# Human Activity Modeling as Brownian Motion on Shape Manifold

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**Abstract.** In this paper we propose a stochastic modeling of human activity on a shape manifold. From a video sequence, human activity is extracted as a sequence of shape. Such a sequence is considered as one realization of a random process on shape manifold. Then Different activities are modeled by manifold valued random processes with different distributions. To solve the problem of stochastic modeling on a manifold, we first regress a manifold values process to a Euclidean process. The resulted process then could be modeled by linear models such as a stationary incremental process and a piecewise stationary incremental process. The mapping from manifold to Euclidean space is known as a stochastic development. The idea is to parallelly transport the tangent along curve on manifold to a single tangent space. The advantage of such technique is the one to one correspondence between the process in Euclidean space and the one on manifold. The proposed algorithm is tested on database [5] and compared with the related work in [5]. The result demonstrate the high accuracy of our modeling in characterizing different activities.

## 1 Introduction

Human activity recognition is of great interest in a wide range of applications, spanning areas such as security surveillance, person identification and content-based image retrieval. In addition to security applications, the explosively increasing daily usage of video cameras has and continues to motivate a increasing interest in motion analysis and understanding in video for diverse applications. Recent progress on human activity analysis from video data has been well documented in [9] [8].

Of the various possible representations of human activity [12], we choose to view any given activity of interest as a shape sequence [5] [11]. Different shape representations will lead to different shape manifolds. In Kendall's shape theory [13], a shape is considered to be a set of land marks on the boundary of an object. Due to the simple geoemtry, the Kendall pre-shape space is the popular platform for different modelings, which is invariant to translation and scaling, and geometrically is a hyper sphere. For example, an AR/ARMA model of human activities was proposed in [11] by projecting the shape sequences onto the tangent space of Kendall's preshape space. To overcome the problem of systematically picking consistent landmarks of shapes, we consider a shape as a simple

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and closed planar curve. Such a shape formulation was proposed in [1] with a numerically efficient computation for tangent space of each point on manifold and the geodesic path between any two shapes.

With a similar goal of classifying the shape sequences as in [11], our goal in this paper is to build stochastic model for the process on shape manifold and then in return use the estimated model parameter to classify different activities. The idea is to develop a stochastic model of a process on the shape manifold (representing a shape sequence trajectory on a manifold) on a non-linear space by regressing the problem onto a linear space. In [11] a shape on a preshape sphere is projected onto a tangent space at the mean shape. Adopting such a tangent approximation is, however, valid in only a sufficiently small neighborhood. The invertibility of such a projection on pre-shape sphere only holds when the shape sequence does not cross the "north or south poles" of the hypersphere. Generally on a smooth manifold, the condition of such an orthogonal projection is restricted to a local area of the manifold. In contrast, our proposed regression is intrinsically constructed by a curve development [3] as a 1-1 mapping of an evolution curve on any smooth manifold to a curve in a flat space.

We exploit the afore-described approach to develop in this paper, an intrinsic stochastic model with a goal to classify activities. Assuming a proper human silhouette segmentation[1] of each frame in a video sequence of interest, a specific activity may be summarized by a sequence of individual closed curves/shapes in form of an evolution curve on the underlying shape manifold. Any reasonable modeling for a activity, for instance like "running", is expected to describe the different data samples of "running". In this paper, the set of the different representative curves of "running" are viewed as the realizations of the "running process", which more precisely is a random process on the shape manifold. As a result, any activity process of interest may hence be modeled as a manifold valued random process.

In the balance of this paper, we first provide a brief (but sufficient for this development) introduction to manifold geometry and to stochastic analysis on manifolds. The preprocessing on shapes is introduced in Section 2, to make the shape manifold finite dimensional. In Section 3, we introduce the stochastic curve development for a human activity process as a mapping from a manifold to a flat space. In Sections 4 and 5, we construct a connection on the shape manifold, and derive the corresponding curve development result for a given human activity.

## 2 Background

In this section we first provide a brief review of the required background in differential geometry and stochastic differential equation to allow us to define a shape manifold as a working space. We also describe the required tools of parallel transportation and curve development on a shape manifold.

[1] Note that errors in segmentation clearly imply errors down the processing stream, and this investigation is a research topic in and of itself, and is left for future work.

### 2.1 A Infinite Dimensional Shape Manifold

According to [1], a planar shape is a simple and closed curve $\alpha(s)$ in $\mathbb{R}^2$,

$$\alpha(s) : I \rightarrow \mathbb{R}^2, \tag{1}$$

where an arc-length parameterization is adopted. A shape is represented by a *direction index function* $\theta(t)$. With such a parameterization, $\theta(s)$ may be associated to the shape by

$$\frac{\partial \alpha}{\partial s} = e^{j\theta(s)}. \tag{2}$$

Due to the fact that the rotation index for simple closed curve is restricted to 1 in [1], the ambient space of the manifold of $\theta$ is an affine space based on $\mathbb{L}^2$,

$$\theta \in A(\mathbb{L}^2). \tag{3}$$

Further more, according to the restriction for a planar curve to be a closed curve, and invariant over rigid Euclidean transformations. The shape manifold $M$ is defined as a level set of function $\phi : A(\mathbb{L}^2) \rightarrow R^3$,

$$\phi(\theta) = (\int_0^{2\pi} \theta ds, \int_0^{2\pi} \cos(\theta) ds, \int_0^{2\pi} \sin(\theta) ds) \tag{4}$$

Using the function $\phi$ defined above, in [1] the shape manifold $M$ is defined as following

$$M = \phi^{-1}(\pi, 0, 0) \tag{5}$$

One of the most important properties of $M$ is that the tangent space $TM$ is well defined. Such a property not only simplifies the analysis, but also makes possible the numerical computation,

$$T_\theta M = \{f \in \mathbb{L}^2 | f \perp span\{1, \cos(\theta), \sin(\theta)\}\} \tag{6}$$

### 2.2 Connection on Manifold

To study a random process on manifold, we need to overcome the difficulty resulted from the curvature of the manifold. The Riemannian structure of the manifold can be defined by the connection in the principle bundle.In this paper the connection is defined in the frame bundle $\mathbb{F}(M)$, which is a special case of principle bundle.

**Definition 1 (Principal Fiber Bundle).** *A principal fiber bundle is a set* $(P, G, M)$*, where* $P, M$ *are* $C^\infty$ *manifolds, and* $G$ *is a Lie group such that*
*(1)* $G$ *acts freely on the right of* $P$*,* $P \times G \rightarrow P$*. For* $g \in G$*, we shall also write* $R_g$ *for the map* $g : P \rightarrow P$
*(2)* $M$ *is the quotient space of* $P$ *by an equivalence relation under* $G$ *(any shape subjected to a* $g \in G$ *is equivalent to itself), and the projection* $\pi : P \rightarrow M$ *is* $C^\infty$*, so for* $m \in M$*,* $G$ *is simply transitive on* $\pi^{-1}(m)$
*(3)* $P$ *is locally trivial. Thus for any open set* $U \subset M$*,* $\pi^{-1}(U) \sim U \times G$

A point $u$ in $\mathbb{F}(M)$ can be written as, $u = (x, b)$, where $m \in M$ and $b = e_1, e_2, ..., e_n$ is a orthogonal basis of the associated tangent space $T_m M$. The group $G$ acting on a fibre is $SO(n)$). Referring the definition of principle bundle, the equivalent class for each point $m \in M$ is all the orthogonal basis for tangent space $T_m M$. The rotation matrix can be utilize to transform one basis to another.

**Definition 2 (Connection).** *A connection on the principal bundle* $(P, G, M)$ *is a* $n$*-dimensional distribution* $H$ *on* $P$*, where* $n = dim(M)$*, such that*
*(1)* $H \in C^\infty$
*(2) for every* $p \in P$*,* $H_p + V_p = T_p P$*, where* $V_p$ *is a vertical space and* $H_p$ *is a horizontal space of* $T_p P$*. A vector* $Y \in T_p P$ *is vertical if* $\pi_*(Y) = 0$
*(3) for every* $p \in P$*,* $g \in G$*,* $(R_g)_*(H_p) = H_{pg}$*.*

With the definition of a connection on a manifold in hand, we can achieve a horizontal lift from a manifold to a linear frame bundle.A more expanded and detailed discussion of connections may be found in [3] [4].

**Definition 3 (Horizontal Lift).** *Let* $\gamma$ *be a piecewise* $C^\infty$ *curve in* $M$*,* $\gamma : [0, 1] \rightarrow M$*. Let* $p \in \pi^{-1}(\gamma(0))$*. Then there exists a unique lift* $\tilde{\gamma}$ *of* $\gamma$ *such that* $\tilde{\gamma}_*(t) \in H_{\tilde{\gamma}(t)}$ *and* $\tilde{\gamma}(0) = p$*. We say that* $\tilde{\gamma}$ *is the horizontal lift of* $\gamma$ *that starts at* $p \in P$

## 3 Dynamics of Human Activity on a Shape Manifold

Hsu in [2] proposes *an efficient analysis framework* to construct an invertible mapping from a manifold-valued random process to a Euclidean-valued random process. The essence of the mapping is to compute a Euclidean process that can drive a stochastic differential equation (SDE) to generate a manifold-valued random process. In a Euclidean space the random process have been extensively studied and there are many tools available for modeling. In contrast to the orthogonal projection method onto a tangent space around a mean, Hsu's theory provides a *one to one correspondence* between a process on a manifold and one on a Euclidean space. This improved accuracy of representation is primarily due to the so-called *"rolling without sliding" property* of a parallel transport.
As in [2], any random process on the shape manifold, may then be written as a solution to some $SDE(X_0, V, Z)$. Generally we have,

$$X_t = X_0 + \int_0^t \sum_i V_i(X_s) \circ dZ_s^i, \tag{7}$$

where, $X_0$ is the initial condition, $V_i$ is a smooth vector field defined on $M$ and $Z_s^i$ is a Euclidean valued random process driving Equation (7). The stochastic integration here is the Stratonovich integration. More intuitively Equation (7) can be understood as $dX_t = \sum_i V_i(X_s) \circ dZ_s^i$. Thus the dynamic described by Equation (7) is characterized by both a vector field $V$ and a driving process $Z_t$.

However, the form of the Euclidean process $Z$ is varies with different choices of $V$. In contrast to our goal to construct a $1-1$ mapping from manifold process to Euclidean process, there is no one one correspondence $X \leftrightarrow Z$ without proper knowledge about $V$ . In [2] this problem is solved by setting $V$ equal to the horizontal lift of $X_t$, which in unique for a given connection. Provided the uniqueness of horizontal lift, the resulting driving process $Z_t$ will have the one one correspondence to $X_t$. Let the vector field $U_t$ be the horizontal lift of $X_t$ in $F(M)$, Equation (7) may be rewritten as

$$X_t = X_0 + \int_0^t \sum_i U^i(X_s) \circ dW_s^i. \tag{8}$$

According to the definition of the orthogonal frame bundle and stochastic horizontal lift in Section 2 we know the horizontal vector field $U(t)$ can be written as,

$$U_t = \{e_1, e_2, \cdots, e_i, \cdots, e_n\} \tag{9}$$

where $e_i$ is the basis of $T_{X_t}M$. In Equation 8 the differential $dX_t$ is represented in a selected basis $U_t$ with corresponding driving process $W_t$. For an orthogonal basis one can write , Consequently, the stochastic development of $X_t$ is,

$$dW_t = U_t^{-1} \circ dX_t \tag{10}$$

$\forall i = 1, 2, 3, \cdots$ Equation 10 can be represented in vector as

$$dW_t^i =< e_i, dX_t > \tag{11}$$

Such rewriting of Equation (7) provides a representation of the random process $X_t$ on a manifold with the Euclidean random process $W_t$, which generate the original process $X_t$ by acting on vector field $U_t$ as in Equation (8). In the above discussion, we provide a $1-1$ mapping from $X_t \in M$ to $W_t \in R^{dim(M)}$. The critical point for implementing such a mapping is the specific form of the connection $H$ which we discuss in Section 4.

## 4 Flat Connection on a Shape Manifold

The construction of connection $H$ is critical to the implementation of the curve development. Theoretically there may exist many different choices of $H$ for a given manifold. Once the exact form of $H$ is determined, the geometry of a manifold is specified accordingly. Among different kind of connections, we adopt the flat connection $H$ for the efficiency of calculation. Flat connection do not always exist. However if the frame bundle $F(M)$ of a manifold $M$ has a global section then the flat connections are easy to define. In the following, we provide a constructive proof of the existence of the global section. Thus the implementation of the flat connection proceeds by constructing a smooth global section $\sigma : M \to \mathbb{F}(M)$ of the linear frame bundle $\mathbb{F}(M)$. Then for each $m \in M$ we define $H_{\sigma(m)}$

to be the tangent space of the submanifold $\sigma(M)$ at $\sigma(m) \in F(M)$. Let $u$ be any point of fiber over $m \in M$. Then there is a unique $g \in GL(n)$ such that,

$$u = R_g(\sigma(m)) \tag{12}$$

The horizontal subspace $H_u$ is then defined as

$$H_u = R_{g*}(H_{\sigma(m)}) \tag{13}$$

In the construction of the smooth section $\sigma$, we smoothly assign to each point $\theta \in M$ a basis $\{E_k\}_{k=1,2,3...}$ for the tangent space $T_\theta M$. From Section 2, we know that the tangent space of $M$ can be written as

$$T_\theta \hat{M} = \{v \in S | v \perp span\{1, \cos(\theta), \sin(\theta)\}\} \tag{14}$$

To construct smoothly distributed basis $\{E_k\}_{k=1,2,3...}$ on manifold, we first construct a Fourier-like global section $\tilde{\sigma}$ in the ambient space $A(L^2[0, 2\pi])$,

$$\tilde{\sigma} : \theta \rightarrow \{1, cos\theta, sin\theta, ..., cosi\theta, sini\theta, \ldots\} \tag{15}$$

Then $\tilde{\sigma}$ is properly projected to the tangent space $TM$ as $\sigma : \theta \rightarrow \{E_k\}_{k=1,2,3...}$ following the geometry defined by Equation (14). The details of this procedure implementation are as as follows, Firstly, one can easily show that the following set of continuous functions is a linearly independent set,

$$\{1, cos\theta, sin\theta, ..., cosi\theta, sini\theta, ...\} \tag{16}$$

Let $B_i$ be the result of a Gram Schmidt orthogonalization of the above basis in ambient space.

$$\begin{aligned}&\{v_1, v_2, v_3, B_{i=1,2,3,\cdots}\} = \\ &ON\{1, cos\theta, sin\theta, ..., cosi\theta, sini\theta, ...\}\end{aligned}$$

where $\{v_1, v_2, v_3\}$ are the first three basis vectors from the Gram Schmidt procedure which correspond to the normal space of the tangent space,

$$span\{v_1, v_2, v_3\} = span\{1, \cos(\theta), \sin(\theta)\} \tag{17}$$

These basis vectors are excluded because they are orthogonal to the tangent space of $M$. Then $B_i$ is the ambient representation of the basis of $T_\theta M$. The orthogonal projection from $L^2$ onto $S$ can be written as a Fourier approximation of $B_i$ and denoted by $\hat{B}_i$. Letting $\phi_j$ denote the Fourier basis functions, it follows that

$$\hat{B}_i = \sum_{j=1}^{N} < B_i, \phi_j > \phi_j^* \tag{18}$$

where $< B_i, \phi_j >$ is the inner product defined in $L^2$ In such setting, we would smooth assign a basis for $T_m\tilde{M}$ by a Gram Schmidt procedure applied to $\hat{B}_{i(k)=k}$.

$$E_k = ON\{\hat{B}_1, \hat{B}_2, \hat{B}_3, \cdots \hat{B}_N\} \tag{19}$$

Thus the resulted global section used to define the connection $H$ is,

$$\sigma : \theta \in M \rightarrow E_k \tag{20}$$

In the shape manifold $M$ equipped with the flat connection $H$ as defined in Equation (21), the horizontal lift $U_t$ of $X_t$ with initial condition $U_0$ is computed as following.

$$U_t = R_g \circ \sigma(X_t) \tag{21}$$

where $g \circ \sigma(X_0) = U_0$.

In the ambient space $A(S)$, $\forall t, U_t, \sigma(X_t)$ can be represented by $N \times N$ invertible matrix. For example, $U_t = [e_1, e_2, \cdots, e_N]$, where $e_i \in R^N$ span the tangent space $T_m M$. In such setting $U_t$ can be calculated as a matrix multiplication in the ambient space.

$$(U_t)_{ij} = \sum_k g_{ik} \dot{\sigma}(X_t)_{kj} \tag{22}$$

where $g \cdot \sigma(X_0) = U_0$. Consequently the development $W_t$ of $X_t$ can be written as,

$$W_t = \int_0^t (U_t)^{-1} dX_t \tag{23}$$

In Figure (1), a few numerical results are demonstrated for $W_t$ for three activities: walking, running and bending.

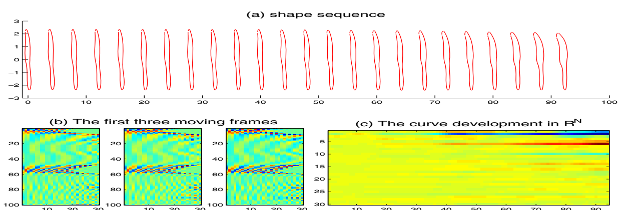
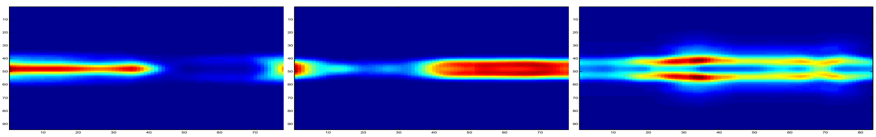



**Fig. 1.** curve development of $X_t \in M$ in $\mathbb{R}^{N=30}$: (a) the original shape sequence represented by angle functions $X_t$ (b) the horizontal lift $U_1, U_2, U_3$. (c) the development $(U_t)^{-1} dX_t$

## 5 Stochastic Analysis in a Euclidean Space

As discussed in the Section 3, the random process $X_t$ on the shape manifold is now mapped to a Euclidean random process $W_t$. The recognition for human activity is thus reduced to comparing different processes in the flat space. In this section, we show that the resulted $W_t$ exhibits a strong non-stationarity trend. A stationarity test is performed on $W_t$ with the “double windows” method as proposed in [6]. The evolutionary spectrum shows that $\|W_t\|^2$ is non-stationary for most of the activity in the motion data base in [5]. The evolution spectrum $Y_t$is estimated by a double sliding window method.

Figure 2 shows several results of the evolutionary spectrum, $T_1 = 11, T_2 = 51$.



(a1) (a2) (a3)

**Fig. 2.** The evolutionary spectrum of $\|W_t\|$: a1, a2, a3 are the EPSD corresponding to the original shape sequence (a), (b), (c) in figure 1;

Given the non-stationary Euclidean process $W_t$, which is a stochastic development of $X_t$, we first analyze it as a Brownian motion. As in Section 5.1, a self-covariance matrix $K$ of the increment $dW_t$ of $W_t$ is estimated from observations. We subsequently proceed to discuss activity classification by introducing a metric for $K$. Computing the increments of $W_t$ to achieve stationarity also carries a potential of increasing the noise, particularly when the process is non-homogeneous.

According to the comparison in5.1, the performance of the Brownian Motion Model is sufficient for the classification of human activities. However, from the view of model fitting, the Brownian motion model still assume the first order incremental $dW_t$ to be stationary, which is still not necessarily truth for all the data. Instead of imposing the strong assumption of higher order stationarity, in Subsection 5.2, we further introduce random process segmentation according to the local stationarity. While additional computational cost is incurred to segment the process $W_t$, we develop the piece-wise Brownian model to further relax the assumption of global stationarity and the resulted modeling can be better fitted to different data.

In the experiments, we test both of the two model on the activity classification database in [5]. The experiment result is compared with [5] and [11] for each of the database. The activity data in [5] includes 10 different activities. Each one has 9 video sequence for 9 different actors. We perform level set segmentation to extract the contour of shape as in [14].

### 5.1 Human Activity as a Brownian Motion on Manifold

In this section we model $W_t$ as a Euclidean Brownian motion. Consequently, the model for $X_t$ is a Brownain motion on the shape manifold $M$, which can be written as the following stochastic differential equation on $M$,

$$X_t = X_0 + \int_0^t \sum_i U^i(X_s) \circ dW_s^i. \tag{24}$$

where $W_t$ is a high dimensional Euclidean Brownian motion and $U(X)$ is the horizontal field calculated in Section 4.

Then by the distribution of $dW_t$ we can characterize different activities. Since $dW_t$ is IID and Gaussian, the time sampling is used as the sampling for random variable $dW$. We next proceed to estimate the covariance matrix $K(dW^i, dW^j)$ as the feature of choice for the underlying distribution.

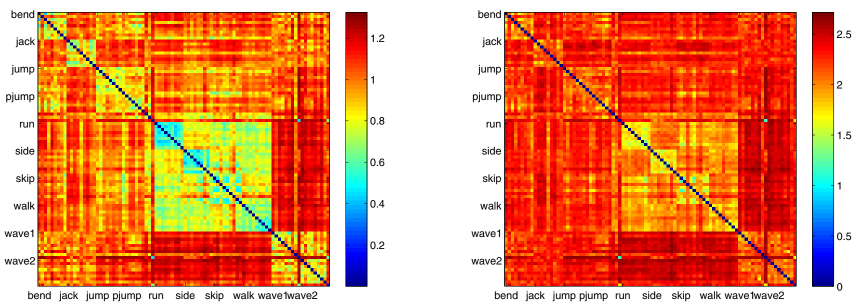
$$K(dW^i, dW^j) = E((dW^i - E(dW^i))(dW^j - E(dW^j))) \tag{25}$$

where $dW^i$ is the $i^{th}$ element of the vector $dW$. The distance between two different covariance matrices is defined by the Frobenius norm as,

$$D(K1, K2) = \|K1 - K2\|_F. \tag{26}$$

The results of $D(K1, K2)$ for the data base in [5] is shown in Figure 3. Using the distance matrix $D$, we may carry the recognition/classification task by using, for example, the leave-one-out algorithm. The nearest neighborhood algorithm is used for classification. If we let $N_B$ be the total number of realizations of B. and $N(B, A)$ the number of realizations of B classified as A activity, we have

$$P(A|B) = \frac{N(B, A)}{N(B)} \tag{27}$$




(a1) (a2)

**Fig. 3.** (a1) Distance matrix for database in [5] with Brownian Motion (a2) Distance matrix for database in [5] with Piece-wise Brownian Motion

To perform a consistent comparison of results published in [5], we need to change our experiment to the same setting. In [5] the number of activity observations is increased by segmenting any video sequence for a given activity into many overlapped chunks. The segments are assumed independent and the classification is carried out. The performance of our proposed method is summarized in the following tables.

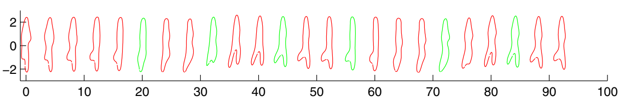
**Table 1.** Table of recognition rate. the number in () is the result in [5].

| $P(act1\|act2)$ | bend | jack | jump | pjump | run | side | skip | walk | wave1 | wave2 |
|---|---|---|---|---|---|---|---|---|---|---|
| bend | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| jack | 0 | 1 (0.98) | 0 (0.02) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| jump | 0 | 0(0.02) | 1 (0.971) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| pjump | 0.0556(0) | 0 | 0 | 0.944(1) | 0 | 0 | 0 | 0 | 0 | 0 |
| run | 0 | 0 | 0 | 0(0.108) | 0.944(0.892) | 0.0556 (0) | 0 | 0 | 0 | 0 |
| side | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| skip | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| walk | 0 | 0 | 0 | 0(0.09) | 0 | 0(0.09) | 0 | 1(0.948) | 0(0.35) | 0 |
| wave1 | 0 | 0 | 0 | 0(0.09) | 0 | 0 | 0 | 0(0.019) | 1(0.972) | 0 |
| wave2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0(0.09) | 1(0.991) |

From the above comparison, our manifold valued Brownian motion model achieve better performance for almost all the activities except slightly lower for case "pJump".

### 5.2 Human Activity as a Piecewise Brownian Motion on Manifold

Alternatively to assuming $dW_t$ to be stationary, we directly address the non-stationarity by segmenting the $W_t$ into several local stationary segments. For each segment we carry out the Brownian motion modeling. Such a topic has been extensively investigated in time series analysis [6]. We apply a sliding window computation of an evolution spectrum $Y_{t,\omega}$ of $dW_t$ as in [6] and detect transient points.




(a)

**Fig. 4.** Nonstationary time series segmentation: (a),(b),(c) is the segmentation results of activity bending, running, skipping

For the $i^{th}$ segment of $dW_t$, we estimate $K^i$ according to Equation 25. We next define the distance between two sequence $dW1_t$ and $dW2_t$ is defined by

$$D_{seg}(K1, K2) = median(min_i(D_{cov}(K1^i, K2^j)))+ \\ median(min_j(D_{cov}(K1^i, K2^j))).$$

The distance matrix is calculated for both the databases according to the above equation. The result is illustrated as in figure 3.

In Figure 3, we show that the distance matrix $D_{cov}$ calculated from the same data set as in [5]. By then using the conditional probability of recognition, as a performance metric as in Equation (27 ), we demonstrate the classification performance in table 5.2. However here we provide no comparison with the result

**Table 2.** Table of recognition rate for data base in [5]

| $P(act1\|act2)$ | bend | jack | jump | pjump | run | side | skip | walk | wave1 | wave2 |
|---|---|---|---|---|---|---|---|---|---|---|
| bend | 0.7778 | 0 | 0 | 0.1111 | 0 | 0 | 0 | 0 | 0.1111 | 0 |
| jack | 0 | 0.7778 | 0 | 0 | 0.1111 | 0 | 0 | 0 | 0 | 0.1111 |
| jump | 0 | 0 | 0.5556 | 0.2222 | 0 | 0.1111 | 0 | 0.1111 | 0 | 0 |
| pjump | 0.2222 | 0 | 0.1111 | 0.3333 | 0 | 0.1111 | 0.1111 | 0 | 0.1111 | 0 |
| run | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| side | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| skip | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| walk | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| wave1 | 0 | 0 | 0 | 0.2222 | 0 | 0 | 0 | 0 | 0.5556 | 0.2222 |
| wave2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.4444 | 0.5556 |

in [5]. Because as mentioned in the previous section, in [5] the data sample is increase by segment the original sequence into small overlapping chunks. Such setting is making our stationary segmentation trivial. If following the same way as in [5], then each small chunk would be a signal stationary segment. Therefore as for the modified database, the piecewise brownian model is the same as the global brownian model. So here we only provide our result on the original database.

## 6 Conclusion

In this paper, we provide a systematic framework for the stochastic modeling of human activity on shape manifold. In theory, such framework is one one mapping from random process on manifold to the random process in Euclidean space. In the resulted flat space, the representative random process of activity is modeled as both global and local Brownian Motion process.The experiment well demonstrate the performance of the proposed modelings of activities.

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