Adaptive Nonlinear Guidance Law Considering Autopilot Dynamics

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Abstract. A third-order state equation with consideration of autopilot dynamics is formulated. A nonlinear coordinate transformation is used to change the state equation into the normal form. Adaptive sliding mode control theory is depicted and its stability analyses are proved. Applying this theory on the normal form equation, an adaptive nonlinear guidance law is proposed. The presented law adopts the sliding mode control approach can effectively solve the guidance problem against target maneuver and effects caused by autopilot dynamics. Simulation results show that under the circumstance of target escaping with high acceleration and big autopilot dynamics, the proposed guidance law still has high precision.

Keywords: adaptive nonlinear guidance law, autopilot dynamics, sliding mode control.

1 Introduction

In the guidance area, there are, in general, two approaches based on either the classical approach or on modern control theory. The proportional navigation guidance (PNG) [1] used in classical approach need a feedback with constant gain from the angle rate of line of sight. This approach is easy to implement and efficient. Nevertheless, owing to its degradation with target maneuvering and inefficient in some situations, many improved methods have been proposed, such as augmented proportional navigation (APN) [2], generalized true proportional navigation (GTPN) [3], and realistic true proportional navigation (RTPN) [4].All these approaches have been approved to improve control performance. It's evident that they also result in the complexity in designing and analysis of the system simultaneously.

Research on modern methods relative to guidance law is increasingly active. Lots of papers have been proposed in this research area. The optimal control theory has been used to develop the proportional navigation [5]. Sliding mode control method has been proposed applying in guidance of homing missile [6]. Using these methods, the integrated missile guidance and control system can be designed easily. Compared with traditional methods, these approaches obtain excellent robustness, when the system exists disturbance and parameter perturbation, and are adaptive to the target maneuvering and guidance parameter changes.

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In this paper, an adaptive nonlinear approach considering autopilot dynamics is proposed. In the guidance area, missile autopilot dynamics is one of the main factors affecting Precision-guided. In the proposed paper, the target maneuvering is taken into account as bounded disturbances. The angle of LOS acts as zero output state variable. Lyapunov stability theory is used to design an adaptive nonlinear guidance (ANG) law.

The rest of this paper is organized as follows. Section 2 derives the model considering autopilot dynamics. In Section 3, an adaptive guidance law considering target uncertainties and autopilot dynamics is presented. Also the analysis of the stability of the design is derived. In Section 4, the simulation for the presented guidance law is provided. The conclusions are given in Section 5.

2 Model Derivation

The three-dimensional pursuit geometry is depicted in Fig.1.The Line of Sight (LOS) coordinate system ($OX_4Y_4Z_4$) is chosen as reference coordinate system. The original point O is at the missile's mass center, $OXYZ$ is the inertial coordinate system. OX_4 is the line of sight of the initial moment of terminal guidance, the missile to the target direction is positive[7].

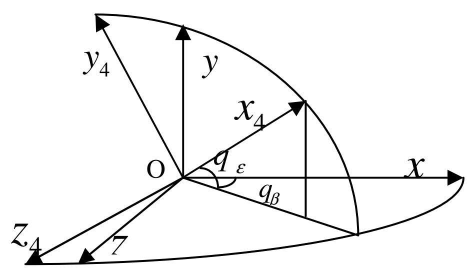

Fig. 1. Three-dimensional pursuit geometry

In the homing process, the acceleration vectors exert on missile and target only change the direction rather than the magnitude of the speed. If missile doesn't rotate, the relative motion between missile and target can be decoupled into two relative independent movements. In this paper, the vertical plane OX_4Y_4 is chosen as an example. Suppose that, during Δt , the incremental of the LOS angle is q. If the time interval is small enough, q is very small. There exists an approximate equation

$$q(t) \approx \sin q(t) = \frac{y_4(t)}{R(t)} \tag{1}$$

In this equation, $R(t)$ represents the relative distance between missile and target.

$y_4(t)$ represents the relative displacement on OY_4 direction during Δt .

The quadratic differential form of (1) can be expressed as

$$\ddot{q} = -a_{g1} q - a_{g2}\dot{q} - b_g a_M + b_g a_T \tag{2}$$

where $a_{g1} = \ddot{R}(t)/R(t), a_{g2} = 2\dot{R}(t)/R(t)$, $b_g = 1/R(t)$, a_M and a_T are the missile and target acceleration.

The first-order autopilot's dynamic characteristic is briefly described as follows

$$\dot{a}_M = -\frac{1}{\tau} a_M + \frac{1}{\tau} a_{MC} \tag{3}$$

where a_{MC} is guidance command provided to autopilot, τ is time-constant of autopilot.

A state space equation is formulated by considering equation (2) and (3) as follows

$$\dot{X}_M = \begin{pmatrix} 0 & 1 & 0 \\ -a_{g1} & -a_{g2} & -b_g \\ 0 & 0 & -\frac{1}{\tau} \end{pmatrix} X_M + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{pmatrix} u + \begin{pmatrix} 0 \\ f_M \\ 0 \end{pmatrix} \tag{4a}$$

$$Y_M = X_{M2} = \dot{q} \tag{4b}$$

where $X_M = \begin{pmatrix} q & \dot{q} & a_M \end{pmatrix}^T$, $u = a_{MC}$, $f_M = b_g a_T$.

In order to apply nonlinear control theory in this problem, the state equation must be transformed into normal form. The output is

$$y = Y_M =: x_1 \tag{5}$$

Differentiating the equation (5) yields

$$\dot{x}_1 = -a_{g1} X_{M1} - a_{g2} x_1 - b_g X_{M3} + b_g a_T \triangleq x_2 + \overline{\Delta}_1 \tag{6}$$

where

$$x_2 = -a_{g1} x_{M1} - a_{g2} x_1 - b_g x_{M3}, \overline{\Delta}_1 = b_g a_T \ . \tag{7}$$

Differentiating x_2 yields

$$\dot{x}_2 = -(a_{g1} + \dot{a}_{g2}) x_1 - a_{g2} x_2 - \dot{a}_{g1} x_{M1} - (\dot{b}_g - b_g \frac{1}{\tau}) x_{M3} b_g \frac{1}{\tau} u + \overline{\Delta}_2 \tag{8}$$

where $\overline{\Delta}_2 = -a_{g2} \overline{\Delta}_1$.

Assign $x_3 = x_{M1} = q$. From equation (7), x_{M3} can be expressed in terms of x_1, x_2 and x_3 ,therefore x_{M3} can be eliminated from the equation (8)

$$\dot{x}_2 = a_1 x_1 + a_2 x_2 + a_3 x_3 - b_g \frac{1}{\tau} u + \overline{\triangle}_2 \tag{9}$$

where $a_1 = -(a_{g1} + \dot{a}_{g2} - \frac{\dot{b}_g}{b_g} a_{g2} + \frac{1}{\tau} a_{g2})$, $a_2 = -(a_{g2} - \frac{\dot{b}_g}{b_g} + \frac{1}{\tau})$, $a_3 = -(\dot{a}_{g1} - \frac{\dot{b}_g}{b_g} a_{g1} + \frac{1}{\tau} a_{g1})$.

Denoting $\tilde{X} = (x_1 \quad x_2 \quad x_3)^T$, a state space equation is formulated. Then assign $X = (x_1 \quad \dot{x}_1 \quad x_3)^T$, a feedback linearization technique can be applied to have

$$\dot{X} = \begin{pmatrix} 0 & 1 & 0 \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{pmatrix} X + \begin{pmatrix} 0 \\ -\frac{1}{\tau} b_g \\ 0 \end{pmatrix} u + \begin{pmatrix} 0 \\ f \\ 0 \end{pmatrix} \tag{10a}$$

$$y = x_1 \tag{10b}$$

where $f = b_g \dot{a}_T + \frac{\mathrm{b}_g}{\tau} a_T$.

3 Adaptive Nonlinear Guidance Law

In this section, a general approach of adaptive sliding mode control is depicted for multi-variable nonlinear system. Then its application on guidance law is formulated.

3.1 Adaptive Sliding Mode Control

The state space equation of the nonlinear system is

$$\dot{X} = F(X,t) + \triangle F(X,t) + G_1(X,t)U(t) + G_2(X,t)W(t) \tag{11}$$

where $X \in R^n, U \in R^m, W \in R^l$, they are the state variables, control input and disturbance separately. $\triangle F(X,t)$ is the structure perturbation of the system.

Assume the above nonlinear equation satisfies the following conditions

1) $0 < \| W(t) \| < a$, a is a positive constant.

2) $\triangle F(X,t) = E(X,t)\delta(X,t)$, $E(X,t) \in R^{n\times n}$, $\| \delta(X,t) \| \le b, b > 0$, $E(X,t)$ is known, b is a constant.

Define the sliding surface

$$s = CX \tag{12}$$

where $C \in R^{l\times n}$ is a constant matrix.

Then control law is designed based on Lyapunov stability theory. First a Lyapunov function is proposed

$$V = \frac{1}{2}s^T s + \frac{1}{2\gamma_1}\tilde{a}^2 + \frac{1}{2\gamma_2}\tilde{b}^2 \tag{13}$$

where $\tilde{a}$ and $\tilde{b}$ are the error estimates of a and b separately. $\tilde{a} = a - \hat{a}, \tilde{b} = b - \hat{b}$, $\gamma_1, \gamma_2 > 0$ are design parameters.

Take time derivative of (13) and consider equation (11) and (12),

$$\dot{V} = s^T \dot{s} - \frac{1}{\gamma_1}\tilde{a}\dot{\hat{a}} - \frac{1}{\gamma_2}\tilde{b}\dot{\hat{b}} = s^T(CF + C\triangle F + CG_1U + CG_2W) - \frac{1}{\gamma_1}\tilde{a}\dot{\hat{a}} - \frac{1}{\gamma_2}\tilde{b}\dot{\hat{b}} \tag{14}$$

If CG_1 is nonsingular, define U as follows

$$U = (CG_1)^{-1}(-CF - Ks - \varepsilon sign(s)) \tag{15}$$

Substituting (15) to (14) and combining the assumptions yields

$$\dot{V} \le -Ks^T s - \varepsilon \| s \| + \| s \| \, \| CE \| b + \| s \| \, \| CG_2 \| a - \frac{1}{\gamma_1}\tilde{a}\dot{\hat{a}} - \frac{1}{\gamma_2}\tilde{b}\dot{\hat{b}} \tag{16}$$

Denote $\varepsilon = \| CE \| \hat{b} + \| CG_2 \| \hat{a}$, it can be derived that

$$\dot{V} \le -Ks^T s + \| s \| \, \| CG_2 \| \tilde{a} + \| s \| \, \| CE \| \tilde{b} - \frac{1}{\gamma_1}\tilde{a}\dot{\hat{a}} - \frac{1}{\gamma_2}\tilde{b}\dot{\hat{b}} \tag{17}$$

Take the adaptive law as

$$\dot{\hat{a}} = \gamma_1 \| s \| \, \| CG_2 \| \tag{18}$$

$$\dot{\hat{b}} = \gamma_2 \| s \| \, \| CE \| \tag{19}$$

Thus $\dot{V} \le -Ks^T s < 0$, that means $\lim_{t\to\infty} V(t) = 0$, and then $s \to 0$, $\tilde{a}, \tilde{b} \to 0$ the sliding mode is asymptotically reached. The final control law can be expressed as

$$U = -(CG)^{-1}[CF + Ks + (\| CE \| \hat{b} + \| CG_2 \| \hat{a}) sign(s)] \tag{20}$$

3.2 Application on Guidance Law

Select the sliding model s as

$$s = c_1 x_1 + c_2 \dot{x}_1 + c_3 x_3 \tag{21}$$

Assume $|f| \le M$, apply the above theory on equation (12) yields

$$u = \frac{\tau}{b_g c_2}[(a_1c_2 + c_3)x_1 + (a_2c_2 + c_1)\dot{x}_1 + a_3c_2x_3 + Ks + \varepsilon sign(s)] \tag{22}$$

where $\varepsilon = c_2\hat{M}$, and the adaptive law is $\dot{\hat{M}} = c_2\gamma_2 |s|$.

If the guidance law and adaptive law are designed as the above equations, missile can hit the target in a uniform speed. The angle of LOS and acceleration can be stable.

Actually, in the process of terminal guidance, $\dot{R} \approx \dot{R}_0, \ddot{R} \approx 0$, so the final guidance law can be simplified by substituting these two equations.

4 Simulation

In this section, a simulation comparison between the proposed guidance law and PNG in the vertical plane is presented. The target maneuvering is $a_T = 20g\sin(0.4\pi t)$.

During the terminal guidance, the velocity of target and missile are invariable. $V_T = 300m/s$, $V_M = 500m/s$ .The initial distance of missile and target is $6000m$. The angle between the missile velocity and LOS is 60° , the angle between the target velocity and LOS is 120° .Design parameters of guidance are $K = -2\dot{R}_0/R$, $c_1 = 2$, $c_2 = 1$, $c_3 = 0$, $\tau = 0.5s$, $\gamma_2 = 6$ [8]. If the time-constant of autopilot is small enough, the guidance law can be simplified as $a_M = -N\dot{R}_0\dot{q}$. That is PNG.

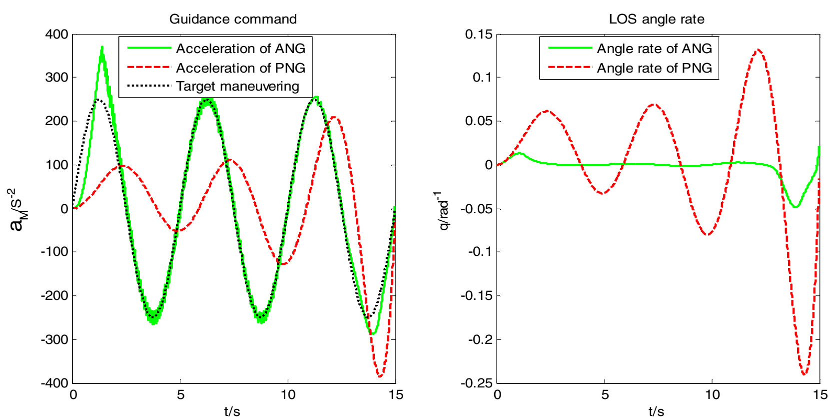

Fig. 2. Acceleration of ANG and PNG), LOS angle rate of ANG and PNG

The first figure of Fig. 2 shows that the normal overload of proportional navigation guidance (PNG) law lags behind target maneuvering significantly. The trend of response property is up. That's because that the PNG reaction to the target maneuvering is slow and indecisive. What's more, the time delay existing in the system also has an effect on the response speed of PNG. However, the normal overload of adaptive nonlinear guidance (ANG) law can keep up with the target maneuvering in real-time after stable. The result shows that ANG has the ability to predict the acceleration of the target. Therefore the proposed guidance law has good adaptability. The results in the second figure of Fig.2 shows that the LOS angle rate become different between ANG and PNG. From the second figure, it is found that the LOS angle rate of ANG stays closely with zero, thus the guidance accuracy is improved. Compared with ANG, the LOS angle rate's amplitude of PNG changes greatly because of the impact of target maneuvering and inertia of autopilot. It can be confirmed that the proposed guidance law can effectively compensate for the target maneuvering and can improve the robustness of the system.

5 Conclusion

In this paper, a guidance law with consideration of autopilot dynamics is proposed. The target acceleration is considered as uncertainties which have bounded disturbances. As the proposed guidance law adopts sliding mode control, the disturbances can be come over. The adaptive law based on Lyapunov stability theory is an estimation of uncertainties. This adaptive law can not only make the system stable but also improve the control performance and the precision of the missile guidance.

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