n}$, and output variable y as linguistic variables [15], given on universal sets (2.2), (2.3) or (2.4), (2.5).

To make an estimation of the linguistic variables x_i, $i = \overline{1,n}$, and y we use qualitative terms from the following term-sets:

$A_i = \{a_i^1, a_i^2, ..., a_i^{l_i}\}$ is the term-set of variable x_i, $i = \overline{1,n}$,

$D = \{d_1, d_2, ..., d_m\}$ is the term-set of variable y,

where a_i^p is the p-th linguistic term of variable x_i, $p = \overline{1,l_i}$, $i = \overline{1,n}$;

d_j is the j-th linguistic term of variable y,

m is the number of various solutions in the considered region.

Cardinalities of term-sets A_i, $i = \overline{1,n}$, in the general case can be various, that is $l_1 \neq l_2 \neq \ ...\ \neq l_n$.

The names of separate terms $a_i^1, a_i^2, ..., a_i^{l_i}$ can also differ for various linguistic variables x_i, $i = \overline{1,n}$.

For example, VEHICLE SPEED { low, average, high, very high }, CONVERSION TEMPERATURE { psychrophilic, mesophilic, thermophilic }, PULSE FREQUENCE { delayed, normal, increased }.

Linguistic terms $a_i^p \in A_i$ and $d_j \in D$, $p = \overline{1,l_i}$, $i = \overline{1,n}$, $j = \overline{1,m}$, are considered as fuzzy sets given on universal sets U_i and Y defined by relations $(2.2) \div (2.5)$.

In the case of quantitative variables x_i, $i = \overline{1,n}$, and y fuzzy sets a_i^p and d_j are defined by relations:

$$a_i^p = \int_{\underline{x_i}}^{\overline{x_i}} \mu^{a_i^p}(x_i) / x_i \quad , \tag{2.6}$$

$$d_j = \int_{\underline{d}}^{\overline{d}} \mu^{d_j}(d) / d \quad , \tag{2.7}$$

where $\mu^{a_i^p}(x_i)$ is the membership function of the input variable $x_i \in [\underline{x_i}, \overline{x_i}]$ value to the term $a_i^p \in A_i$, $p = \overline{1,l_i}$, $i = \overline{1,n}$;

$\mu^{d_j}(d)$ is the membership function of the output variable $y \in [\underline{y}, \overline{y}]$ to the term - solution $d_j \in D$, $j = \overline{1,m}$.

In the case of qualitative variables x_i, $i = \overline{1,n}$ and y fuzzy sets a_i^p and d_j are defined as:

$$a_i^p = \sum_{k=1}^{q_i} \mu^{a_i^p}(v_i^k) / v_i^k \quad , \tag{2.8}$$

$$d_j = \sum_{r=1}^{q_m} \mu^{d_j}(y^r) / y^r \ , \tag{2.9}$$

where $\mu^{a_i^p}(v_i^k)$ is the membership degree of the element $v_i^k \in U_i$ to the term $a_i^p \in A_i$, $p = \overline{1,l_i}$, $i = \overline{1,n}$, $k = \overline{1,q_i}$;

$\mu^{d_j}(y^r)$ is the membership degree of the element $y^r \in Y$ to the term - solution $d_j \in D$, $j = \overline{1,m}$;

U_i and Y are defined by relations (2.4) and (2.5).

Note that integral and summation signs in relations (2.6) – (2.9) designate joining of pairs $\mu(u)/u$.

This stage of fuzzy model construction is named *fuzzification* of variables in fuzzy logic literature [9]. At this stage the linguistic estimations of variables and the membership functions necessary for their formalization are defined.

2.1.3 Fuzzy Knowledge Base

Let us take N experimental data connecting inputs and output of the identification object, and distribute it in the following way:

$$N = k_1 + k_2 + ... + k_m ,$$

where k_j is the number of experimental data corresponding to output solution d_j, $j = \overline{1,m}$, m is the number of output decisions where in the general case $k_1 \neq k_2 \neq ... \neq k_m$.

It is supposed that $N < l_1 \cdot l_2 \cdot ... \cdot l_n$, that is, the number of the selected experimental data is smaller than the complete set of various combinations of object input variables change levels $(l_i, i = \overline{1,n})$.

Let us number N experimental data in the following way:

11, 12, ..., $1\,k_1$ – numbers of input variables combinations for solution d_1;

...

$j\,1$, $j\,2$, ..., $j\ k_j$ – numbers of input variables combinations for solution d_j;

...

$m\,1, m\,2$, ..., $m\ k_m$ – numbers of input variables combinations for solution d_m.

Let us designate Table 2.1 as a knowledge matrix formed according to such rules:

1) Dimension of this matrix is equal to $(n+1)\times N$, where $(n+1)$ is the number of columns and $N = k_1 + k_2 + ... + k_m$ is the number of rows.

2) The first n columns of the matrix correspond to input variables x_i, $i = \overline{1,n}$, and the $(n+1)$-th column corresponds to values d_j of output variable y ($j = \overline{1,m}$).

3) Each row of the matrix represents some combination of input variables values referred to one of possible output variable y values. In this connection: the first k_1 rows correspond to output variable $y = d_1$ value, the second k_2 rows correspond to $y = d_2$ value, . . . , the last k_m rows correspond to value $y = d_m$.

4) Element a_i^{jp}, placed at the crossing of i-th column and jp-th row, corresponds to the linguistic estimation of parameter x_i in row number jp of the fuzzy

knowledge base, where linguistic estimation a_i^{jp} is selected from a term-set corresponding to variable x_i, that is $a_i^{jp} \in A_i$, $i=\overline{1,n}$, $j=\overline{1,m}$, $p=\overline{1,k_j}$.

Thus introduced knowledge base defines some system of logical expressions of the type «IF - THEN, OTHERWISE», interconnecting input variables values $x_1 \div x_n$ with one of the possible types of solution d_j, $j=\overline{1,m}$:

Table 2.1. Knowledge base

Number of the input combination of values	Input variables				Output variable
	x_1	x_2	$\dots x_i \dots$	x_n	y
11 12 ... $1k_1$	a_1^{11} a_1^{12} $a_1^{1k_1}$	a_2^{11} a_2^{12} $a_2^{1k_1}$	$\dots a_i^{11} \dots$ $\dots a_i^{12} \dots$ $\dots a_i^{1k_1} \dots$	a_n^{11} a_n^{12} $a_n^{1k_1}$	d_1
...					
$j1$ $j2$ ... jk_j	a_1^{j1} a_1^{j2} $a_1^{jk_j}$	a_2^{j1} a_2^{j2} $a_2^{jk_j}$	$\dots a_i^{j1} \dots$ $\dots a_i^{j2} \dots$ $\dots a_i^{jk_j} \dots$	a_n^{j1} a_n^{j2} $a_n^{jk_j}$	d_j
...					
$m1$ $m2$ ... mk_m	a_1^{m1} a_1^{m2} $a_1^{mk_m}$	a_2^{m1} a_2^{m2} $a_2^{mk_m}$	$\dots a_i^{m1} \dots$ $\dots a_i^{m2} \dots$ $\dots a_i^{mk_m} \dots$	a_n^{m1} a_n^{m2} $a_n^{mk_m}$	d_m

IF $(x_1 = a_1^{11})$ AND $(x_2 = a_2^{11})$ AND $\dots$ AND $(x_n = a_n^{11})$ OR
$(x_1 = a_1^{12})$ AND $(x_2 = a_2^{12})$ AND $\dots$ AND $(x_n = a_n^{12})$ OR $\dots$
$(x_1 = a_1^{1k_1})$ AND $(x_2 = a_2^{1k_1})$ AND $\dots$ AND $(x_n = a_n^{1k_1})$,
THEN $y = d_1$, OTHERWISE
IF $(x_1 = a_1^{21})$ AND $(x_2 = a_2^{21})$ AND $\dots$ AND $(x_n = a_n^{21})$ OR
$(x_1 = a_1^{22})$ AND $(x_2 = a_2^{22})$ AND $\dots$ AND $(x_n = a_n^{22})$ OR $\dots$
$(x_1 = a_1^{2k_2})$ AND $(x_2 = a_2^{2k_2})$ AND $\dots$ AND $(x_n = a_n^{2k_2})$,
THEN $y = d_2$, OTHERWISE $\dots$

$$\begin{aligned}
&\text{IF} \quad (x_1 = a_1^{m1}) \text{ AND } (x_2 = a_2^{m1}) \text{ AND } \ldots \text{ AND } (x_n = a_n^{m1}) \text{ OR} \\
&\quad (x_1 = a_1^{m2}) \text{ AND } (x_2 = a_2^{m2}) \text{ AND } \ldots \text{ AND } (x_n = a_n^{m2}) \text{ OR } \ldots \\
&\quad (x_1 = a_1^{mk_m}) \text{ AND } (x_2 = a_2^{mk_m}) \text{ AND } \ldots \text{ AND } (x_n = a_n^{mk_m}), \\
&\text{THEN } y = d_m, \qquad (2.10)
\end{aligned}$$

where d_j ($j = \overline{1,m}$) is a linguistic estimation of output variable y defined from term-set D ;

a_i^{jp} is a linguistic estimation of input variable x_i in p-th row of j-th disjunction selected from the corresponding term-set A_i, $i = \overline{1,n}$, $j = \overline{1,m}$, $p = \overline{1,k_j}$;

k_j is the number of rules defining output variable value $y = d_j$.

Let us call the system of logic statements like this one *the fuzzy knowledge base* system.

Using operations $\bigcup$ (OR) and $\bigcap$ (AND) the system of logical statements (2.10) can be rewritten in a more compact form:

$$\bigcup_{p=1}^{k_j}\left[\bigcap_{i=1}^{n}(x_i = a_i^{jp})\right] \longrightarrow y = d_j, \; j = \overline{1,m} \, . \qquad (2.11)$$

Thus, the required relation (2.1) defining interconnection between input parameters x_i and output variable y , is formalized in the form of fuzzy logical statements (2.11) system, which is based on the above introduced knowledge matrix.

2.1.4 Membership Functions

According to definition [15], membership function $\mu^T(x)$ characterizes some subjective measure (in the range of [0, 1]) of expert certainty in the fact that crisp value x corresponds to fuzzy term T . The most spread in practical applications [9] are triangle, trapezoidal and bell shape Gaussian membership functions, parameters of which allow us to change function shapes.

We suggest an analytical model of a variable x membership function to an arbitrary fuzzy term T in the form of:

$$\mu^T(x) = \frac{1}{1+\left(\dfrac{x-b}{c}\right)^2} \, , \qquad (2.12)$$

which is simple and convenient for tuning, where *b* and *c* are tuning parameters: *b* is the function maximum coordinate, $\mu^T(b) = 1$; *c* is the function concentration-extension ratio (Fig. 2.1). For fuzzy term T number *b* represents the most possible value of variable x .

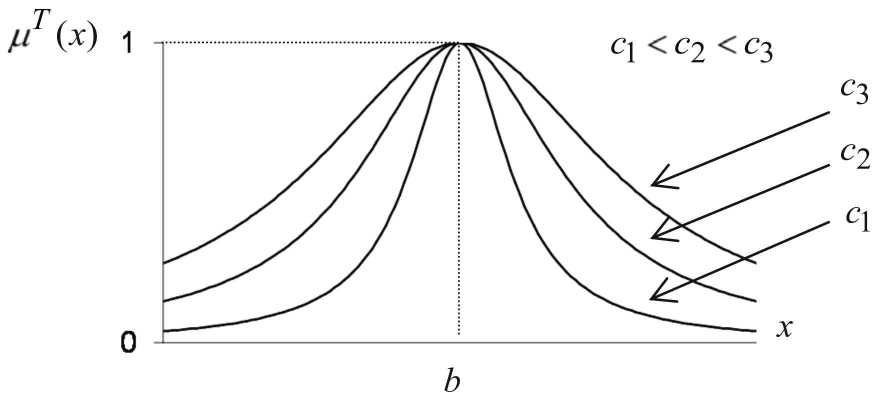

Fig. 2.1. Membership function model

2.2 Fuzzy Approximator for System with Discrete Output

2.2.1 Problem Statement

Let us consider the following as known:

* the set of decisions $D = \{d_1, d_2, ..., d_m\}$, corresponding to output variable y,

* the set of input variables $\mathbf{X} = (x_1, x_2, ..., x_n)$,

* the ranges of quantitative change of each input variable $x_i \in [\underline{x_i}, \overline{x_i}]$, $i = \overline{1,n}$,

* the membership functions allowing to represent variables x_i, $i = \overline{1,n}$, in the form of fuzzy sets (2.6) or (2.8),

* the knowledge matrix defined according to the rules introduced in Section 2.1.3.

It is thus required to design such an algorithm of decision making which allows us to bring the fixed vector of input variables $\mathbf{X}^* = \left\langle x_1^*, x_2^*, ..., x_n^* \right\rangle$, $x_i^* \in [\underline{x_i}, \overline{x_i}]$, into correspondence with decision $y \in D$.

The task of object approximation with a discrete output is shown in the form of a diagram in Fig. 2.2, where it is emphasized that the object inputs are given by three methods: 1- by number, 2- by linguistic term, 3- by thermometer principle.

The idea behind the method suggested below for the solution of this task consists of using fuzzy logic equations. These equations are constructed on the basis of a knowledge matrix or of some system of logical statements (2.10) which is isomorphic to this matrix and allow us to calculate the values of membership functions of various decisions (solutions) for fixed values of object input variables. The solution with the greatest value of membership function is chosen as the required one.

2.2.2 Fuzzy Logical Equations

Linguistic estimations a_i^{jp} of variables $x_1 \div x_n$, contained in logic statements about decisions d_j (2.10), are considered as fuzzy sets defined on universal sets $U_i = [\underline{x_i}, \overline{x_i}]$, $i = \overline{1,n}$, $j = \overline{1,m}$.

Let $\mu^{a_i^{jp}}(x_i)$ be the membership function of parameter $x_i \in [\underline{x_i}, \overline{x_i}]$ to fuzzy term a_i^{jp}, $i = \overline{1,n}$, $j = \overline{1,m}$, $p = \overline{1,k_j}$;

$\mu^{d_j}(x_1, x_2, ..., x_n)$ is the membership function of input variables $X = (x_1, x_2, ..., x_n)$ vector to the value of output variable $y = d_j$, $j = \overline{1,m}$.

Interconnection between these functions is defined by fuzzy knowledge base (2.11) and can be represented in the form of the following equations:

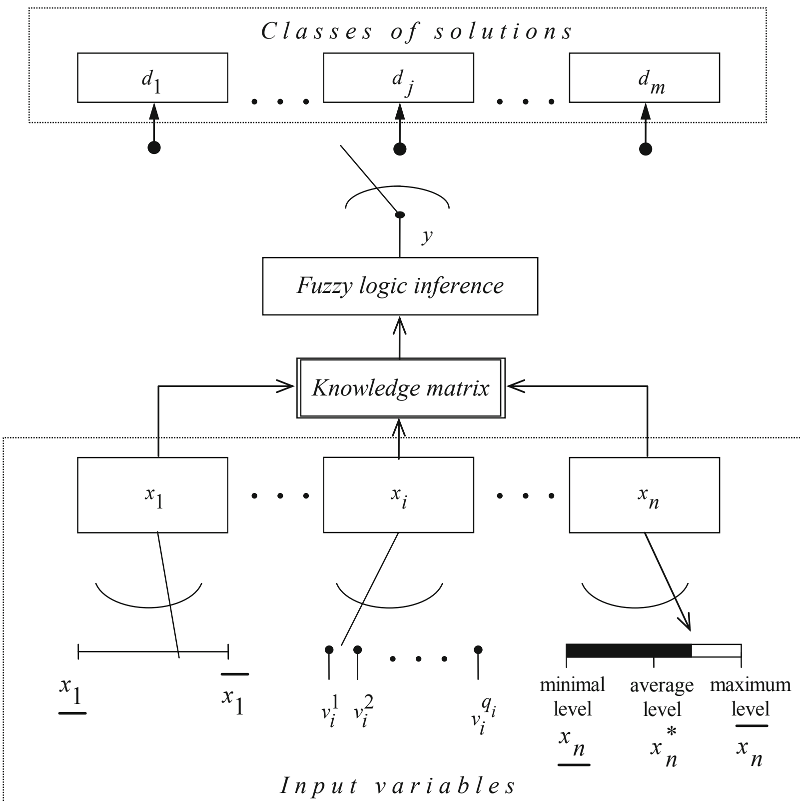

Fig. 2.2. Approximation of a nonlinear object with discrete output

$$\mu^{d_1}(x_1,x_2,...,x_n)=\mu^{a_1^{11}}(x_1)\wedge\mu^{a_2^{11}}(x_2)\wedge\ldots\wedge\mu^{a_n^{11}}(x_n)\vee$$
$$\vee\,\mu^{a_1^{12}}(x_1)\wedge\mu^{a_2^{12}}(x_2)\wedge\ldots\wedge\mu^{a_n^{12}}(x_n)\vee\ldots$$
$$\ldots\vee\mu^{a_1^{1k_1}}(x_1)\wedge\mu^{a_2^{1k_1}}(x_2)\wedge\ldots\wedge\mu^{a_n^{1k_1}}(x_n)\quad,$$

$$\mu^{d_2}(x_1,x_2,...,x_n)=\mu^{a_1^{21}}(x_1)\wedge\mu^{a_2^{21}}(x_2)\wedge\ldots\wedge\mu^{a_n^{21}}(x_n)\vee$$
$$\vee\,\mu^{a_1^{22}}(x_1)\wedge\mu^{a_2^{22}}(x_2)\wedge\ldots\wedge\mu^{a_n^{22}}(x_n)\vee\ldots$$
$$\ldots\vee\mu^{a_1^{2k_2}}(x_1)\wedge\mu^{a_2^{2k_2}}(x_2)\wedge\ldots\wedge\mu^{a_n^{2k_2}}(x_n)\quad,$$

. . .

$$\mu^{d_m}(x_1,x_2,...,x_n)=\mu^{a_1^{m1}}(x_1)\wedge\mu^{a_2^{m1}}(x_2)\wedge\ldots\wedge\mu^{a_n^{m1}}(x_n)\vee$$
$$\vee\,\mu^{a_1^{m2}}(x_1)\wedge\mu^{a_2^{m2}}(x_2)\wedge\ldots\wedge\mu^{a_n^{m2}}(x_n)\vee\ldots$$
$$\ldots\vee\mu^{a_1^{mk_m}}(x_1)\wedge\mu^{a_2^{mk_m}}(x_2)\wedge\ldots\wedge\mu^{a_n^{mk_m}}(x_n)\quad,$$

where $\vee$ is the logic OR operation, $\wedge$ is the logic AND operation.

These fuzzy logical equations are derived from fuzzy knowledge base (2.11) by way of replacing linguistic terms a_i^{jp} and d_j by corresponding membership functions, and operations $\bigcup$ and $\bigcap$ by operations $\vee$ and $\wedge$.

The logical equation system can be briefly written in the following way:

$$\mu^{d_j}(x_1,x_2,...,x_n)=\bigvee_{p=1}^{k_j}\left[\bigwedge_{i=1}^{n}\mu^{a_i^{jp}}(x_i)\right],\ j=\overline{1,m}\ . \tag{2.13}$$

2.2.3 Approximation Algorithm

The making of decision $d^*\in D=\{d_1,d_2,...,d_m\}$, which corresponds to the fixed values vector of input variables $\mathbf{X}^*=\left\langle x_1^*,x_2^*,...,x_n^*\right\rangle$, is performed in the following sequence.

1°. Let us fix the input variables values vector

$$\mathbf{X}^*=(x_1^*,x_2^*,...,x_n^*)\ .$$

2°. Let us assign fuzzy terms membership functions used in the fuzzy knowledge base (2.11) and define values of these functions for the given values of input variables $x_1^*\div x_n^*$.

3°. Using logical equations (2.13) we calculate multidimensional membership functions $\mu^{d_j}(x_1^*, x_2^*, ..., x_n^*)$ of vector $\mathbf{X}^*$ for all the values d_j, $j=\overline{1,m}$ of output variable y. Logic operations AND ($\wedge$) and OR ($\vee$) performed on membership functions are replaced by the operations *min* and *max*.

$$\mu(a) \wedge \mu(b) = \min[\mu(a), \mu(b)] \ ,$$

$$\mu(a) \vee \mu(b) = \max[\mu(a), \mu(b)] \ .$$

4°. Let us define value d_j^*, the membership function of which is maximal:

$$\mu^{d_j}(x_1^*, x_2^*, ..., x_n^*) = \max_{j=\overline{1,m}} \left(\mu^{d_j}(x_1^*, x_2^*, ..., x_n^*) \right).$$

It is this solution that is required for the input variables values vector $\mathbf{X}^* = (x_1^*, x_2^*, ..., x_n^*)$.

Thus, the suggested algorithm uses the idea of linguistic term identification by membership function maximum and generalizes this idea over the entire knowledge base.

The computational part of the suggested algorithm is easily realized with the membership functions values matrix derived from the knowledge matrix by way of doing *min* and *max* operations (Fig. 2.3).

The suggested algorithm of finding discrete values $\{d_1, d_2, ..., d_m\}$ of output variable y by the given input variables fixed values vector $\mathbf{X}^* = \left\langle x_1^*, x_2^*, ..., x_n^* \right\rangle$ and by the knowledge matrix allows to approximate the object $y = f_y(x_1, x_2, ..., x_n)$ with a discrete output.

2.3 Fuzzy Approximator for System with Continuous Output

Let us break interval $[\underline{y}, \overline{y}]$, with which object output y changes, into m parts:

$$[\underline{y}, \overline{y}] = \underbrace{[\underline{y}, y_1)}_{d_1} \cup \underbrace{[y_1, y_2)}_{d_2} \cup ... \cup \underbrace{[y_{j-1}, y_j)}_{d_j} \cup ... \cup \underbrace{[y_{m-1}, \overline{y}]}_{d_m} \ . \tag{2.14}$$

Known expert information about the object with continuous output we give in the form of fuzzy logical expressions system:

$$\begin{array}{ll}
\text{IF} & [(x_1 = a_1^{j1}) \text{ AND } (x_2 = a_2^{j1}) \text{ AND ... } (x_n = a_n^{j1})] \\
\text{OR} & [(x_1 = a_1^{j2}) \text{ AND } (x_2 = a_2^{j2}) \text{ AND ... } (x_n = a_n^{j2})] \quad ... \\
\text{... OR} & \left[(x_1 = a_1^{jk_j}) \text{ AND } (x_2 = a_2^{jk_j}) \text{ AND ... } (x_n = a_n^{jk_j})\right] , \\
\text{THEN} & y \in d_j = [y_{j-1}, y_j) \ \text{, for all } j = \overline{1,m},
\end{array} \tag{2.15}$$

where a_j^p is the linguistic term by which variable x_i in the row with number $p = k_j$ is estimated;

k_j is the number of rows-conjunctions corresponding to interval d_j, $j = \overline{1,m}$.

$\mu^{11}(x_1)$	$\mu^{11}(x_2)$	...	$\mu^{11}(x_n)$	min		
$\mu^{12}(x_1)$	$\mu^{12}(x_2)$	...	$\mu^{12}(x_n)$	min		
...	...	...	...		max	
$\mu^{1k_1}(x_1)$	$\mu^{1k_1}(x_2)$	...	$\mu^{1k_1}(x_n)$	min		
...	...	...	...			
$\mu^{21}(x_1)$	$\mu^{21}(x_2)$	...	$\mu^{21}(x_n)$	min		
$\mu^{22}(x_1)$	$\mu^{22}(x_2)$	...	$\mu^{22}(x_n)$	min	max	max
...	...	...	...			
$\mu^{2k_2}(x_1)$	$\mu^{2k_2}(x_2)$	...	$\mu^{2k_2}(x_n)$	min		
...	...	...	...			
$\mu^{m1}(x_1)$	$\mu^{m1}(x_2)$	...	$\mu^{m1}(x_n)$	min		
$\mu^{m2}(x_1)$	$\mu^{m2}(x_2)$	...	$\mu^{m2}(x_n)$	min	max	
...	...	...	...			
$\mu^{mk_m}(x_1)$	$\mu^{mk_m}(x_2)$	...	$\mu^{mk_m}(x_n)$	min		

Fig. 2.3. Matrix realization of decision making algorithm

2.3.1 Problem Statement

Let us consider the following as known:

* the interval of change $[\underline{y}, \overline{y}]$ of output variable y,
* the input variables set $\mathbf{X} = (x_1, x_2, ..., x_n)$,
* the ranges of quantitative change of each input variable $x_i \in [\underline{x_i}, \overline{x_i}]$, $i = \overline{1,n}$,
* the membership functions allowing to represent variables x_i, $i = \overline{1,n}$, in the form of fuzzy sets (2.6) or (2.8),
* the system of logical expressions of form (2.15), which can be represented in the form of the knowledge base from Section 2.1.3.

It is thus required to design such a decision making algorithm that allows to bring the fixed vector of input variables $\mathbf{X}^* = \left\langle x_1^*, x_2^*, ..., x_n^* \right\rangle$, $x_i^* \in [\underline{x}_i, \overline{x}_i]$ into correspondence with decision $y \in [\underline{y}, \overline{y}]$.

The fuzzy logic evidence algorithm presented in Section 2.2.3 allows us to calculate the output value y in the form of a fuzzy set:

$$\tilde{y} = \left\{ \frac{\mu^{d_1}(y)}{[\underline{y}, y_1)}, \frac{\mu^{d_2}(y)}{[y_1, y_2)}, \ldots, \frac{\mu^{d_m}(y)}{[y_{m-1}, \overline{y}]} \right\}. \tag{2.16}$$

To obtain a crisp number corresponding to the fuzzy value (2.16) from interval $[\underline{y}, \overline{y}]$ it is necessary to use the defuzzification operation [9]. Defuzzification is the operation of transforming fuzzy information into its crisp form. Let us define a crisp number y^* which corresponds to fuzzy set (2.16) such that:

$$y^* = \frac{\underline{y}\mu^{d_1}(y) + y_1\mu^{d_2}(y) + \ldots + y_{m-1}\mu^{d_m}(y)}{\mu^{d_1}(y) + \mu^{d_2}(y) + \ldots + \mu^{d_m}(y)}. \tag{2.17}$$

Where there is probability interpretation of membership degrees, formula (2.17) can be considered as an analog to mathematical expectation of a discrete random value.

If we break interval $[\underline{y}, \overline{y}]$ into m equal parts, that is,

$$y_1 = \underline{y} + \Delta, \; y_2 = \underline{y} + 2\Delta, \; \ldots, \; y_{m-1} = \overline{y} - \Delta, \; \Delta = \frac{\overline{y} - \underline{y}}{m-1},$$

then formula (2.17) is simplified and takes the form which is convenient for calculations:

$$y^* = \frac{\sum_{j=1}^{m} [\underline{y} + (j-1)\Delta] \, \mu^{d_j}(y)}{\sum_{j=1}^{m} \mu^{d_j}(y)}. \tag{2.18}$$

2.3.2 Approximation Algorithm

To solve the stated problem of the approximation of a nonlinear object with continuous output we use the fuzzy logic evidence algorithm from Section 2.2.3 and the defuzzification operation (2.18). Then the value of the output variable $y^* \in [\underline{y}, \overline{y}]$, which corresponds to the vector of input variables fixed values $\mathbf{X}^* = \left\langle x_1^*, x_2^*, ..., x_n^* \right\rangle$, is found in such a sequence.

1°. Using the fuzzy logic evidence algorithm from Section 2.2.3 we calculate multi-dimensional membership functions $\mu^{d_j}(x_1^*, x_2^*, ..., x_n^*)$ of vector $\mathbf{X}^*$ for all the subintervals $d_j = [y_{j-1}, y_j)$, $j = \overline{1,m}$, into which interval $[\underline{y}, \overline{y}]$ of output variable y is broken.

2°. Using defuzzification operation (2.18) we obtain the required value y^*.

Approximation of a nonlinear object with continuous output is shown in Fig. 2.4.

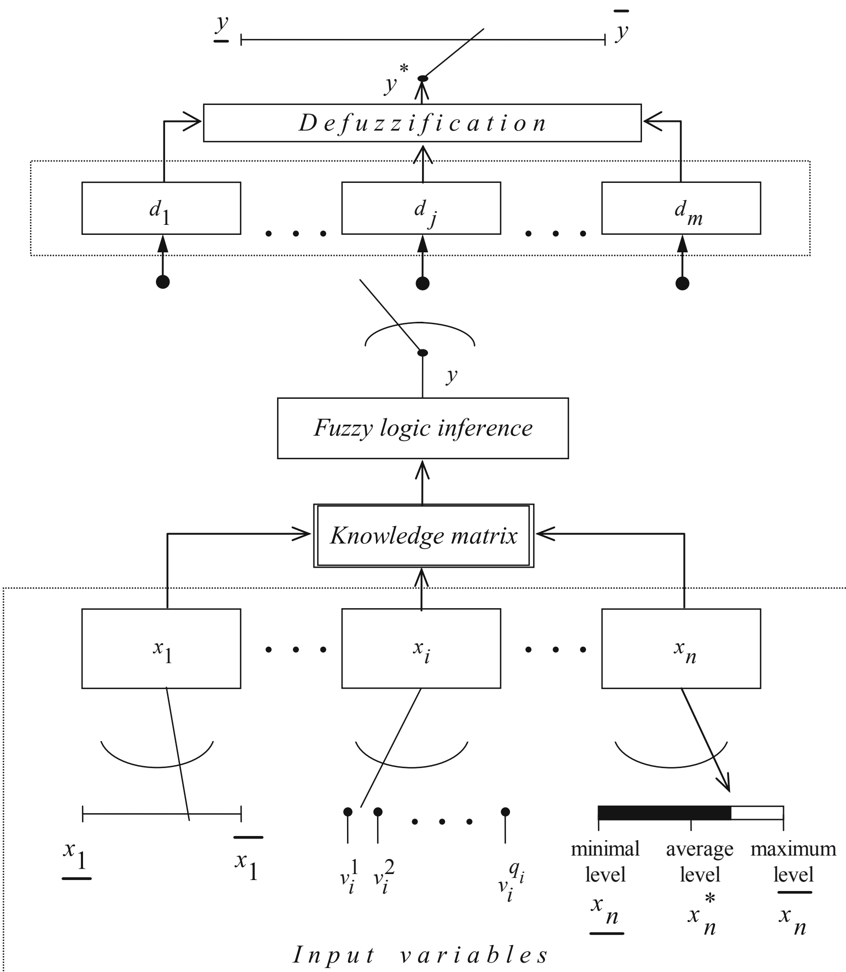

Fig. 2.4. Approximation of a nonlinear object with continuous output

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