8

Strip with a Circular Hole under Tension and Compression

8.1 Introduction

Elasto-plastic analysis of a strip with a hole under tension is a classical problem in computational plasticity. It was studied experimentally by Theocaris and Marketos (1964) and was first analyzed using finite element methods by Marcal and King (1967) and Zienkiewicz, Valliappan and King (1969). This problem was also studied by Narisawa (1991) and Yu and Zeng (Yu, 1998). The results were described by many authors, such as Zienkiewicz (1971), Zienkiewicz and Taylor (2000; 2009) and Yu (1998; 2004). The Huber-von Mises criterion for non-SD materials was used before 1994. The results can be adopted only for those materials which have identical strength both in tension and compression and the shear strength equals $\tau_y = 0.577\sigma_y$. It cannot be used for most materials, especially geomaterials. The twin-shear strength theory was used for elasto-plastic analysis of a strip with a hole under tension and compression for non-SD materials and SD materials by Yu and Zeng (Yu, 1998). The unified yield criterion was studied for elasto-plastic analysis of a strip with a hole under tension for non-SD materials by Yu (Yu, 2004). The analytical results obtained using various yield criteria are very different. The shape and size of the plastic zone as well as the slip angle are influenced strongly by the choice of the yield criterion. It is necessary to use a new efficient criterion. The effect of the yield criterion on analytical results in plasticity were observed by Humpheson and Naylor (1975), Zienkiewicz and Pande (1977), Mean and Hutchinson (1985), Tvergaard (1987), Narisawa (1991), Lee and Ghosh (1996), Hopperstad et al. (1998), Moin and Pankaj (1998), Wang and Fan (1998), Frieman and Pan (2000), Kuroda and Tvergaard (2000).

The unified strength theory and its implementation in a computer code provides us with a very effective approach for studying the effect of yield criterion for various engineering problems. A series of new computational results can be obtained by using the unified strength theory (see Chapter 5). These serial results can be adapted for more materials and structures.

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Elasto-plastic analysis of a strip with a circular hole for non-SD materials (having identical strength both in tension and compression), SD materials (strength difference in tension and compression) and concrete plates are described in this chapter. A simple experimental method is used for comparison. The analytical and experimental results clearly show the difference in plastic regions of a strip with a circular hole between tension and compression when SD materials are involved.

8.2 Plastic Analysis of a Strip with a Circular Hole for Non-SD Material

The finite element mesh and spread of the plastic zone using the Huber-von Mises criterion was given by Zienkiewicz (1971), as shown in Fig. 8.1.

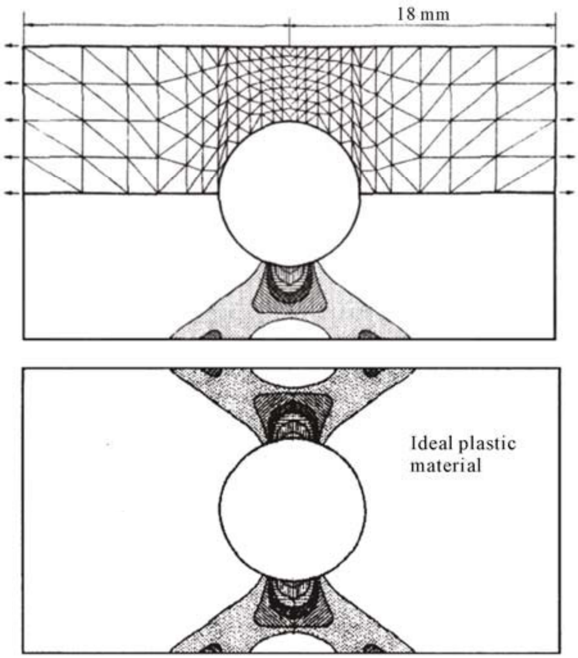


Fig. 8.1 FEM mesh and plastic zone of a strip with a hole (Zienkiewicz, 1971)

The growth in yielding around a circular hole of polymer alloys material was studied by Narisawa (1991). The spread of the plastic zone using FEM plasticity is shown in Fig. 8.2. It is obtained also by using the Huber-von Mises criterion. The shear yielding grows toward a region near θ=45º, as shown in Fig. 8.2. The extension of the plastic zone is the same as in Fig. 8.1.

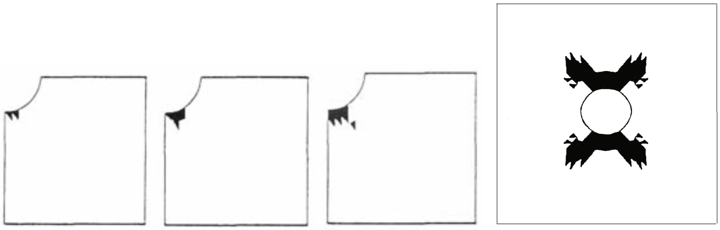


Fig. 8.2 The onset and growth of yielding around the circular hole (Narisawa, 1991)

A similar problem was also analyzed by Yu and Zheng based on the unified strength theory for comparison, as shown in Fig. 8.3. The material behaviour is E=7×10^4 MPa, μ=0.2, yield stress σ_s=243 MPa. The strip is considered as a plane stress problem.

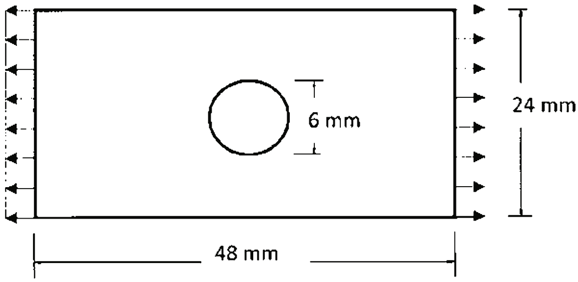
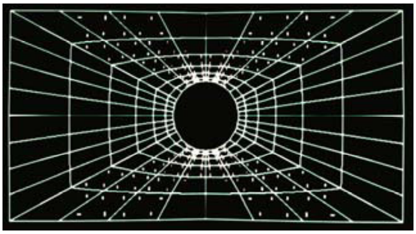


Fig. 8.3 A strip with a circular hole under tension

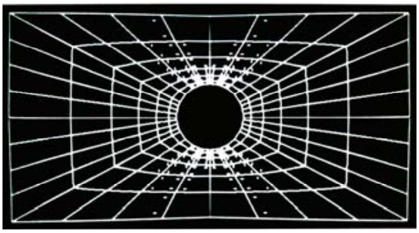
The configuration and the division into 96 isoparametric elements are shown in Fig. 8.4, in which the 4-nodes isoparametric element is used. The number of the nodes is 119. The elastic limit of a strip with a circular hole in terms of the unified yield criterion can be obtained after the first iteration. It requires a large number of iterations of non-linear computation for the plastic limit, until no further increase in the reaction at that point is achieved (furthermore, the numerical solution process will be divergent). The plastic limit load is approached progressively until that limit point where no convergence is achieved. The strip with a circular hole reaches the plastic limit in terms of the unified yield criterion with b=0 (i.e. the Tresca criterion) when p=140 MPa.

The size and shape of the plastic zone is of importance to the understanding of the failure of structures. The spread figures of plastic zones with a different parameter b of the unified yield criterion (i.e. different criteria) at a same load p=140MPa for a strip with a circular hole are shown in Figs. 8.4(a)~8.4(e).

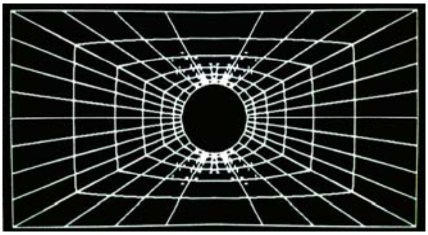
Figure 8.4(a) shows the spread of the plastic zone obeying the unified yield criterion when b=0 and $\alpha=\sigma_t/\sigma_c$=1, i.e. the single-shear yield criterion or the Tresca yield criterion. Fig. 8.4(b) shows the spread of the plastic zone obeying the unified yield criterion when b=1/4 and α= σ_t/σ_c=1; this is a new yield criterion introduced from the unified yield criterion. Figure 8.4(c) shows the spread of the plastic zone obeying the unified yield criterion when b=1/2 and α= σ_t/σ_c=1; this is a new yield criterion introduced from the unified yield criterion. Figure 8.4(d) shows the spread of the plastic zone obeying the unified yield criterion when b=3/4 and α= σ_t/σ_c=1; this is a new yield criterion introduced from the unified yield criterion. Figure 8.4(e) shows the spread of the plastic zone obeying the unified yield criterion when b=1 and α=σ_t/σ_c=1, i.e. the twin-shear yield criterion (Yu, 1961; 1983).



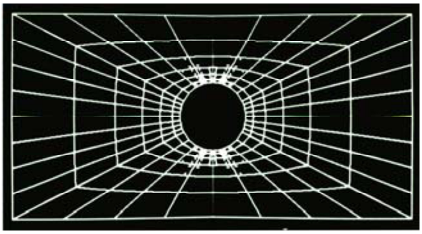
(a) UST with b=0 and α =1(Single-shear; criterion, Tresca criterion)



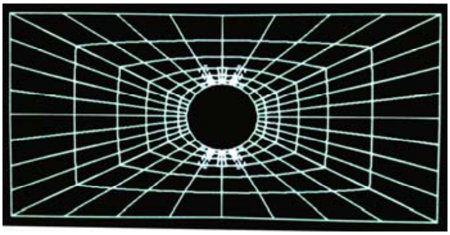
(b) UST with b=1/4 and α =1 (New criterion introduced from the unified yield criterion)



(c) UST with b=1/2 and α =1 (New criterion introduced from the unified yield criterion)



(d) UST with b=3/4 and α =1 (New criterion introduced from the unified yield criterion)



(e) UST with b=1 and α =1 (Twin-shear criterion)

Fig. 8.4 Distribution of plastic zone around a circular hole with different yield criteria

It is of interest to note that the spread of the plastic region is different under the same load for different yield criteria. It means the effect of the yield criterion on the spread of the plastic zone is evident. The plastic zone of twin-shear material (the unified yield criterion with b=1) is smallest. The plastic zone of single-shear material (the unified yield criterion with b=0), i.e. the Tresca yield criterion, is largest, as shown in Fig. 8.4. Obviously, the plastic zone of the unified yield criterion with b=1/4 is the median between the plastic zone of the unified yield criterion with b=0 and b=1/2. The plastic zone of the unified yield criterion with b=3/4 is the median between the plastic zone of the unified yield criterion with b=1/2 and b=1.

The unified yield criterion with b=1/2 and $\alpha=\sigma_t/\sigma_c=1$ may be regarding as a linear approximation to the Huber-von Mises yield criterion. The result of the unified yield criterion with b=1/2 and $\alpha=\sigma_t/\sigma_c=1$ is equivalent to the result of the Huber-von Mises criterion.

8.3 Elasto-Plastic Analysis of a Strip with a Circular Hole for SD Material under Tension

A similar example is examined by using the unified strength theory (Yu, 1991) and associated flow rule for a pressure sensitive material (SD material, $\sigma_t \neq \sigma_c$). Material parameters are chosen for the comparison with experimental materials. The experimental material is a hard blue polymer, the color of which can change to white when it reaches the plastic state. Material parameters are as follows: tensile yield stress σ_t=5.89 kN/cm^2, compressive yield stress σ_c=7.58 kN/cm^2, the ratio of tensile strength to compressive strength is $\alpha=\sigma_t/\sigma_c$=0.777.

The plastic zones of a strip with a circular hole for SD material under tension were also tested and calculated using the unified strength theory and unified elasto-plastic constitutive rule (Yu, 1998). The plastic zone based on the unified strength theory with b=0 (single-shear theory, i.e. the Mohr-Coulomb theory) is shown in Fig. 8.5. The plastic zone based on the unified strength theory with b=1 (twin-shear theory) is illustrated in Fig. 8.6. The plastic zones are different for single-shear material and twin-shear material.

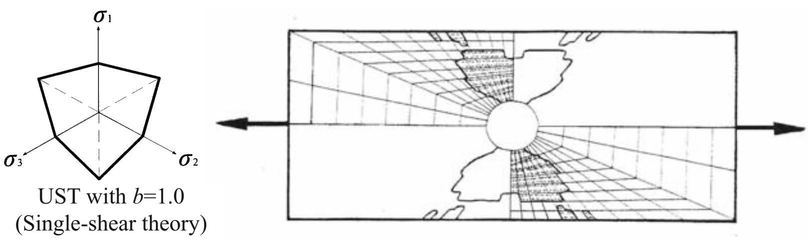


Fig. 8.5 Plastic zone based on the unified strength theory with b=0 (Single-shear theory)

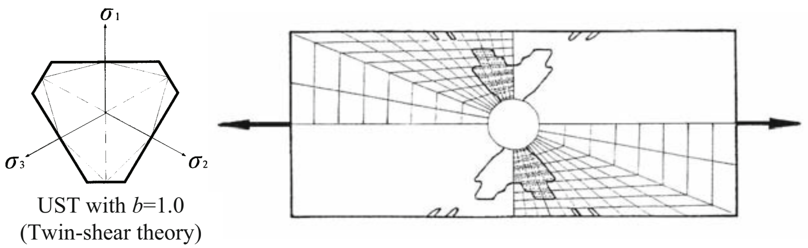
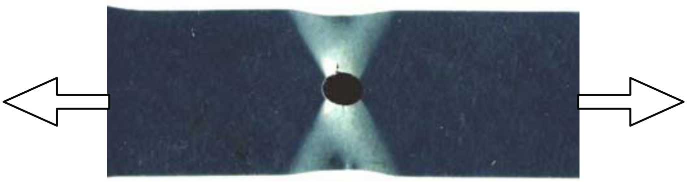
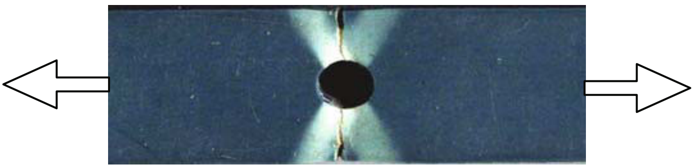


Fig. 8.6 Comparison of plastic zones of strip in tension with b=1 (Twin-shear theory)

The test results of two specimens are shown in Fig. 8.7. It can also be seen that the point of the plastic zone in the test result is close to the result of twin-shear theory.



(a) Smaller circular hole



(b) Bigger circular hole

Fig. 8.7 Test result of plastic zones of strip with two different circular holes under tension

The experimental results also present the difference in plastic zones under tension and under compression.

8.4 Plastic Zone of a Strip with a Circular Hole for SD Material under Compression

The plastic zones of a strip with a circular hole for SD material under compression were calculated by using the unified strength theory and the unified elasto-plastic constitutive rule. The plastic zone based on the unified strength theory with b=0 (single-shear theory, i.e. the Mohr-Coulomb theory, 1900) and b=1 (twin-shear strength theory (Yu, 1985)) are illustrated in Fig. 8.8. It can be seen that the plastic zones of a strip with a circular hole under compression are different for single-shear material and twin-shear material.

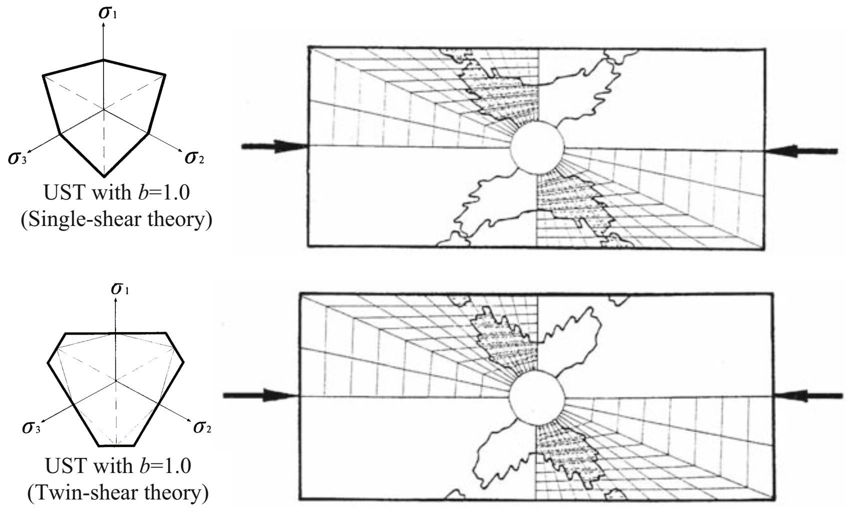


Fig. 8.8 Comparison of plastic zones of strip under compression using two theories

8.5 Comparison of Numerical Analysis with Experiments

As can be seen, the plastic zones under tension and compression are different for SD (strength difference in tension and in compression) materials. Figure 8.9 shows the computational results of the spread of the plastic zone of a strip using the twin-shear strength theory under tension. Figure 8.10 shows the computational results of the spread of the plastic zone of a strip using the twin-shear strength theory under compression.

An experimental study using polymer and computer image techniques was carried out by Yu et al. (1992). A strip with a circular hole subjects a tensile load, as show in Fig. 8.9 and Fig. 8.10. Sometimes deformation is accompanied by a change in color; the material deforms and changes color when it reaches a plastic state. The different colors and patterns of a strip in different regions show up clearly, which can be seen in Fig. 8.8. The plastic zone begins at the edge of the hole and spreads in four directions, as show in Fig. 8.9 and Fig. 8.10. The plastic zones are similar to the calculated results using the unified strength theory with b=1 (Twin-shear strength theory). In general, the slip angle is a compromise between the two angles of single-shear theory and twin-shear theory. Obviously, the slip angle is close to that of twin-shear theory, as shown in Fig. 8.6.

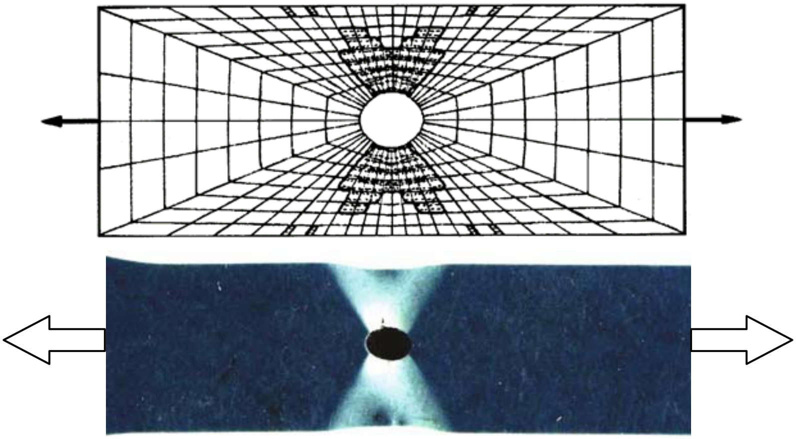


Fig. 8.9 Comparison of the computational results with experiments on plastic Zones under tension

A strip with a circular hole subjecting a load to compression is shown in Fig. 8.12. The plastic zone begins at the edge of circular hole and spreads in four directions, as shown in Fig. 8.12. The plastic zones are similar to the calculated results of a strip under compression using the unified strength theory with b=1 (Twin-shear strength theory).

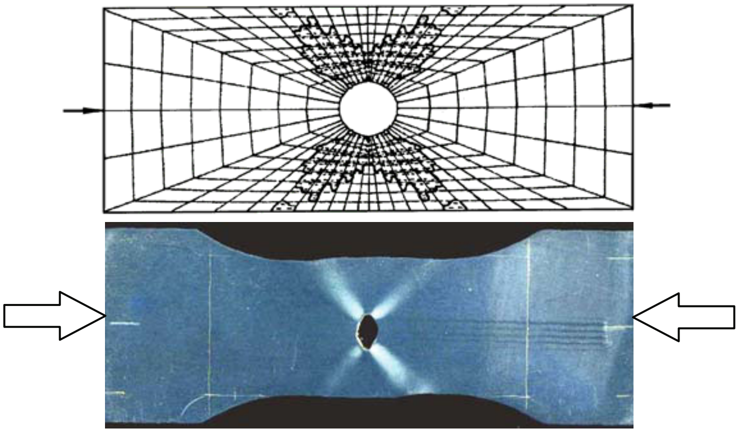


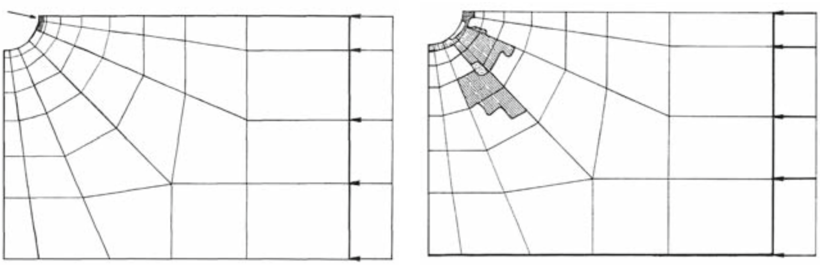
Fig. 8.10 Comparison of computational results with experiments on plastic zones under compression

The analytical result can be obtained by unified slip line field theory (Yu et al., 1997; Yu et al., 2006). The computational results are very close to those results obtained from the experiments and from slip line theory.

8.6 Elasto-Plastic Analysis of a Strip with a Circular Hole for a Special SD Material: Concrete

Concrete is an SD material. The compressive strength is much larger than the tensile strength. A concrete plate with a circular hole under pressure q is shown in Fig. 8.11. The thickness of the plate is 6cm, so it is considered to be a plane stress problem. The material parameters are: σ_t=2.7 MPa, σ_c=27 MPa, E=2.65×10^4 MPa, ν=0.19.

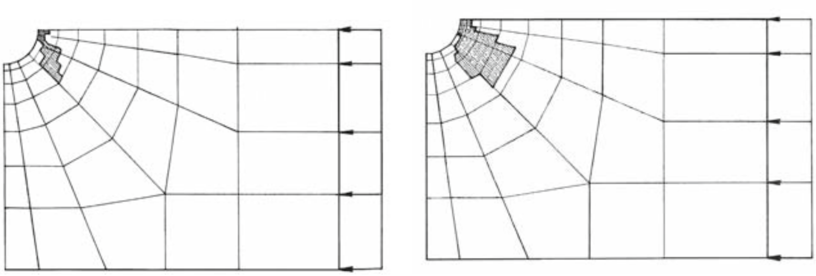
The single-shear theory (Mohr-Coulomb strength theory) and the twin-shear strength theory are used as the yield criteria for concrete (This analysis was done by Zheng in 1990). Due to symmetry, only a quarter of the plate is considered. The beginning and the spread of the plastic zone for single-shear material under q=228.6 kg/cm and q=400 kg/cm are shown in Fig. 8.11.



(a) q=228.6 kg/cm (b) q=400 kg/cm

Fig. 8.11 Spread of plastic zones for Mohr-Coulomb material

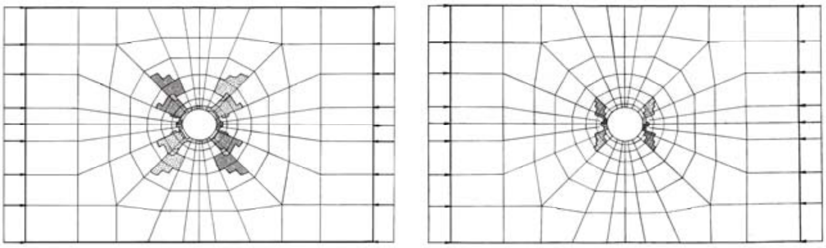
According to the twin-shear theory, no plastic zone occurs when q=228.6 kg/cm. The spread of the plastic zone for twin-shear material under q=400 kg/cm and q=571.4 kg/cm is shown in Fig. 8.12.



(a) q=400 kg/cm (b) q=571.4 kg/cm

Fig. 8.12 Spread of plastic zones for twin-shear material

Figure 8.13 shows the comparison between the two computational results for single-shear material and twin-shear material under the same load. It is shown that the plastic zone of the twin-shear material is smaller than that of single-shear material under the same load. The bearing capacity of twin-shear material is higher than that for single-shear material.



(a) Single-shear material (b) Twin-shear material

Fig. 8.13 Comparison of the spread of plastic zones for a concrete plate with a circular hole using the single-shear criterion and the twin-shear criterion under the same load

8.7 Brief Summary

The shape and size of the plastic zone, as well as the slip angle, are influenced strongly by the choice of the yield criterion. This can be seen from the tension and compression of a strip with a circular hole, as shown in Fig. 8.14 and Fig. 8.15. These results are also different for SD materials for the same strip under tension and under compression. The single-parameter criterion can only be used for non-SD materials. SD materials, however, have to use the two-parameters criterion.

The computational results for a strip under compression and the experimental results are summarized in Fig. 8.14. The comparisons show that the results for twin-shear theory are closer to the experimental results than the results for single-shear theory.

The computational results for a strip under tension and the experimental results are summarized in Fig. 8.15. The comparisons show that the results for twin-shear theory are closer to the experimental results than the results for single-shear theory.

It is very important how we choose a reasonable strength theory (yield criteria or material model in FEM code) in research and design. We have to determine the bounds and region of the failure criteria before the research. The two bounds are important. The two bounds and region of the yield loci for non-SD materials and SD materials are shown in Fig. 8.16. The lower bound (inner bound) is the yield locus of the single-shear theory, and the upper bound (outside bound) is the twin-shear theory.

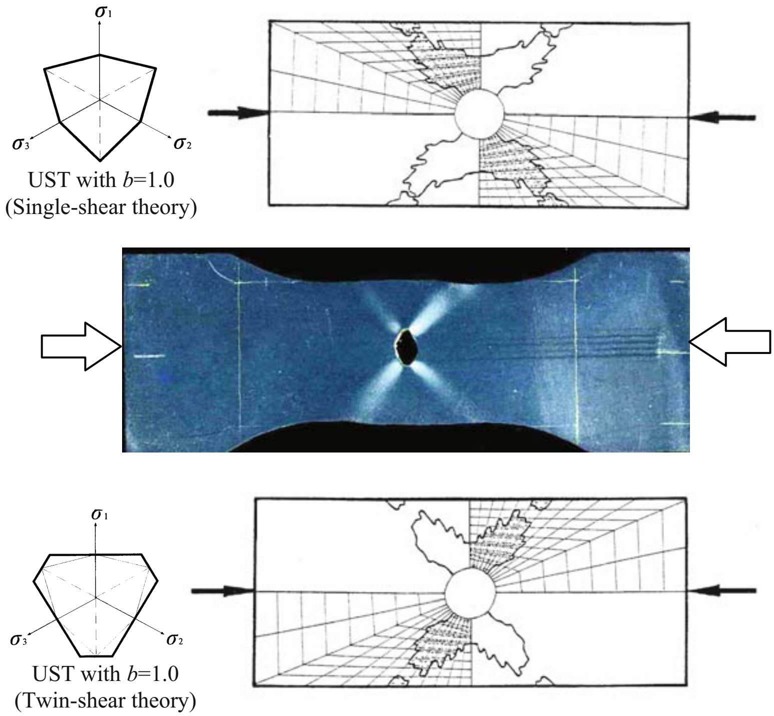


Fig. 8.14 Plastic zones of a strip with a circular hole under compression based on single-shear theory and twin-shear theory

The results of research and design depend strongly on the choice of strength theory in most cases. The selection of the correct strength theory becomes even more important than the calculations, as indicated by Sturmer, Schulz and Wittig (1991) and others. The bearing capacity of structures, the forming limit of FEM simulations, the size of plastic zones, the orientation of the shear band and plastic flow localization *etc*. will be much affected by the application of strength theory. More experimental results regarding the strength of materials in a complex stress state and a more precise choice of the strength theory applied are necessary in research and engineering applications.

As use of FEM and other numerical analysis expands in engineering design with increased access to computers, it becomes important that strength theory (yield criterion, failure criterion) related stress be carefully chosen. In adopting a criterion for use it is important that at least as much concern be directed to the physics of the problem and to the limitation of the criteria. When it becomes necessary to adopt a criterion for use, it is important to experimentally check the criterion, or to investigate the experimental data in the literature. If this is not done, then very exact numerical procedures or commercial codes can lead to completely worthless results (Hopperstad et al., 1998).

The unified yield criterion and the unified strength theory provide us with systematic yield criteria, an effective approach and a powerful tool for studying these effects. More results can be obtained using strength theory and the associated flow rule, which can be adapted for different materials and structures.

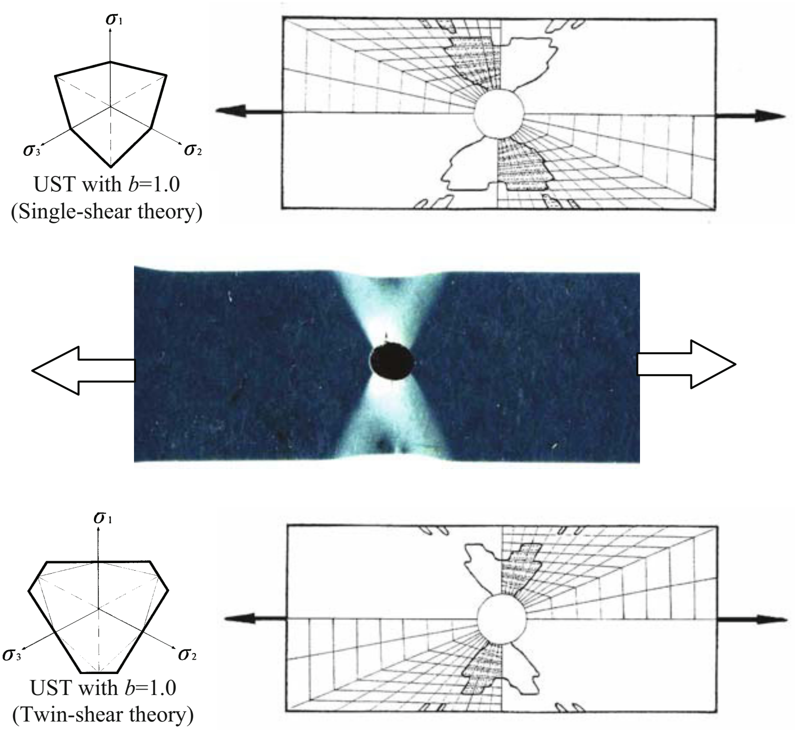
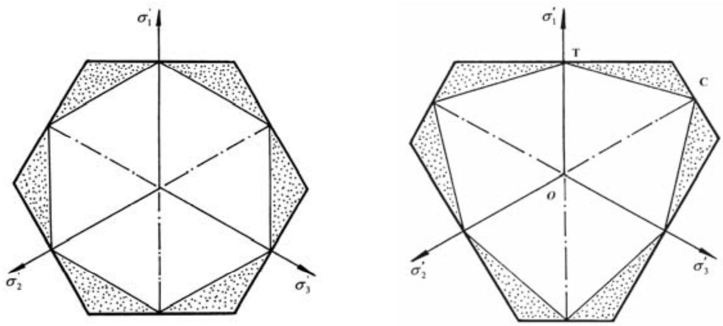


Fig. 8.15 Plastic zones of a strip with a circular hole under tension based on single-shear theory and twin-shear theory



(a) For non-SD materials (b) For SD materials

Fig. 8.16 Bounds and region of yield loci

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