Chapter 7
Searching Musical Representative Phrases Using Decision Making Based on Fuzzy Similarities

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Abstract. A new method to find representative phrases from a musical score is given in this paper. It is based on the computation and use of a fuzzy proximity relation on a set of phrases. This relation is computed as a conjunction of values given by a W-indistinguishability on a set of variation of notes in the phrases, where W is the Lukasiewicz t-norm. Different fuzzy logics are used and compared in order to show their influence on the final decision. The proposed method to find the most representative phrase has been proved successful on different musical scores.

1 Introduction

The concept of musical "motif" stands for a short musical phrase on which a composer develops the whole musical score. The "motif" is a melodic element that is important throughout the work and that can be varied to generate more musical phrases. This work uses a practical approximation to the criteria of Overill [8] for searching musical motifs based on the analysis of the different phrases. The motif of a score is found using a "fuzzy pattern machine model" that uses indistinguishability operators and proximity fuzzy relations to compare the phrases.

In [8] the author discusses the importance in music analysis of establishing the occurrences of a musical motif and its variants. He presents it as a tedious and time-consuming process; therefore, it is a task that can be carried out by a computer using several models that must include the design of which variants are to be included in the search. The number of variants that are considered have been found to have a profound effect on the computer time required. He presents two models that are based on recurrence relations and closed analytic expression of fuzzy pattern matching.

Each one of the Overill [8] models assumes the existence of an atomic exact matching operation that can be represented in a formula to be evaluated and tabulated as a function of some independent parameters. These results allow a prior estimation of the relative run times of different music searches. Both proposed models are also equally capable of handling inversion, retrogradation or inverted retrogradation of a motif [1]. Nevertheless, both models are only concerned with pitch, without taking into account other musical issues such as the duration.

Finally, Overill concludes that from the music analysis, the traditional approaches such as the two models analyzed in his paper, have many drawbacks and limitations,

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mainly due to the complexity and the computer time they require, which makes them less useful for practical applications.

Some other approaches present a method to find representative phrases from a musical score [3][4]. They are based on the computation of a fuzzy proximity relation on the set of phrases. Two musical phrases can be considered ‘similar’ when the variations between the first and the second notes are ‘equivalent’, AND the variations between the second and the third notes are ‘equivalent’, AND …, so on and so forth. That is, two phrases are similar if the conjunction of the distances between couples of consecutive notes are similar. This conjunction can be modeled using different mathematical operators, specifically triangular norms. Therefore, two phrases can be similar even if the starting tone is different, because the comparison system works by evaluating relative distances between notes allowing transportations on the scale.

Once the fuzzy relation “proximity” has been computed on a set of phrases, a method that automatically selects some phrases is defined by computing a fuzzy set representing the characteristic of being ‘similar to the other phrases’ on the set of phrases. The chosen representative phrase is the one with highest membership degree in such fuzzy set. As an example, this method is applied to find the representative phrase of the musical score shown in Figure 1.

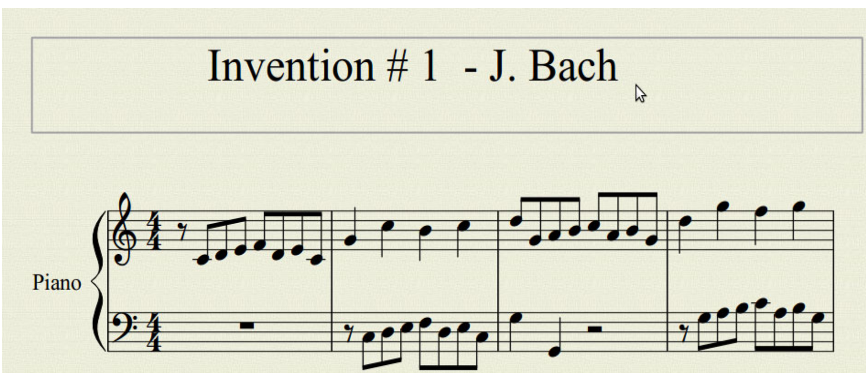

Fig. 1. A few phrases of Invention # 1 of J. Bach

This chapter is organized as follows. Section 2 includes some preliminaries about fuzzy logic and decision making in uncertain environments with imprecise information. Also the concepts of proximity and similarity which are basis for the proposed searching methodology are defined. Section 3 presents the concept of specificity measurement that is used to evaluate the reliability of results produced by the decision method. The definitions that are used to compute the distance between consecutive notes of a phrase are described in section 4. It is also proven that the negation of a distance is a W-indistinguishability operator. This is applied to the variations of consecutive notes for each couple of phrases in order to compute the proximity on the set of phrases. It is then possible to obtain how similar the phrases sound. The experimental procedure, step by step experiments and results are explained in sections 5 to 8. An example to show how the algorithm for searching musical motifs performs starts in section 5. The details on how to build a proximity relationship on the set of phrases using 3 different t-norms and the OWA operator with 3 levels of tolerance are shown in section 6, in order to compare different aggregations' operators.

Section 7 details the method to choose a representative phrase, and finally, section 8 includes the last step of the application related to the calculation of the specificity measurement. The chapter finalizes with the conclusions in section 9.

2 Fuzzy Logic in Decision Making with Uncertainty

Fuzzy logic is useful when dealing with vague, uncertain, and complex environments. The imprecise information that characterizes the elements of a universe can be interpreted as a linguistic variable and modeled with fuzzy sets.

Given a universe of discourse E, a fuzzy set [13] is a mapping $\mu: E \to [0, 1]$ gives a membership degree to every element of E in the interval [0, 1].

A semantic label is assigned to this fuzzy set and its membership degree is used to measure a characteristic of the elements of the universe E.

It is also well known that an algebra on fuzzy sets allows to define an extension of the logic operators AND, OR, and NOT, using triangular norms (t-norms), triangular conorms (t-conorms) and negation operators respectively [7]. The t-norm can be defined as follows [9][10],

For all x, y, z in [0,1], a binary operation $T: [0, 1] \times [0, 1] \to [0, 1]$ is a t-norm if it satisfies the following axioms:

$$T(1, x) = x, T(0, x) = 0 \text{ for all } x \text{ in } [0, 1] \quad (1)$$

$$T(x, y) = T(y, x) \text{ –symmetry-} \quad (2)$$

$$T(x, T(y, z)) = T(T(x, y), z) \text{ –associativity-} \quad (3)$$

$$\text{If } x \leq x' \text{ and } y \leq y' \text{ then } T(x, y) \leq T(x', y') \text{ –monotonicity-} \quad (4)$$

The t-conorm operator can be defined in a similar way, but having S(1, x) = 1, S(x, 0) = x, and similar axioms (2), (3), and (4).

The most common continuous logic operators are shown in Table 1.

Table 1. Most used t-norms and t-conorms in fuzzy logic

Logic	t-norm	t-conorm
Zadeh	min(x,y)	max(x,y)
Product	x*y	x + y - xy
Łukasiewicz	max(0, x+y-1)	min(1, x+y)

The usual negation operator is defined as a mapping N: [0, 1] $\rightarrow$ [0, 1] with N(x) = 1-x for any x $\in$ [0, 1]. It is possible to define the fuzzy set "NOT A" from a fuzzy set "A" and a negation operator N as follows:

$$\mu_{NOT\ A}(x) = N(\mu_A(x)) \text{ for every x in E.} \quad (5)$$

When it is necessary to gather several concepts, informations or fuzzy sets in a single fuzzy set, aggregation operators are useful. For example, operators between t-norms and t-conorms, such as averages, are aggregation operators [2].

On the other hand, fuzzy relations R: E×E$\rightarrow$ [0, 1] have many applications to make fuzzy inference with uncertain, imprecise, or incomplete knowledge. For example, it can be used to model similarity or implication relations in order to generate rules and therefore to infer some conclusions [13]. A fuzzy rule can be expressed as,

IF "x is A" THEN "y is B"

where A and B are fuzzy sets (may be defined as a conjunction, aggregations or disjunction of other fuzzy sets), and *x* and *y* are measurable variables of the elements of the universe E. The main characteristic of the approximate reasoning is that from the knowledge of "x is A' ", for example 'x is almost A', and the rule stated above, and by applying the compositional rule of inference it is possible to learn that "y is B' " [14], for example, inferring that "x is almost B".

2.1 Proximity and Similarity

Let E = {e1, ..., en} be a finite set and R a fuzzy relation on E. The relation degree for every pair of elements ei and ej in E is denoted eij. So eij = R(ei, ej). Some common fuzzy relations properties are the following:

- A fuzzy relation R is reflexive if $e_{ii} = 1$ for all $1 \leq i \leq n$.
- A fuzzy relation R is α -reflexive if $e_{ii} \geq \alpha$, for all $1 \leq i \leq n$.
- The relation R is symmetric if $e_{ij} = e_{ji}$ for all $1 \leq i, j \leq n$.

A reflexive and symmetric fuzzy relation is called a fuzzy **proximity** relation.

Let T be a t-norm [9]. A fuzzy relation R: E× E $\rightarrow$ [0, 1] is **T-transitive** if and only if T(R(a, b), R(b, c)) $\leq$ R(a, c) for every a, b, c in E. So R is T transitive if $T(e_{ik}, e_{kj}) \leq e_{ij}$ for every $1 \leq i, j, k \leq n$.

A **T-indistinguishability** relation is a reflexive, symmetric and T-transitive fuzzy relation.

Finally, a fuzzy **similarity** is a reflexive, symmetric and min-transitive fuzzy relation.

Similarities are therefore particular cases of T-indistinguishabilities, where the t-norm is the minimum [11].

It is possible to establish the following proposition: Let d be a normalized distance in E. Let N: [0, 1] $\rightarrow$ [0, 1] be the usual negation operator. Then S: E× E $\rightarrow$ [0, 1] defined as S(x, y) = N(d(x, y)) is a **W-indistinguishability**, where W is the Łukasiewicz t-norm.

Proof

If d is a distance,

$$d(x, x) = 0 \text{ for all } x \text{ in } E \tag{6}$$

$$d(x, y) = d(y, x) \text{ for all } x, y \text{ in } E. \tag{7}$$

$$d(x, y) \leq d(x, z) + d(z, y) \text{ (triangular inequality)} \tag{8}$$

Then S is a W-indistinguishability because:

$$S(x, x) = N(d(x, x)) = N(0) = 1 \text{ for all } x \text{ in } E \text{ (S is reflexive)} \tag{9}$$

$$S(x, y) = N(d(x, y)) = N(d(y, x)) = S(y, x) \text{ for all } x, y \text{ in } E \text{ (symmetry)} \tag{10}$$

$$\text{By the triangular inequality of d, } d(x, y) \leq W^*(d(x, z), d(z, y)) \tag{11}$$

where W* is the Łukasiewicz t-conorm, that is, the dual t-conorm of W [9]. Then applying the operator N,

$$N(d(x, y)) \geq N(W^*(d(x, z), d(z, y))) = W(N(d(x, z)),N(d(z, y))) \tag{12}$$

and then

$$S(x, y) \geq W(S(x, z), S(z, y)) \text{ for all } x, y, z \text{ in } E. \tag{13}$$

Then S is W-transitive and therefore is a W-indistinguishability.

3 Measure of Specificity on Fuzzy Sets

The concept of specificity provides a measurement of the degree of having just one element in a fuzzy set or a possibility distribution. It is strongly related to the inverse of the cardinality of a set.

Specificity values were introduced by Yager showing their usefulness as a measure of "tranquility" when making a decision. Yager stated the specificity-correctness tradeoff principle. The output information of an expert system or any other knowledge based system should be both specific and correct if it is to be useful. Yager suggested the use of specificity in default reasoning, in possibility qualified statements and in data mining processes, giving several possible manifestations of this measure.

Let X be a set with elements $\{x_i\}$ and let $[0, 1]^X$ be the class of fuzzy sets of X. A measure of specificity [12] is a mapping Sp: $[0, 1]^X \rightarrow [0, 1]$ such that:

$$Sp(\mu) = 1 \text{ if and only if } \mu \text{ is a singleton } (\mu = \{x_1\}) \tag{14}$$

$$Sp(\emptyset) = 0 \tag{15}$$

$$\text{If } \mu \text{ and } \eta \text{ are normal fuzzy sets in X and } \mu \subset \eta, \text{ then } Sp(\mu) \geq Sp(\eta) \tag{16}$$

Given a measurement of specificity Sp on a fuzzy set and given a T-indistinguishability S, the expression Sp(μ/S) is a **measure of specificity under T-indistinguishabilities** [6] when it verifies the following four axioms:

$$Sp(\{x\} / S) = 1 \tag{17}$$

$$Sp(\varnothing / S) = 0 \tag{18}$$

$$Sp(\mu / Id) = Sp(\mu) \tag{19}$$

$$Sp(\mu / S) \geq Sp(\mu) \tag{20}$$

Yager introduced the linear measurement of specificity on a finite space X as:

$$Sp(\mu) = a_1 - \sum_{j=2}^{n} w_j a_j \tag{21}$$

where a_j is the j_{th} greatest membership degree of μ and $\{w_j\}$ is a set of weights that verifies,

$$w_j \in [0, 1] \sum_{j=2}^{n} w_j = 1 \tag{22}$$

$w_j \geq w_i$ for all $1< j<i$.

This is the operator which is going to be applied to musical composition to know how useful the decision of choosing a phrase is. If there is only one representative phrase, the specificity of the fuzzy set "similar to other phrases" is maximal. By adding some more information on the proximity on phrases, some groups of similar phrases can be clustered in just one phrase.

4 Intelligent Algorithm for Searching Musical Motifs

A first step to find musical motifs is to separate a musical score into phrases. Then, the phrases are compared to each other in order to evaluate the proximity degree of every couple of phrases. From the proximity and the concept "similar to other phrases' defined in a fuzzy set, it is possible to identify the motifs from the set of candidate phrases [5].

An approximation of the pre-searching method described by [8] is used as a starting point for searching the musical motifs. The algorithm in pseudo code can be written as:

a) A score is separated into phrases.
b) The proximity degree of every couple of phrases is computed.

c) A fuzzy set *'candidate to be a motif'* is computed on the set of phrases by aggregating the proximity degree of each phrase with other phrases.
d) The most representative phrase is the one with highest membership degree on the fuzzy set 'candidate to be a motif'.

The selection of the set of phrases can be done in different ways. The whole process depends on the chosen way of separation of phrases.

The pre-searching method takes into account the following criteria:

1) The variation of tones into a phrase.
2) The distance of the intervals between notes into a phrase.

The following definitions are given to establish a notation and a description of the given methodology.

4.1 Phrases and Variation Points

A phrase of a score is a sorted set of notes. For each couple of consecutive notes there is a measurable variation. It is possible to define a point for each couple of consecutive notes represented by (x, y).

A variation point for a note is the pair p_i = [tone, variation], where the tone is represented using the positive integer in the standard MIDI for the respective note, and the variation is the difference between two tones (number of semitones of difference with the next note).

4.2 Distance between Ordered Notes

Let $P_n = \{ p_1, p_2, \ldots, p_n\}$ be a set of notes, that is, a musical phrase.

A function that computes the n-1 distances between consecutive notes is defined as follows:

$$D(Pn) = [\, d(p_1, p_2), d(p_2, p_3), \ldots, d(p_{n-1}, p_n)\,] \tag{23}$$

where d is a distance, for example, the Euclidean distance.

4.3 A W-Indistinguishability S of Consecutive Notes

Let Sr be a W-indistinguishability operator $Sr : R \times R \rightarrow [0,1]$ defined by

$$Sr(a, b) = (r_{max} - d(|a - b|)) / r_{max} \text{ where the range } r_{max} \text{ is } |a_{max} - b_{max}| \tag{24}$$

A function of real numbers $Sv: R_n \times R_n \rightarrow R_n$ is used to compute the W-indsitinguishabilities between the (n-1) variation points of two phrases X and Y. It is defined as follows,

$$Sv(\{x_1,x_2,...,x_{n1}\},\{y_1,y_2,...y_{n1}\})=\{Sr(x_1,y_1),Sr(x_2,y_2), ..., Sr(x_n, y_n)\} \tag{25}$$

To define a proximity degree S: $P_n \times P_{n'} \rightarrow [0,1]$ of two musical phrases P_n and $P_{n'}$, a conjunction operator T on the W-indistinguishability degrees of the distances between variation points given by D is computed as follows.

$$S(P_{ni}, P_{nj}) = T(Sv(D(P_{ni}), D(P_{nj}))) \tag{26}$$

Note that T is an n-ary t-norm operator (a conjunction operator) defined from a binary t-norm through the associative property:

$$T(x_1, x_2, ..., x_n) = T(x_1, T(x_2, T(..., T(..., x_n)))) \tag{27}$$

4.4 Choosing Operators for Different Meanings of "Representative Phrases"

Once a proximity relationship on the set of phrases is obtained, it is necessary to translate it into musical concepts. The way in which a fuzzy set describing the representative concept on the set of phrases based on the proximity relation can be done using different operators. Depending on how is it understood the concept of representative phrase:

- If a phrase is representative when it is similar to ALL other phrases, it is possible to define the representativeness of a phrase by using a conjunction of the proximity values with the rest of phrases through a t-norm. So a phrase is representative when it is similar to a phrase 1 AND it is similar to phrase 2 AND…it is similar to phrase n.
- If the phrase is representative when it is similar to ANY other phrase, then it is possible to use a disjunction, for example, the MAX t-conorm. So a phrase is representative when it is similar to a phrase 1 OR it is similar to phrase 2 OR…it is similar to phrase n (all other phrases but itself).
- If the phrase is representative when it is similar to SOME other phrase, it is possible to use aggregation operators, which are in between conjunction and disjunctions. For example, an Ordered Weighted Averaging function (OWA), or an average.

R. R. Yager´s Ordered weighted averaging functions (OWAs) are also a class of averaging aggregation functions. The difference in the weighted arithmetic mean is that the weights are not associated with the inputs but with the magnitude. In some applications all inputs are equivalent, and the importance of one specific input is determined by its absolute value.

5 Experiments and Results

In this section, a step by step example is developed to show how the proposed algorithm performs. The eight first notes of Figure 1 are considered to evaluate if there is a possible motif. The score is divided into phrases of 8 notes.

The first phrase P^n_1 includes the first 8 notes in the score; the initial silence is omitted. The rest of the phrases $P^{n'}_i$ are formed taking 8 consecutive notes starting from the 9^{th} note in the superior line (first voice); then it is shifted a note for every phrase for the two voices separately until there are 10 more phrases. All the resulted phrases after the normalization of the durations of their notes are shown in Figure 2.

Note that in same way that the models described in [8], this method is concerned only with pitch, without taking into account the real duration of the notes in order to simplify the pre-searching. On the other hand, based on the combination of tools and techniques, this method is capable of handling inversion, retrogradation and inverted retrogradation of motifs.

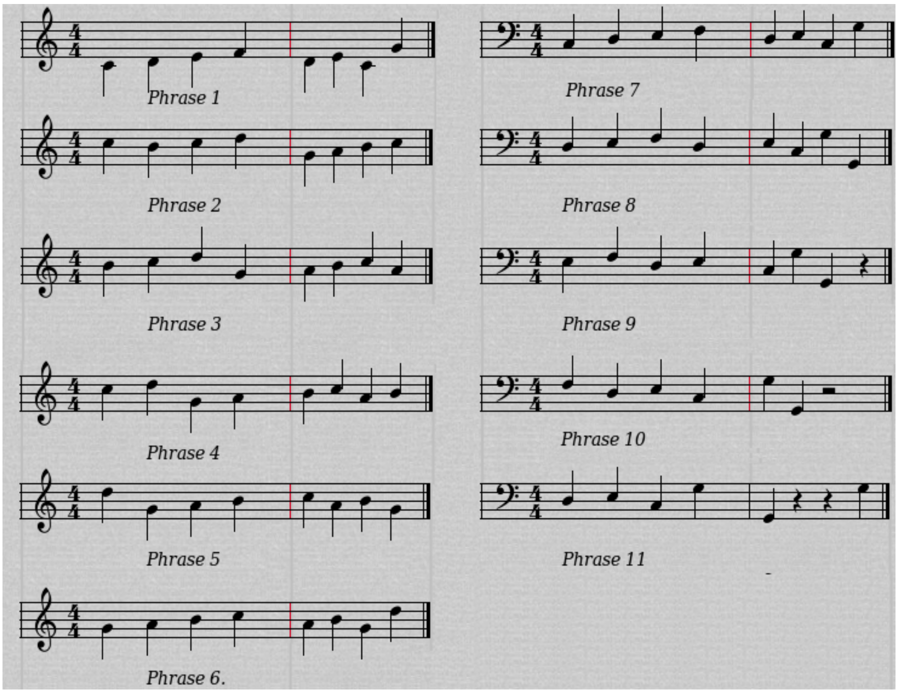

Fig. 2. Normalized phrases of invention #1

The 8 notes represented by their scale and duration of the 11 phrases are:

P^n_1 = [(C5 , 2) , (D5 , 2) , (E5 , 1) , (F5 , -3) , (D5, 2) , (E5 , -4) , (C5 , 7) , (G5 , 0)]
P^n_2 = [(C6 , -1) , (B5 , 1) , (C6 , 2) , (D6 , -7) , (G5, 2) , (A5 , 2) , (B5 , 1) , (C6 , -3)]
P^n_3 = [(B5 , 1) , (C6 , 2) , (D6 , -7) , (G5 , 2) , (A5 , 2) , (B5 , 1) , (C6 , -3) , (A5 , 0)]
P^n_4 = [(C6 , 2) , (D6 , -7) , (G5 , 2) , (A5 , 2) , (B5 , 1) , (C6 , -3) , (A5 , 2) , (B5 , 0)]
P^n_5 = [(D6 , -7) , (G5 , 2) , (A5 , 2) , (B5 , 1) , (C6, -3) , (A5 , 2) , (B5 , -4) , (G5 , 0)]
P^n_6 = [(G5 , 2) , (A5 , 2) , (B5 , 1) , (C6 , -3) , (A5 , 2) , (B5 , -4) , (G5 , 7) , (D6 , 0)]
P^n_7 = [(C4 , 2) , (D4 , 2) , (E4 , 1) , (F4 , -3) , (D4 , 2) , (E4 , -4) , (C4 , 7) , (G4 , 0)]
P^n_8 = [(D4 , 2) , (E4 , 1) , (F4 , -3) , (D4 , 2) , (E4 , -4) , (C4 , 7) , (G4 , -12), (G3 , 0)]
P^n_9 = [(E4 , 1) , (F4 , -3) , (D4 , 2) , (E4 , -4) , (C4 , 7) , (G4 , -12 , (G3 , -) , (- , 0)]
P^n_{10} = [(F4 , -3) , (D4 , 2) , (E4 , -4) , (C4 , 7) , (G4 , -12) , (G3 , -) , (- , -) , (- , 0)]
P^n_{11} = [(D4 , 2) , (E4 , -4) , (C4 , 7) , (G4 , -12), (G3 , -) , (- , -) , (- , -) , (G4 , 0)]

In this case, only eleven of the initial phrases for the two voices are analyzed in order to simplify the example and to show some details. This procedure can be applied to the whole musical score (more than 400 phrases), using different lengths to find the best motif.

The variation points of the eleven normalized phrases of Invention # 1 of J. Bach (Figure 2) are represented in the following matrix using the traditional musical notation for the different classes of pitch, with the first seven letters of the Latin alphabet and a number after the letter which represents the octave. The selected phrases include notes between 4 th and 6 th octaves.

The n-1 distances D(P^n) between the variation points of every phrase are calculated and shown in the following Table 2:

Table 2. Distance D(P^n) between notes of every phrase

$D(P^{n}{}_{0}) =$	[2,00	2,24	4,12	5,83	6,32	11,70	15,65]
$D(P^{n'}{}_{1}) =$	[2,24	1,41	9,22	11,40	2,00	2,24	1,41]
$D(P^{n'}{}_{2}) =$	[1,41	9,22	11,40	2,00	2,24	4,12	5,83]
$D(P^{n'}{}_{3}) =$	[9,22	11,40	2,00	2,24	4,12	5,83	2,24]
$D(P^{n'}{}_{4}) =$	[11,40	2,00	2,24	4,12	5,83	6,32	11,70]
$D(P^{n'}{}_{5}) =$	[2,00	2,24	4,12	5,83	6,32	11,70	15,65]
$D(P^{n'}{}_{6}) =$	[2,00	2,24	4,12	5,83	6,32	11,70	15,65]
$D(P^{n'}{}_{7}) =$	[2,24	4,12	5,83	6,32	11,70	20,25	22,47]
$D(P^{n'}{}_{8}) =$	[4,12	5,83	6,32	11,70	20,25	16,97	9,00]
$D(P^{n'}{}_{9}) =$	[5,83	6,32	11,70	20,25	16,97	0,00	10,00]
$D(P^{n'}{}_{10}) =$	[6,32	11,70	20,25	16,97	0,00	0,00	7,00]

The next step is to normalize Table 2 using the maximum distance, which in this case is 22.47 (see row 8, last column). The normalized table is shown in Table 3.

Table 3. Normalized distances D(Pn) between notes of every phrase

Phrase	***Normalized Values***						
1	0,09	0,1	0,18	0,26	0,28	0,52	0,7
2	0,1	0,06	0,41	0,51	0,09	0,1	0,06
3	0,06	0,41	0,51	0,09	0,1	0,18	0,26
4	0,41	0,51	0,09	0,1	0,18	0,26	0,1
5	0,51	0,09	0,1	0,18	0,26	0,28	0,52
6	0,09	0,1	0,18	0,26	0,28	0,52	0,7
7	0,09	0,1	0,18	0,26	0,28	0,52	0,7
8	0,1	0,18	0,26	0,28	0,52	0,9	1
9	0,18	0,26	0,28	0,52	0,9	0,76	0,4
10	0,26	0,28	0,52	0,9	0,76	0	0,45
11	0,28	0,52	0,9	0,76	0	0	0,31

The next step is to compute the W-indistinguishability of every phrase with respect to the other 10 phrases. This is obtained by computing the distance of every variation point of the phrases (values of Table 3) and the other 10 rows, and applying the usual negation operator $N(x) = 1 - x$ of distances, to obtain W-indistinguishabilities between variation points of different phrases.

For example, Table 4 shows the proximity of phrase 6 with the rest of the phrases using different continuous conjunction operators (t-norms of Table 1). Every value in the rows of Table 4 is calculated using the negation operator N(x) = 1 – x to the subtraction of 2 values from Table 3. Those are obtained by comparing phrase by phrase and variation point by variation point for each one of the eleven phrases. For example, the value in column 1 and row 5 of Table 4 (0.58) is computed using the negation N(x) = 1 – x, where x= (0.09-0.51), normalized values of the first variations points of phrases 6 and 5 respectively (Table 3).

The last 3 columns of Table 4 are the values for every t-norm of Table 1 (product, minimum, Łukasiewicz) applied to each one of the rows. For example, the value obtained by the t-norm product for the phrase 5 (0.299) is the product of (0.58 * 0.99 * 0.92 * 0.92 * 0.98* 0.76 * 0.82), that is, the multiplication of all the values of the corresponding row.

Table 4. W-indistinguishabilities of the variation points of Phrase 6 with the other phrases

	Phrase 6							*product*	*min*	*w*
1	1	1	1	1	1	1	1	1	1	1
2	0,99	0,96	0,77	0,75	0,81	0,58	0,37	0,095	0,366	0,000
3	0,97	0,69	0,68	0,83	0,82	0,66	0,56	0,115	0,563	0,000
4	0,68	0,59	0,91	0,84	0,9	0,74	0,4	0,082	0,403	0,000
5	0,58	0,99	0,92	0,92	0,98	0,76	0,82	0,299	0,582	0,000
7	1	1	1	0,92	1	1	1	1	1	1
8	0,99	0,92	0,92	0,92	0,76	0,62	0,7	0,254	0,620	0,000
9	0,91	0,84	0,9	0,92	0,38	0,77	0,7	0,130	0,380	0,000
10	0,83	0,82	0,66	0,92	0,53	0,48	0,75	0,078	0,479	0,000
11	0,81	0,58	0,28	0,92	0,72	0,48	0,61	0,026	0,282	0,000

6 Building a Proximity Relationship on the Set of Phrases

Two phrases can be considered 'similar' when the variation between the first and the second notes are 'similar', AND the variation between the second and the third notes are 'similar', AND …, so on and so forth. Such concept of 'similar' is replaced by 'W-indistinguible' in this paper's proposal.

The calculation of the conjunction of W-indistinguishabilities of the variation points for each phrase regarding the others defines a proximity relationship on the set of phrases. The final values of the proximity on the set of phrases are shown in Tables 5, 6 and 7 where the conjunction (AND) is implemented by the three different t-norms of Table 1. Tables 8, 9 and 10 also show the proximity values using the OWA operator with different percentages. Table 8 shows the OWA operator at 85%, taking out the least significant variation point, Table 9 shows the OWA operator at 71% taking out the two least significant variation points, and finally, Table 10 shows the OWA operator at 57%, taking out the three least significant variation points.

These three cases are equivalent to using a vector of weighs, W_i=1/6, for the 6 highest membership degrees values of each one of the phrases. The second case would correspond to using a vector of weighs Wi=1/5 for the 5 highest membership degree values of every phrase, and the last case uses a vector W_i=1/4 for the 4 highest membership degree values of every phrase.

Table 5. Proximity of phrases using the t-norm minimum

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	1	0,366	0,563	0,403	0,582	1	1	0,620	0,380	0,358	0,282
2	0,366	1	0,582	0,556	0,542	0,366	0,366	0,063	0,188	0,334	0,509
3	0,563	0,582	1	0,582	0,556	0,563	0,563	0,259	0,198	0,188	0,334
4	0,403	0,556	0,582	1	0,579	0,403	0,403	0,100	0,282	0,198	0,188
5	0,582	0,542	0,556	0,579	1	0,582	0,582	0,380	0,358	0,282	0,198
6	1	0,366	0,563	0,403	0,582	1	1	0,620	0,380	0,358	0,282
7	1	0,366	0,563	0,403	0,582	1	1	0,620	0,380	0,358	0,282
8	0,620	0,063	0,259	0,099	0,380	0,620	0,620	1	0,400	0,099	0,099
9	0,380	0,188	0,198	0,282	0,358	0,380	0,380	0,400	1	0,245	0,099
10	0,358	0,334	0,188	0,198	0,282	0,358	0,358	0,099	0,245	1	0,245
11	0,282	0,509	0,334	0,188	0,198	0,282	0,282	0,099	0,099	0,245	1

The proximity values using the t-norm minimum are in the range of 0.063 to 1, with an average value of 0.46; t-norm minimum gives the highest values in the set of proximities.

Table 6. Proximity of phrases using the t-norm product

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	1	0,095	0,115	0,082	0,299	1	1	0,269	0,104	0,030	0,014
2	0,095	1	0,240	0,113	0,099	0,095	0,095	0,004	0,027	0,066	0,105
3	0,115	0,240	1	0,241	0,113	0,115	0,115	0,019	0,024	0,030	0,098
4	0,082	0,113	0,241	1	0,249	0,082	0,082	0,007	0,027	0,015	0,027
5	0,299	0,099	0,113	0,249	1	0,299	0,299	0,059	0,050	0,033	0,016
6	1	0,095	0,115	0,082	0,299	1	1	0,269	0,104	0,030	0,014
7	1	0,095	0,115	0,082	0,299	1	1	0,269	0,104	0,030	0,014
8	0,269	0,004	0,019	0,007	0,059	0,269	0,269	1	0,133	0,007	0,002
9	0,104	0,027	0,024	0,027	0,050	0,104	0,104	0,133	1	0,085	0,004
10	0,030	0,066	0,030	0,015	0,033	0,030	0,030	0,007	0,085	1	0,084
11	0,014	0,105	0,098	0,027	0,016	0,014	0,014	0,002	0,004	0,084	1

The proximity values using the t-norm product are in the interval 0.002 to 1 with an average value of 0.23; t-norm product gives medium values of proximities.

Table 7. Proximity of phrases using the t-norm Lukasiewicz

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	1	0,000	0,000	0,000	0,000	1	1	0,000	0,000	0,000	0,000
2	0,000	1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
3	0,000	0,000	1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
4	0,000	0,000	0,000	1	0,000	0,000	0,000	0,000	0,000	0,000	0,000
5	0,000	0,000	0,000	0,000	1	0,000	0,000	0,000	0,000	0,000	0,000
6	1	0,000	0,000	0,000	0,000	1	1	0,000	0,000	0,000	0,000
7	1	0,000	0,000	0,000	0,000	1	1	0,000	0,000	0,000	0,000
8	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1	0,000	0,000	0,000
9	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1	0,000	0,000
10	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1	0,000
11	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1

The proximity values calculated by the t-norm of Lukasiewicz are only 0s and 1s. It gives the lowest values of proximities.

Table 8. Proximity of phrases using OWA 85%

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	1	0,579	0,663	0,592	0,761	1	1	0,697	0,704	0,479	0,479
2	0,579	1	0,653	0,592	0,592	0,579	0,579	0,198	0,344	0,606	0,542
3	0,663	0,653	1	0,653	0,592	0,663	0,663	0,282	0,428	0,344	0,606
4	0,592	0,592	0,653	1	0,582	0,592	0,592	0,358	0,504	0,428	0,344
5	0,761	0,592	0,592	0,582	1	0,761	0,761	0,521	0,526	0,504	0,428
6	1	0,579	0,663	0,592	0,761	1	1	0,697	0,704	0,479	0,479
7	1	0,579	0,663	0,592	0,761	1	1	0,697	0,704	0,479	0,479
8	0,697	0,198	0,282	0,358	0,521	0,697	0,697	1	0,620	0,380	0,311
9	0,704	0,344	0,428	0,504	0,526	0,704	0,704	0,620	1	0,620	0,245
10	0,479	0,606	0,344	0,428	0,504	0,479	0,479	0,380	0,620	1	0,620
11	0,479	0,542	0,606	0,344	0,428	0,479	0,479	0,311	0,245	0,620	1

The proximity values obtained by applying the OWA 85% are in the range of 0.198 to 1, with an average of 0.608. It eliminates the first level of the lowest values.

Table 9. Proximity of phrases using OWA 71%

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	1	0,752	0,676	0,679	0,824	1	1	0,761	0,739	0,526	0,504
2	0,752	1	0,803	0,679	0,676	0,752	0,752	0,568	0,662	0,618	0,751
3	0,676	0,803	1	0,840	0,679	0,676	0,676	0,579	0,568	0,803	0,781
4	0,679	0,679	0,840	1	0,903	0,679	0,679	0,663	0,579	0,568	0,741
5	0,824	0,676	0,679	0,903	1	0,824	0,824	0,592	0,663	0,579	0,568
6	1	0,752	0,676	0,679	0,824	1	1	0,761	0,739	0,526	0,504
7	1	0,752	0,676	0,679	0,824	1	1	0,761	0,739	0,526	0,504
8	0,761	0,568	0,579	0,663	0,592	0,761	0,761	1	0,761	0,445	0,358
9	0,739	0,662	0,568	0,579	0,663	0,739	0,739	0,761	1	0,761	0,380
10	0,526	0,618	0,803	0,568	0,579	0,526	0,526	0,445	0,761	1	0,761
11	0,504	0,751	0,781	0,741	0,568	0,504	0,504	0,358	0,380	0,761	1

The proximity values using the OWA operator at 71% gives values between 0.36 and 1 with an average value of 0.71, OWA 71%. This operator eliminates the first and second levels of the lowest values.

Table 10. Proximity of phrases using OWA 57%

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	1	0,773	0,689	0,739	0,916	1	1	0,916	0,766	0,663	0,579
2	0,773	1	0,903	0,689	0,689	0,773	0,773	0,774	0,803	0,781	0,752
3	0,689	0,903	1	0,903	0,739	0,689	0,689	0,752	0,774	0,814	0,817
4	0,739	0,689	0,903	1	0,916	0,739	0,739	0,676	0,699	0,655	0,788
5	0,916	0,689	0,739	0,916	1	0,916	0,916	0,739	0,676	0,719	0,719
6	1	0,773	0,689	0,739	0,916	1	1	0,916	0,766	0,663	0,579
7	1	0,773	0,689	0,739	0,916	1	1	0,916	0,766	0,663	0,579
8	0,916	0,774	0,752	0,676	0,739	0,916	0,916	1	0,854	0,739	0,479

Table 10. (*continued*)

9	0,766	0,803	0,774	0,699	0,676	0,766	0,766	0,854	1	0,854	0,739
10	0,663	0,781	0,814	0,655	0,719	0,663	0,663	0,739	0,854	1	0,854
11	0,579	0,752	0,817	0,788	0,719	0,579	0,579	0,479	0,739	0,854	1

The proximity values obtained by OWA 57% are in the range of 0.48 to 1 with an average value of 0.79, OWA 57%. This operator eliminates the first, second, and third levels of the lowest values.

7 A Method to Choose a Representative Phrase

Once all the previous measurements have been calculated, the proposed method tries to find the representative phrases from the information of Tables 5, 6, 7, 8, 9 and 10. By the aggregation of every row in the proximity matrix and by using the arithmetic mean, a fuzzy set: *'proximity with the rest of phrases'* is defined on the set of phrases. Then, the phrase or phrases with certain membership degree (that exceed a threshold), are chosen as the most representative phrases. The normalized mean values are presented in Tables 11 and 12 and Figures 3 and 4.

Table 11. Fuzzy set "proximity with other phrases" on the set of phrases using t-norms

Phrase	*Avg Product*		*Avg Min*		*Avg W*	
1	0,301	1,000	0,555	1,000	0,200	1,000
2	0,094	0,312	0,387	0,697	0,000	0,000
3	0,111	0,369	0,439	0,790	0,000	0,000
4	0,093	0,308	0,369	0,665	0,000	0,000
5	0,152	0,504	0,464	0,835	0,000	0,000
6	0,301	1,000	0,555	1,000	0,200	1,000
7	0,301	1,000	0,555	1,000	0,200	1,000
8	0,104	0,345	0,326	0,587	0,000	0,000
9	0,066	0,220	0,291	0,524	0,000	0,000
10	0,041	0,137	0,267	0,480	0,000	0,000
11	0,038	0,125	0,252	0,453	0,000	0,000

Table 12. Fuzzy set "proximity with other phrases" on the set of phrases using OWAs

Phrase	*Avg OWA85%*		*Avg OWA71%*		*Avg OWA57%*	
1	0,695	1,000	0,746	1,000	0,804	1,000
2	0,526	0,757	0,701	0,940	0,771	0,959
3	0,555	0,798	0,708	0,949	0,777	0,966
4	0,524	0,753	0,701	0,939	0,754	0,938
5	0,603	0,867	0,713	0,956	0,794	0,988
6	0,695	1,000	0,746	1,000	0,804	1,000
7	0,695	1,000	0,746	1,000	0,804	1,000
8	0,476	0,685	0,625	0,837	0,776	0,965
9	0,540	0,777	0,659	0,883	0,770	0,957
10	0,494	0,711	0,611	0,819	0,740	0,921
11	0,453	0,652	0,585	0,785	0,688	0,856

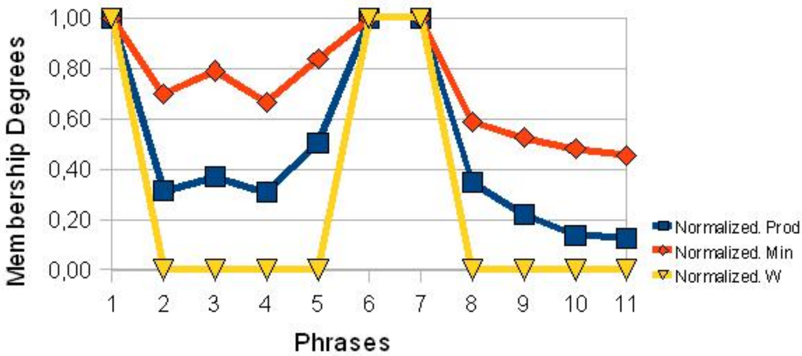

Fig. 3. Fuzzy set "proximity of every phrase i with the rest of phrases", using t-norms connectives

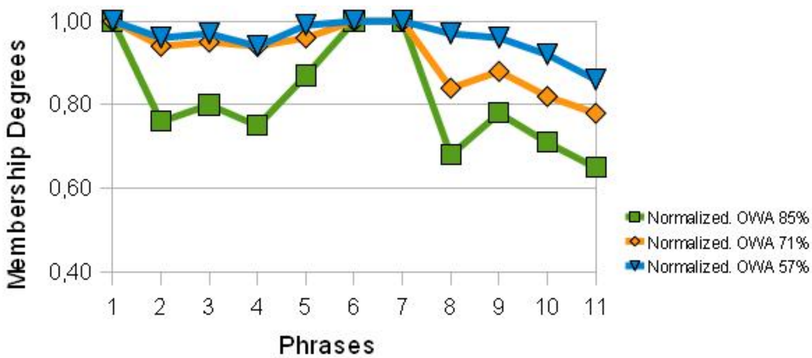

Fig. 4. Fuzzy set "proximity of every phrase i with the rest of phrases", using OWAs aggregations

It is possible to conclude that the representative phrases in this case are 1, 6, and 7, with mean values over 30%, 55% and 20% respectively for each t-norm. These phrases are shown in Figure 5.

It is also possible to identify the second set of representative phrases, 3 and 5, by looking at Tables 11 and 12. They are also representative with values over 40% and 10% for the t-norms Min and Product respectively. A musical representation of phrases 3 and 5 is shown in Figure 6.

The case using OWAs is similar to the t-norms, but in this case, the membership degrees for each phrase have undergone a big increment that is inversely proportional to the percentage of the OWA. This is because the elements with lower values are taken off from the calculations. In the case of OWA 85% the representative phrases are still 1, 6, and 7; phrases 3 and 5 are an additional subset of representative phrases with values over 79%. In the cases of OWA 71% and OWA 57% all of the values are over 78% as they tend to 1, so the important differences have been lost especially for the range of phrases between 1 and 7.

Fig. 5. A musical representation of phrases 1, 6 and 7

Fig. 6. A musical representation of phrases 3 and 5

Phrase 7 is descending an octave and phrase 6 is ascending in 7 semitones (see how they confirm proximity relations in Tables 5, 6, 7). On the other hand, phrases 3 and 5 have a high level of proximity, and in practice, it is easy to see that these phrases contain an important part of representative phrases 1, 6 and 7.

8 Computing the Specificity Measure of the Fuzzy Set "Similar to Other Phrases" and the Inference Independent Sets Using the Proximity on Phrases

After choosing the representative phrases, following the presented procedure, the next step consists in computing the specificity measure of every one of the fuzzy sets obtained in section 7 (Table 11 and Table 12). This is a mechanism to evaluate the decision's reliability from the perspective of the data that have been used in the selection of phrases. When the specificity is one, there is just one representative phrase to choose.

Figure 7 shows all the fuzzy sets obtained in section 7. Every fuzzy set is obtained by the aggregation of the proximity values of each one of the t-norm tables (Tables 5 to 7) and each one of the OWA tables (Tables 8 to 10).

Table 13 shows the 11 values for every one of the 6 fuzzy sets and their calculated values of specificity (last column), using the formula of lineal specificity (21).

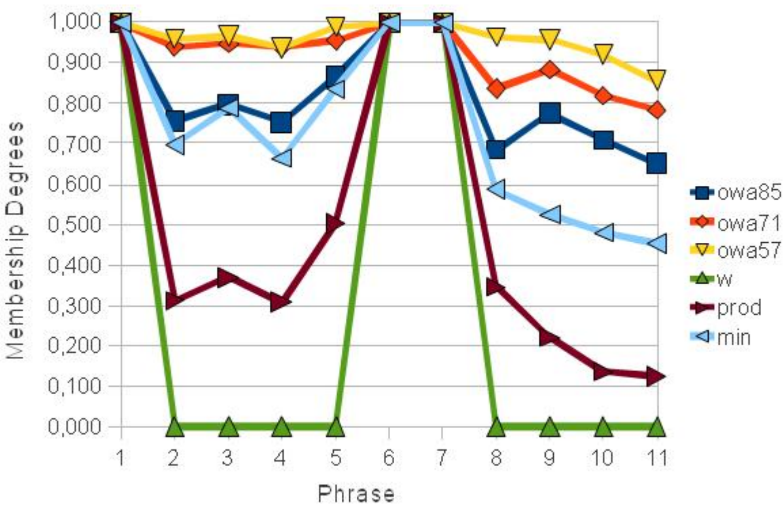

Fig. 7. Fuzzy set of values of proximity of phrases, OWA and t-norm cases

Table 13. Normalized membership degree "proximity with other phrases" on the set of phrases, OWA and t-norm cases

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*Sp*
owa85	1,00	0,75	0,79	0,75	0,86	1,00	1,00	0,68	0,77	0,71	0,65	0,2
owa71	1,00	0,94	0,94	0,93	0,95	1,00	1,00	0,83	0,88	0,81	0,78	0,08
owa57	1,00	0,95	0,96	0,93	0,98	1,00	1,00	0,96	0,95	0,92	0,85	0,04
w	1,00	0,00	0,00	0,00	0,00	1,00	1,00	0,00	0,00	0,00	0,00	0,8
prod	1,00	0,31	0,36	0,30	0,50	1,00	1,00	0,34	0,22	0,13	0,12	0,56
min	1,00	0,69	0,79	0,66	0,83	1,00	1,00	0,58	0,52	0,48	0,45	0,29

The inference independent sets [6] aims to gather in one class all the similar phrases, so the decision is easier, as we now choose between a few cases representing some similar phrases. Those sets of phrases are calculated for every one of the initial fuzzy sets that are shown in Table 13. For each case, one of the proximity matrixes of Table 5 to 10 is used. The result is a table of 6 new fuzzy sets obtained from the original fuzzy sets that in some cases have changes in their values.

Table 14 is the result of computing the proximity values from Table 8 with the fuzzy sets in Table 13. This is how the inference independent sets for the OWA 85% case are obtained. A graphical view of the results is shown in Figure 8. These results should be compared with Figure 7. In this specific case, all the values begin with at least 25%. This is a special case where the fuzzy sets obtained by product t-norm and Lukasiewicz t-norm have the same membership degrees.

Table 14. New membership degree “proximity with other phrases” on the set of phrases for OWA 85%

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11Sp*	
owa85	1,00	0,75	0,79	0,75	0,86	1,00	1,00	0,69	0,77	0,71	0,65	0,19
owa71	1,00	0,94	0,94	0,93	0,95	1,00	1,00	0,83	0,88	0,81	0,78	0,08
owa57	1,00	0,95	0,96	0,93	0,98	1,00	1,00	0,96	0,95	0,92	0,85	0,04
w	1,00	0,57	0,66	0,59	0,76	1,00	1,00	0,69	0,70	0,47	0,47	0,30
prod	1,00	0,57	0,66	0,59	0,76	1,00	1,00	0,69	0,70	0,47	0,47	0,30
min	1,00	0,69	0,79	0,66	0,83	1,00	1,00	0,69	0,70	0,48	0,47	0,26

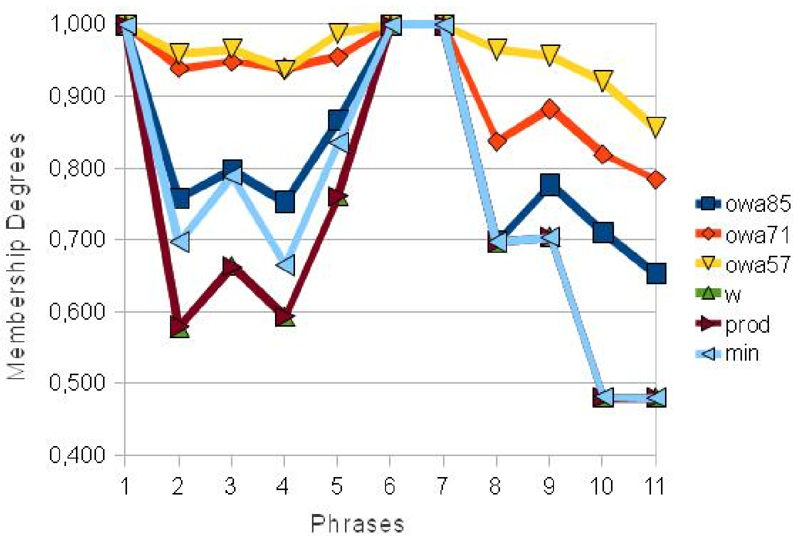

Fig. 8. New fuzzy sets of values of proximity under OWA 85% proximity

Table 15 and Figure 9 show the results of computing the proximity values from Table 9 with the fuzzy sets in Table 13 for the OWA 71% operator. In this case, all values are over 50%. Again, the fuzzy sets of product t-norm and Lukasiewicz t-norm have the same membership degrees.

Table 15. New membership degree “proximity with other phrases” on the set of phrases under OWA 71% proximity

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*Sp*
owa85	1,00	0,75	0,79	0,77	0,86	1,00	1,00	0,76	0,77	0,71	0,65	0,19
owa71	1,00	0,94	0,94	0,93	0,95	1,00	1,00	0,83	0,88	0,81	0,78	0,08
owa57	1,00	0,95	0,96	0,93	0,98	1,00	1,00	0,96	0,95	0,92	0,85	0,04
w	1,00	0,75	0,67	0,67	0,82	1,00	1,00	0,76	0,73	0,52	0,50	0,25
prod	1,00	0,75	0,67	0,67	0,82	1,00	1,00	0,76	0,73	0,52	0,50	0,25
min	1,00	0,75	0,79	0,73	0,83	1,00	1,00	0,76	0,73	0,59	0,57	0,22

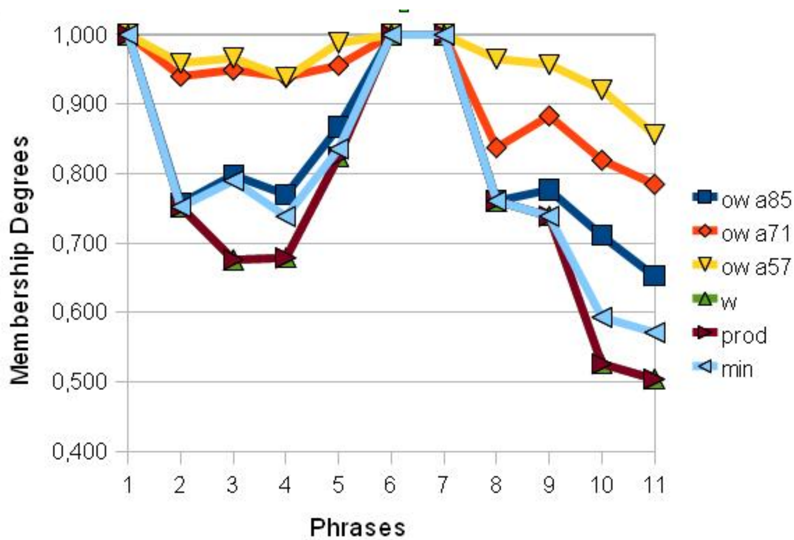

Fig. 9. New fuzzy sets of values of proximity under OWA 71% proximity

Table 16 and Figure 10 show the result of computing the proximity values from Table 10 with the fuzzy sets in Table 13 for the OWA 57% case. All the values are over 57%; again they are special cases where the fuzzy sets of product t-norm and Lukasiewicz t-norm have the same membership degrees.

Table 16. New membership degree "proximity with other phrases" on the set of phrases under OWA 57% proximity

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*Sp*
owa85	1,00	0,77	0,79	0,78	0,91	1,00	1,00	0,91	0,77	0,71	0,65	0,16
owa71	1,00	0,94	0,94	0,93	0,95	1,00	1,00	0,91	0,88	0,81	0,78	0,08
owa57	1,00	0,95	0,96	0,93	0,98	1,00	1,00	0,96	0,95	0,92	0,85	0,04
W	1,00	0,77	0,68	0,73	0,91	1,00	1,00	0,91	0,76	0,66	0,57	0,19
prod	1,00	0,77	0,68	0,73	0,91	1,00	1,00	0,91	0,76	0,66	0,57	0,19
min	1,00	0,77	0,79	0,75	0,91	1,00	1,00	0,91	0,76	0,66	0,60	0,18

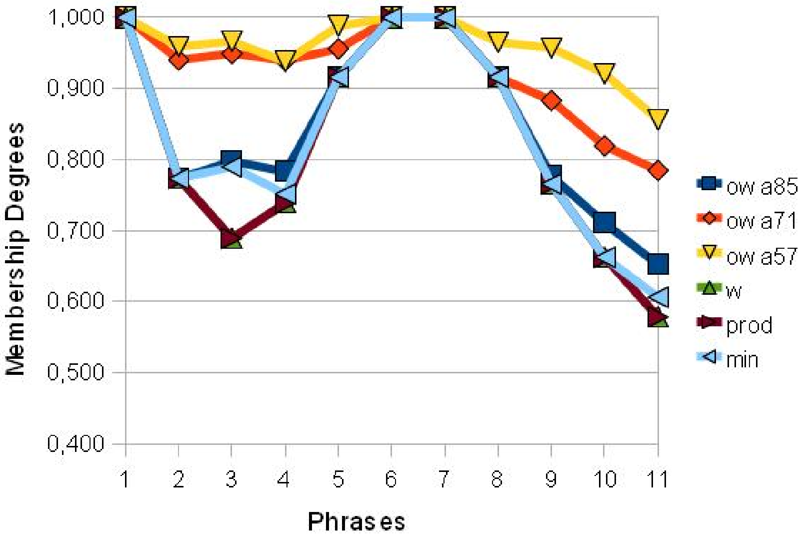

Fig. 10. New fuzzy sets of values of proximity under OWA 57% proximity

Table 17 and Figure 11 show the result of computing the proximity values from Table 7 with the fuzzy sets in Table 13 for the Lukasiewicz t-norm case. The values are still the same, distributed between 0 and 1.

Table 17. New membership degree "proximity with other phrases" on the set of phrases under Lukasiewicz t-norm % proximity

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*Sp*
owa85	1,00	0,75	0,79	0,75	0,86	1,00	1,00	0,68	0,77	0,71	0,65	0,20
owa71	1,00	0,94	0,94	0,93	0,95	1,00	1,00	0,83	0,88	0,81	0,78	0,08
owa57	1,00	0,95	0,96	0,93	0,98	1,00	1,00	0,96	0,95	0,92	0,85	0,04
W	1,00	0,00	0,00	0,00	0,00	1,00	1,00	0,00	0,00	0,00	0,00	0,80
prod	1,00	0,31	0,36	0,30	0,50	1,00	1,00	0,34	0,22	0,13	0,12	0,56
min	1,00	0,69	0,79	0,66	0,83	1,00	1,00	0,58	0,52	0,48	0,45	0,29

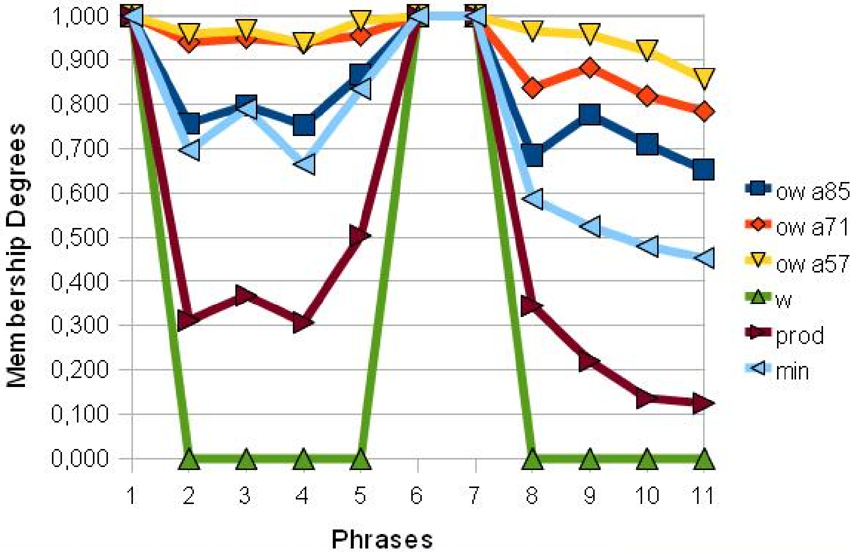

Fig. 11. New fuzzy sets of values of proximity under Lukasiewicz t-norm proximity

Table 18 shows the results of computing the proximity values from Table 6 with the fuzzy sets in Table 13 for the product t-norm proximity. The fuzzy sets are shown in Figure 12. The values are distributed between 0 and 1. Note that in this case the only fuzzy set that changes is the Lukasiewicz t-norm.

Table 18. New membership degree “proximity with other phrases” on the set of phrases under product t-norm % proximity

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*Sp*
owa85	1,00	0,75	0,79	0,75	0,86	1,00	1,00	0,68	0,77	0,71	0,65	0,20
owa71	1,00	0,94	0,94	0,93	0,95	1,00	1,00	0,83	0,88	0,81	0,78	0,08
owa57	1,00	0,95	0,96	0,93	0,98	1,00	1,00	0,96	0,95	0,92	0,85	0,04
W	1,00	0,09	0,11	0,08	0,29	1,00	1,00	0,26	0,10	0,03	0,01	0,69
prod	1,00	0,31	0,36	0,30	0,50	1,00	1,00	0,34	0,22	0,13	0,12	0,56
Min	1,00	0,69	0,79	0,66	0,83	1,00	1,00	0,58	0,52	0,48	0,45	0,29

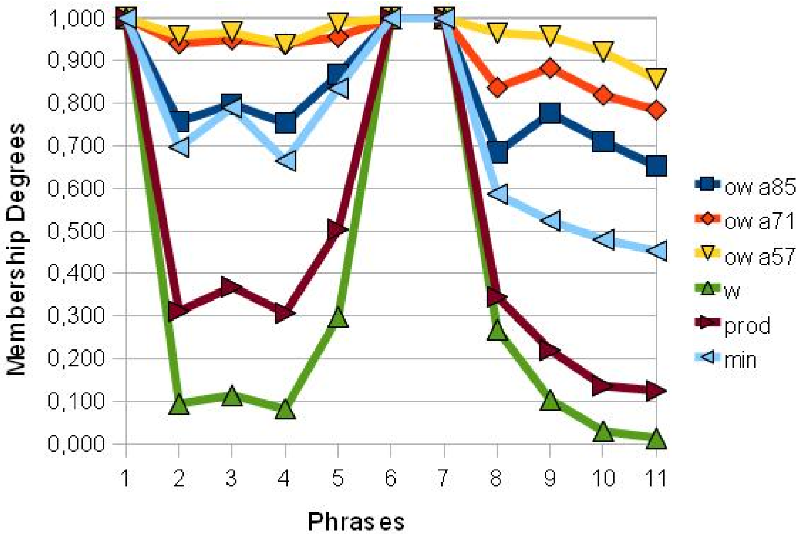

Fig. 12. New fuzzy sets of values of proximity under Product t-norm proximity

Table 19 and Figure 13 summarize the proximity values obtained from Table 5 with the fuzzy sets in Table 13 for the minimum t-norm proximity. The values in this case are distributed between 29% and 100%. Again, the fuzzy sets of product t-norm and Lukasiewicz t-norm have same membership degrees.

Table 19. New membership degree "proximity with other phrases" on the set of phrases under min t-norm % proximity

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*Sp*
owa85	1,00	0,75	0,79	0,75	0,86	1,00	1,00	0,68	0,77	0,71	0,65	0,20
owa71	1,00	0,94	0,94	0,93	0,95	1,00	1,00	0,83	0,88	0,81	0,78	0,08
owa57	1,00	0,95	0,96	0,93	0,98	1,00	1,00	0,96	0,95	0,92	0,85	0,04
W	1,00	0,36	0,56	0,40	0,58	1,00	1,00	0,62	0,38	0,35	0,28	0,44
prod	1,00	0,36	0,56	0,40	0,58	1,00	1,00	0,62	0,38	0,35	0,28	0,44
Min	1,00	0,69	0,79	0,66	0,83	1,00	1,00	0,62	0,52	0,48	0,45	0,29

Additional information is the number of times that every of the membership degrees has increased during each one of the inference independent sets calculation. It allows us to identify the most susceptible elements to be inferred from the proximity

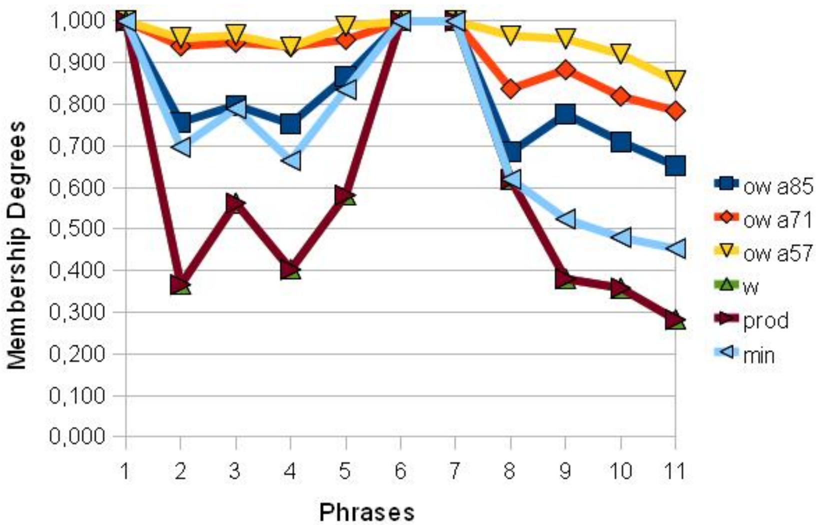

Fig. 13. New fuzzy sets of values of proximity under Minimum t-norm proximity

relations and other elements in the fuzzy set. Those elements are considered susceptible to be inferred because their increments are part of an inference independent set.

The phrase with the highest number of variations is phrase 8 with a total of 17 variations, followed by the set of phrases 2, 9 and 11 with 12 variations. The set composed by phrases 5 and 10 has 11 variations, and phrase 3 with just 9 variations. All of these are listed in Table 20.

Figure 14 is a graphical representation of the number of variations of every phrase for every case of inference independent set calculation process and an accumulate total of variations per phrase.

Table 20. Number of variations in the calculation of inference independent sets

Num of Variations	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
owa85	0	2	2	2	2	0	0	4	3	2	3
owa71	0	3	2	4	2	0	0	4	3	3	3
owa57	0	4	2	4	4	0	0	5	3	3	3
w	0	0	0	0	0	0	0	0	0	0	0
pro	0	1	1	1	1	0	0	1	1	1	1
min	0	2	2	2	2	0	0	3	2	2	2
Total	0	12	9	13	11	0	0	17	12	11	12

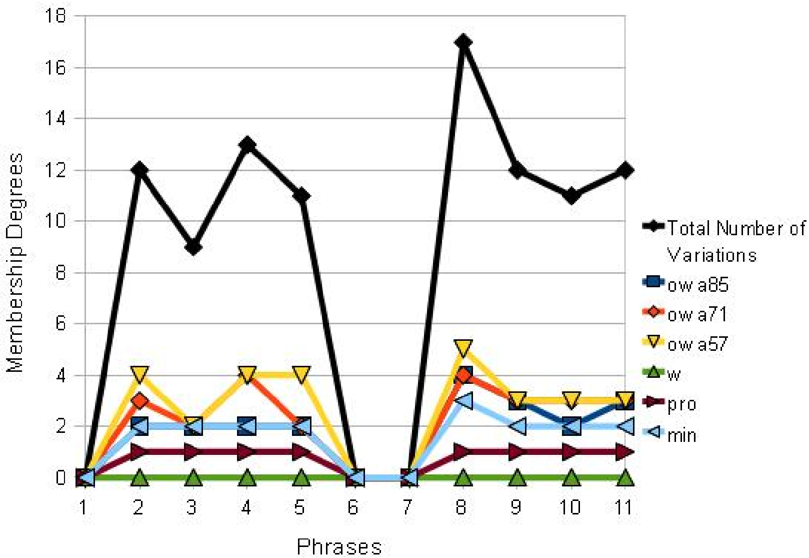

Fig. 14. Variations in membership degrees during inference independent sets calculation

9 Conclusions and Remarks

A method to search musical representative phrases using a W-indistinguishability operator and fuzzy proximity relations on a set of phrases is proposed and illustrated.

An algorithm for searching musical motifs is followed step by step. A musical score is separated in phrases, six cases of proximity relations are calculated, six fuzzy set of phrases candidates to be a motif are computed by aggregating the proximities with different operators. Finally, the results are evaluated by the calculation of the independent inference sets to choose among classes of similar phrases, instead of choosing single phrases, by the determination of the specificity measurements under the knowledge of the proximities. Three of the proximities are computed using t-norms and the other three were computing using different OWA operators.

On the other hand, Yager's specificity measure of fuzzy sets is considered in order to evaluate the reliability in the decisions of selecting the representative phrases. Different fuzzy logic operators were applied to compute each one of the proximities, and the determination of the representative phrases process was successfully carried out.

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