# 17

# Stability of Slope

## 17.1 Introduction

There are more and more side slope problems and slope engineering problems every day along with the increase in human activity. These will be the most common geotechnical engineering problems needing to be addressed in the natural geological environment. The side slope in civil engineering, water conservation engineering as well as traffic engineering is extremely common. Many excellent monographs are devoted to this field (Baker and Gather, 1978; Bishop, 1955; Brand, 1989; Chen, 2003; Chen et al., 2005; Cividin, 2001; Dong et al., 2004; Duncan, 1996; Fredinnd, 1984; Hudson and Harrison, 1997; Janbu, 1973; Leshchinsky, 1990, 1992; Morgenstem, 1992; Morgenstem and Price, 1965; Pan, 1980; Zhang and Zhou, 1997; Zhou and Yang, 2005).

The high slope of the ship-lock of the Three Gorges Project is an example of one of the key technologies used in this great project. Figure 17.1 is a bird's-eye view of the ship-lock of the Three Gorges Project (Encyclopedia of Water Resources in China, Second Edition, 2006). The numerical simulation of deformation for the high slope is shown in Fig. 17.2.

The unified strength theory has been used in slip line field and characteristics analysis (Yu et al., 2006). The slip line fields of two examples are illustrated in Figs. 17.3 and 17.4. A series of results of the bearing capacity for a trapezoid structure can be obtained by using the unified slip field theory, as shown in Fig. 17.5.

M.-H. Yu et al., *Computational Plasticity*

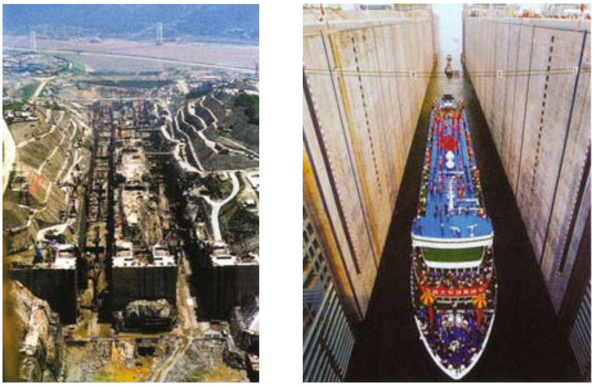


**Fig. 17.1** Superb view over ship-lock of the Three Gorges Project

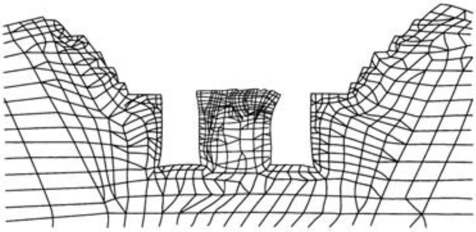


**Fig. 17.2** Numerical simulation of deformation for the high slope

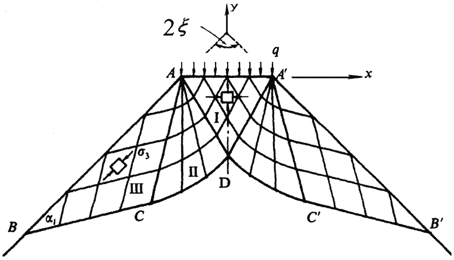


**Fig. 17.3** Slip field of a trapezoid structure

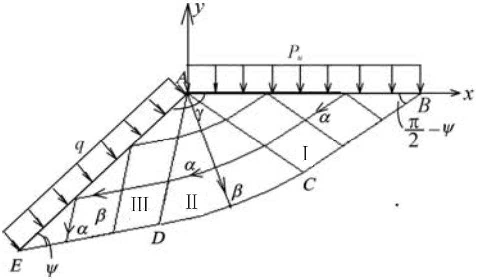


**Fig. 17.4** Slip field of obtuse wedge

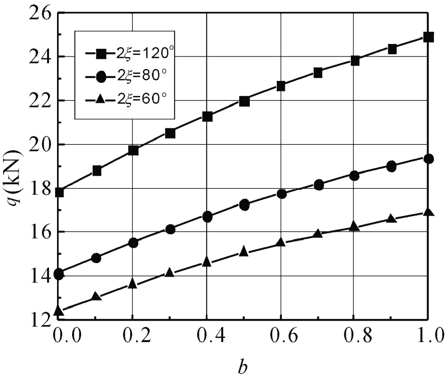


**Fig. 17.5** Bearing capacity of a trapezoid structure

The UST (unified strength theory) and the slip line field theory are also implemented into ANSYS by Li and Chen (2010). According to the stress fields derived by finite element calculations, the slip line fields with UST material parameters in elastic and plastic zones were simulated. The critical slip surface on slopes was searched for through the slip line fields and the safety factor was derived so as to solve problems of slope stability. The results can be employed to analyze differences in safety factors and positions of the critical slip surfaces for various unified yield criteria, as shown in Fig. 17.6 (Li K and Chen ER, 2010). It is seen that the slip surface is dependent on the unified strength theory parameter *b*. The critical safety factor is also dependent on the unified strength theory parameter *b*.

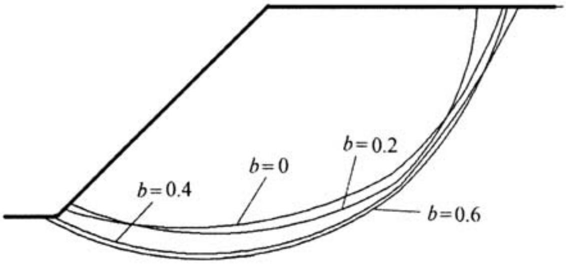


**Fig. 17.6** Critical slip surfaces for various unified strength theory parameters *b*

Stability of a slope has been studied widely. It still remains open to debate, however, including the choice of yield criterion. The effect of yield criteria is rarely studied in this regard in the context of the safety factor, stability and excavation of vertical faces etc. The single-shear theory (Mohr-Coulomb strength theory) does not consider the effect of intermediate principal stress. A more comprehensive strength theory is also very significant.

The effect of yield criteria on the analysis of a slope, the excavation analysis of the Three Gorges ship lock (carried out by the Yangtze River Academy of

Sciences), the excavation simulation of a vertical surface face, and the numerical simulation of the slope of a highway in Guangxi Province, China (Bai, 2007) are described in this chapter. Using the unified strength theory in the computer analysis can also be a good application. A series of results are obtained by using the unified strength theory, which encompasses the Mohr-Coulomb criterion as a special case. Using the unified strength theory (Yu, 1992; 2004) it is possible for us to adopt different results of the numerical analysis to handle the effects of different materials and different structures.

## 17.2 Effect of Yield Criterion on the Analysis of a Slope

Stability of a slope is analysed by Dr. Ma ZY with respect to the unified strength theory, as shown in Fig. 17.7.

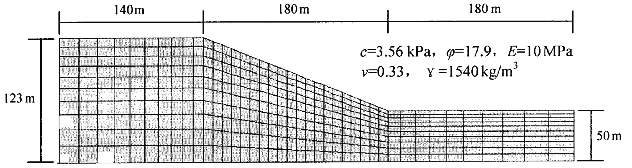


**Fig. 17.7** A slope problem in Shaanxi province, China

The reason for using the unified strength theory is explained by the fact that the single-shear theory of Mohr-Coulomb or the three-shear theory of the Drucker-Prager criterion do not completely match with experimental data. It has been shown that the yield criteria of geomaterials depend not only on the maximum shear stress, but also on the intermediate principal shear stress (also on the intermediate principal stress $\sigma_2$) and the third invariant of the deviatoric stress tensor $J_3$. The reason that the Mohr-Coulomb theory and the Drucker-Prager criterion are not in good agreement with the experimental data is that the effect of $J_3$ and the effect of $\sigma_2$ are neglected.

The unified strength theory with $b$=0, $b$=0.25, $b$=0.5, $b$=0.75 and $b$=1 are used. The plastic displacements of the slope with different yield criteria under the same condition are shown in Fig. 17.8. The plastic strain based on the unified strength theory with $b$=0, $b$=0.25, $b$=0.5, $b$=0.75 and $b$=1 are given in Fig. 17.9. The difference is obvious.

The configurations of displacement vectors of a slope using different yield criteria are shown in Fig. 17.10. The configurations of deformations of finite elements of a slope are shown in Fig. 17.11. Five results using the unified strength theory with $b$=0, $b$=0.25, $b$=0.5, $b$=0.75 and $b$=1 are presented. Another three results using the Mohr-Coulomb criterion and the Drucker-Prager criterion are also presented, which are obtained by using the original material model in FLAC-2D. The differences are obvious.

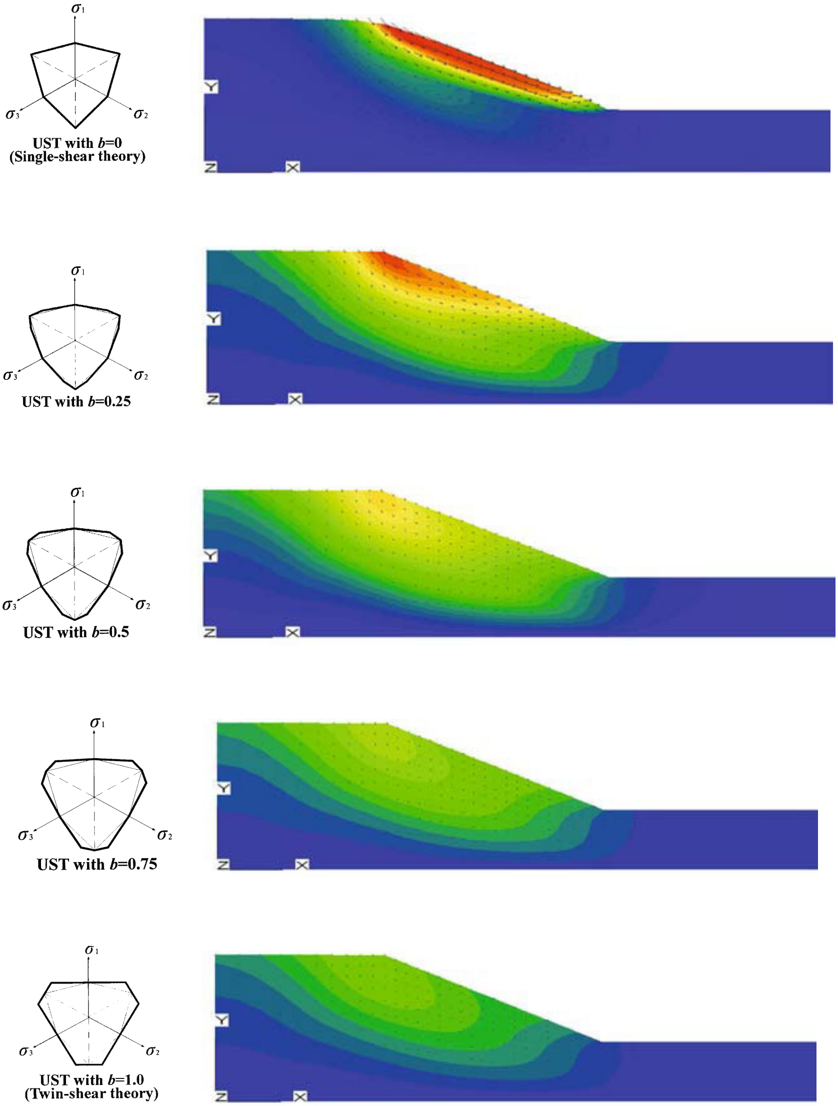


**Fig. 17.8** Displacements of the slope with different yield criteria

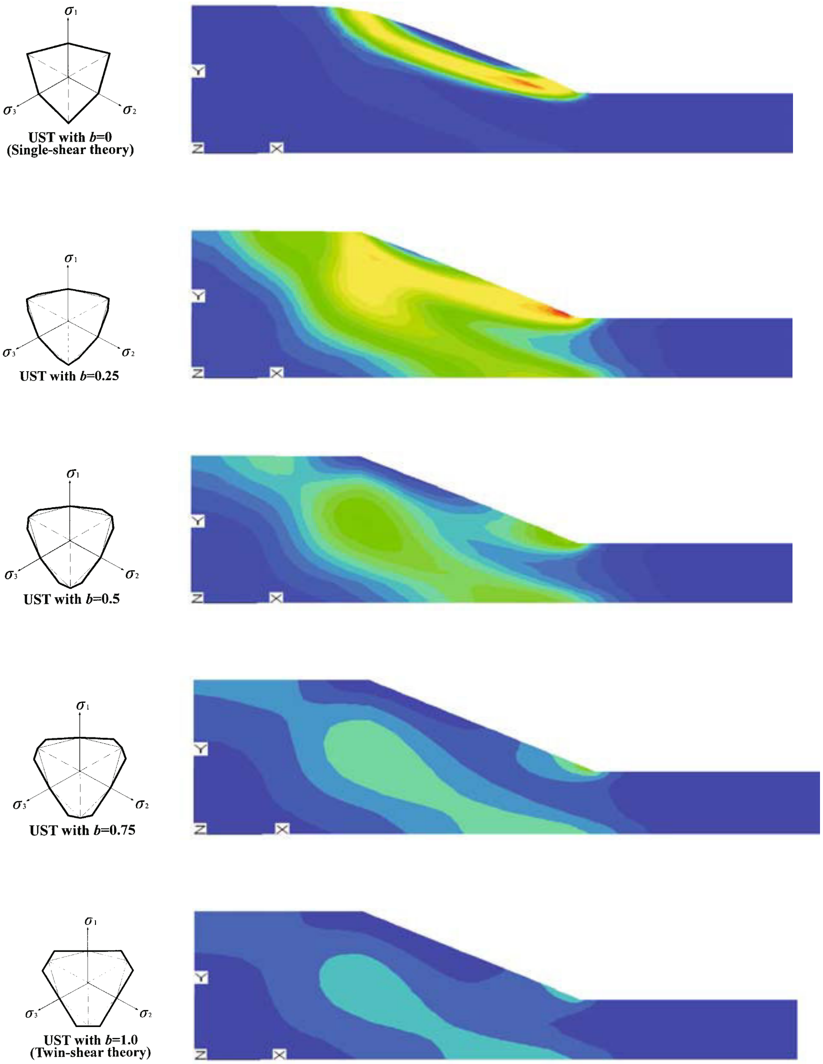


**Fig. 17.9** Configuration of plastic strain of the slope with different yield criteria

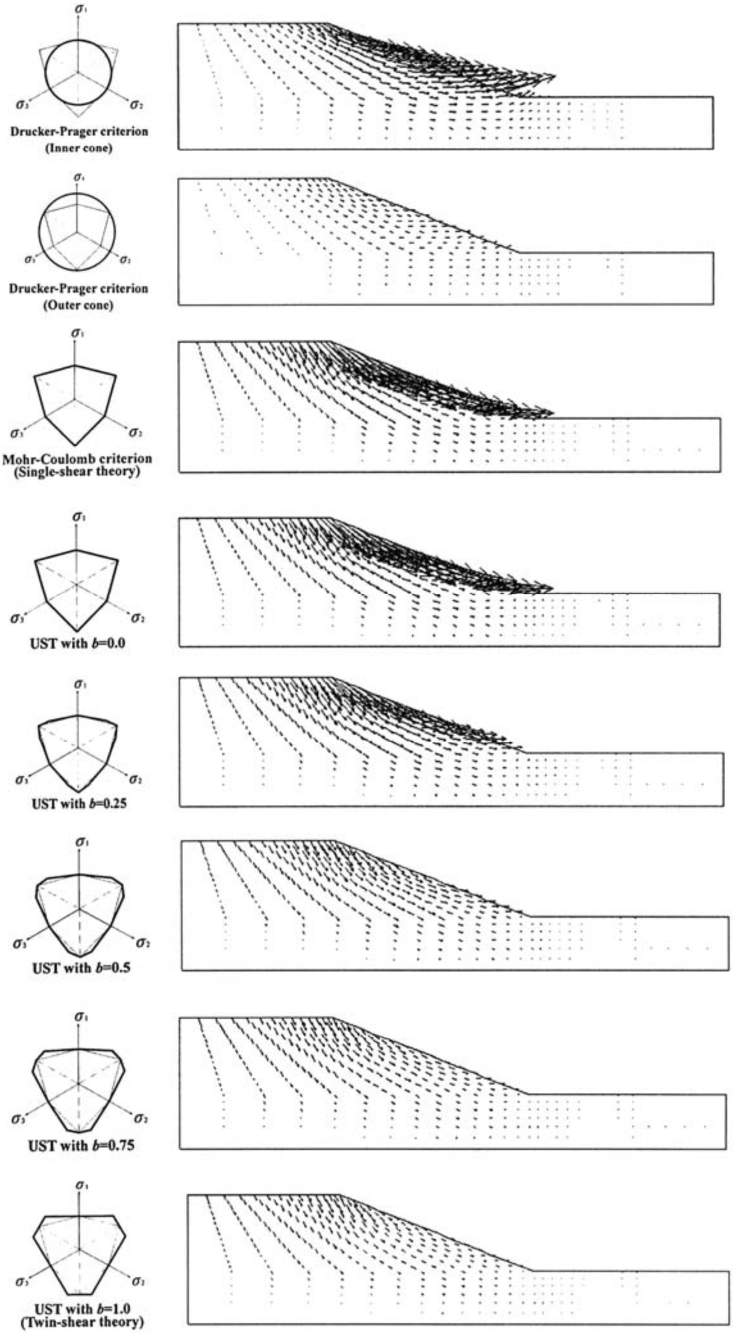


**Fig. 17.10** Configuration of displacement vectors of slope using different yield criteria

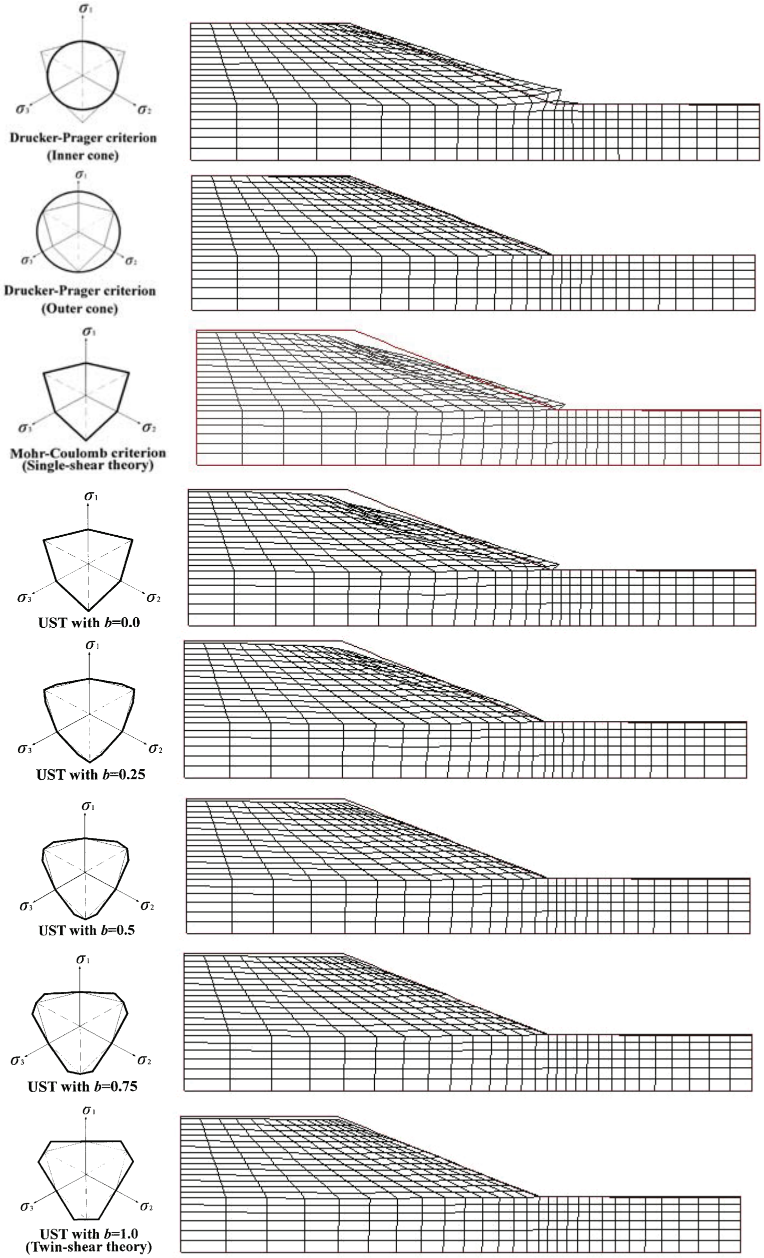


**Fig. 17.11** Configuration of deformation of finite elements mesh of slope

We can see the following:

1) A series of new results are obtained using the unified strength theory with *b*=0, *b*=0.25, *b*=0.5, *b*=0.75 and *b*=1.

2) The result obtained by using the unified strength theory with $b$=0 implemented in FLAC-2D is the same as the result that is obtained from the oriented material model of the Mohr-Coulomb criterion (Single-shear theory) in FLAC-2D.

3) The outer cone of the Drucker-Prager criterion gives the minimum deformation and displacement; the inner cone of the Drucker-Prager criterion gives the maximum deformation and displacement; however, these two results are not in agreement with the real condition.

## 17.3 Stability of Three Gorges High Slope

The Three Gorges Project is the biggest water conservation and energy project in the world. Its five levels of continual sluices are open to navigation. After excavation, the construction will reach as high as 170 meters on the steep side slope. It is an important waterway for the future. Extremely high stability is required (Sheng et al., 1997; Zhang and Zhou, 1997; Dong et al., 1999; Kou et al., 2001). To guarantee the stability of the Three Gorges high side slope sluice is one of the project's important research topics. Therefore the Yangtze River Academy of Science, the Wuhan Rock and Soil Institute of Chinese Academia, Qinghua University, HeHai University, Hong Kong University, Wuhan University etc. carried out stability research of the side slope (Research Report of the Yangtze River Academy of Science, 1997-260). The excavation of the Three Gorges sluice is shown in Fig. 17.12.

**Fig. 17.12** Excavation of the Three Gorges sluice

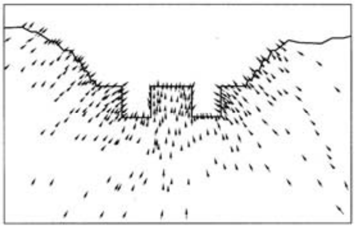


**Fig. 17.13** Displacement vector field

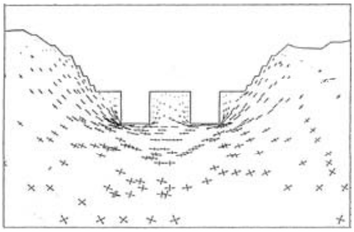


**Fig. 17.14** Principal stress vector fields

The stress analysis of the ship sluice was carried out by the Yangtze River Academy of Science. The displacement vector field and principal stress fields are shown in Figs. 17.13 and 17.14 (Sheng et al., 1997; Dong et al., 1999).

The stability of the high slope of the ship lock of the Three Gorges Project has been studied (Sheng et al., 1997; Dong et al., 1999; Zhou and Yang, 2005). The plastic zone and limit equilibrium analysis for the stability of the high slope of the ship lock were presented (Sheng et al., 1997). Figure 17.15 is the plastic zone of the high slope of the ship lock using the single-shear theory (Mohr-Coulomb Theory). Figure 17.16 is the plastic zone of the high slope of the ship lock using the twin-shear theory (Yu, 1985). Figure 17.17 is the plastic zone of the high slope of the ship lock using the Drucker-Prager criterion.

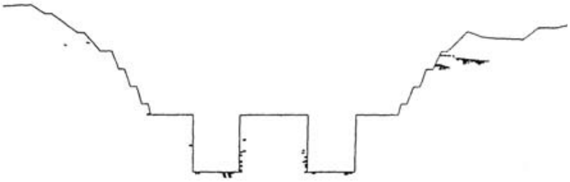


**Fig. 17.15** Plastic zone for single-shear theory (Sheng et al., 1997)

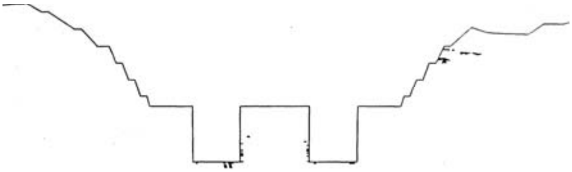


**Fig. 17.16** Plastic zone for the twin-shear theory (Sheng et al., 1997)

**Fig. 17.17** Drucker-Prager yield criterion (Sheng et al., 1997)

The results indicated that the deformation shape and stress field obtained from the three yield criteria show no significant difference. However, the differences in the plastic zones are larger (Sheng et al., 1997). It is seen that the spread of plastic zones using the single-shear yield criteria and the twin-shear yield criterion is similar. However, the size of the plastic zones varied widely. The difference can be illustrated from the yield surface on the deviatoric plane, as shown in Fig. 17.18.

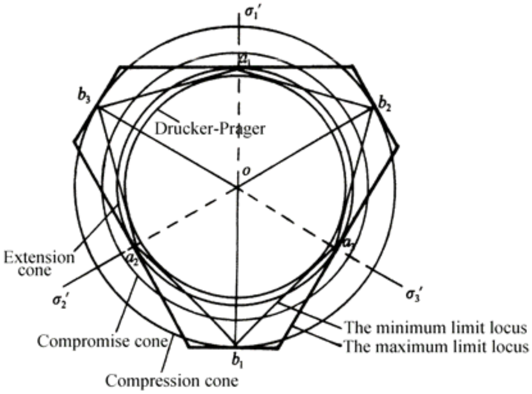


**Fig. 17.18** Limit loci of various failure criteria on the deviatoric plane

As pointed out by Zienkiewicz and Pande (1977), the choice of the best limit surface is still in the hands of the analyst who has modeled the strength behavior in the best possible manner. They also indicated that the Drucker-Prager criterion and the limit loci of extension circular cones give a very poor approximation of, the real failure conditions (Humpheson and Naylor, 1975; Zienkiewicz and Pande. 1977). The choice of yield criteria has a marked effect on the prediction of the load-bearing capacities of structures. It is obvious that the Drucker-Prager criterion or other circular criterion cannot match the two experimental points *a* and *b*, as shown in Fig. 17.18.

The comparison of the single-shear theory (Mohr-Coulomb criterion) and the twin-shear theory is shown in Fig. 17.19. In fact, various yield criteria must be situated between the bounds if the convexity is considered. The lower bound is the single-shear strength theory (the Mohr-Coulomb strength theory 1900) and the upper bound is the twin-shear strength theory (Yu et al., 1985). Other failure criteria are situated between these two bounds. The limit loci of the unified strength theory cover all the regions of the convex limit loci, as shown in Fig. 17.19.

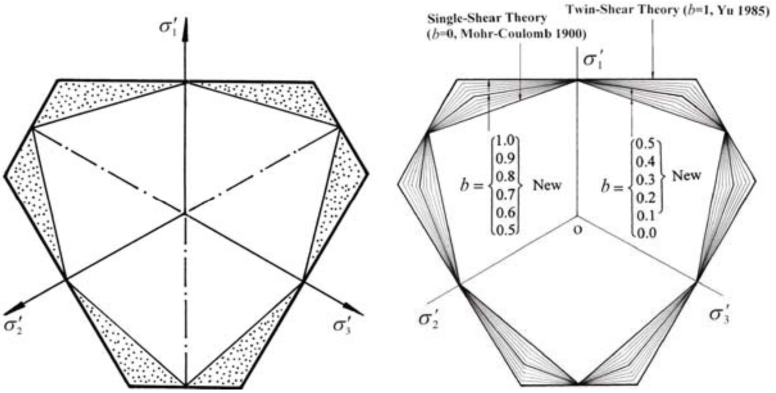


**Fig. 17.19** Bounds and region of limit loci

## 17.4 Stability of a Vertical Cut

In the natural state, the loess soil is so resistant to erosion and so stable as to maintain indefinitely almost vertical faces in cuts, as shown in Fig. 17.20 (Hogentogler, 1937). Figure 17.21 shows an artificial vertical cut at the Green Dragon Temple near Xi'an Jiaotong University in Xi'an, China (Yoshimine, 2001). The stability of loess soil becomes troublesome when the height of the vertical cut increases. The critical height of the vertical cut was studied in the literature by using the Mohr-Coulomb theory. The computational result using the Drucker-Prager criterion with size-adjustment using the Mohr-Coulomb criterion was given by Zimmermann and Commend (2001). New bounds for the height limit of a vertical slope are given by Pastor et al. (2000). As pointed out by Pastor et al., when addressing the classic problem of the height limit of a Tresca or Mises criterion vertical slope subjected to the action of gravity, the exact solution to this problem remains unknown.

**Fig. 17.20** Vertical faces of loess soil in cut

**Fig. 17.21** Vertical faces of loess in Xi'an

We use a vertical cut where the material parameters are $\gamma$=1.6×10$^4$ N/m$^3$, $C$=18 kPa, $\varphi$=30°. The stress field of the vertical cut is shown in Fig. 17.22. The critical cut heights using the three basic criteria of the unified strength theory with $b$=0, $b$=1/2 and $b$=1 are calculated and shown in Fig. 17.23. The calculated critical heights are: $H_p$=6.4 m ($b$=0), $H_p$=7.9 m ($b$=1/2) and $H_p$=9.2 m ($b$=1). The last one is close to the height of the vertical cut of the loess as shown in Fig. 17.21.

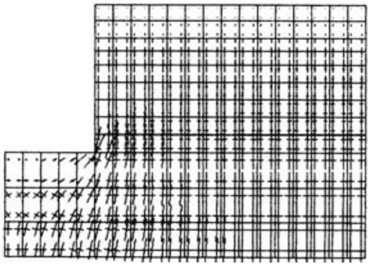
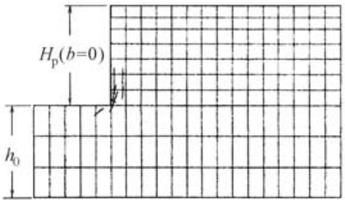
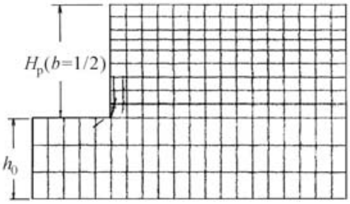


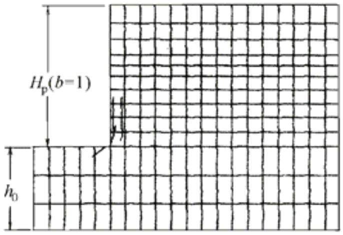
**Fig. 17.22** Stress field of a vertical cut slope



(a) UST with $b$=0, $H_p$=6.4 m



(b) UST with $b$=1/2, $H_p$=7.9 m



(c) UST with $b$=1, $H_p$=9.2 m

**Fig. 17.23** Three critical heights obtained using the three basic criteria

## 17.5 Stability for a Slope of a Highway

The stability of the slope for the Baise-Luo highway in Guangxi province, China, was studied by Bai in 2005, in which the unified strength theory was used. The slope is considered as a plane strain problem. A special section of the slope and its mesh for numerical analysis is shown in Fig. 17.24. The parameters of the soil and the rock on the slope for various values of *b* in UST are listed in Table 17.1 (Bai, 2005; Fan W et al., 2007).

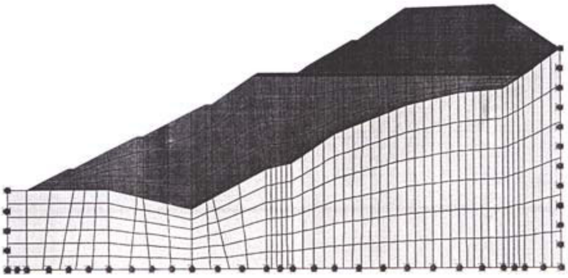


**Fig. 17.24** Mesh of slope at Baise-Luo highway

**Table 17.1** Parameters for various values of *b* in UST

| Materials | Unified slip field | Unified strength theory parameter *b* | | | | |
|---|---|---|---|---|---|---|
| | | *b*=0 | *b*=1/4 | *b*=1/2 | *b*=3/4 | *b*=10 |
| The clay loam mixes with the gravel (above water level) | $C_{\mathrm{uni}}$ (kPa) | 19.37 | 20.84 | 21.94 | 22.8 | 23.49 |
| | $\varphi_{\mathrm{uni}}$ (°) | 30.57 | 33.05 | 34.97 | 36.49 | 37.73 |
| The clay loam mixes with the gravel (under water level) | $C_{\mathrm{uni}}$ (kPa) | 14.67 | 15.87 | 16.8 | 17.53 | 18.13 |
| | $\varphi_{\mathrm{uni}}$ (°) | 24.1 | 26.14 | 27.76 | 29.05 | 30.10 |
| Rock base | $C_{\mathrm{uni}}$ (kPa) | 30 | 32.07 | 33.64 | 34.88 | 35.88 |
| | $\varphi_{\mathrm{uni}}$ (°) | 35 | 36.84 | 38.16 | 39.17 | 39.97 |

The contours of the security rate for different yield criteria are shown in Figs. 17.25 to 17.29. It can be seen from Figs. 17.25 to 17.29 that the contours of the safety rate at the same stress level are different at different values of *b*.

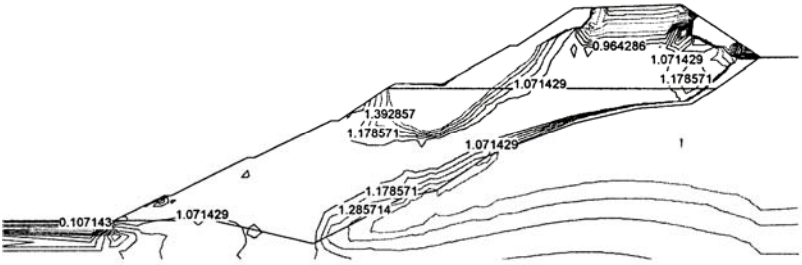


**Fig. 17.25** Contours of security rate (UST with *b*=0)

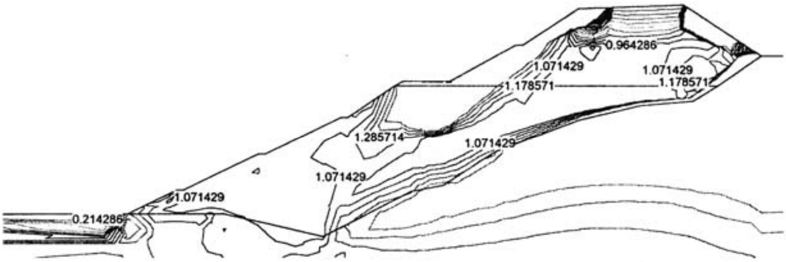


**Fig. 17.26** Contours of security rate (UST with *b*=0.25)

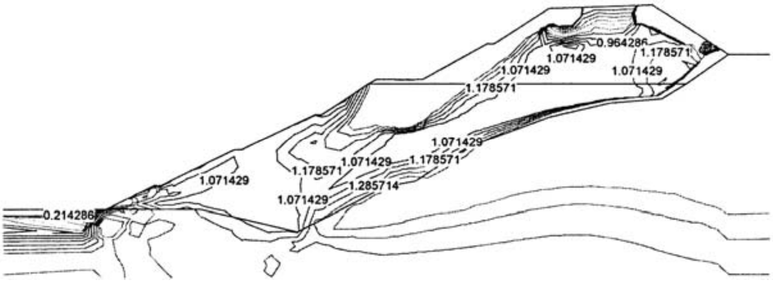


**Fig. 17.27** Contours of security rate (UST with *b*=0.5)

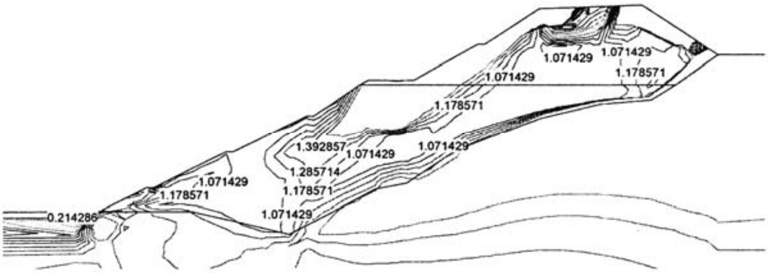


**Fig. 17.28** Contours of security rate (UST with *b*=0.75)

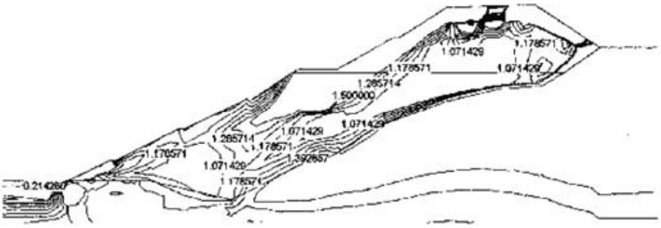
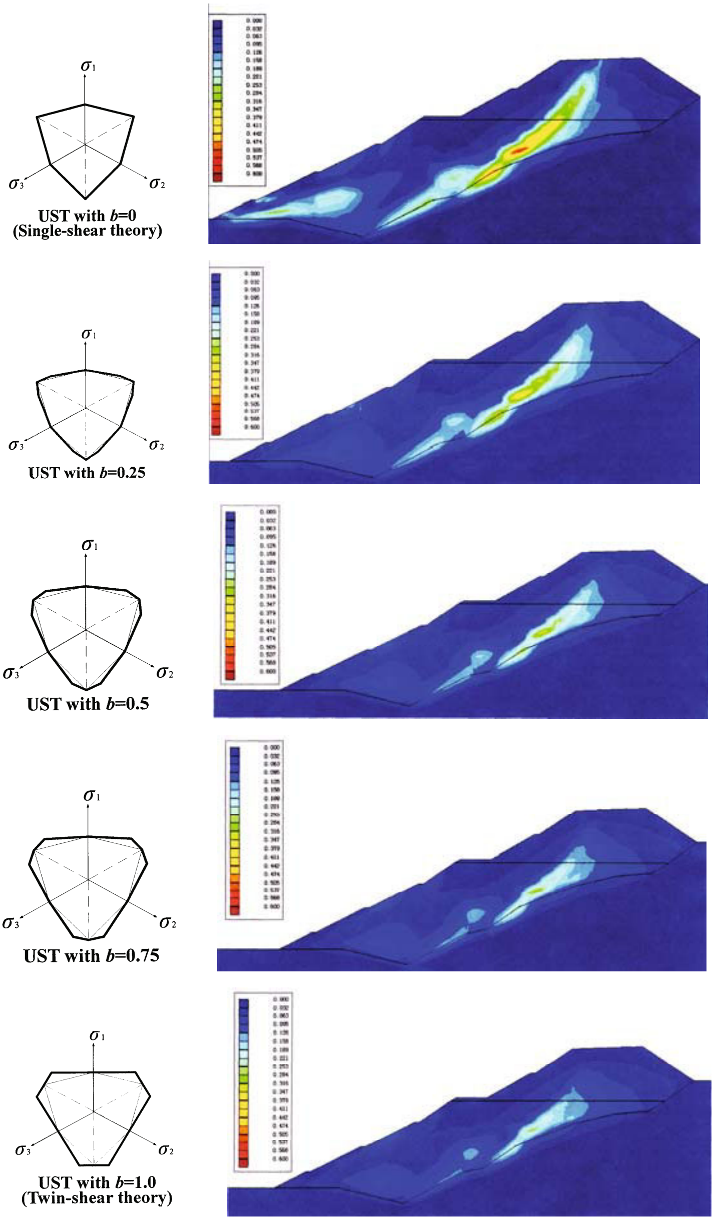


**Fig. 17.29** Contours of security rate (UST with *b*=1.0)

The charts of the shear strain of the slope based on UST with different values of *b* are shown in Fig. 17.30 (Bai, 2005; Fan et al., 2007).



**UST with *b*=0 (Single-shear theory)**

**UST with *b*=0.25**

**UST with *b*=0.5**

**UST with *b*=0.75**

**UST with *b*=1.0 (Twin-shear theory)**

**Fig. 17.30** Charts of shear strain based on UST with different values of *b*

Overall, along with the increase in the value of the UST parameter *b*, in the region of the side slope, safety coefficient changes are obviously small, the shear strain changes are obviously small and the strength of the side slope increases gradually. Therefore, the effect of the intermediate principal stress is obvious in side slope stability analysis. The potential of the strength of materials can be played with using the unified strength theory in which the intermediate principal stress is taken into account.

## References

Bai XY (2005) The use of the unified strength theory in the stability analysis of slopes. Master Thesis, Chang'An University, Xi'an, China (in Chinese).

Baker R and Gather M (1978) Theoretical analysis of the stability of slopes. Geotechnique, 28(4): 395-413.

Bishop A W(1955) The use of the slip circle in the stability analysis of slopes. Geotechnique, 5(1): 7-17.

Brand EW (1989) Occurrence and significance of landslides in Southeast Asia. In: Landslides: extant and economic significance, Brabb & Hm'rod (eds), Balkema.

Chen WF (1975) Limit Analysis and Soil Plasticity. Elsevier.

Chen ZY (2003) Soil Slope Stability and Analysis: Theory, Methods and Programs (in Chinese). China Hydraulic and Hydropower Press: Beijing, China.

Chen ZY and Morgenstera NR(1983) Extensions to the generalized method of slices for stability analysis. Canadian Geotechnical Journal, 20(1): 104-119.

Chen ZY, Wang XG, Yang J et al. (2005) Rock Slope Stability Analysis: Theory, Methods and Programs (in Chinese). China Hydraulic and Hydropower Press: Beijing.

Cividin A (2001) Application of numerical methods to slope stability analysis. In: VaIliappan S and Khalili N editors, Computational Mechanics: New Frontiers for New Millennium. Elsevier Science Ltd: Amsterdam. pp 323-332.

Dong XS, Tian Y, Wu AQ (2004) Rock Mechanics in Hydraulic Engineering. Collection of Yangtze River Hydro-Conservancy Committee, Large and middle scale water conservation water and electricity project technology, Chinese Water Conservation and Water Electricity Press: Bejing pp 228-233 (in Chinese).

Janbu N (1973) Slope stability computations. Embankment Dam Engineering. John Wiley and Sons: New York.

Duncan JM (1996). State of the art: Limit equilibrium and finite element analysis of slopes. J. of Geotechnical Engineering, 122(7): 577-596.

Encyclopedia of Water Resources in China (2006) Second Edition. China Hydraulic and Hydropower Press: Beijing (in Chinese).

Fan W, Deng LS, Bai XY and Yu MH (2007) Application of unified strength theories to the stability of slope analysis. Coal Geology & Exploration, 35(1): 63-66.

Fredinnd DC (1984) State of the Art: Analytical methods for slope stability analysis. Proceedings of the 46th International Symposium on Landslides, Toronto, pp 229-250.

Hogentogler CA (1937) Engineering Properties of Soil. McGraw-Hill : New York.

Hudson JA and Harrison JP (1997) Engineering Rock Mechanics: An Introduction to the Principles. Elsevier Science Ltd.

Humpheson C and Naylor DJ (1975) The importance of the form of the failure criterion. C/R/243/75, University of Wales, Swansea, UK.

Janbu N (1973) Slope stability computations. Embankment Dam Engineering, 47-86.

Kou XD, Zhou WY, and Yang RQ (2001) Stability analysis on the high slopes of Three-Gorges ship-lock using FA\LAC-3D, Chinese J. Rock Mech. Eng., 20(1): 6-0.

Li K and Chen GR (2010) Finite element analysis of slope stability based on slip line field theory.J. of Hohai University (Natural Sciences),38(2): 191-195.

Morgenstem NR and Price V (1965) The analysis of the stability of general slip surface. Geotechnique, 15 (1): 79-93.

Morgenstem NR (1992) Keynote paper：The role of analysis in the evaluation of slope stability, Proceedings of Sixth Int. Symposium on Landslides, 3: 1615-1629.

Pan JZ (1980) Stability and Landslide Analysis. Hydraulic Press: Beijing (in Chinese).

Pastor J, Thai T-H and Francescato P (2000) New bounds for the height limit of a vertical slope. Int. J. for Numerical and Analytical Methods in Geomechanics, 24(2): 165-182.

Sheng Q, Xu NF and Ren F (1997) Application of the concept of plastic zone and limit equilibrium method for the stability analysis of the high slope of ship lock of Yangtze River. Three Gorges Project. Science Research Report of Yangtze River Academy of Sciences, 97-260 (in Chinese).

Yoshimine M (2001) Mechanical Properties of Loess Soil and Failure Criteria of Materials. Post-Doctor Research Report, Xi'an Jiaotong Universituy, Xi'an, China (in English).

Yu MH, He LN and Song LY (1985) Twin shear stress theory and its generalization. Scientia Sinica Series A, 28(11): 1174-83 (English edn.).

Yu MH (1992) New System of Strength Theory. Xi'an Jiaotong University Press: Xi'an (in Chinese).

Yu MH (2004) Unified Strength Theory and its Applications. Springer: Berlin.

Yu MH et al. (2006) Generalized Plasticity. Springer: Berlin.

Zienkiewicz OC and Pande GN (1977) Some useful forms of isotropic yield surfaces for soil and rock mechanics. Finite Elements in Geomechanics. Gudehus G ed. Wiley: London, pp 179-190.

Zimmermann T and Commend S (2001) Stabilized finite element applications in geomechanics. In: Valliappan S and Khalili N eds., Computational Mechanics: New Frontiers for the New Millennium. Elsevier: Amsterdam, pp 533-538.

Zhang YT and Zhou WY (1997) Deformation and Stability of Rock High Slope. China Hydraulic and Hydropower Press: Beijing (in Chinese).

Zhou WY and Yang Q (2005) Numerical Computational Methods for Rock Mechanics. China Electric Power Press: Beijing (in Chinese).