Existence and Simulations of Periodic Solution of a Predator-Prey System with Holling-Type Response and Impulsive Effects

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Abstract. The principle aim of this paper is to explore the existence of periodic solution of a predator-prey model with functional response and impulsive perturbations. Sufficient and realistic conditions are obtained by using Mawhin's continuation theorem of the coincidence degree. Further, some numerical simulations show that our model can occur in many forms of complexities including periodic oscillation and chaotic strange attractor.

Keywords: Periodic Solution, Predator-prey system, Impulses, Coincidence degree theory.

1 Introduction

In this paper, we will consider the following T-periodic Holling-type functional response predator-prey system [1,2,3,4] with diffusion and impulsive effects:

$$\begin{cases} \left.\begin{aligned} \dot{x}_1(t) &= x_1(t)(b_1(t) - d_1(t)x_1(t)) - a_1(t)y(t)\tfrac{\alpha(t)x_1(t)}{N(t)+x_1(t)} \\ &\quad + D_1(t)(x_2(t) - x_1(t)), \\ \dot{x}_2(t) &= x_2(t)(b_2(t) - d_2(t)x_2(t)) + D_2(t)(x_1(t) - x_2(t)), \\ \dot{y}(t) &= y(t)(b_3(t) - d_3(t)y(t)) + a_2(t)y(t)\tfrac{\alpha(t)x_1(t)}{N(t)+x_1(t)}, \end{aligned}\right\} t \neq t_n, \\ \left.\begin{aligned} x_1(t_n^+) &= (1 + h_{1n})x_1(t_n), \\ x_2(t_n^+) &= (1 + h_{2n})x_2(t_n), \\ y(t_n^+) &= (1 + g_n)y(t_n), \end{aligned}\right\} t = t_n, n \in Z^+. \end{cases} \tag{1}$$

where $x_1(t)$ and $y(t)$ are the densities of prey species and predator species in patch I at time t, $x_2(t)$ is the density of prey species in patch II, prey species $x_1(t)$, $x_2(t)$ can diffuse between two patches while the predator species $y(t)$ is confined to patch I. $a_1(t)$ is the maximum of prey that can be eaten by a predator per unit of time, $a_2(t)$ a conversion efficiency, $b_i(t)\,(i = 1, 2, 3)$ intrinsic growth rate, $d_i(t)\,(i = 1, 2, 3)$ the rate of intra-specific competition, $D_i(t)\,(i = 1, 2)$ the dispersal rate of prey species, and h_{in} and g_n represent the annual birth pulse of population $x_i(t)$, $y(t)$ at t_n $(i = 1, 2)$, $n \in Z^+$. In this paper, we will assume that the following conditions are fulfilled:

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(A1) $a_i(t)$, $D_i(t)$, $b_i(t)$, $d_i(t)$ $(i = 1, 2, 3)$ and $\alpha(t)$, $N(t)$ are continuous positive T-periodic functions;
(A2) h_{1n}, h_{2n}, g_n are constants and there exists a positive integer q such that $h_{1(n+q)} = h_{1n}$, $h_{2(n+q)} = h_{2n}$, $g_{n+q} = g_n$, $t_{n+q} = t_n + T$.

With model (1) we can take into account the possible exterior effects under which the population densities change very rapidly. For instance, impulsive reduction of the population density of a given species is possible after its partial destruction by catching, a natural constraint in this case is $1+h_{1n} > 0,\ 1+h_{2n} > 0,\ 1+g_n > 0,\ n \in Z^+$.

2 Notations and Preliminaries

Let $J \subset R$, denote by $PC(J, R)$ the set of functions $\psi : J \to R$, which are piecewise continuous in $[0, T]$, have points of discontinuity $t_n \in [0, T]$, where they are continuous from the left. Let $PC^1(J, R)$ denote the set of functions ψ with derivative $\dot{\psi}(t) \in PC(J, R)$. Throughout this paper we deal with the Banach space of T-periodic functions

$$PC_T = \{\psi \in PC([0, T], R) | \psi(0) = \psi(T)\}$$

with the supremum norm:

$$\|\psi\|_{PC_T} = \sup\{|\psi(t)| : t \in [0, T]\}$$

and

$$PC_T^1 = \left\{\psi \in PC^1([0, T], R) | \psi(0) = \psi(T)\right\}$$

with the supremum norm:

$$\|\psi\|_{PC_T^1} = \max\{\|\psi\|_{PC_T}, \|\dot{\psi}\|_{PC_T^1}\}.$$

we will also consider the product space $PC_T \times PC_T$ which is also a Banach space with the norm

$$\|(\psi_1, \psi_2)\|_{PC} = \|\psi_1\|_{PC} + \|\psi_2\|_{PC}.$$

Moreover, for any $y \in C_T$ or $y \in PC_T$, define average value of y as follows:

$$\overline{y} := \frac{1}{T} \int_0^T y(t)dt$$

and the minimum, maximum of y respectively are:

$$y^L := \min_{t \in [0,T]} y(t), \qquad y^M := \max_{t \in [0,T]} y(t).$$

Give $\alpha, \beta \in PC_T$, $\beta > 0$, we consider the following Logistic equation with impulsive effects.

$$\begin{cases} \dot{\omega}(t) = \alpha(t)\omega(t) - \beta(t)\omega^2(t), & t \neq t_n, n \in Z^+, \\ \omega(t^+) = (1 + c_n)\omega(t_n), & t = t_n, n \in Z^+. \end{cases} \tag{2}$$

where $c_n(n \in Z^+)$ is constant, there exists an integer $q > 0$ such that $c_{n+q} = c_n$, $t_{n+q} = t_n + T$, and assume that $1 + c_n > 0$ $(n \in Z^+)$.

Lemma 1. *System (2) admits a unique positive solution if and only if* $\bar{\alpha} + \frac{1}{T}\sum_{n=1}^{q} \ln(1 + c_n) > 0$.

Let $\theta_{[\alpha,\beta]}$ denote the unique positive periodic solution to (2). Dividing $\dot{\theta}_{[\alpha,\beta]} = \alpha\theta_{[\beta,\beta]} - \beta\theta^2_{[\alpha,\beta]}$ by $\theta_{[\alpha,\beta]}$ and integrating over intervals $(0, T]$, we have

$$\bar{\alpha} + \frac{1}{T}\sum_{n=1}^{q} \ln(1 + c_n) = \frac{1}{T}\int_0^T \beta\theta_{[\alpha,\beta]}dt = \overline{\beta\theta_{[\alpha,\beta]}}.$$

To shorten notation, we rewrite $\theta_\alpha := \theta_{[\alpha,\beta]}$.

We denote by $\Phi_{[a,b]}(t, t_0, \omega_0)$ the unique solution of Cauchy problem

$$\begin{cases} \dot{\omega}(t) = \alpha(t)\omega(t) - \beta(t)\omega^2(t), & t \geq t_0(t \neq t_n), \\ \omega(t_n^+) = (1 + c_n)\omega(t_n), & t = t_n, \\ \omega(t_0^+) = \omega_0. \end{cases} \tag{3}$$

Lemma 2. *Give* $\alpha, \beta \in PC_T$, *with* $\beta > 0$, *for any* $\omega_0 > 0$ *we have*

$$\lim_{t\to\infty} |\Phi_{[a,b]}(t, t_0, \omega_0) - \theta_\alpha| = 0$$

provided that $\bar{\alpha} + \frac{1}{T}\sum_{n=1}^{q} \ln(1 + c_n) > 0$ *and* $1 + c_n > 0$ *for* $n \in Z^+$.

Lemma 3. *Given a positive* $x_0 \in R$, *consider two functions* $a, b \in PC((t_0, \infty), R)$ *with* $b > 0$, *suppose that* $x(t) \in PC_T^1$ *such that*

$$\begin{cases} \dot{x}(t) \geq ax(t) - bx^2(t), & t \geq t_0(t \neq t_n), \\ x(t_n^+) \geq (1 + c_n)x(t_n), & t = t_n, \\ x(t_0^+) \geq x_0. \end{cases} \tag{4}$$

Then $x(t) \geq \Phi_{[a,b]}(t, t_0^+, x_0)$ *for all* $t \geq t_0$. *Similarly* $x(t) \leq \Phi_{[a,b]}(t, t_0^+, x_0)$ *for all* $t \geq t_0$ *if all the sign of inequalities in (4) are converse.*

In order to obtain the existence of positive T-periodic solution to system (1), we must use the following lemma, named as the continuation theorem of coincidence degree theory [5].

Let X, Z be normed vector spaces, $L : \mathrm{Dom}L \subseteq X \to Z$ be a linear mapping, $N : X \to Z$ be a continuous mapping. If $\dim\mathrm{Ker}L = \mathrm{comdim}\mathrm{Im}L < +\infty$ and $\mathrm{Im}L$ is closed in Z, then the mapping L will be called a Fredholm mapping of index zero. If L is a Fredholm mapping of index zero, there exist continuous projects $P : X \to X$ and $Q : Z \to Z$ such that ImP = Ker$L$, ImL = Ker$Q$ = Im$(I - Q)$. It follows that $L|_{\text{Dom}L \bigcap \text{Ker}P} : (I - P)X \to \text{Im}L$ has an inverse which is denoted by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ provided that $QN(\overline{\Omega})$ is bounded and $K_p(I - Q)N : \overline{\Omega} \to X$ is compact. Since ImQ is isomorphic to Ker$L$ there exists an isomorphism $F : \text{Im}Q \to \text{Ker}L$.

Lemma 4. *Let L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$. Suppose that*

(a) For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ such that $x \notin \partial\Omega$;
(b) $QNx \neq 0$ for each $x \in \text{Ker}L \bigcap \partial\Omega$;
(c) deg $\{FQN, \Omega \bigcap \text{Ker}L, 0\} \neq 0$.

Then the equation $Lx = Nx$ has at least one solution lying in $\text{Dom}L \bigcap \overline{\Omega}$.

3 Existence of Positive Periodic Solution

In this section, we study the existence of positive periodic solution to (1).

Theorem 1. *If system (1) satisfies*

1. $b + \frac{1}{T}\sum_{n=1}^{q} \ln(1 + m_n) > 0$, $\bar{b}_3 + \frac{1}{T}\sum_{n=1}^{q} \ln(1 + g_n) > 0$,
2. $b_i(t) > D_i(t)(i = 1, 2)$, $\bar{b}_2 - \bar{D}_2 + \frac{1}{T}\sum_{n=1}^{q} \ln(1 + h_{2n}) > 0$.

here $b = \max\{b_1^M, b_2^M\}$, $m_n = \max\{1 + h_{1n}, 1 + h_{2n}\}$, $d = \min\{d_1^L, d_2^L\}$.
Then system (1) has at least one T-periodic positive solution.

Proof. Let $x_1(t) = e^{u_1(t)}$, $x_2(t) = e^{u_2(t)}$, $y(t) = e^{u_3(t)}$ then system (1) is reformulated as

$$\begin{cases} \left.\begin{array}{l} \dot{u}_1(t) = b_1(t) - D_1(t) - d_1(t)e^{u_1(t)} - \frac{a_1(t)\alpha(t)e^{u_3(t)}}{N(t)+e^{u_1(t)}} + D_1(t)e^{u_2(t)-u_1(t)}, \\ \dot{u}_2(t) = b_2(t) - D_2(t) - d_2(t)e^{u_2(t)} + D_2(t)e^{u_1(t)-u_2(t)}, \\ \dot{u}_3(t) = b_3(t) - d_3(t)e^{u_3(t)} \frac{a_2(t)\alpha(t)e^{u_1(t)}}{N(t)+e^{u_1(t)}}, \end{array}\right\} t \neq t_n, \\ \left.\begin{array}{l} u_1(t_n^+) = u_1(t_n) + \ln(1 + h_{1n}), \\ u_2(t_n^+) = u_2(t_n) + \ln(1 + h_{2n}), \\ u_3(t_n^+) = u_3(t_n) \ln(1 + g_n), \end{array}\right\} t = t_n. \end{cases} \quad (5)$$

If system (5) has a T-periodic solution $(u_1(t), u_2(t), u_3(t))^T$, then

$$(e^{u_1(t)}, e^{u_2(t)}, e^{u_3(t)})^T = (x_1^*(t), x_2^*(t), y^*(t))$$

is a positive T-periodic solution to system (1). So, in the following,we discuss the existence of T-periodic solution to system (5).

In order to use Lemma 2.4, we set $\mathbf{u} = (u_1(t), u_2(t), u_3(t))^T$. Define
$X = \{x \in PC(R, R^3) : x(t + T) = x(t)\}$, $Z = X \times R^{3q}$, then it is standard to show both X and Z are Banach space when they are endowed with the

norms $\|x\|_c = \sup_{t\in[0,\omega]} |x(t)|$ and $\|(x,c_1,c_2,c_3)\| = (\|x\|_c^2 + |c_1|^2 + |c_2|^2 + |c_3|^2)^{1/2}$. Let $\mathrm{Dom}L \subset X = \left\{x \in C^1[0,\omega; t_1,...,t_m] \big| x(0) = x(\omega)\right\}$, $L\colon \mathrm{Dom}L \to Z$, $L\mathbf{u} = (\mathbf{u}', \Delta\mathbf{u}(t_1),...,\Delta\mathbf{u}(t_q))$; $N : X \to Z$, $N\colon \mathrm{Dom}L \to Z$, $N\mathbf{u} = (\mathbf{u}', \Delta\mathbf{u}(t_1),...,\Delta\mathbf{u}(t_q))$. It is easy to prove that L is a Fredholm mapping of index zero.

Consider the operator equation

$$L\mathbf{u} = \lambda N\mathbf{u}, \quad \lambda \in (0,1). \tag{6}$$

Integrating (6) over the interval $[0,T]$, we obtain

$$\begin{cases} B_1 = \int_0^T [d_1(t)e^{u_1(t)} + \frac{a_1(t)\alpha(t)e^{u_3(t)}}{N(t)+e^{u_1(t)}} - D_1(t)e^{u_2(t)-u_1(t)}]dt, \\ B_2 = \int_0^T [d_2(t)e^{u_2(t)} - D_2(t)e^{u_1(t)-u_2(t)}]dt, \\ B_3 = \int_0^T [d_3(t)e^{u_3(t)} - \frac{a_2(t)\alpha(t)e^{u_1(t)}}{N(t)+e^{u_1(t)}}]dt, \end{cases} \tag{7}$$

here $B_i = \bar{b}_iT - \bar{D}_iT + \sum_{n=1}^q \ln(1+h_{in})(i=1,2)$,
$B_3 = \bar{b}_3 + \sum_{n=1}^q \ln(1+g_n)$.
From (6) and (7), we have

$$\int_0^T |\dot{u}_1(t)|dt \le 2(\bar{b}_1 - \bar{D}_1)T + \left|\sum_{n=1}^q \ln(1+h_{1n})\right|, \tag{8}$$

$$\int_0^T |\dot{u}_2(t)|dt \le 2(\bar{b}_2 - \bar{D}_2)T + \left|\sum_{n=1}^q \ln(1+h_{2n})\right|, \tag{9}$$

$$\int_0^T |\dot{u}_3(t)|dt \le 2\bar{b}_3T + \left|\sum_{n=1}^q \ln(1+g_n)\right|. \tag{10}$$

Since $u_i(t) \in PC_T$, there exist $\xi_i, \eta_i \in [0,T](i=1,2,3)$ such that

$$u_i(\xi_i) = \min_{t\in[0,T]} u_i(t), \qquad u_i(\eta_i) = \max_{t\in[0,T]} u_i(t). \tag{11}$$

Let $v(t) = \max\{u_1(t), u_2(t)\}$, then $v(t) \in PC_T$, moreover

1. if $u_1(t) \ge u_2(t)$ but $\dot{u}_1(t) \ge \dot{u}_2(t)$, then $v(t) = u_1(t)$ and $\dot{u}_1(t) \le \lambda(b_1(t) - d_1(t)e^{u_1(t)}) \le \lambda(b_1^M - d_1^L e^{u_1(t)})$,
2. if $u_2(t) \ge u_1(t)$ but $\dot{u}_2(t) \ge \dot{u}_1(t)$, then $v(t) = u_2(t)$ and $\dot{u}_2(t) \le \lambda(b_2(t) - d_2(t)e^{u_2(t)}) \le \lambda(b_2^M - d_2^L e^{u_2(t)})$.

Denote $b = \max\{b_1^M, b_2^M\}$, $d = \min\{d_1^L, d_2^L\}$, $m_n = \max\{h_{1n}, h_{2n}\}$, then

$$\begin{cases} D^+v(t) \le \lambda(b - de^{v(t)}),\ t \ne t_n, \\ \Delta v(t_n) \le \lambda\ln(1+m_n),\ t = t_n. \end{cases} \tag{12}$$

Integrating (12) over $[0,T]$, we have

$$-\sum_{n=1}^q \ln(1+m_n) \le bT - d\int_0^T e^{v(t)}dt,$$

therefore,

$$\int_0^T e^{u_i(\xi_i)} dt \leq \frac{bT + \sum_{n=1}^{q} \ln(1+m_n)}{d} \quad (i=1,2),$$

so

$$u_i(\xi_i) \leq \ln \left[\frac{bT + \sum_{n=1}^{q} \ln(1+m_n)}{dT} \right] (i=1,2),$$

then

$$\begin{aligned} u_i(t) \leq & u_i(\xi_i) + \int_0^T |\dot{u}_i(t)| dt + \left| \sum_{n=1}^{q} \ln(1+h_{in}) \right| \\ \leq & \ln \left[\frac{bT + \sum_{n=1}^{q} \ln(1+m_n)}{dT} \right] + 2(\bar{b}_i - \bar{D}_i)T \\ & + 2 \left| \sum_{n=1}^{q} \ln(1+h_{in}) \right| = M_i \ (i=1,2). \end{aligned} \tag{13}$$

From (7) and (11), we have

$$\int_0^T d_2(t) e^{u_2(\eta_2)} dt \geq \int_0^T d_2(t) e^{u_2(t)} dt \geq B_2,$$
$$\int_0^T d_3(t) e^{u_3(\eta_3)} dt \geq \int_0^T d_3(t) e^{u_3(t)} dt \geq B_3.$$

that is

$$u_2(\eta_2) \geq \ln \left(\frac{B_2}{\bar{d}_2 T} \right); \ u_3(\eta_3) \geq \ln \left(\frac{B_3}{\bar{d}_3 T} \right).$$

Then

$$\begin{aligned} u_2(t) \geq & \ u_2(\eta_2) - \textstyle\int_0^T |\dot{u}_2(t)| dt - \left| \sum_{n=1}^{q} \ln(1+h_{2n}) \right| \\ & \geq \ln \left(\tfrac{B_2}{\bar{d}_2 T} \right) - 2(\bar{b}_2 - \bar{D}_2)T - 2 \left| \sum_{n=1}^{q} \ln(1+h_{2n}) \right| = M_3, \\ u_3(t) \geq & \ u_3(\eta_3) - \textstyle\int_0^T |\dot{u}_3(t)| dt - \left| \sum_{n=1}^{q} \ln(1+h_{3n}) \right| \\ & \geq \ln \left[\tfrac{B_3}{\bar{d}_3 T} \right] - 2(\bar{b}_3 - \bar{D}_3)T - 2 \left| \sum_{n=1}^{q} \ln(1+h_{3n}) \right| = M_4. \end{aligned}$$

So we have

$$B_3 \geq \int_0^T [d_3(t)e^{u_3(\xi_3)} - \frac{a_2(t)\alpha(t)e^{M_1}}{N(t)+e^{M_1}}]dt = \bar{d}_3 T e^{u_3(\xi_3)} - \overline{\left(\frac{a_2\alpha e^{M_1}}{N(t)+e^{M_1}}\right)}T,$$

that is

$$u_3(\xi_3) \leq \ln\left[\frac{B_3 + \overline{\left(\frac{a_2(t)\alpha(t)e^{M_1}}{N(t)+e^{M_1}}\right)}T}{\bar{d}_3 T}\right],$$

Then

$$\begin{aligned} u_3(t) &\leq u_3(\xi_3) + \int_0^T |\dot{u}_3(t)|dt + \left|\sum_{n=1}^{q} \ln(1+g_n)\right| \\ &\leq \ln\left[\frac{B_3 + \overline{\left(\frac{a_2(t)\alpha(t)e^{M_1}}{N(t)+e^{M_1}}\right)}T}{\bar{d}_3 T}\right] + 2\bar{b}_3 T + 2\left|\sum_{n=1}^{q} \ln(1+g_n)\right| \\ &= M_5. \end{aligned}$$

Similarly,

$$B_3 \geq \int_0^T [d_3(t)e^{M_5} - \frac{a_2(t)\alpha(t)e^{u_1(\eta_1)}}{N(t)}]dt,$$

that is

$$u_1(\eta_1) \geq \ln\left[\frac{\bar{d}_3 e^{M_5}T - B_3}{\overline{\left(\frac{a_2\alpha}{N}\right)}T}\right],$$

then

$$\begin{aligned} u_1(t) &\geq u_1(\eta_1) - \int_0^T |\dot{u}_1(t)|dt - \left|\sum_{n=1}^{q} \ln(1+h_{1n})\right| \\ &\geq \ln\left[\frac{\bar{d}_3 e^{M_5}T - B_3}{\overline{\left(\frac{a_2\alpha}{N}\right)}T}\right] - 2(\bar{b}_1 - \bar{D}_1)T - 2\left|\sum_{n=1}^{q} \ln(1+h_{1n})\right| = M_6. \end{aligned}$$

Thus, we have

$$\begin{aligned} \sup_{t\in[0,T]} |u_1(t)| &\leq \max\{|M_1|, |M_6|\} = N_1, \\ \sup_{t\in[0,T]} |u_2(t)| &\leq \max\{|M_2|, |M_3|\} = N_2, \\ \sup_{t\in[0,T]} |u_1(t)| &\leq \max\{|M_4|, |M_5|\} = N_3. \end{aligned}$$

Obviously, there exists a constant $N_4 > 0$ such that $\max\{|u_1|, |u_2|, |u_3|\} < N_4$. Take $r > N_1+N_2+N_3+N_4$, $\Omega = \{x \in X | \|x\|_c < r\}$, then N is L-compact on $\overline{\Omega}$. So, for $\forall\, \mathbf{u} = (u_1, u_2, u_3)^T \in \partial\Omega \bigcap \mathrm{Ker}L$, we have $QN\mathbf{u} \neq 0$. Let $J : \mathrm{Im}Q \to x$, $(d, 0, \ldots, 0) \to d$. Then when $\mathbf{u} \in \Omega \bigcap \mathrm{Ker}L$, in view of the assumptions in Mawhin's continuation theorem, one obtains, $deg\{FQN, \Omega \bigcap \mathrm{Ker}L, 0\} \neq 0$. By now we have proved that Ω satisfies all the requirements in Mawhin's continuation theorem. Hence, (5) has at least one T-periodic solution in $\mathrm{Dom}L \cap \overline{\Omega}$. □

4 Some Simulations

In this section, we shall discuss an example to illustrate main results. For system (1), we take: $t_n = n\omega$, $b_1(t) = 1 + 0.2\sin(t)$, $d_1(t) = 0.8 + 0.2\sin(t)$, $a_1(t) = 1 + 0.8\cos(t)$, $\alpha(t) = 1 + 0.2\sin(t)$, $N(t) = 2 + \sin(t)$, $D_1(t) = 0.2 + 0.1\sin(t)$, $b_2(t) = 1 + 0.1\cos(t)$, $d_2(t) = 0.9 + 0.1\cos(t)$, $D_2(t) = 1 + 0.2\sin(t)$, $b_3(t) = 1 + 0.2\cos(t)$, $d_3(t) = 0.8 + 0.2\cos(t)$, $a_2(t) = 1 + 0.6\sin(t)$, $h_{1n} = 0.2$, $h_{2n} = 0.2$, $g_n = 0.2$. Obviously, all conditions of Theorem 1 are satisfied.

If $\omega = \pi/2$, then system (1) under the above conditions has a unique 2π-periodic solution (In Fig.1-Fig.4, we take $[x_1(0), x_2(0), y(0)]^T = [0.5, 0.6, 1]^T$). We find the occurrence of sudden changes in the figures of the time-series and phase portrait. The influence of pulse is obvious.

If $\omega = 2$, then (A2) is not satisfied. Periodic oscillation of system (1) under the above conditions will be destroyed by impulsive effect. Numeric results (see Fig.5) show that system (1) under the above conditions has Gui chaotic strange attractor [6].

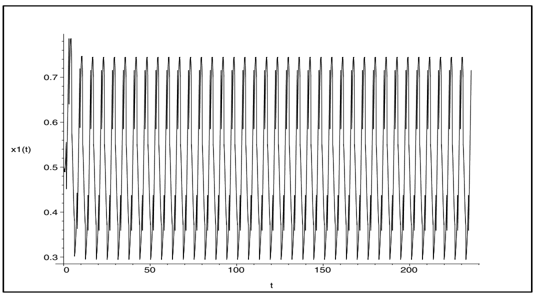

Fig. 1. Time-series of $x_1(t)$ evolved in system (1) with $\omega = \pi/2$

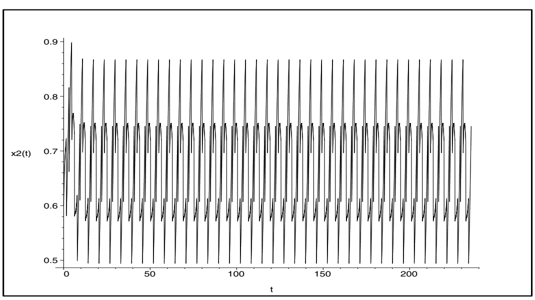

Fig. 2. Time-series of $x_2(t)$ evolved in system (1) with $\omega = \pi/2$

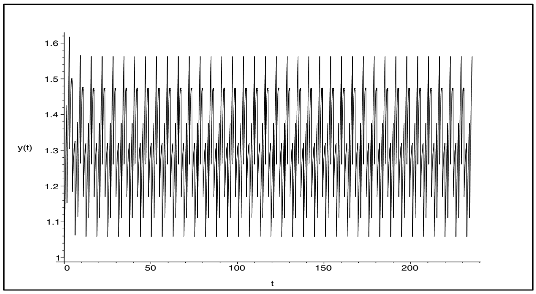

Fig. 3. Time-series of $y(t)$ evolved in system (1) with $\omega = \pi/2$

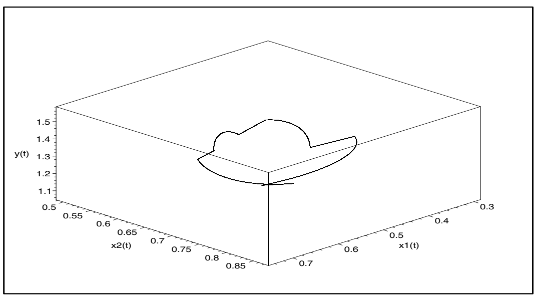

Fig. 4. Phase portrait of 2π-periodic solution of system (1) with $\omega = \pi/2$

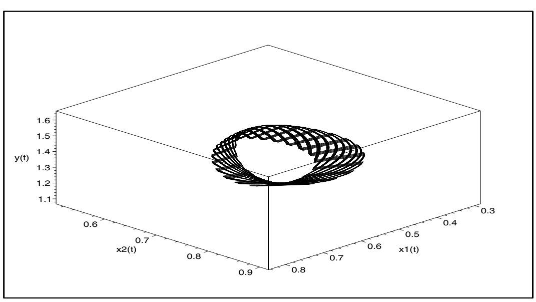

Fig. 5. Phase portrait of chaotic strange attractor of system (1) with $\omega = 2$

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References

1. Ye, D., Fan, M., Zhang, W.P.: Existence of Positive Periodic Solution of a Predator-Prey System. Journal of Biomathematies 19, 161–168 (2004)
2. Meng, X.Z., Wang, X.L.: Permanence and Global Stability for Nonautonomous Predator-Prey Systems with Peridic Coefficients and Type Holling Functional Response. College Mathematics 19, 14–19 (2003)
3. Gao, Y.L., Gui, G.Z.: The Qualitative Analysis of a Model of Predator-Prey System with Holling III Functional Response. Journal of Information Engineering University 6, 19–22 (2005)
4. Zhang, B.L., Yuan, L.W.: Periodic Solutions of Predator-Prey System with Impulsive Effects and Functional Response. Mathematics in Practice and Theory 39, 109–115 (2009)
5. Gaines, R.E., Mawhin, J.L.: Coincidence degree and nonlinear differential equations. Springer, Berlin (1977)
6. Zhang, J., Gui, J.: Periodic solutions of nonautonomous cellular neural networks with impulses and delays. Nonlinear Analysis: Real World Applications 19, 1891–1903 (2009)