Numerical Study of Shock Induced Mixing in a Cylindrical Shell

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1 Introduction

The Richtmyer-Meshkov instability (RMI) develops when a shock wave traverses a density interface separating two gases. The miss-alignment of the pressure gradient across the shock and the local density gradient at the contact during shock passage leads to vorticity production at the interface. Subsequently the flow driven by the deposited vorticity leads to interfacial instability growth and eventually to turbulence mixing. RMI is important in many areas of physics, from geophysical and astrophysical problems to industrial applications. In particular, attention has recently focused on RMI and RM mixing in the converging geometry such as that occurs in an imploding inertial confinement fusion (ICF) capsule. When a stratified cylindrical shell with initial perturbations is driven by a convergent shock wave, the effect of convergence tends to enhance the perturbation growth compared with that in a planar geometry. The convergent incident shock wave reflects at the cylinder center and the succedent reflected shock waves move to and fro within the whole region. Besides, the Rayleigh-Taylor instability (RTI) also occurs whenever the light fluid accelerates the heavy one during the evolution. All these factors make the mixing procedure in a stratified cylindrical shell driven by shock wave much complex than that in the planar geometry. Although many models have been proposed to predict the instability growth in the linear, weakly nonlinear, and turbulent regimes, each of these models has limitations and a restricted domain of applicability. For this complex mixing process with strongly nonlinear transition stage, the direct numerical simulation with high resolution is the common way to study its evolution. In this paper a hybrid scheme combined with the finite-difference and the weighted essentially non-oscillatory (WENO) method, combined with high order strong-stability preserving Runge-Kutta scheme for the time integration, is used to simulate the mixing due to the interfacial instability in a stratified cylindrical shell driven by convergent shock wave. Growth and mixing properties of the turbulent mixing zone

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(TMZ) have been investigated using the simulation results. And the effect of initial perturbation on the mixing has been discussed.

2 Equations of Motion

We use the N-species Navier-Stokes equations to simulate the motion as follows

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial (\rho u_i u_j + p\delta_{ij})}{\partial x_j} = \frac{\partial d_{ij}}{\partial x_j} \tag{2}$$

$$\frac{\partial E}{\partial t} + \frac{\partial (E+p)u_j}{\partial x_j} = \frac{\partial d_{ij} u_i}{\partial x_j} \tag{3}$$

$$\frac{\partial \rho \varphi_m}{\partial t} + \frac{\partial \rho \varphi_m}{\partial x_j} = 0, \text{ for } m = 1, N-1 \tag{4}$$

where repeated indices denote summation and φ_m denotes the m-th species mass fraction. Pressure is determined from the ideal equation of state for a mixture of gases,

$$P = \frac{\rho R T}{\bar{m}} \tag{5}$$

where R denotes the ideal gas constant, while $\bar{m}$ denotes the mean molecular weight which is given by

$$\frac{1}{\bar{m}} = \sum_{i=1}^{N} \frac{\varphi_i}{m_i} \tag{6}$$

m_i is the molecular weight of the i-th species of the mixture.

The Newtonian stress tensor d_{ij} of the mixture is expressed as

$$d_{ij} = \mu \left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right] \tag{7}$$

where the shear viscosity μ is calculated as follows

$$\mu = \frac{\sum_{i=1}^{N} \mu_i \varphi_i m_i^{-1/2}}{\sum_{i=1}^{N} \mu_i m_i^{-1/2}} \tag{8}$$

3 Numerical Method

For the turbulent mixing we examined in this paper, simulation demands two mutually exclusive numerical approaches. On one hand, the presence of shocks, whose length scale is of the order of the mean free path, implies that the numerical method must be of a shock-capturing type. On the other hand, turbulence is better simulated when the numerical method is non-dissipative. Since all shock-capturing methods are dissipative, two mutually orthogonal numerical requirements arise. To address this difficulty, we used the TCD-WENO hybrid method proposed by Hill and Pullin[1] to approximate the derivatives in the advection terms, in conjunction with a third-order strong-stability preserving Runge-Kutta scheme for the time integration. The fluxes of the viscous and diffusion transport terms are computed using explicit center-difference operator.

The WENO method is a successful shock-capturing scheme with high order precision. But its up-winding strategy makes it too dissipative for the smooth turbulent regions. In the TCD-WENO hybrid method, the tuned centre-difference scheme with low numerical dissipation and good wave-dispersion properties is adopted in the regions away from shocks and material interfaces, while the WENO scheme[4] based on the characteristic decomposition is used around discontinuities(shocks and material interfaces) .We utilized a discontinuity detection criterion to switch the two different schemes suggested by Ref.[2]. In the tuned centre-difference scheme, the skew-symmetric form is employed to improve the numerical stability. For details the reader is referred to Ref. [2, 3].

4 Numerical Results

The calculation model is shown schematically in figure 1. The shell is driven by a convergent incident shock wave. There are three materials and two material interfaces within the calculation region. The main parameters we used are as follows:

$$\rho_{in} = 1.0 \tag{9}$$
$$\rho_{shell} = 10.0 \tag{10}$$
$$\rho_{out} = 1.0 \tag{11}$$
$$P_0 = 80000.0 \tag{12}$$
$$Ma = 1.5 \tag{13}$$

For the simulations in this paper, broadband random perturbations have been set on the initial outer interface. Figure 2 shows the evolution of the mixing zone in a typical case. During the interval corresponding to fig. 2(1) and fig. 2(3), the shell is compressed by the incident shock wave. RMI occurs and both the shell's and the inner material's densities increased. Then the shell begins to slow down and then rebound when the shock wave reaches the center and reflects outwards, and mixing grows due to both RMI and RTI. We can see mixing near both the outer interface and the inner interface in fig. 2(5). If the shell is thin enough or the mixing has undergone enough time, the shell may be shredded.

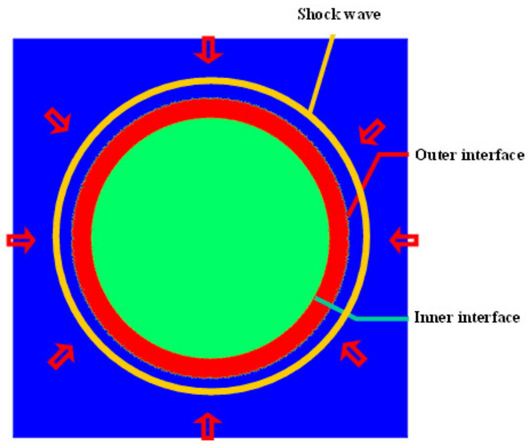

Fig. 1 Schematic showing the calculation model.

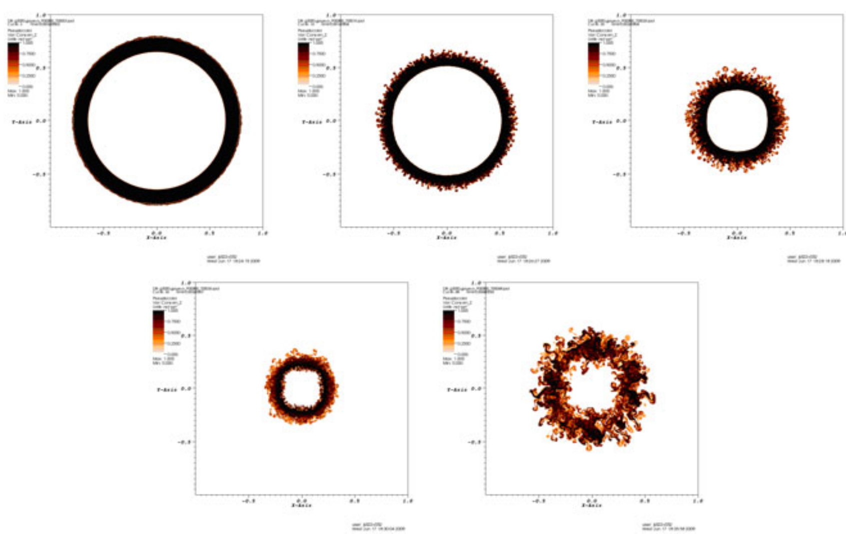

Fig. 2 Evolution of the mixing zone.

In order to investigate the effect of initial perturbation, various initial spectra were considered in our calculations including the flat distribution and the Gauss distribution. Random phases were assigned to each mode in every simulation case. And the averaged perturbation amplitudes of all cases are the same.

case 1: flat spectrum within [0,200]
case 2: flat spectrum within [100 , 300]
case 3: flat spectrum within [200, 400]
case 4: flat spectrum within [300,500]
case 5: flat spectrum within [0,400]
case 6: flat spectrum within [200,600]
case 7: flat spectrum within [400,800]
case 8: Gauss spectrum within [400, 800], half-width: 100
case 9: Gauss spectrum within [400, 800], half-width: 50

Figure 3 shows the mixing zone's width near the outer interface with time. We can see that the mixing zone's growth is quite sensitive to the initial perturbation scale. There is no apparent approach to a self-similar regime independent of the initial conditions. And the mixing zone grows more slowly with smaller scale perturbations.

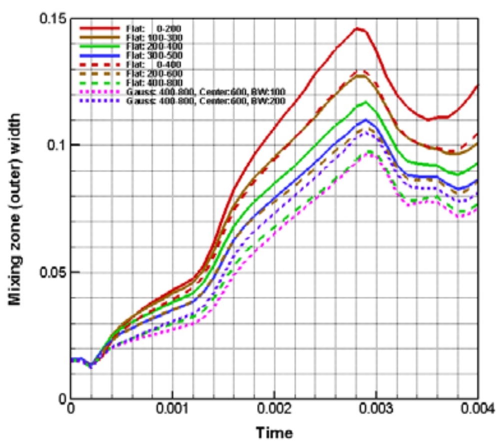

Fig. 3 Width of the mixing zones vs time.

For better understanding of the mixing evolution, we used some statistical quantities of the mixing zone as follows. The reader is referred to Ref. [5]for details. Figure 4 shows the mixing fraction within the mixing zone, which indicates the atomic mixing degree. For this measure, values near unity correspond to complete mixing, while values near zero correspond to little atomic mixing. In fig. 4, the mixing fractions approach an asymptotic value in range [0.6, 0.8] at late time in all cases. It means that as refer to the atomic mixing degree, the imprint of initial perturbation tends to be lost at late time, although it affects the mixing zone's width obviously.

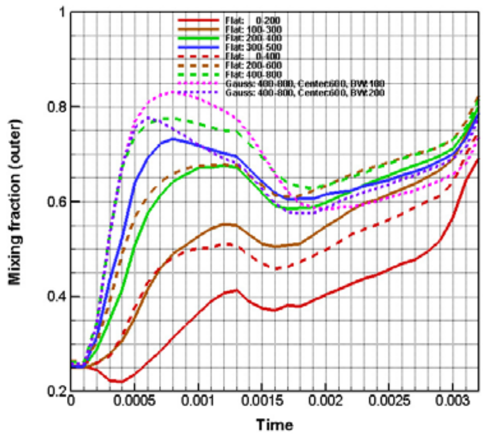

Fig. 4 Mixing fraction within the mixing zone

5 Conclusions

In this paper the mixing process in a stratified cylindrical shell driven by a convergent shock wave is numerically studied by using a hybrid method combined with the weighted essentially non-oscillatory shock-capturing method and the tuned center difference scheme. We investigate the mixing according to the mixing zone width and the mixing fraction. It was found that the mixing zone's growth is quite sensitive to the initial perturbation scale, while the atomic mixing degree tends to be independent of the initial perturbation at late time.

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