6

Implementation of the Unified Strength Theory into FEM Codes

6.1 Introduction

The yield criteria and various material models have been implemented into elasto-plastic programs and most current commercial FEM systems. In some systems, only the Huber-von Mises criterion, Drucker-Prager criterion and the Mohr-Coulomb criterion were implemented. Sometimes, the multi-parameters criteria for geomaterials and concrete structures are also used. The twin-shear strength theory has been implemented into special finite element programs since 1990 (Yu and Meng, 1990; Yu and Li, 1991; Yu, 1992; Yu et al., 1992). Only some single models, however, are used in several programs, and only one result can be obtained by using the single material model, which can be adopted only for one kind of material. Such models as the Tresca model can be used only for non-SD materials (those materials with the same strength both in tension and in compression), and the shear strength equals half of the tensile strength $\tau_y=0.5\sigma_y$. The Huber-von Mises model can be used for non-SD materials with the shear strength $\tau_y=0.577\sigma_y$. The twin-shear yield criterion (Yu, 1961) or the maximum deviatoric stress criterion (Haythornthwaite, 1961), the shape distortion criterion (Schmidt-Ishilinsky, 1932-1940), or the matched circular criterion (Hill, 1950) can be used only for non-SD materials, and with the shear strength $\tau_y=0.667\sigma_y$. There is no relationship between these material models.

The unified yield criterion and the unified strength theory have been implemented and applied to several plasticity and engineering problems (Yu et al., 1992; Yu et al., 1993; Yu and Zeng, 1994; Yu et al., 1997; 1999). The singularities at the corners of the single-shear series of strength theory, twin-shear series of strength theory and the singularity of the unified strength theory have been overcome by using a unified numerical procedure.

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Yu and his research group wrote a special elasto-plastic FEM program. It is called the UEPP-Unified Elasto-Plastic Program (Yu et al., 1993; Yu et al., 1992; Yu and Zeng, 1994; Yu and Lu, 1994; Yu and Yang, 1997; Yu et al., 1999). The feature of the UEPP is that the unified strength theory was implemented into the finite element method code. UEPP includes two codes, i.e. UEPP-2D for plane stress, plane strain and axial-symmetric problems and UEPP-3D for three-dimensional problems. The material models are increasing and forming a series of systematical and effective constitutive relations for practical use. A detailed description of the unified strength theory and UEPP can be seen in the books “New System of Strength Theory” (Yu, 1992, in Chinese) and “Twin-Shear Theory and its Applications” (Yu, 1998, in Chinese). Some examples can be found in the papers in English (Yu et al., 1992; Yu and Zeng, 1993; Yu et al., 1994, Yu et al., 1999; Yu, 2001; Yu et al., 2001) and Chinese papers (Yu and Zeng, 1994; Yu and Lu, 1994; Yu et al., 1997).

Recently, the unified strength theory was also implemented into the general FEM code, such as ABAQUS, AutDYN and FLAC-3D at Nangyang Technological University, Singapore; Griffith University in Australia, the National Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Science; Beijing Sci. and Tech. University, Sichuan University, Jinan University in China. The work was conducted by Shen (1993), Quint Co. (1993; 1994), Li and Ishii (1994; 1998), Liu et al. (1994), Wang (1998), Fan and Qiang (2001), Zhang et al. (2001), Zhou (2002), Zhang CQ (2005), Shao and Qian (2007), Shao et al. (2007), Yang (2008), Li et al. (2008), Li (2008), Wang et al. (2008), Zhang et al. (2008). The details can be seen in chapter 1. Table 1.1 gives some cases of yield criteria in FEM codes

The unified strength theory can be applied and implemented into various elasto-plastic programs. It is worth showing that most parts of the elasto-plastic program are the same as the conventional program, only the subroutine of yield criteria (subroutine “INVAR” to calculate equivalent stresses), the subroutine of flow vector and the subroutine of the corner (subroutine “YIELD” and “FOLWPL” to calculate flow vector) are different. The details of the finite element method in plasticity can be seen in Hinton and Owen (1977), Owen and Hinton (1980), Lewis and Schrefler (1987), Owen et al., (1989), Smith and Griffiths (2004). An elasto-plastic program in 2D and an elasto-viscoplastic program in 2D are presented in chapter 7 and chapter 8 of the book by Owen and Hinton (1980). A 2D non-linear thermo-elastoplastic consolidation program, PLASCON, is described in detail in chapter 9 by Majorana in the book of Lewis and Schrefler (1987). The unified strength theory is easy to implement in these programs.

A series of results can be obtained by using the unified strength theory for various problems. It can be applied in various materials such as metal, plastic,rock, soil and concrete. Therefore, it can not only be employed in strength calculation of metal structures and machine parts in mechanical engineering, electrical engineering, chemical engineering, aeronautical engineering and railway engineering, but can also be used in elastic and plastic analysis of geological and concrete structures in civil engineering, geological engineering and hydraulic engineering. In the meantime, it can be applied in the computer-aided teaching of courses such as mechanics of materials, plasticity, plastic ultimate analysis of structures, finite element methods and geomechanics.

It contains:

1) Elastic limit analysis;
2) Elasto-plastic analysis of structures;
3) Plastic limit analysis of structures;
4) Elasto-visco-plastic analysis;
5) Eigenvalue analysis;
6) Elasto-plastic transient analysis;
7) Earthquake response analysis.

6.2 Bounds of the Single Criteria for Non-SD Materials

The stress state of an arbitrary infinitesimal element can be described in three principal stresses. We call the two or three principle stresses state the complex stress state. The strength of material under the complex stress state is an important and complicated problem. A large amount of research was conducted by researchers all over the world, and various yield or failure criteria were proposed. The three main yield criteria for metal materials with the same tensile and compressive strength are:

1). Single-shear yield criterion (Tresca, 1864) used for those materials: $\tau_y = 0.5\sigma_y$;

2). Three-shear yield criterion (Huber-von Mises, 1904-1913), used for those materials: $\tau_y = 0.577\sigma_y$;

3). Twin-shear yield criterion (Yu, 1961) or maximum deviatoric stress criterion (Haythornthwaite, 1961) used for those materials: $\tau_y = 0.667\sigma_y$

These three yield loci in π-plane are shown in Fig. 6.1.

The intermediate principle stress σ_2 was not taken into account in the Tresca yield criterion. Many studies were devoted to the research of the effect of the intermediate principle stress. The intermediate principle stress σ_2 was taken into account through the consideration of the intermediate principle shear stress by Yu Maohong in 1961. The mathematical modeling of the twin-shear stress yield criterion is as follows:

$$f = \tau_{13} + \tau_{12} = C \qquad \text{when} \quad \tau_{12} \geq \tau_{23} \tag{6.1a}$$

$$f' = \tau_{13} + \tau_{23} = C \qquad \text{when} \quad \tau_{12} \leq \tau_{23} \tag{6.1b}$$

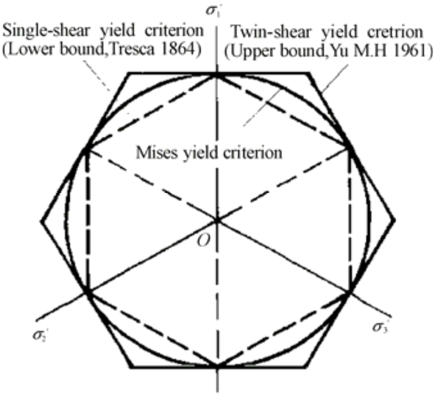


Fig. 6.1 Three yield loci in π plane (for non-SD materials)

where $\tau_{13}=\dfrac{\sigma_1-\sigma_3}{2}$, $\tau_{12}=\dfrac{\sigma_1-\sigma_2}{2}$, $\tau_{23}=\dfrac{\sigma_2-\sigma_3}{2}$, C is material parameter.

The twin shear yield criteria can be represented in principal stresses $\sigma_1,\sigma_2,\sigma_3$:

$$f=\sigma_1-\frac{1}{2}(\sigma_2+\sigma_3)=\sigma_{\mathrm{y}} \qquad \text{when} \quad \sigma_2\le\frac{1}{2}(\sigma_1+\sigma_3) \tag{6.2a}$$

$$f'=\frac{1}{2}(\sigma_1+\sigma_2)-\sigma_3=\sigma_{\mathrm{y}} \qquad \text{when} \quad \sigma_2\le\frac{1}{2}(\sigma_1+\sigma_3) \tag{6.2b}$$

Here σ_{y} is the yield stress under tension and compression. Single-shear yield criterion, three-shear yield criterion and the twin-shear yield criterion are suitable for materials with the same strength both in tension and in compression. Only one parameter is needed for such materials.

6.3 Bounds of the Failure Criteria for SD Materials

There are four kinds of failure criteria for SD materials. They are suitable for materials with different strengths in tension and in compression such as rocks and concrete. i.e.

(1) Single-shear failure criterion (Mohr, 1900; Coulomb, 1773) is the lower bound of the convex criteria.

(2) Twin-shear failure criterion (Yu, 1983) is the upper bound of the convex criteria.

(3) Drucker-Prager criterion (1952) is a circle.

(4) Curved criteria.

Actually, Mohr-Coulomb's single-shear criterion and the Drucker-Prager criterion are the generalizations of the Tresca criterion and Huber-von Mises criterion. It is shown that the Mohr-Coulomb single-shear criterion only considers two principle stresses σ_1 and σ_3, the intermediate principle stress σ_2 is not taken into account. In the meantime, the Drucker-Prager criterion is not in good agreement with experiments for geomaterials. Yu (1983) generalized Eq. (6.1a) and (6.1b) and applied them to SD materials. The mathematical modeling of the generalized twin-shear criterion is

$$F = \tau_{13} + \tau_{12} + \beta(\sigma_{13} + \sigma_{12}) = C, \quad \text{when}\ \tau_{12} + \beta\sigma_{12} \geq \tau_{23} + \beta\sigma_{23} \tag{6.3a}$$

$$F' = \tau_{13} + \tau_{23} + \beta(\sigma_{13} + \sigma_{23}) = C, \quad \text{when}\ \tau_{23} + \beta\sigma_{23} \leq \tau_{23} + \beta\sigma_{23} \tag{6.3b}$$

The mathematical formulae in terms of the three principal stresses are

$$F = \sigma_1 - \frac{\alpha}{2}(\sigma_2 + \sigma_3) = \sigma_t, \quad \text{when}\ \ \sigma_2 \leq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \tag{6.4a}$$

$$F' = \frac{1}{2}(\sigma_1 + \sigma_2) - \alpha\sigma_3 = \sigma_t, \quad \text{when}\ \ \sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}\ \text{时} \tag{6.4b}$$

where $\sigma_{13} = \dfrac{\sigma_1 + \sigma_3}{2}$, $\sigma_{12} = \dfrac{\sigma_1 + \sigma_2}{2}$, $\sigma_{23} = \dfrac{\sigma_2 + \sigma_3}{2}$, β and C are material constants. The comparison of failure loci in the π plane of twin-shear strength theory (TS theory) and Mohr-Coulomb criterion (Single-shear strength theory, SS theory), three-shear theory (Octahedral-shear theory, OS theory) is shown in Fig. 6.2.

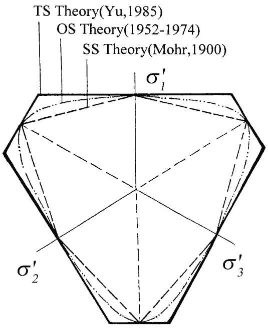


Fig. 6.2 Yield loci of three main theories in π plane (SD materials)

It can be seen that two parameters are needed for the SD material. It is noted that only some materials can be applied for each of the yield criteria.

6.4 Unification of the Yield Criteria for Non-SD Materials and SD Materials

The mathematical modeling of the unified strength theory can be expressed as follows:

$$F = \tau_{13} + b\tau_{12} + \beta\left(\sigma_{13} + b\sigma_{12}\right) = C, \quad \text{when} \quad \tau_{12} + \beta\sigma_{12} \geq \tau_{23} + \beta\sigma_{23} \tag{6.5a}$$

$$F' = \tau_{13} + b\tau_{23} + \beta\left(\sigma_{13} + b\sigma_{23}\right) = C, \quad \text{when} \quad \tau_{12} + \beta\sigma_{12} \leq \tau_{23} + \beta\sigma_{23} \tag{6.5b}$$

The mathematical expression of the unified strength theory in terms of principal stresses σ_1, σ_2 and σ_3 is:

$$F = \sigma_1 - \frac{\alpha}{1+b}\left(b\sigma_2 + \sigma_3\right) = \sigma_t, \quad \text{when} \quad \sigma_2 \leq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \tag{6.6a}$$

$$F' = \frac{\alpha}{1+b}\left(\sigma_1 + b\sigma_2\right) - \alpha\sigma_3 = \sigma_t, \quad \text{when} \quad \sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \tag{6.6b}$$

where b is the unified strength theory parameter, the limit loci of the unified strength theory in the π plane is shown in Fig. 6.3. The figure of the yield loci shows the eleven special cases of the unified yield criterion.

There are some special cases of Eqs. (8.5) and (8.5′) as follows:

(a) $b = 1, \quad \beta \neq 0\ (\alpha \neq 1)$, Yu twin-shear strength theory (1983);

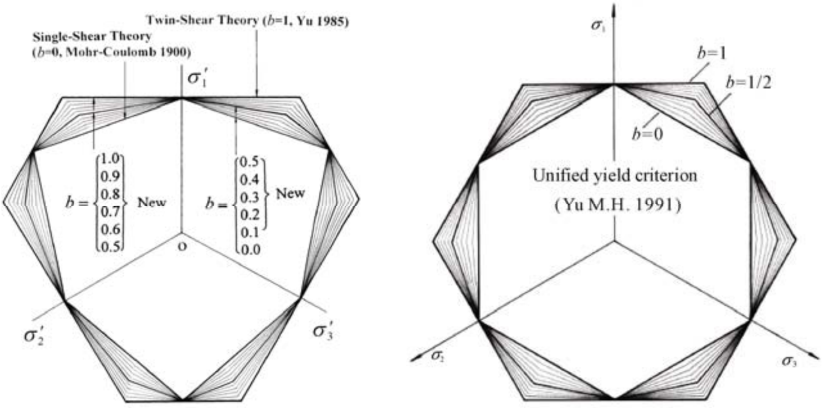
(b) $b = 0, \ \beta \neq 0\ (\alpha \neq 1)$, Mohr-Coulomb single-shear strength theory (1900);

(c) $b = 1, \beta = 0\ \ (\alpha = 1)$, Twin-shear yield criterion (Yu, 1961), or the maximum deviatoric stress criterion (Haythornthwaite, 1961), or the sharp distortion criterion (Schmidt, 1932; Ishilinski, 1940);

(d) $b = 0, \beta = 0\ (\alpha = 1)$, Tresca single-shear yield criterion (1864);

(e) $b = (\sqrt{3} - 1)/2, \ \beta = 0\ (\alpha = 1)$, Approximated Huber-von Mises criterion (1913).

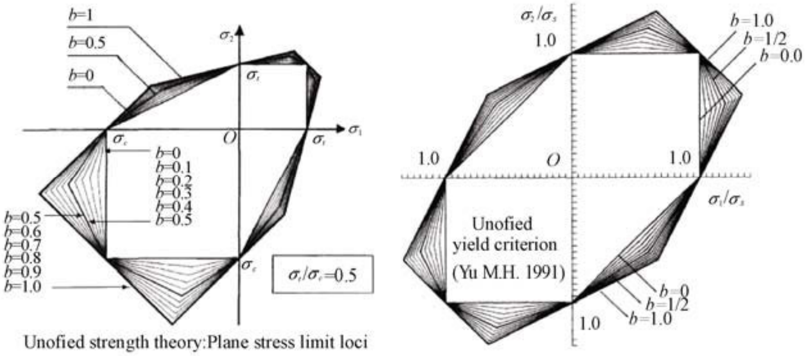
It can be seen that all the commonly used strength criteria are special cases of the unified strength theory. There is a series of yield criteria suitable for non-SD materials when b varies from 0 to 1. These criteria lie between the Mohr-Coulomb single-shear theory and Yu Mao-hong (1985) twin-shear strength theory.



(a) $\alpha \neq 1$ ($\sigma_t \neq \sigma_c$) (b) $\alpha = 1$ ($\sigma_t = \sigma_c$)

Fig. 6.3 Limit loci of unified strength theory in π-plane

Yield loci of the unified strength theory in the plane stress state (σ_1-σ_2 plane) are shown in Fig. 6.4.



(a) $\alpha \neq 1$ ($\sigma_t \neq \sigma_c$) (b) $\alpha = 1$ ($\sigma_t = \sigma_c$)

Fig. 6.4 Limiting loci of the unified strength theory in plane stress state (σ_1-σ_2 plane)

It is shown that the unified strength theory has a relatively clear physical meaning and simple mathematical formulae. The frequently used criteria are its special cases or approximation forms. The theory is suitable for most material and is readily used in engineering. The material models of unified strength elasto-plastic programs are based on the unified strength theory.

6.5 Material Models

The material models can be implemented and used as follows.

1) Tresca yield criterion (single-shear criterion) is equal to the unified yield criterion with b=0.

2) Huber-von Mises yield criterion (three-shear criterion) is equivalent to the unified yield criterion with α=1 and b=0.366.

3) Twin-shear yield criterion (twin-shear criterion) is equal to the unified yield criterion with b=1.

4) Mohr-Coulomb failure criterion (single-shear theory) is equal to the unified strength theory with b=0.

5) Drucker-Prager failure criterion (three-shear theory)

6) Twin-shear failure criterion (twin-shear theory) is equal to the unified strength theory with b=1.

7) UST (unified strength theory) with any α, b, $0\leqslant\alpha\leqslant 1$, $0\leqslant b\leqslant 1$.

8) UST with α=1, b=0 is equal to the Tresca criterion.

9) UST with α=1, b=0.25 is a new one.

10) UST with $\alpha = 1, b = (\sqrt{3}-1)/2$ is equal to the Huber-von Mises criterion.

11) UST with α=1, b=0.5 is equivalent to the Huber-von Mises criterion.

12) UST with α=1, b=0.75 is a new one.

13) UST with α=1, b=1 is equal to the twin-shear yield criterion, or maximum deviatoric stress yield criterion.

14) UST with any α and b=0 is equal to the single-shear theory (the Mohr-Coulomb theory).

15) UST with any α and b=1/4 is a new one.

16) UST with any α and b=1/2 is a new one.

17) UST with any α and b=3/4 is a new one.

18) UST with any α and b=1 is equal to the twin-shear strength theory.

19) Twin-shear three parameter criterion.

20) Three parameter unified strength theory.

21) Others.

The yield loci of the five typical criteria of the unified yield criterion with α=1 and b=0, b=1/4, b=1/2, b=3/4 and b=1 for non-SD materials (material models 8, 9, 11, 12 and 13) are illustrated in Fig. 6.5. The yield loci of the five typical criteria of the unified strength theory with b=0, b=1/4, b=1/2, b=3/4 and b=1 for SD materials (material models 14 to 18) are illustrated in Fig. 6.6.

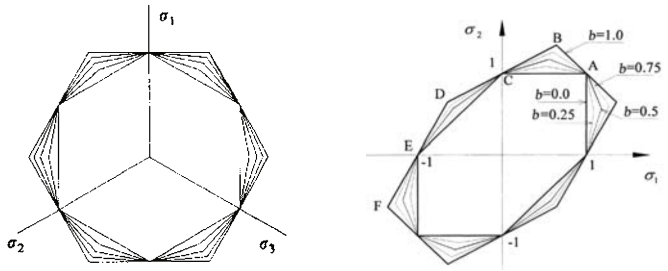


Fig. 6.5 Yield loci of the five typical criteria of the unified yield criterion for non-SD materials

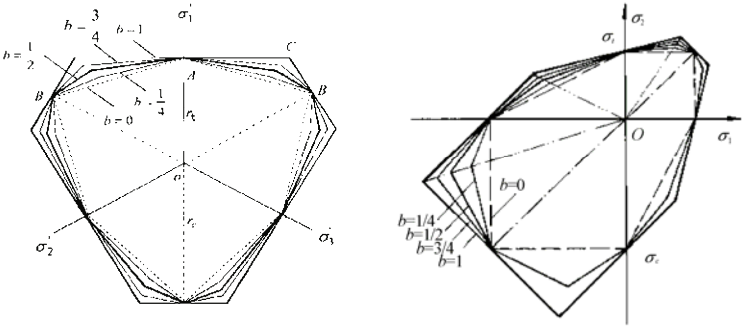


Fig. 6.6 Yield loci of five typical criteria of the unified strength theory for SD materials

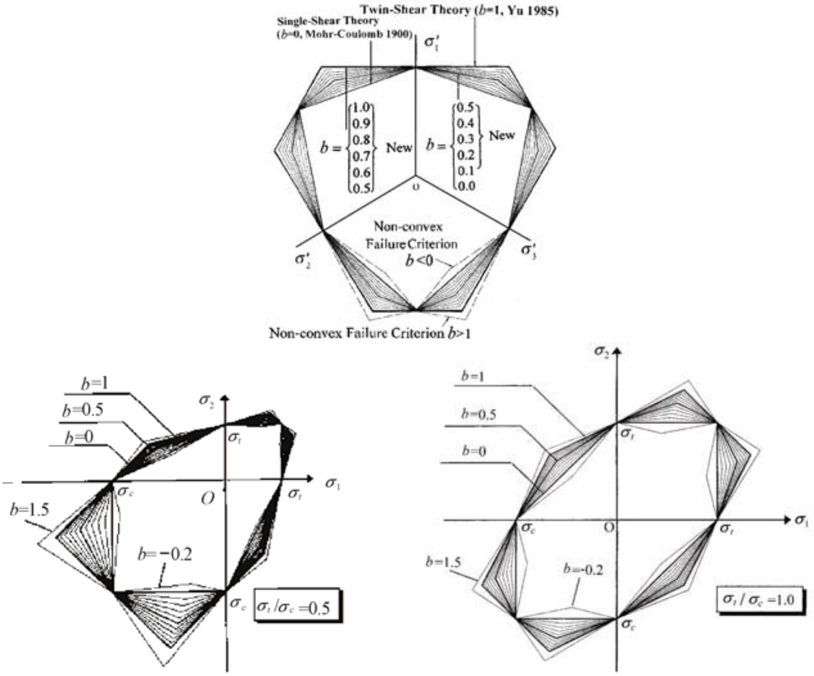


Fig. 6.7 Convex yield loci extend to non-convex yield loci of the unified strength theory

6.6 Program Structure and its Subroutines Relating to the Unified Strength Theory: INVARY, YIELDY, FLOWVP

The flow chart for calculating the elasto-plastic problems is shown in Fig. 6.8.

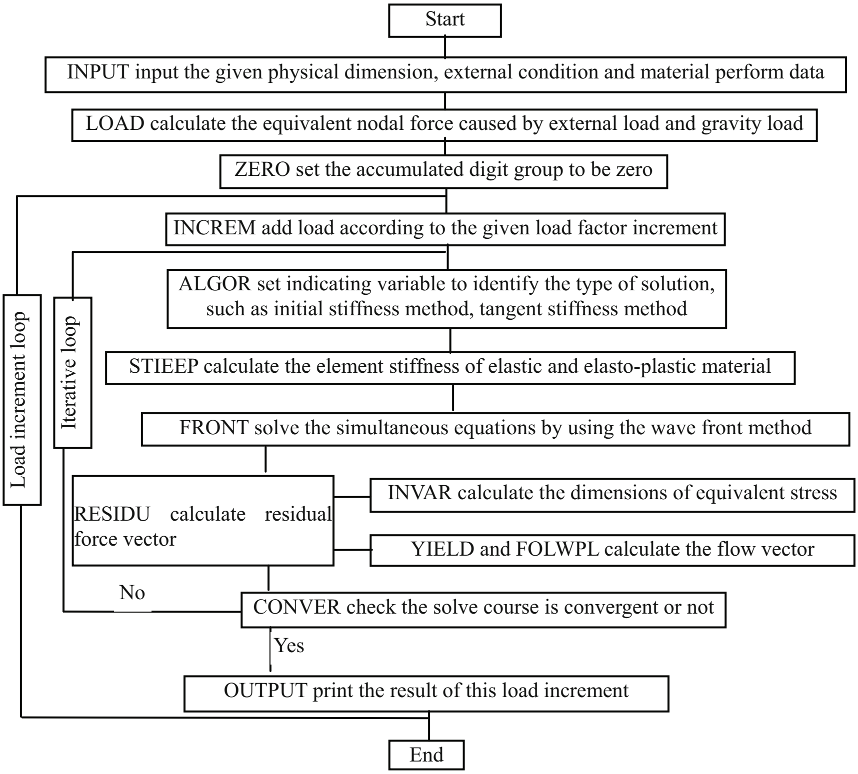


Fig. 6.8 The flow chart for calculating the elasto-plastic problems

Several subroutines about the unified strength theory and its flow vector will be introduced in this section, including INVAR, INVARY, YIELDY and CRITEN. When these subroutines are implemented in any elasto-plastic programs, the unified strength elasto-plastic analysis can be conducted. Therefore, a series of ordered results can be obtained.

6.6.1 Subroutine "Invar"

The purpose of this subroutine is to calculate the stress functions which indicate whether it is the initial or succeeding plastic deformation when considering various yield criteria. The yield criteria include various criteria such as Huber-von Mises, Drucker-Prager, Tresca, Mohr-Coulomb and a series of the yield criteria introduced from the unified strength theory.

```
SUBROUTINE    "INVAR" (LPROP,NCRIT,PROPS,STEFF,SMEAN,NPROP,THETA,
VARJ2,VARJ3,YIELD,IND,IELEM,GASH,MATNO)
C************************************************************
C***    EVALUATES THE STRESS INVARIANTS AND THE CURRENT VALUE
C************************************************************
IMPLICIT REAL*8(A-H,O-Z)
COMMON/CMN01/NELEM,NPOIN,NVFIX,NMATS,NDOFN,
.                        NGAUS, NGAS2,NALGO,NINCS,NTOTV,
.                        NTOTG,NSTRE, NSTR1,NTYPE,NREST
COMMON/CMN06/FACTDP
DIMENSION NCRIT(NMATS),PROPS(NMATS,NPROP),MATNO(NELEM,3)
PI=3.1415926535
ROOT3=1.73205080757
STEFF=DSQRT(VARJ2)
IF(STEFF.EQ.0.0) GO TO 31
COST3=3.0*ROOT3*VARJ3/(2.0*VARJ2*STEFF)
GO TO 20
31          COST3=0.0
20          CONTINUE
IF(COST3.LT.-1.0) COST3=-1.0
IF(COST3.GT.1.0) COST3=1.0
THETA=DACOS(COST3)/3.0
NCRT1=NCRIT(LPROP)
NTMP=MATNO(IELEM,2)/10
IF((NTMP.EQ.3).OR.(NTMP.EQ.4)) GO TO 1
YIELD=DSQRT(GASH)
RETURN
C
C*** VON MISES
C
1           IF(NCRT1.EQ.2) THEN
YIELD=ROOT3*STEFF
RETURN
ENDIF
C
C*** DRUCKER-PRAGER
C
IF(NCRT1.EQ.5) THEN
PHIRA=PROPS(LPROP,7)*0.017453292
SNPHI=DSIN(PHIRA)
C*** COMPRESSIVE CONE
C               YIELD=6.0*SMEAN*SNPHI/(ROOT3*(3.0-SNPHI))+STEFF
C*** TENSILE CONE
YIELD=6.0*SMEAN*SNPHI/(ROOT3*(3.0+FACTDP*SNPHI))+STEFF
RETURN
ENDIF
```

```
C
C*** CALL UNIFIED TWINSHEAR STRESS CRITERION
C
IF(NCRT1.LE.18) THEN
BTWIN=PROPS(LPROP,8)
ARLFA=PROPS(LPROP,7)/PROPS(LPROP,5)
SMEAM=SMEAN*(1.0-ARLFA)
CALL INVARY(SMEAM,ARLFA,STEFF,THETA,BTWIN,IND,YIELD)
RETURN
ENDIF
IF(NCRT1.EQ.19) THEN
BTWIN=PROPS(LPROP,8)
CTWIN=(1.0+BTWIN)*PROPS(LPROP,5)*PROPS(LPROP,7)/
.     (PROPS(LPROP,5)+PROPS(LPROP,7))
ATWIN=CTWIN*(PROPS(LPROP,9)-PROPS(LPROP,5))/(PROPS(LPROP,5)*
.     PROPS(LPROP,9))
BEITA=2.0*(CTWIN/PROPS(LPROP,7)-ATWIN)/(1.0+BTWIN)-1.0
ARLFA=(1.0-BEITA)/(1.0+BEITA)
SMEAM=((1.0-ARLFA)+3.0*(1.0+ARLFA)*ATWIN/(1.0+BTWIN))*SMEAN
CALL INVARY(SMEAM,ARLFA,STEFF,THETA,BTWIN,IND,YIELD)
RETURN
ENDIF
IF(NCRT1.GE.20) CALL USERINVR(LPROP,NCRIT,PROPS,SMEAN,
.                          NPROP,NMATS,THETA,VARJ2,VARJ3,YIELD,IND)
END
```

6.6.2 Subroutine “Invary”

The aim of this subroutine is to calculate the invariants of the unified strength theory.

```
SUBROUTINE INVARY(SMEAM,ARLFA,STEFF,THETA,BTWIN,IND,YIELD)
C************************************************************
C
C***   EVALUATES THE STRESS INVARIANTS AND THE CURRENT VALUE OF
C***   THE UNIFIED STRENGTH THEORY
C
C************************************************************
IMPLICIT REAL*8(A-H,O-Z)
PI=3.1415926535
ROOT3=1.73205080757
TWINB=1.0/(1.0+BTWIN)
VARJA=2.0*STEFF/ROOT3
F1=SMEAM+VARJA*(DCOS(THETA)-ARLFA*TWINB*BTWIN*DCOS(THETA-2.0*P
I/3.0)-ARLFA*TWINB*DCOS(THETA+2.0*PI/3.0))
F2=SMEAM+VARJA*(TWINB*DCOS(THETA)+TWINB*BTWIN*DCOS(THETA-2.0*
PI/3.0)-ARLFA*DCOS(THETA+2.0*PI/3.0))
YIELD=F1
IND=1
```

```
IF(F1.LT.F2) YIELD=F2
IF(F1.LT.F2) IND=2
IF(DABS(F1-F2).LE.1.0E-6) IND=3
RETURN
END
```

6.6.3 Subroutine "Yieldy"

The aim of this subroutine is to conduct the corner point process for the Tresca criterion, the Mohr-Coulomb criterion and the unified strength theory.

```
SUBROUTINE YIELDY(THETA,IND,ARLFA,VARJ2,LPROP,PROPS,CONS2,CONS3)
C****************************************************************
C
C***   EVALUATES CONSTANTS OF FLOW VECTORS OF UNIFIED STRENGTH
THEORY
C
C****************************************************************
IMPLICIT REAL*8(A-H,O-Z)
COMMON/CMN01/NELEM,NPOIN,NVFIX,NMATS,NDOFN,
.                     NGAUS, NGAS2,NALGO,NINCS,NTOTV,
.                     NTOTG,NSTRE, NSTR1,NTYPE,NREST
COMMON/CMN02/MBUFA,MFRON,MSTIF,NPROP
DIMENSION PROPS(NMATS,NPROP)
ROOT3=1.73205080757
PI=3.1415926535
BTWIN=PROPS(LPROP,8)
TWINB=1.0/(1.0+BTWIN)
ATATH=1.0/DTAN(THETA)
SINTH=DSIN(THETA)
COSTH=DCOS(THETA)
SINT3=DSIN(3.0*THETA)
IF(IND.EQ.1.OR.IND.EQ.3) THEN
ABTHE=THETA*57.29577951308
IF(ABTHE.LT.0.1) THEN
CONA3=(1.0+0.5*ARLFA)/VARJ2
CONA2=4.0*(1.0+0.5*ARLFA)/3.0/ROOT3
ELSE
ATAT3=1.0/DTAN(3.0*THETA)
C1=2.0*(1.0+0.5*ARLFA)/ROOT3
C2=ARLFA*(1-BTWIN)/(1.0+BTWIN)
CONA2=C1*COSTH+C2*SINTH+ATAT3*(-C1*SINTH+C2*COSTH)
CONA3=-0.5*ROOT3*(-C1*SINTH+C2*COSTH)/(VARJ2*SINT3)
ENDIF
ENDIF
IF(IND.EQ.2.OR.IND.EQ.3) THEN
ABTHE=THETA*57.29577951308
IF(ABTHE.GT.59.9) THEN
CONA3=(0.5+ARLFA)/VARJ2
```

```
CONA2=4.0*(0.5+ARLFA)/3.0/ROOT3
ELSE
ATAT3=1.0/DTAN(3.0*THETA)
C1=(ARLFA+(2.0-BTWIN)/(1.0+BTWIN))/ROOT3
C2=(ARLFA+BTWIN/(1.0+BTWIN))
CONB2=C1*COSTH+C2*SINTH+ATAT3*(-C1*SINTH+C2*COSTH)
CONB3=-0.5*ROOT3*(-C1*SINTH+C2*COSTH)/(SINT3*VARJ2)
ENDIF
ENDIF
IF(IND.EQ.1) THEN
CONS2=CONA2
CONS3=CONA3
ELSE IF(IND.EQ.2) THEN
CONS2=CONB2
CONS3=CONB3
ELSE
CONS2=(CONA2+CONB2)/2.0
CONS3=(CONA3+CONB3)/2.0
ENDIF
RETURN
END
```

6.6.4 Subroutine “Criten”

The aim of this subroutine is to calculate the flow vectors for various yield criteria. For Tresca, Mohr-Coulomb and unified strength theories, invoke YIELDY to conduct the corner point process.

```
SUBROUTINE CRITEN(LPROP,NCRIT,PROPS,THETA,SMEAN,VARJ2,
.                              VARJ3,CONS1,CONS2,CONS3,IND)
C**********************************************************
C
C**** THIS SUBROUTINE EVALUATES THE FLOW VECTOR
C
C***********************************************************
IMPLICIT REAL*8(A-H,O-Z)
COMMON/CMN01/NELEM,NPOIN,NVFIX,NMATS,NDOFN,
.                      NGAUS, NGAS2,NALGO,NINCS,NTOTV,
.                      NTOTG,NSTRE, NSTR1,NTYPE,NREST
COMMON/CMN02/MBUFA,MFRON,MSTIF,NPROP
COMMON/CMN06/FACTDP
DIMENSION PROPS(NMATS,NPROP),NCRIT(NMATS)
ROOT3=1.73205080757
NCRT1=NCRIT(LPROP)
FRICT=PROPS(LPROP,7)
C
C*** VON MISES
C
IF(NCRT1.EQ.2) THEN
```

```
CONS1=0.0
CONS2=ROOT3
CONS3=0.0
GO TO 40
ENDIF
C
C*** DRUCKER-PRAGER
C
IF(NCRT1.EQ.5) THEN
SNPHI=DSIN(FRICT*0.017453292)
C*** COMPRESSIVE CONE
C               CONS1=2.0*SNPHI/(ROOT3*(3.0-SNPHI))
C*** TENSILE CONE
CONS1=2.0*SNPHI/(ROOT3*(3.0+FACTDP*SNPHI))
CONS2=1.0
CONS3=0.0
GO TO 40
ENDIF
C
C*** TWIN SHEAR STRESS ALSO TRESCA MOHR-COULOMB
C
IF(NCRT1.LE.18) THEN ARLFA=PROPS(LPROP,7)/PROPS(LPROP,5)
CONS1=(1.0-ARLFA)/3.0
CALL YIELDY(THETA,IND,ARLFA,VARJ2,LPROP,PROPS,CONS2,CONS3)
GO TO 40
ENDIF
C
C***   TWIN SHEAR
C
IF(NCRT1.EQ.19) THEN
BTWIN=PROPS(LPROP,8)
CTWIN=(1.0+BTWIN)*PROPS(LPROP,5)*PROPS(LPROP,7)/
.               (PROPS(LPROP,5)+PROPS(LPROP,7))
ATWIN=CTWIN*(PROPS(LPROP,9)-PROPS(LPROP,5))/(PROPS(LPROP,5)*
.               PROPS(LPROP,9))
BEITA=2.0*(CTWIN/PROPS(LPROP,7)-ATWIN)/(1.0+BTWIN)-1.0
ARLFA=(1.0-BEITA)/(1.0+BEITA)
CONS1=(1.0+ARLFA)*ATWIN/(1.0+BTWIN)+(1.0-ARLFA)/3.0
CALL YIELDY(THETA,IND,ARLFA,VARJ2,LPROP,PROPS,CONS2,CONS3)
GO TO 40
ENDIF
IF(NCRT1.GE.20) CALL USERCRIT(LPROP,NCRT1,PROPS,THETA,SMEAN,
VARJ2,VARJ3,NPROP,NMATS,CONS1,CONS2,CONS3,IND)
40        CONTINUE
RETURN
END
```

6.7 Brief Summary

The unified strength theory and its associated flow rule are implemented in finite element codes UEPP, which are described in this chapter. Most parts of the elasto-plastic program are the same as the conventional elasto-plastic program, only the **subroutine** of yield criteria (subroutine “INVAR” to calculate equivalent stresses), **subroutine** flow vector and the **subroutine** of the corner (subroutine “YIELD” and “FOLWPL” to calculate flow vector) are different. Several subroutines for the unified strength theory and its flow vector are given, including INVAR, INVARY, YIELDY and CRITEN. When these subroutines are implemented in any elasto-plastic programs, the unified strength elasto-plastic analysis can be conducted. Therefore, a series of ordered results for elasto-plastic analysis of structures can be obtained.

The unified strength theory is also implemented into several commercial FEM codes, and used for engineering problems (Shen, 1993; Quint Co., 1993; 1994; Li and Ishii, 1994; 1998; Liu et al., 1994; Wang, 1998; Fan and Qiang, 2001; Zhang et al., 2001; Zhou, 2002; Sun et al., 2004a; 2004b; Liu and Wang, 2004; Zhang et al., 2005; Shao and Qian, 2007; Shao et al., 2007; Yang, 2008; Li et al., 2008; Li, 2008; Wang et al., 2008; Zhang et al., 2008; Yang and Zhang, 2008; 2009; Wang and Lu, 2009; Fen and Du, 2010; Li and Chen, 2010; Li and Qiao, 2010; Ma and Liao, 2010; Pan et al., 2010). Serial results are obtained that can be adapted for more materials and structures.

References

Belytschko T, Liu WK and Moran B: (2000) Nonlinear Finite Elements for Continua and Structures. John Wiley, Chichester: New York

Fan SC and Qiang HF (2001)Normal high-velocity impaction concrete slabs- a simulation using the meshless SPH procedures. In: Computational Mechanics–New Frontiers for New Millennium, Valliappan S. and Khalili N. eds. Elsevier Science Ltd, pp 1457-1462.

Hinton E and Owen DRJ (1977) Finite Elements Programming. Academic Press: London.

Lewis RW and Schrefler BA (1987) The Finite Element Method in the Deformation and Consolidation of Porous Media. John Wiley & Sons: Chichester.

Fen TG and Du B (2010) Dynamic analysis of the 500 kV underground transformer substation for Shanghai World EXPO, to be published.

Li J, Lu XY and Zhao XW (2008) Topology optimization based on bi-direction evolutionary structural of unified strength. Journal of Shandong Jianzhu

University, 23(1): 65-69.

Li K and Chen GR (2010) Finite element analysis of slope stability based on slip line field theory. Journarl of Hohai University (Natural Sciences),38(2): 191-195.

Li Y (2008) In-situ Stress Measurement and Stability Analysis Based on the Unified Strength Theory in Large Scale Underground Caverns Zone. Doctoral Dissertation, Beijing Scientific and Technical University.

Li Y and Qiao L (2010) Stability Analysis of Underground Caverns based on the Unified Strength Theory, Chapter16 in this monograph.

Li YM, Ishii K, Nakazato C and Shigeta T (1994) Prediction of safety rate and multi-slip direction of slip failure under complex stress state. Advances Eng. Plasticity and its Applications. Xu BY and Yang W eds. International Academic Publishers: Beijing, pp 349-354.

Li YM and Ishii K (1998) The Evaluation of the Elasto-plastic Behavior of Composite Materials under Biaxial Stress with Homogenization Method, Proc. of the Conference on Computational Engineering and Science, 3: 1023-1026. JSCES.

Li YM. and Ishii K(1998) The Evaluation of Strength for the Composite Materials. In : Strength Theory: Application, Development and Prospect for 21st Century, Yu MH and Fan SC eds. Science Press: New York and Beijing, pp 337-342.

Liu F, Li LY and Mei ZX (1994) Elasto-visco-plastic finite element analysis of self-enhanced thick cylinder. Chinese J. Appl. Mechanics, 11(3): 133-137 (in Chinese, English abstract).

Liu GH and Wang ZY (2004) Dynamic response and blast-resistance analysis of a tunnel subjected to blast loading. Journal of Zheiiang University (Engineering Science), 38(2):204-209 (In Chinese).

Ma ZY and Liao HJ (2010) Unified strength theory and FLAC-3D. Chapter 18 in this monograph.

Owen DRJ and Hinton E (1980)Finite Elements in Plasticity: Theory and Practice. Pineridge Press: Swansea.

Owen DRJ, Hinton E and Onate E (1989) Computational Plasticity: Models, Software and Applications. Pineridge Press Limited, pp 1460.

Pan XM, Kong J, Yang Z and Liu C (2010) Secondary development and application of unified elastoplastic constitutive model to ABAQUS. Rock and Soil Mechanics, 31(4): 1092-1098.

Quint Co. (1993a) COMPMAT-Analysis system for composite materials. FEM codes of Quint Corporation: Japan.

Quint Co. (1993b) COMPMAT-Analysis system for composite materials. FEM codes of Quint Corporation, Japan.

Quint Co. (1994a) PREMAT/POSTMAT-Pre and Post Processor for Composite Materials. FEM Codes of Quint Corporation, Japan.

Quint Co. (1994b) STAMPS-Structural Analysis Program for Civil Engineering. FEM Codes of Quint Corporation, Japan.

Shao CJ and Qian YJ (2007) Load bearing capability of concrete-filled steel tube component considering effect of intermediate principal stress. Chinese Journal of Applied Mechanics, 24(03): 396-399.

Shao CJ, Wu YH and Qian YJ (2007) Plastic seismic damage of concrete structure based on five-parameter unified strength theory. Chinese Journal of Applied Mechanics, 24(1): 97-101.

Shen ZJ (1993) Comparison of several yield criteria. Rock and Soil Mechanics, 14(1): 41-50 (in Chinese, English abstract).

Smith IM and Griffiths DV (2004) Programming the Finite Element Method, 4th edition. John Wiley & Sons， Ltd.

Sun HY, Shang YQ, Zhang CS, Ying HP, Li SQ (2004) 3D numerical modeling of possible failure zone with underground excavation. Chinese Journal of rock mechanics and engineering, 23(13): 2192- 2196.

Sun HY, Shang YQ and Zhang CS (2004) Numerical modeling analysis for surrounding rockmass stability of large underground cavities. Journal of Zheiiang University (Engineering Science), 38(1): 70-74.

UEPP User's Manual, Version 3.0, Research Division of Structural Strength, Dept. of Civil Eng., Xi'an Jiaotong University.

Wang D, Chen JK, Wang QZ, Chen Y, Huang Y (2008) The new method of structural reliability analysis by Monte-Carlo stochastic finite element. Journal of Sichuan University (Engineering Science Edition), 40(3): 20-26.

Wang D, Chen JK, Wang QZ, Li YL (2008) Structural Reliability Analysis by Monte-Carlo Based on Conditional Expectation Variance Reduction and Antithetic Variable Sampling. China Rural Water and Hydropower, (5):66-70.

Wang F and Fan SC (1998) Limit pressures of thick-walled tubes using different yield criteria. Strength Theory: Applications, Developments and Prospects for the 21st Century. Yu MH and Fan SC eds. Science Press: New York, Beijing, pp 1047-1052.

Wang F (1998) Nonlinear finite element analysis of RC plate and shell using the unified strength theory. PhD thesis, Nanyang Technological University, Singapore.

Wang JQ and Lu F (2010) Unified strength theory constitutive model embedded software ABAQUS and its application in tunnel engineering. Journal of Yangtze River Scientific Research Institute, 27(2): 68-74 (in Chinese).

Wang JQ and Lu F, (2010) Implementation of the Unified Strength Theory into ABAQUS and its Application for Tunnel Engineering. Chapter11 in this monograph.

Yang LQ, Zhang SR (2008) Analysis of textural stress and rock failure of diversion tunnels. Chinese Journal of Geotechnical Engineering, 30(6): 813-817.

Yang LQ, Li YQ and Chen ZP (2008) Sudden-crack Phenomenon and Simulation of Surrounding Rock-mass in Diversion Tunnel. China Rural Water and Hydropower, (5):53-56.

Yang LQ, Zhang SR, Chen ZP. (2009) Adaptive arithmetic of arch dam cracking analysis. Journal of Hydraulic Engineering, 40(2): 214-219.

Yu-MH, He LN, Song LY (1985) Twin shear strength theory with extension research, Sciences in China (Series.A), 28(11): 1113-1120 (English Edn);

28(12): 1174-1183(Chinese Edn).

Yu MH and Meng XM (1990) Research in the protection and utilization of ancient city wall in Xi’an (in Chinese). Research Report of Xian Jiaotong University, pp 90-338, In: Fourth Symposium of Ancient Architecture and Gardens of China, Xi’an China.

Yu MH and Li YM (1991) Twin shear constitutive theory and its computational implementation. In: Computational Mechanics, Ed. by CheungYK LeeJ,H,W. and LeungA.Y.T. Balkema: Rotterdam, pp 875-879.

Yu MH (1992) New System of Strength Theory. Xian Jiaotong University Press: Xian, China (in Chinese).

Yu MH, He LN and Zeng WB (1992) A new unified yield function : Its model, computational implementation and engineering application. Computational Methods in Engineering: Advances and Applications. Tay AAO and Lam KY eds. World Scientific: Singapore, pp 157-162.

Yu MH, He LN, Liu CY (1992) Generalized twin shear yield criteria with extension research. Bulletin of Science, 37(2):182-186.

Yu MH and Meng XM (1992) Twin shear elasto-plastic model and its application in geotechnical engineering. Chin. J. Geotech. Eng., 14(3): 71-75 (in Chinese, English abstract).

Yu MH and Meng XM (1993) Researches on the stability of ancient city wall in Xi’an (in English). Research in the City Wall in Xi’an. Yu MH, Zhang XP and Fang TP eds. Xi’an Jiaotong University Press: Xi’an China, pp 168-174.

Yu MH, Meng XM and Xie S (1993) Research on the protection and utilization of ancient city wall in Xi’an (in Chinese). Research on the City Wall in Xian. Yu MH, Zhang XP and Fang TP eds. Xian Jiaotong University Press: Xi’an, China, pp 94-126.

Yu MH and Zeng WB (1993) Mesomechanical simulation of failure criterion for a composite material. Macro-Meso-micro Mechanical Properties of Materials. Tokuda M and Xu BY eds. Mie Academic Press: Mie, Japan, pp 571-576.

Yu MH and Zeng WB (1994) New theory of engineering structural analysis and its application. J. Eng. Mech., 11(1): 9-20 (in Chinese, English abstract).

Yu MH, Zeng WB, Ma GW, Yang SY, Wang F and Wang Y (1993) Unified Elasto-Plastic Program—UEPP. Xian Jiaotong University, English version.

Yu MH, Zeng WB, Ma GW, Yang SY, Wang F and Wang Y (1993) Unified Elasto-Plastic Program—UEPP. Xian Jiaotong University, Chinese version.

Yu MH (1994) Unified strength theory for geomaterials and its application. Chinese J. Geotech. Eng., 16(2): 1-9 (in Chinese, English abstract).

Yu MH and Zeng WB (1994) New theory of engineering structural analysis and its application. J. Eng. Mech., 11(1): 9-20 (in Chinese, English abstract).

Yu MH, Yang SY, Fan SC, et al. (1997) Twin shear unified elasto-plastic constitutive model and its applications. Chin. J. Geotech Eng., 21(6): 9-18 (in Chinese, English abstract).

Yu MH (1998) Twin Shear Theory and its Applications. The Science Press: Beijing (in Chinese).

Yu MH, Yang SY, Fan SC and Ma GW (1999) Unified elasto-plastic associated and non-associated constitutive model and its engineering applications. Int.Journal of Computers & Structures, 71: 627-636.

Yu MH (2004) Unified Strength Theory and its Applications. Springer: Berlin.

Yu MH, Ma GW et al. (2006) Generalized Plasticity. Springer: Berlin.

Yu MH, Ma GW and Li JC (2009) Structural Plasticity: Limit, Shakedown and Dynamic Plastic Analyses of Structures. ZJU Press and Springer: Hangzhou and Berlin.

Zhang CQ (2005) Study on method of safety evaluation for rock engineering based on failure approach index. Doctoral Dissertation of Chinese Academy of Sciences, Institute of Rock & Soil Mechanics.

Zhang CQ, Zhou H and Feng XT (2008) Numerical format of elastoplastic constitutive model based on the unified strength theory in FLAC~(3D). Rock and Soil Mechanics, 29(3): 596-602 (in Chinese, English Abstract).

Zhang XS, Guan H, Yew-Chaye Loo (2001) UST (Unified Strength Theory) failure criterion for punching shear analysis of reinforcement concrete slab-column connections. In: Computational Mechanics–New Frontiers for New Millennium**,** Valliappan S. and Khalili N. eds. Elsevier Science Ltd, pp 299-304.

Zhou XQ (2002) Numerical Analysis of Reinforced Concrete using Multi-surface Strength Model. Doc. thesis of Nanyang Technological University, Singapore.