Curvature Minimization for Surface Reconstruction with Features

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Abstract. A new surface reconstruction method is proposed based on graph cuts and local swap. We novelly integrate a curvature based variational model and Delaunay based tetrahedral mesh framework. The minimization task is performed by graph cuts and local swap sequentially. The proposed method could reconstruct surfaces with important features such as sharp edges and corners. Various numerical examples indicate the robustness and effectiveness of the method.

1 Introduction

Reconstructing a surface from an unorganized point data set is a significant and challenging problem in the field of computer graphics. The development of scanner techniques and their wide applications in the areas such as animation industry, medical imaging, and archeology have boosted the demand of a good reconstruction method. Extensive research has been conducted and tremendous advances have been made. Therefore, a robust reconstruction method which could recover the surface with the sharp features motivates this study.

Most surface reconstruction methods could be classified into two groups, explicit methods and implicit methods. Explicit methods are local geometric approaches based on Delaunay triangulation and dual Voronoi diagram [3, 2, 4, 13]. One advantage of these methods is their theoretical guarantee that there exists a sub-complex of Delaunay triangulation of the data set, which is homeomorphic to the ground truth surface given a sufficient sampling. However, due to the insufficient sampling density at the sharp features, the explicit approaches could not reconstruct the desired features. Sharp features are high frequency portion in the signal processing language, which means the normal data acquisition resolution could not fulfill the sufficient requirement.

In the last two decades, some researchers turned to the implicit methods to gain flexibility of representation and mathematical facilities [16, 35, 34, 15, 26, 27, 5, 22]. The success of the weighted minimal surface model in [35] and its variants prove the effectiveness of this methodology. The most popular regularization term added in the variational model is based on the area, which is designed

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for noise removal but not for feature preservation. The application of Euler's elastica model in image processing inspires the graphic community and some works oriented to curvature have been proposed, see [15]. However, most of implicit reconstruction methods utilize the regular grid to discretize the energy functional. The consequence of this framework is the staircasing observed in the reconstructed surface. Some smoothing post-processing is needed more or less, but the procedure weakens the feature sharpness.

Graph cuts techniques from combinational optimization have been used in vision problems to find the global minimum of energy functionals for a long time [7,9,8]. Recently, it is also widely used in the field of solving of higher order models [19,6] and surface reconstruction problem [17,18,20,21,25,28,32]. It is a useful tool that can minimize energy functions over implicitly defined surfaces. Compared with the iterative ways such as gradient descent, the main advantages of graph cuts are the efficiency and ability to find global minima.

In this study, we propose a novel method for surface reconstruction. The weighted minimal surface model in [35] has been added with a curvature term. The variational model is first discretized on the tetrahedral mesh. A graph is constructed dual to the mesh and graph cuts are applied. The high order curvature term as well as the closeness term are assigned to the graph edge weights. The energy is calculated based on the last iteration result and graph cuts are performed iteratively. Local swap will be applied on each element of the explicit surface, which is regarded as the mesh partition. The curvature based energy functional is then calculated on 2-manifolds and the change of the energy will be recorded.

Our method integrates Delaunay-based tetrahedral mesh and curvature based variational model. It also takes the advantages of both. The Delaunay triangulation guarantees the existence of reliable recovered surface to the ground truth given sufficient sampling. The curvature based model helps to preserve the features. More important is that the tetrahedral mesh guarantees the better capability of representing piecewise smooth surfaces with sharp corners and cusps. The earlier works based on grid intrinsically could not obtain the sharp features. The input data points are represented by grid data at first place. The precise information is coarsened. Consequently the ground truth are difficult to be reconstructed exactly. In our method, ground truths could be reconstructed exactly, which will be seen in our examples.

The rest of this paper is organized as follows. In Section 2, we review some related works and give an overview of our proposed method. In Section 3, we propose a graph cuts based method as the first stage, global minimization. In Section 4, the local swap method is proposed to recover the remaining features. In Section 5, various numerical experiments are conducted and the results are shown.

2 An Overview of the Proposed Method

We proposed a new method based on the surface model by Zhao et al. in [35], which is solved by a gradient descent method. In [15], Franchini et al. also solved this model. The signed distance function $d(x)$ and the curvature $\kappa(x)$ are calculated on the regular mesh grid. The level set was computed by local RBF reconstruction. In the mean time, models that minimize curvature based functionals have been demonstrated to perform particularly well to avoid the staircasing effect. The Euler's elastica model is of central importance such curvature based model, which was first introduced in image processing in [24, 11, 23] and later in [6, 29].

Inspired by the performance of Euler's elastica model in imaging, we introduce the curvature term into the model for surface reconstruction. We also calculate curvature on tetrahedral mesh to avoid staircasing.

Zhao et al. proposed the weighted minimal surface model as follows:

$$E_{Zhao}(\Gamma) = \left[\int_{\Gamma} d^p(x) ds \right]^{\frac{1}{p}} . 1 \leq p \leq \infty, \tag{1}$$

where Γ is an arbitrary surface and ds is the surface area. $d(x) = d(x, P)$ here is the distance function from the point x to the nearest point of data set P.

The Euler's elastica of a curve C is given by the energy

$$E_{EL}(C) = \int_{C} (a + b \cdot |k|^{\beta}(x)) dl, \tag{2}$$

where a and b are two parameters and k is the curvature of C at position x. By setting $b = 0$, $E_{EL}(C)$ measures the total length of the curve. If $a = 0$, $E_{EL}(C)$ measures the total curvature of the curve. For solving this kind of curvature based model, traditionally, the Euler-Lagrange or gradient descent equations are derived. In [6], in order to accelerate the convergence of solution, based on the general formulation of energy functional, we can solve the problem via graph cuts by the connection between minimization problems and binary MRFs.

Our method has been motivated by these methods which adopt the weighted surface area and the curvature function. We firstly introduce our model which can recover not only the smooth parts but also the features such as sharp edges and corners as follows:

$$E(\Gamma) = \int_{\Gamma} (d(x) + \lambda |\kappa(x)|) ds \,. \tag{3}$$

Here distance function for each point $d(x)$ is the fidelity term and $\lambda|\kappa(x)|$ is the regularization term which replaced area term in the Zhao et al.'s model, $\kappa(x)$ is the mean curvature at position x. Given the input data set P, we add the non-geometric background points Q; generate mesh in a Delaunay way in order to have the reasonable Delaunay triangulations $P \cup Q$. Then we proposed a two stage strategy:

1. We use graph cuts to minimize the energy functional based on the primal mesh and dual graph. Assign the graph weight according to the energy functional to some extent to get the surface initialization which is for the curvature based evolution;

2. Based on the explicit surface obtained by the first stage, we use local swap here to recover the features without oscillation.

The flowchart of our method is as shown in Fig. 1, and we will describe the details in the following two sections.

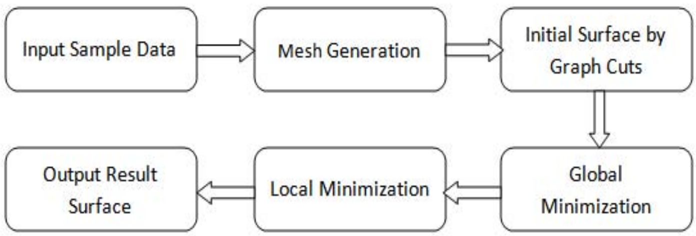


Fig. 1. Flowchart of the proposed method

3 Global Minimization for Surface Reconstruction via Graph Cuts

In this section, a curvature based variational model will be proposed for surface reconstruction and solved by graph cuts. This new energy functional is a generalization from that of the weighted minimal surface model, which is also related to the geodesic active contours approaches [10, 12]. This functional is minimized on an unstructured tetrahedral mesh framework. The method can handle many reconstruction difficulties such as noise, undersampling and non-uniformity.

In this method, the unstructured tetrahedral mesh $\mathcal{T}_h$ is used instead of structured grids, which provides more flexibility and effectiveness. Normally, we use $\{K_i\}_{i=1}^N$ to denote all N tetrahedra in $\mathcal{T}_h$. In such mesh framework, the surface Γ can be approximated by Γ_h, a sub-complex of $\mathcal{T}_h$. Therefore, our energy functional (3) can be approximated by:

$$E(\Gamma_h) = \int_{\Gamma_h} (d(x) + \lambda|\kappa(x)|)ds\,.$$

For convenience reason, we do not distinct Γ and Γ_h in the rest of this paper. The surface triangulation Γ_h can be thought of as the union of the triangular faces shared by tetrahedra in different partitions. In this section, we only discuss two phase problems, in which the ground truth surface $\mathcal{S}$ simply separates the embedding domain $X \subset R^3$ into two connected regions, inside and outside.

We define the level set function:

$$\phi_{\Gamma_h}(K_i) = \begin{cases} c_1 & \text{if } K_i \text{ inside } \Gamma_h, \\ c_2 & \text{if } K_i \text{ outside } \Gamma_h. \end{cases}$$

If we denote $\Gamma_{i,j} = K_i \cap K_j$, which means the shared face of the two neighboring tetrahedrons K_i and K_j, then we have $\Gamma_h = \bigcup \Gamma_{i,j}$.

We define indication function $\mathbf{1}_{\{\mathbf{T}\}}$ as:

$$\mathbf{1}_{\{\mathbf{T}\}} = \begin{cases} 1 & \text{if the statement } \mathbf{T} \text{ is true,} \\ 0 & \text{if the statement } \mathbf{T} \text{ is false.} \end{cases}$$

Hence the energy formulation can be discretized as follows:

$$E(\Gamma_h) = \sum_{i,j} (d_{i,j} + \lambda|\kappa_{i,j}|) S_{i,j} \mathbf{1}_{\{\phi_{\Gamma_h}(K_i) \neq \phi_{\Gamma_h}(K_j)\}}\,, \tag{4}$$

where

$$d_{i,j} = \frac{\int_{\Gamma_{i,j}} d(x)ds}{\int_{\Gamma_{i,j}} ds}\,,\; S_{i,j} = \int_{\Gamma_{i,j}} ds\,,\, \kappa_{i,j} = \frac{\int_{\Gamma_{i,j}} \kappa(x)ds}{\int_{\Gamma_{i,j}} ds}. \tag{5}$$

In level set formulation, the curvature can be calculated by signed distance function as follows:

$$\kappa(x_i) = \nabla \cdot \left(\frac{\nabla d(x_i, \Gamma)}{|\nabla d(x_i, \Gamma)|} \right).$$

In [30], Tong et al. give the corresponding discretization of curvature in details. In order to focus on the steady state solution and not the evolution sequence itself, we first initialize:

$$\kappa^0(x_i) = 0,$$

and

$$\kappa^n(x_i) = \nabla \cdot \left(\frac{\nabla d(x_i, \Gamma^n)}{|\nabla d(x_i, \Gamma^n)|} \right).$$

The energy functional in each iteration is:

$$E(\Gamma^{n+1}) = \int_{\Gamma^{n+1}} (d(x) + \lambda|\kappa^n(x)|)ds = \sum_{i,j} (d_{i,j} + \lambda|\kappa^n_{i,j}|) S^n_{i,j} \mathbf{1}_{\{\phi_{\Gamma_h}(K_i) \neq \phi_{\Gamma_h}(K_j)\}}\,. \tag{6}$$

Therefore, the energy functional can be minimized efficiently by graph cuts, since it is graph representable. A graph dual to the whole mesh is built according to the energy functional and applied with max-flow/min-cut algorithms as in Fig. 2. The two neighboring tetrahedron K_i, K_j can be expressed as two neighboring nodes in the graph i, j respectively. We use triangulation to express the element and small circle is the corresponding graph node for graph cuts computation. In Fig. 3, the weight on the edge (i, j) now is set to $d_{i,j} + \lambda|\kappa^n_{i,j}|$, which can be calculated from (5).

By graph cuts, the proposed energy functional could be minimized globally. However, the iteration result is not satisfactory. The global minimization technique, i.e. graph cuts, is not the main reason for the undesirable result. The reason is the inaccuracy of the curvature calculation. The tetrahedral mesh is intrinsically a much sparser representation compared with the grid representation. The calculation based on such a sparse framework could not obtain a desirable result. From the results of this stage, we can observe that some elements have been recovered. However it is far away from the ground truth. Hence, the local swap based on more precise calculation would be applied sequentially.

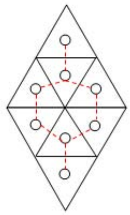


Fig. 2. Primal mesh and dual graph

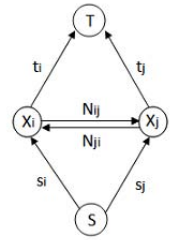


Fig. 3. Graph edge weight assignment

4 Feature Sensitive Local Minimization

Once the tetrahedral mesh is established, finding the embedded surface is equivalent to finding the labeling for all tetrahedra to partition the whole mesh. For each surface Γ, there is one corresponding labeling L. The labeling L is a local minimum with respect to the energy functional in (3) if $E(L) \leq E(L')$ for any L' "near to" L. In the environment of discrete tetrehedral mesh, the labelings near to L are those within the swap of a single tetrahedron. This move is usually referred to by standard moves in computer vision. One good example of the standard moves is simulated annealing [31]. In this section, the object of the swap operation is only changed from image pixels to volumetric tetrahedra.

When the explicit surface expression is obtained, we will adopt the method of [23]. The operator $\mathbf{K}$ maps a point x_i on the surface to the vector:

$$\mathbf{K}(x_i) = 2\kappa(x_i)\mathbf{n}(x_i),$$

where $\mathbf{n}(x_i)$ is the normal vector. The mean curvature normal operator $\mathbf{K}$, known as the Laplace-Beltrami operator for the surface S, is a generalization of Laplacian from flat spaces to manifolds [14]. By using the Gauss' theorem, the integral of the Laplace-Beltrami operator reduces to the following form:

$$\int\int_{\mathcal{A}_M} \mathbf{K}(x)dA = \frac{1}{2}\sum_{j\in N_1(i)} (cot\alpha_{i,j} + cot\beta_{i,j})(x_i - x_j), \tag{7}$$

where $\mathcal{A}_M$ is the 1-ring neighborhood surface area around the point x_i, $\alpha_{i,j}$ and $\beta_{i,j}$ are the two angles opposite to the edge in the two triangles sharing the edge (x_i, x_j) as in Fig. 4, and $N_1(i)$ is the set of 1-ring neighbor vertices of vertex i.

The mean curvature normal operator is:

$$\mathbf{K}(x_i) = \frac{1}{2\mathcal{A}_M}\sum_{j\in N_1(i)} (cot\alpha_{i,j} + cot\beta_{i,j})(x_i - x_j). \tag{8}$$

Therefore, the discretization of energy functional (3) can be written as we mentioned:

$$E(\Gamma_h) = \sum_i (d_i + \lambda|\kappa_i|)\Gamma_i, \tag{9}$$

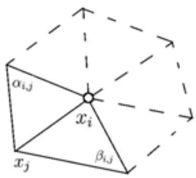


Fig. 4. 1-ring neighbors and angles opposite to an edge

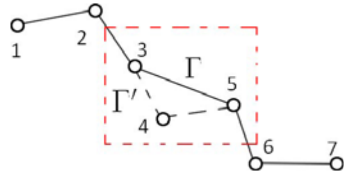


Fig. 5. The change of energy for each local swap

where

$$\Gamma_h = \bigcup \Gamma_i,\ d_i = \frac{1}{3}\sum_{j=1}^{3} d(v_{ij}),\ \kappa_i = \frac{1}{3}\sum_{j=1}^{3} \kappa(v_{ij}). \tag{10}$$

$v_{ij}, j = 1, 2, 3$ are three vertices of Γ_i.

For each known surface, the energy functional could be calculated explicitly by (9). Hence for the labeling swap of a single tetrahedral, the change of energy could also be calculated locally. This swap and comparing procedure is illustrated in Fig. 5. This local swap of a single tetrahedral has the counterpart in image processing field, i.e. stimulated annealing. What is worth mentioning is that stimulated annealing has been questioned of sensitive to the initial labeling. But in our cases, this local swap seldom encounters such problem since the initial labeling is determined by the global minimization stage. This good initial surface makes the local stage work less likely to stuck in a local minimum far away from the global one.

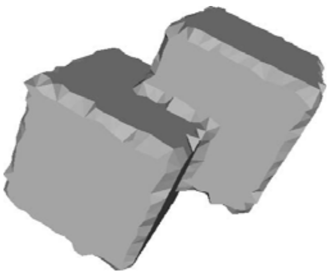
Algorithm 1. Local Swap Procedure

- Step1: Start with the initial surface Γ;
- Step2: For each element, swap it to the other partition and obtain the new surface Γ';
- Step3: Re-calculate the after energy of (9):$E(\Gamma')$, and compare $E(\Gamma')$ with $E(\Gamma)$,
  Step3.1: If $E(\Gamma') < E(\Gamma)$, confirm this swap;
  Step3.2: If $E(\Gamma') \geq E(\Gamma)$, undo this swap.

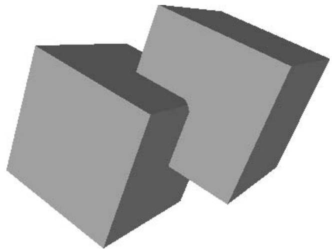
5 Numerical Experiments

In this section, various examples are presented to illustrate the effectiveness, efficiency and robustness of the proposed method. All experiments were conducted on a PC with Intel Pentium 4 CPU of 3.2GHz and 4GB memory and all examples were synthesized by ourselves. In the mesh generation stage, we adopted the incremental insert algorithm implemented by CGAL [1]. All surfaces are rendered by MeshLab. Only points locations were utilized in the algorithm.

We start by giving illustrative reconstruction examples in Fig. 6 which clearly show the advantage of using curvature information over total variation (TV). As is shown, our algorithm perfectly recovers the sharp edges of cubes. Total variation on the other hand, just recovers the smooth faces. Some more identical examples were also approached in [33,15], readers could compare the performance and find we have recovered the most features.



(a) TV reconstruction result



(b) Our reconstruction result

Fig. 6. The comparison of TV result and our result

For all these experiments, we set the value of λ to 0.1. Table 1 gives the sizes of data sets of four surface examples and corresponding CPU time counted in seconds. The first column gives the examples' names. The second column contains the numbers of data points P. The third column is the mesh generation time. The fourth column is the number of generated tetrahedra, the fifth the graph cuts iteration cycles, the sixth the graph cuts time, the seventh the local swap iteration cycles, and the eighth the local swap time.

Table 1. Statistics of four examples

Example	Data Set	Mesh Generation Time	Tetrahedra Number	Global Iteration Cycle	Total Graph Cuts Time	Local Iteration Cycle	Total Swap Time
two cubes	2472	24.1	175851	3	0.33	5	5.57
two spheres	2653	34.2	218119	3	0.42	4	0.31
female symbol	4530	41.8	307968	3	0.63	7	11.57
bolt	2357	34.9	223346	3	0.41	6	2.57

The following figures include data points sets in the first row and the reconstructed surfaces in the second row. In Fig. 7 from left to right, two geometries from basic boolean operation are shown: the unions of two cubes and two spheres. In Fig. 8, surface reconstruction results of four little complicated geometries are shown. In Fig. 9, two platonic solids, i.e. a dodecahedron and an icosahedron, a bucky ball model and a brilliant cut diamond are shown. In Fig. 10, four interesting CAD models are shown, all of which have sharp edges or corners.

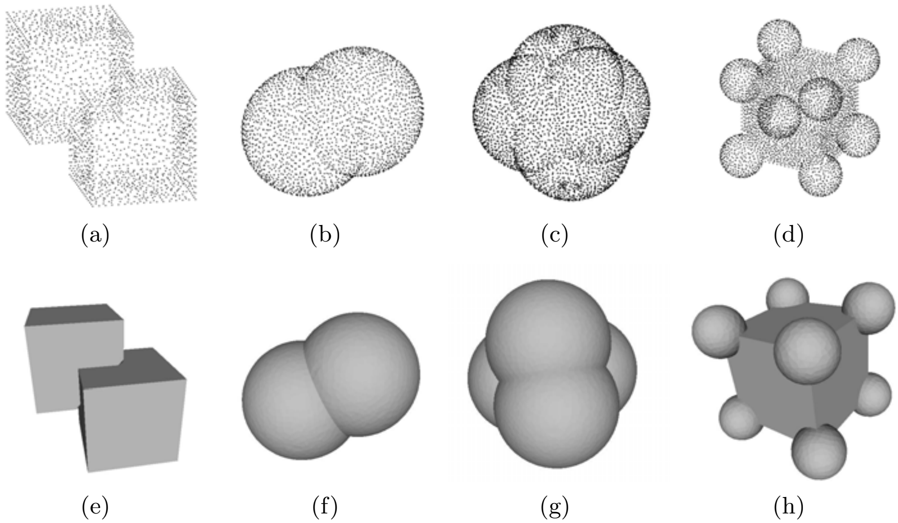


Fig. 7. The unions of geometries

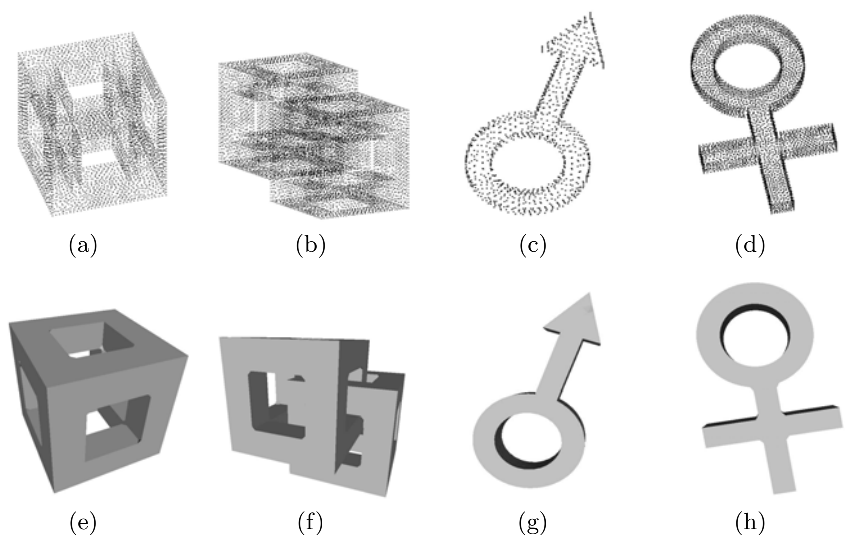


Fig. 8. A perforated cube, two tangling ones, male and female symbol models

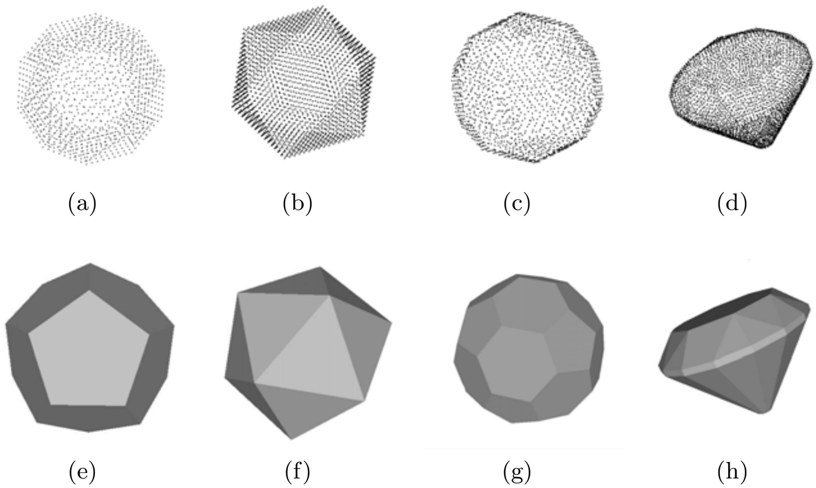


Fig. 9. Two platonic solids: dodecahedron and icosahedron, a buckyball and a brilliant cut diamond

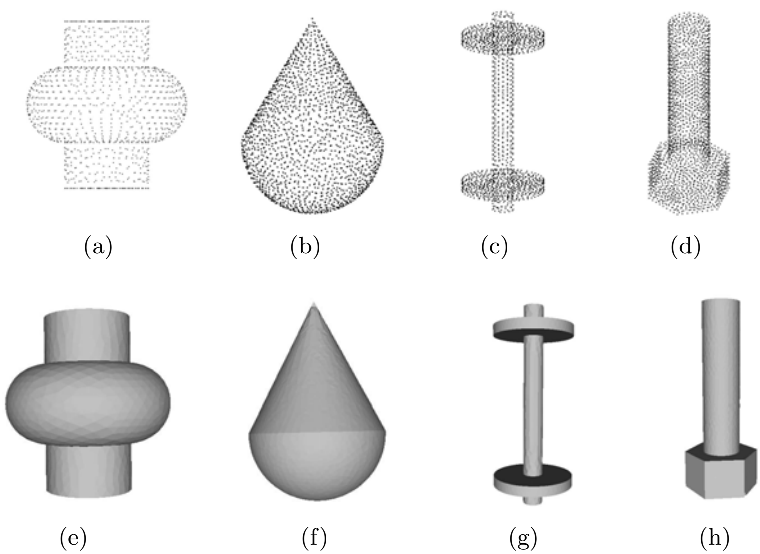


Fig. 10. Four CAD models from left to right are: power wheel, tear drop, dumb bell and bolt

From all our reconstructed examples, most features are recovered especially the sharp edges. The reconstructed surfaces are almost the ground truth surfaces except some place "over-enhancement". The accuracy of our feature-preserving operation will be improved and better performance could be expected in our future works.

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