Hesitant Fuzzy Linguistic Term Sets

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Abstract. Dealing with vague or imprecise information has been always a challenging problem. Different tools have been proposed to manage that uncertainty. A new model based on hesitant fuzzy sets was presented to manage situations where experts hesitate among several values to assess alternatives, variables, etc. Hesitant fuzzy sets models quantitative settings, however, it could occur similar situations but in qualitative settings, where experts think of several possible linguistic values or richer expressions than a single linguistic term to assess alternatives, variables, etc. In this contribution the aim is to introduce the concept of *Hesitant Fuzzy Linguistic Term Sets* (HFLTS) that will provide a linguistic elicitation based on the fuzzy linguistic approach and the use of context-free grammars.

Keywords: Hesitant fuzzy sets, fuzzy linguistic approach, context-free grammar, linguistic information.

1 Introduction

Problems defined in context with uncertainty are quite common in real world, but very challenging due to the difficulty to model and deal with such an uncertainty. Different tools have been proposed to solve those problems, however, there are situations where the uncertainty is not probabilistic in nature, but rather imprecise or vague. Other models as fuzzy logic and fuzzy sets theory [28] have been successfully applied to manage vague and imprecise information [18]. The modeling tools of ordinary fuzzy sets are limited when two or more sources of vagueness appear simultaneously. For this reason, different generalizations and extensions of fuzzy sets have been introduced:

- *Type 2 fuzzy sets* [5,17], and *type n fuzzy sets* [5] that include uncertainty about the membership function in their definition.
- *Intuitionistic fuzzy sets* [1] that extends fuzzy sets by an additional uncertainty degree.
- *Fuzzy multisets* [26] based on multisets that allow elements repeated in the set.
- *Hesitant fuzzy sets* proposed by Torra [23] that try to manage those situations where a set of values are possible in the definition process of the membership of an element.

The previous fuzzy tools suit problems defined in quantitative context, but sometimes, the uncertainty is due to the vagueness of meaning used by experts in problems whose

Y. Wang and T. Li (Eds.): Foundations of Intelligent Systems, AISC 122, pp. 287–295.
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nature is rather qualitative. In such situations, the fuzzy linguistic approach [29] has provided good results in different fields and applications [11,12,13,19]. However, the use of fuzzy linguistic approach also presents some limitations mainly regarding information modeling and computational processes, called processes of *computing with words* (CW) [10,14,16]. Different linguistic models have been proposed to extend and improve the fuzzy linguistic approach:

- *The linguistic model based on type-2 fuzzy sets* representation [15,24] that represents the semantics of the linguistic terms by type-2 membership functions and use interval type-2 fuzzy sets for CW.
- *The linguistic 2-tuple model* [6] that keeps the accuracy in the processes of CW by means of a parameter, so-called *symbolic translation*.
- *The proportional 2-tuple model* [25] that generalizes and extends the 2-tuple model by using two linguistic terms with their proportion to model more accurately the information and perform the processes of CW.
- Other extensions based on the previous ones were presented in [4,9].

Revising the fuzzy linguistic approach, different linguistic extensions and generalizations, it is observed that the modeling of linguistic information is still limited because experts provide their assessments by using single and simple terms over alternatives, variables, etc. However, it might occur that experts are thinking of several linguistic terms at the same time or looking for a more complex linguistic term that are not defined in the linguistic term set.

Therefore, to overcome such limitations and taking into account the concept of hesitant fuzzy sets provided by Torra [23] to deal with several values in a membership function in a quantitative setting, in this paper we present the concept of HFLTS based on the fuzzy linguistic approach that will serve as basis to increase the flexibility of the elicitation of linguistic information. Additionally, different operations and properties of HFLTS are introduced. Afterwards, it is presented their use to improve the elicitation of linguistic information by using the fuzzy linguistic approach and context-free grammars.

The paper is organised as follows: Section 2 reviews some basic concepts necessary to understand easily our proposal. Section 3 defines the concept of HFLTS and different properties and operations to carry out processes of CW. Section 4 presents the use of HFLTS to facilitate and increase the flexibility to elicit the linguistic information. Finally, Section 5 points out some concluding remarks and future works.

2 Preliminaries

In this section, we review briefly some concepts of the fuzzy linguistic approach [29] and hesitant fuzzy sets [23] to understand the proposal of HFLTS and its use.

2.1 Fuzzy Linguistic Approach

In many real decision situations are suitable the use of linguistic information due to the nature of the problem. In these cases, the fuzzy linguistic approach [29] models the linguistic information by using the fuzzy set theory [28] to manage the uncertainty and model the information.

Zadeh [29] introduced the concept of linguistic variable as *a variable whose values are not numbers but words or sentences in a natural or artificial language*. A linguistic value is less precise than a number, but it is closer to the natural language used by human beings.

To deal with linguistic variables, it is necessary to choose the appropriate linguistic descriptors for the linguistic term sets and their semantics. To do so, there are different possibilities [27,29]. The choice of the linguistic descriptors can be carried out as follows:

- Supplying directly the term set by considering all the terms symmetrically distributed on a scale which has an order defined [27]. In these cases, it is usually necessary that exist the following operators: (i) Negation: $\text{Neg}(s_i) = s_j$ such that $j = g - i$ ($g+1$ is the cardinality), (ii) Maximization: $\max(s_i, s_j) = s_i \; if \; s_i \geq s_j$, (iii) Minimization: $\min(s_i, s_j) = s_i \; if \; s_i \leq s_j$.
- Defining the linguistic term set by means of a context-free grammar, G, such that the linguistic terms are sentences generated by G [2,3,29]. A grammar G is a 4-tuple (V_N, V_T, I, P), where V_N is the set of non-terminal symbols, V_T is the set of terminals symbols, I is the starting symbol, and P the production rules defined in an extended Backus Naur Form [3]. Among the terminal symbols of G, we can find primary terms (e.g., low, medium), hedges (e.g., not, very), relations (e.g., lower than, higher than), conjunctions (e.g., and, but), and disjunctions (e.g., or).

And the definition of their semantics can be accomplished as [27,29]:

- A semantics based on membership functions and a semantic rule. It assumes that the meaning of each linguistic term is given by means of a fuzzy subset defined in the interval [0,1], which is described by membership functions [3]. This semantic approach is used when the linguistic descriptors are generated by means of a context-free grammar.
- A semantics based on an ordered structure of the linguistic term set that introduces the semantics from the structure defined over the linguistic term set. So, the users use an ordered linguistic term set to provide their assessments [22,27].
- Mixed semantics that uses elements from the previous approaches.

2.2 Hesitant Fuzzy Sets

Torra presented in [23] the definition of hesitant fuzzy sets to fulfil the management of decision situations in quantitative contexts where the decision makers hesitate among different possible values to assess an alternative or criterion.

A hesitant fuzzy set is defined in terms of a function that returns a set of membership values for each element in the domain [23]:

Definition 1. *[23] Let X be a reference set, a hesitant fuzzy set on X is a function h that returns a subset of values in [0,1].*

$$h : X \to \{[0,1]\}$$

A hesitant fuzzy set can be also defined in terms of the union of their membership degree to a set of fuzzy sets.

Definition 2. *[23] Let $M = \{\mu_1, \mu_2, ..., \mu_n\}$ be a set of n membership functions. The hesitant fuzzy set associated with M, h_M, is defined as:*

$$h_M : M \rightarrow \{[0,1]\}$$

$$h_M(x) = \bigcup_{\mu \in M} \{\mu(x)\}$$

Some basic operations with hesitant fuzzy sets, such as, union, intersection, complement and so on were defined in [23].

3 Hesitant Fuzzy Linguistic Term Sets

In qualitative contexts might occur that experts hesitate among several linguistic values. As it was pointed out in the introduction several proposals have been proposed in the literature [6,25]. However, all of them are still limited and are not adequate to fulfil the necessities and requirements of experts in hesitant situations.

In this section is introduced the concept of HFLTS based on the fuzzy linguistic approach and hesitant fuzzy sets. Additionally, some basic operations of HFLTS are defined.

3.1 Concept and Basic Operations

Definition 3. *Let S be a linguistic term set, $S = \{s_0, \ldots, s_g\}$, a HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S.*

Let S be a linguistic term set, $S = \{s_0, \ldots, s_g\}$, we then define the empty and full HFLTS for a linguistic variable, x, as follows:

- Empty HFLTS: $H_S(x) = \{\}$
- Full HFLTS: $H_S(x) = S$

Any other HFLTS is formed at least with one linguistic term in S.

Example 1. *Let S be a linguistic term set, $S = \{s_0 : nothing, s_1 : very\_low, s_2 : low, s_3 : medium, s_4 : high, s_5 : very\_high, s_6 : perfect\}$, different HFLTS might be:*

$$H_S(x) = \{very\_low, low, medium\}$$

$$H_S(x) = \{high, very\_high, perfect\}$$

Once defined the concept of HFLTS, it is necessary to introduce operations and computations that can be performed on them.

Let S be a linguistic term set, $S = \{s_0, \dots, s_g\}$ and H_S, H_S^1, and H_S^2 three HFLTS:

Definition 4. *The upper bound, H_{S^+}, and lower bound, H_{S^-}, of the HFLTS, H_S, are defined as:*

- $H_{S^+} = max(s_i, s_j) = s_i$, *if* $s_i \geq s_j$; $s_i, s_j \in H_S$
- $H_{S^-} = min(s_i, s_j) = s_i$, *if* $s_i \leq s_j$; $s_i, s_j \in H_S$

Definition 5. *The complement of HFLTS, H_S, is defined as:*

$$H_S^c = S - H_S = \{s_i / s_i \in S \text{ and } s_i \notin H_S\}$$

Proposition 1. *The complement of a HFLTS is involutive:*

$$(H_S^c)^c = H_S$$

Definition 6. *The union between two HFLTS, H_S^1 and H_S^2 is defined as:*

$$H_S^1 \cup H_S^2 = \{s_i / s_i \in H_S^1 \text{ or } s_i \in H_S^2\}$$

the result will be another HFLTS.

Definition 7. *The intersection of two HFLTS, H_S^1 and H_S^2 is:*

$$H_S^1 \cap H_S^2 = \{s_i / s_i \in H_S^1 \text{ and } s_i \in H_S^2\}$$

the result of this operation is another HFLTS.

The comparison between linguistic terms is necessary in many problems and has been defined in different approaches. The comparison between HFLTS is not simple, therefore, we introduce the concept of envelope of a HFLTS.

Definition 8. *The envelope of the HFLTS, $env(H_S)$, is a linguistic interval whose limits are obtained by means of upper bound (*max*) and lower bound (*min*), hence:*

$$env(H_S) = [H_{S^-}, H_{S^+}], \quad H_{S^-} <= H_{S^+}$$

Example 2. *Let $S = \{nothing, very\_low, low, medium, high, very\_high, perfect\}$ be a linguistic term set, and $H_S = \{very\_low, low, medium\}$ be a HFLTS of S, its envelope is:*

$$H_{S^-}(very\_low, low, medium) = \boldsymbol{very\_low}, \quad H_{S^+}(very\_low, low, medium) = \boldsymbol{medium}$$

$$env(H_S) = [very\_low, medium]$$

Definition 9. *The definition of the comparison between two HFLTS is based on the concept of envelope of the HFLTS, $env(H_S)$. Hence, the comparison between, H_S^1 and H_S^2 is defined as follows:*

$$H_S^1(x) > H_S^2(x) \;\; iff \;\; env(H_S^1(x)) > env(H_S^2(x))$$
$$H_S^1(x) = H_S^2(x) \;\; iff \;\; env(H_S^1(x)) = env(H_S^2(x))$$

As consequence the comparison is carried out by interval values. In the literature have been introduced different approaches to comparing intervals [7,8,21]. We use the approach presented by Sengupta in [21] to accomplish the comparison of HFLTS, more detail can be found in [20].

3.2 Properties

To conclude this section some important properties of the HFLTS operations are reviewed.

Let H_S^1, H_S^2 and H_S^3 be three HFLTS and $S = \{s_0, \ldots, s_g\}$, then:

- *Commutativity*

$$H_S^1 \cup H_S^2 = H_S^2 \cup H_S^1$$

$$H_S^1 \cap H_S^2 = H_S^2 \cap H_S^1$$

- *Associative*

$$H_S^1 \cup (H_S^2 \cup H_S^3) = (H_S^1 \cup H_S^2) \cup H_S^3$$

$$H_S^1 \cap (H_S^2 \cap H_S^3) = (H_S^1 \cap H_S^2) \cap H_S^3$$

- *Distributive*

$$H_S^1 \cap (H_S^2 \cup H_S^3) = (H_S^1 \cap H_S^2) \cup (H_S^1 \cap H_S^3)$$

$$H_S^1 \cup (H_S^2 \cap H_S^3) = (H_S^1 \cup H_S^2) \cap (H_S^1 \cup H_S^3)$$

Due to the long limitation of the paper, the demonstrations of these properties can be found in [20].

4 Elicitation of Information Linguistic Based on HFLTS

The main objective of the definition of HFLTS is to improve the flexibility of the elicitation of linguistic expressions when experts hesitate among several linguistic values to assess alternatives or criteria.

So far, it has been introduced the concept of HFLTS that can be directly used by the experts to elicit several linguistic values for a linguistic variable, but such elements are not similar to the human beings way of thinking and reasoning. Therefore, in this section it is proposed the definition of linguistic sentences that are more similar to human beings expressions and semantically represented by means of HFLTS and generated by a context-free grammar.

Definition 10. *Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:*

$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$

$V_T = \{lower\ than, greater\ than, between, and, s_0, s_1, \ldots, s_g\}$

$I \in V_N$

The production rules are defined in an extended Backus Naur Form such that the brackets enclose optional elements and the symbol | indicate alternative elements [3]. For the context-free grammar, G_H, the production rules are the following ones:

$P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle$
$\langle composite\ term\rangle ::= \langle unary\ relation\rangle\langle primary\ term\rangle | \langle binary\ relation\rangle$
$\langle primary\ term\rangle\langle conjunction\rangle\langle primary\ term\rangle$
$\langle primary\ term\rangle ::= s_0|s_1|\ldots|s_g$
$\langle unary\ relation\rangle ::= lower\ than|greater\ than$
$\langle binary\ relation\rangle ::= between$
$\langle conjunction\rangle ::= and\}$

Remark 1. *The* unary relation *has some limitations. If the non-terminal symbol is* lower than*, the* primary term *cannot be* s_0 *and if the non-terminal symbol is* greater than *the* primary term *cannot be* s_g*.*

Remark 2. *In the* binary relation *the* primary term *of the left side must be less than the* primary term *of the right side.*

Example 3. *Let* $S = \{nothing, very\_low, low, medium, high, very\_high, perfect\}$ *be a linguistic term set, some linguistic expressions obtained by means of the context-free grammar,* G_H*, might be:*
$ll_1 = very\_low$
$ll_2 = lower\ than\ low$
$ll_3 = greater\ than\ high$
$ll_4 = between\ high\ and\ very\_high$

It was also defined a transformation function, E_{G_H}, to obtain HFLTS from the linguistic expressions, ll, generated by the context-free grammar, G_H.

Definition 11. *Let* E_{G_H} *be a function that transforms linguistic expressions, ll, obtained by* G_H*, into HFLTS,* H_S*, where S is the linguistic term set used by* G_H*.*

$$E_{G_H} : ll \longrightarrow H_S$$

The linguistic expressions generated by using the production rules are transformed into HFLTS in different ways as follows:

- $E_{G_H}(s_i) = \{s_i/s_i \in S\}$
- $E_{G_H}(\text{less than } s_i) = \{s_j/s_j \in S \text{ and } s_j \leq s_i\}$
- $E_{G_H}(\text{greater than } s_i) = \{s_j/s_j \in S \text{ and } s_j \geq s_i\}$
- $E_{G_H}(\text{between } s_i \text{ and } s_j) = \{s_k/s_k \in S \text{ and } s_k \geq s_i \text{ and } s_k \leq s_j\}$

5 Conclusions and Future Works

This contribution has introduced the concept of HFLTS to increase the flexibility and richness of linguistic elicitation based on the fuzzy linguistic approach and the use of context-free grammars to support the elicitation of linguistic information by experts in hesitant situations in qualitative contexts. Additionally, different operations and properties of HFLTS have been presented.

In the future, it will be studied the application of HFLTS to decision making processes defined under uncertainty where experts will be able to provide their assessments by using linguistic expressions based on HFLTS similar to the expressions used by human beings.

Acknowledgements. This work is partially supported by the Research Project TIN-2009-08286, P08-TIC-3548 and FEDER funds.

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