15

Reinforced Concrete Beam and Plate

15.1 Introduction

Concrete plasticity has been described by Chen (1982; 1998) and Nielsen (1984; 1989). Several material models were discussed. Yu's unified strength theory (Yu's UST or UST) and the UEPP (Unified Elasto-Plastic Program) are successfully used for structural analysis, as has been described in Chapters 6, 7 and 8.

The feature of the UEPP is that the unified strength theory was implemented into the finite element method code. UEPP includes two codes, i.e. UEPP-2D for plane stress, plane strain and axial-symmetric problem and UEPP-3D for three-dimensional problems. The material models are increasing and forming a series of systematic and effective constitutive relations for practical use. UEPP provides us with a very effective approach for studying the effect of failure criterion for various problems.

The unified strength theory and unified elasto-plastic constitutive relations are also implemented in some commercial finite element codes and some special finite element codes in China, Japan, Singapore, Australia, Sweden etc. The multi-parameter unified yield criterion has been applied to analyze two groups of reinforced concrete slabs and a parabolic cylindrical shell, by Wang et al., in Singapore. The nonlinear finite-element analysis code for plates and shells written by Huang (1988) and his predecessors (Owen and Hilton, 1980) is modified to incorporate the unified material model for concrete. The unified strength theory with b=0.6 is used. Elasto-plastic analysis for reinforced concrete slabs and high-strength concrete slabs using the unified strength theory was also successfully studied by Wang, Teng and Fan (2001). Various applications of the UST were presented.

The unified strength theory (UST) with tension cutoff is also adopted as the failure criterion for the analysis of punching shear failure of beams and slab-column connections by Zhang et al. at Griffith University, Australia. The

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results of applications of UST and UST with tension cutoff criterion and the comparisons with other yield criteria in the literature and experimental data are described in this chapter. These studies are focused on the implementation of the UST for the analysis of failure of reinforced concrete structures. Numerical studies are carried out in an attempt to verify the applicability of the UST. The numerical and published experimental results are compared for several RC (Reinforced Concrete) structures.

The UST is also implemented into FLAC-2D, FLAC-3D, AutoDYNA, AutoDYNA2D, ABQUSE, etc. which has been described in the above chapters.

15.2 Elasto-Plastic Analysis for Reinforced Concrete Beams

At present, some failure criteria are already in use to analyze the punching shear strength of slab-column connections and its associated failure behaviour. Gonzalez-Vidosa et al. (1988) used a failure criterion based on the test data under axisymmetric stress conditions (i.e., two of the three principal stresses are equal) for the axisymmetrical punching shear analysis of reinforced concrete circular slabs. For a similar problem, Zhou and Jiang (1991) adopted Ottosen (1977) criterion in the punching shear failure analysis of reinforced concrete circular slabs under axisymmetrical loading. For a series of half-scale reinforced concrete flat plates with edge and comer column connections, Loo and Guan (1997) have analyzed their flexural failure and punching shear behaviour by means of the Ottosen criterion. The abovementioned failure criteria were developed on the basis of experimental results rather than theoretical derivations.

In addition, such criteria involve a large number of parameters which are rather difficult to determine accurately. When dealing with such a complex stress problem, the appropriate constitutive model and failure criterion for concrete have to be utilized.

15.2.1 Material Modelling

For various materials, Yu (1991; 1992) has suggested a unified strength theory (UST) based on the assumption that the plastic flow is controlled by the combination of the two larger principal shear stresses and their corresponding normal stresses. The UST can be presented by two simple mathematical formulae and a set of piecewise linear yield surfaces. A class of convex criteria can be obtained by varying the coefficient *b* in the UST to suit different materials like metal, concrete, rock and soil, etc.

Based on the orthogonal octahedron of the twin shear element model (Yu, l985), the unified strength theory specifies that material fails when a certain function of the two larger principal shear stresses and the corresponding normal stresses on their surfaces reach the limiting value. The mathematical modelling of the UST is

$$\tau_{13}+b\tau_{12}+\beta(\sigma_{13}+b\sigma_{12})=C \qquad \text{when} \quad \tau_{12}+\beta\sigma_{12}\geq\tau_{23}+\beta\sigma_{23} \tag{15.1a}$$

$$\tau_{13}+b\tau_{23}+\beta(\sigma_{13}+b\sigma_{23})=C \qquad \text{when} \quad \tau_{12}+\beta\sigma_{12}\leq\tau_{23}+\beta\sigma_{23} \tag{15.1b}$$

where $\tau_{13},\tau_{12},\tau_{23}$ are the principal shear stresses defined as $\tau_{13}=(\sigma_1-\sigma_3)/2$, $\tau_{12}=(\sigma_1-\sigma_2)/2$, $\tau_{23}=(\sigma_2-\sigma_3)/2$ and $\sigma_{13},\sigma_{12},\sigma_{23}$ are the corresponding normal stresses defined as $\sigma_{13}=(\sigma_1+\sigma_3)/2$, $\sigma_{12}=(\sigma_1+\sigma_2)/2$, $\sigma_{23}=(\sigma_2+\sigma_3)/2$, which are in the stress planes of $(\sigma_1,\sigma_3),(\sigma_1,\sigma_2),(\sigma_2,\sigma_3)$, respectively; $\sigma_1,\sigma_2,\sigma_3$ are the principal stresses and $\sigma_1\geq\sigma_2\geq\sigma_3$, C is a material strength parameter; b and β are the coefficients that reflect the influence of the intermediate principal shear stress τ_{12} (or τ_{21}) and the corresponding normal stresses on the strength of the material, respectively.

For concrete, Eqs. (15.1a) and (15.1b) can be rewritten in terms of three principal stresses as

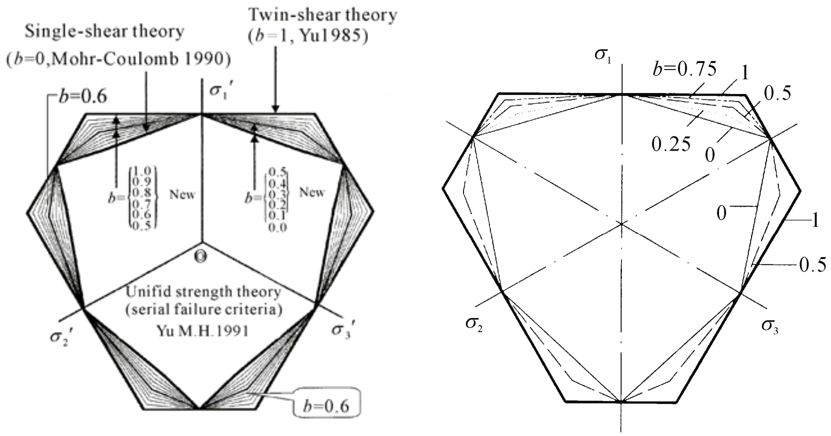
$$\sigma_1-\frac{\alpha}{1+b}(b\sigma_2+\sigma_3)=f_t\,, \quad \text{when}\ \sigma_2\leq\frac{1}{2}(\sigma_1+\sigma_3) \tag{15.2a}$$

$$\frac{1}{1+b}(\sigma_1++b\sigma_2)-\alpha\sigma_3=f_t\,, \quad \text{when}\ \ \sigma_2\geq\frac{1}{2}(\sigma_1+\sigma_3) \tag{15.2b}$$

where f_t is the tensile strength of concrete, α is the ratio of the tensile strength f_t and the compressive strength f_c of concrete; $b=(\tau_0(1+\alpha)-f_t)/(f_t-\tau_0)$ in which τ_0 is the shear strength of concrete.

Figure 15.1 depicts the limited loci of the UST which also indicates that the unified strength theory represents a group of hexagons on a deviatoric plane. A family of convex yield criteria related to a variety of materials is deduced when b varies from 0 to 1. For instance, UST becomes the Mohr-Coulomb criterion if b=0 and the twin-shear yield criterion (Yu, 1983) will be obtained when b=l.

The eleven limit loci with parameter b=0, b=0.1, b=0.2, b=0.3, b=0.9 and b=1.0 cover the whole of the convex region, as shown in Fig. 15.1(a); the five typical limit loci with parameters b=0, b=0.1/4, b=1/2, b=3/4, and b=1.0 and three typical limit loci with parameters b=0, b=1/2 and b=1.0 are as shown in Fig. 15.1(b).



(a) Eleven limit loci cover the whole region (b) Five and three typical limit loci

Fig. 15.1 Limited loci of the unified strength theory

15.2.2 Material Modeling of Concrete

In the analysis, the failure of reinforced concrete is considered to be a result of either tension cracking in concrete or plastic yielding, which leads to the crushing of concrete (Loo and Guan, 1997). Concrete is assumed to be linear elastic and its behaviour is characterised as isotropic until the specified fracture surface determined by the UST is reached. Numerical modelling of either cracking or crushing of concrete involves the modification of material stiffness and the release of the appropriate stresses partially or completely in the fractured elements.

The tensile type of fracture or cracking is governed by a maximum tensile stress criterion, referred to as the tension cut-off. Cracked concrete is treated as an orthotropic material using a smeared crack approach. After cracking has occurred, the tensile and shear stresses acting on the cracked plane are released and redistributed to the neighboring elements. Under subsequent loading, concrete loses its tensile strength normal to the crack direction, but retains the tensile strength in the directions parallel to the crack plane.

The strain-hardening plasticity approach is used to model the concrete in compression. This approach involves loading surfaces, loading function, normality rule and unloading associated with the UST (Unified Strength Theory). After the compression type of fracture occurs, the concrete material is assumed to lose some, but not all, of its strength and rigidity.

15.2.3 Reinforcing Steel

The reinforcing steel is assumed to be an elastic-plastic uniaxial material. The reinforced bars at a given level in an element are modeled as a smeared steel layer of equivalent thickness.

15.2.4 Structural Modeling

The study of Zhang, Guan and Loo is focused on the implementation of the UST for the analysis of beams and punching shear failure of reinforced concrete slab-column connections, which will be described in the next section. Numerical studies are carried out in an attempt to verify the applicability of this criterion. The numerical and published experimental results are compared for two simply supposed beams and two slab-column connections (next section).

In the study, the layered finite element method developed by Guan and Loo (1997a; 1997b) is adopted to model the structure. In the analysis, eight-node degenerate shell elements with bi-quadratic serendipity shape functions are adopted in conjunction with the layered approach. The model makes use of the transverse shear deformations associated with the Mindlin hypothesis. Five d.o.f are specified at each nodal point. They are the in-plane displacements, u and v, lateral bending displacement w and two independent bending rotations about the x and y axes, i. e.θ_y andθ_x respectively.

In the layered approach, each element is subdivided into a chosen number of layers which are fully bonded together The concrete characteristics are specified individually for each layer over its thickness. On the other hand, each layer of the reinforcing bars is represented by a smeared layer of equivalent thickness. In a nonlinear analysis, the material state at any Gauss point located at the mid-surface of a layer can be elastic, plastic or fractured, according to the loading history. To account for the mechanical change in the materials throughout the incremental loading process, cracking and nonlinear material response are traced layer by layer. By incorporating all the in-plane and out-of-plane stress components in the finite element formulation, inclined cracks can be simulated.

15.2.5 Simply Supported Beams

A series of 12 simply supported, reinforced concrete beams were specially designed and tested by Bresler and Scordelis (1963) to determine the crack loads and the ultimate strength characteristics. Each beam was subjected to a concentrated load applied at the mid-span, as illustrated in Fig. 15.2. The test beams were grouped into four series. The first group, the OA-series (without web reinforcement), was analysed by Zhang et al. (2001; 2002) at Griffith University, Australia, with dimensional details shown in Fig. 15.2. Other relevant data can be found elsewhere (Bresler and Seordelis, 1963).

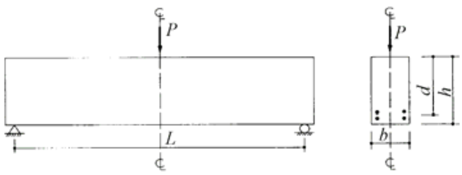


Fig. 15.2 Simply supported beam under central concentrated load

The parameters of the RC beam are: OA1: b=307.3 mm, h=556.3 mm, d=461.0 mm, L=3657.6 mm; OA2: b=304.8 mm, h=561.3 mm, d=466.1 mm, L=4572.0 mm.

The experimental and numerical load-displacement curves are compared in Fig. 15.3. It may be seen that the use of UST is capable of simulating the overall shear failure behavior of the beams without web reinforcement.

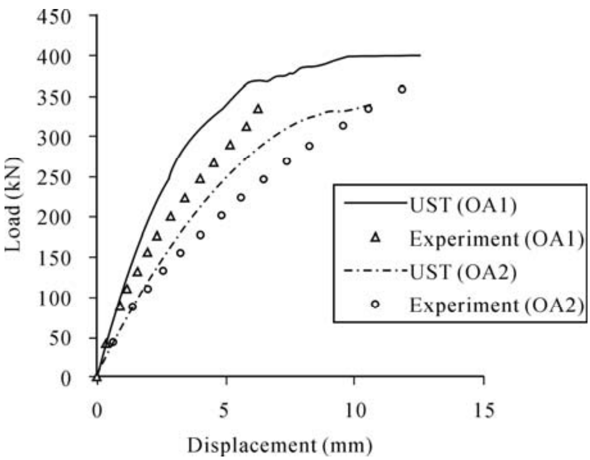


Fig. 15.3 Load versus displacement of beams

The experimental and numerical load-displacement curves are compared in Fig. 15.3. It may be seen that the use of UST (Unified Strength Theory) is capable of simulating the overall shear failure behavior of the beams without web reinforcement.

15.3 Punching Shear Failure Analysis of Flat Slabs by UST

A new analysis of the reinforced concrete slab-column connection was also presented by Zhang, Guan and Loo at Griffith University, Australia, in 2001. The unified strength theory (UST) is adopted as the failure criterion for the analysis of the punching shear failure of slab-column connections. The results described here follow Zhang et al. (2001).

Employing the layered finite element method, both material and geometrical nonlinearities are considered in the analysis. An investigation is carried out on reinforced concrete beams and slab-column connections. The numerical results indicate that the analysis based on UST is capable of simulating the overall failure behavior of slab-column connections.

Punching shear failure is referred to as a local shear failure that could occur around concentrated loads or column heads. In the design of reinforced concrete flat plates, the regions around the columns always pose a critical analysis problem. This is because large bending moments and shear forces are concentrated at the slab-column connections. This in turn complicates the stress distribution at the connections.

15.3.1 Slab-Column Connections

To verify the appropriateness of the UST in predicting the punching shear strength analysis, two typical slab-interior column connections (Slabs A and B) tested by Regan (1986) are analyzed. The dimensions of both slabs were 2 m×2 m×l00 mm; they were simply supported on four sides with l.83m spans and with the comer free to lift up. For each slab, a load was applied through a 200 mm monolithically cast column stub which projected above and below the slab. These two slabs were designed mainly to investigate the effect of the arrangement of flexural reinforcement. Both slabs have the same reinforcement ratios. However, Slab B has a uniform arrangement of flexural reinforcement whereas the steel arrangement in Slab A does not. Figure 15.4 shows the reinforcement details of slab A.

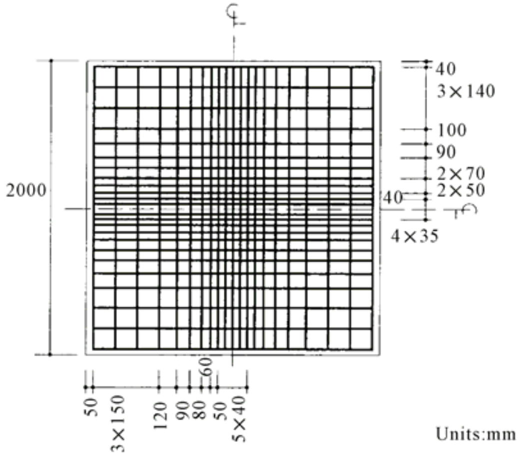


Fig. 15.4 Reinforcement details of slab (Zhang et al., 2001)

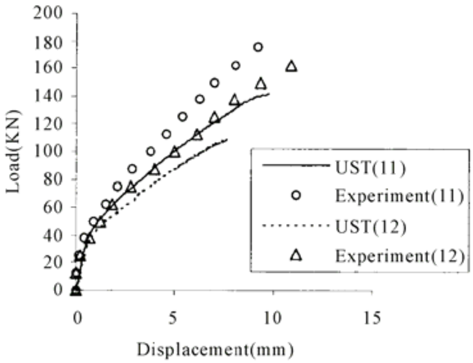


Fig. 15.5 Load versus displacement of slabs (Zhang et al., 2001)

The load-displacement responses shown in Fig. 15.5 demonstrate that the numerical results correlate reasonably well with the experimental outcome except that the numerical analysis somewhat underestimated the punching shear strength of the slab-column connections.

15.3.2 Conclusions

The conclusions of Zhang, Guan and Loo are given as follows:

"The adoption of the unified strength theory (UST) as the failure criterion for the layered finite element shear strength analysis is presented. The capabilities of the UST criterion are checked in a numerical investigation which covers typical reinforced concrete beams failing in shear, as well as slab-column connections failing in punching shear. Comparisons with published experimental data show that the analysis underestimated the failure loads of the slab-column connections".

"The UTS (Unified Strength Theory) is advantageous over other failure criteria because it encompasses all other established criteria as special cases. Or, such criteria are merely the linear approximations of the UST. Moreover, the parameters of the UST are easily obtained by experiments."

15.4 Elasto-Plastic Analysis for an Ordinary RC Beam

Yu's UST was implemented into commercial FEM software DIANA through the facility of a user-defined subroutine by Dr. Zhou at Nanyang Technological University in Singapore (Zhou, 2002). UST with parameter b=0.6 is the choice. The yield locus of the unified strength theory with b=0.6 is shown in Fig. 15.1(a).The descriptions of Zhou are as follows.

DIANA is a commercial software, which is suitable for treating various kinds of problems in finite element analysis. This program provides some kinds of elements for modeling concrete, steel and also the interfaces between different materials. But there is no strong physical meaning in its material models for concrete, in particular the post-failure behavior. With a user-defined subroutine, any kind of material model is permitted to be added into DIANA. In the subroutine, the previous value of strain tensor ε_{ij}^{r}, the increment of strain tensor $\Delta\varepsilon_{ij}^{r+1}$ elasticity matrix D, the previous stress σ_{ij}^{r}, effective stress $\tilde{\sigma}_{ij}^{r}$, equivalent plastic strain $\bar{\varepsilon}_{p}^{r}$ are the input parameters, which can be obtained from the major module. Users are allowed to defined their own constitutive relation and then feed back the current total σ_{ij}^{r} and the tangent stiffness $[D]_{ep}$ to the main module. The theoretical development outlined in the previous chapter was coded in two subroutines and incorporated into DIANA. In Zhou's study, DIANA release 7.1 is used.

This example shows the numerical simulation of an ordinary RC beam. The configurations and the loading on the beam are chosen to simulate the experimental tests by Kotsovos and Pavlovic (1995). The beam was subjected to two one- third-point loads and failed in a ductile, flexural manner. Figure 15.6 shows the dimensions of the beam and the reinforcement details.

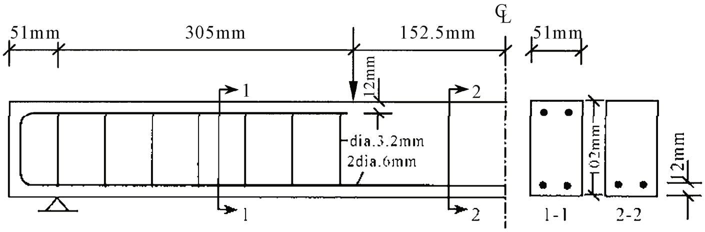


Fig. 15.6 Dimension and details of the RC beam

The ratio of the shear span to the effective depth is equal to 0.33 and the percentage of the longitudinal tension steel is 1.2%. Transverse reinforcement was provided within the shear span so as to prevent a shear failure.

The material parameters used in the finite element model are shown in Table 15.1.

Table 15.1 Material parameters of the ordinary RC beam

Parameters	Values
E_c	29100 MPa
E_s	200000 MPa
f_c	37.8 MPa
f_t	3.38 MPa
f_y	417 MPa
v	0.2
UST parameter b	0.6
A_t	1.0
B_t=0.6	50000

Figure 15.7 shows the mesh and loading condition of the beam. Three-node truss elements and twenty-node brick elements are used to model the reinforced bars and the concrete, respectively. The concrete cover to the tension steel is ignored. All the steel elements (truss elements) lie on edges of brick elements (concrete). A perfect bond is assumed throughout this analysis.

With the plastic damage models proposed, the nonlinear response of the beam is calculated and the deflection at the mid-span of the beam is evaluated. Figure 15.8 shows the load-deflection curves obtained from Model I and Model II, which are the combination of UST and Kotsovos's experimental data.

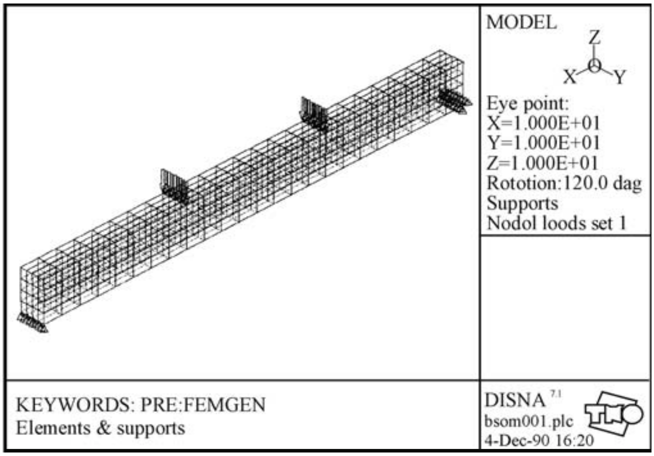


Fig. 15.7 Elements, supports and loading condition of the beam (Zhou, 2002)

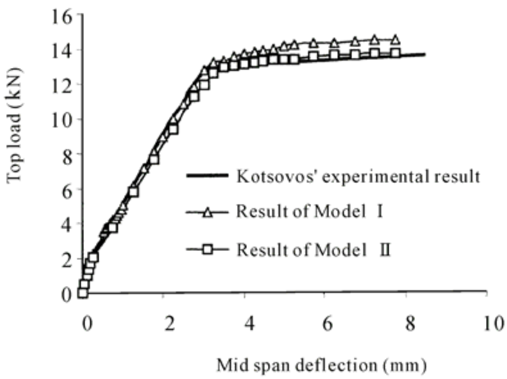


Fig. 15.8 Load-deflection curves of the RC beam (Zhou, 2002)

It can be seen that Model II yields results in very good agreement with the experimental ones, while the results obtained from Model I are inferior to those of Model II. Regarding the ultimate capacity of the beam, the value obtained from the test is 13.6 kN and the analytical values are 14.5 kN for Model I and 13.7 kN for Model II, with an error of 6.6 % and 0.7%, respectively.

15.5 Elasto-Plastic Analysis of an RC Deep Beam

This example shows the numerical simulation of an RC deep beam, which was reportedly tested in the School of Civil and Structural Engineering, Nanyang Technological University, Singapore (Poh and Susanto, 1996). The details of the experimental beam are shown in Fig. 15.9.

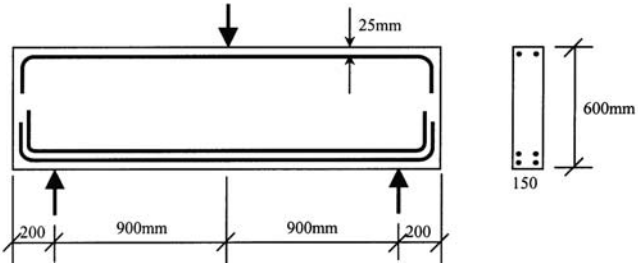


Fig. 15.9 Details of the deep beam (Zhou, 2002)

There is no web reinforcement at all. Only longitudinal reinforcement T22 is provided, 2 numbers near the top surface and 4 numbers near the bottom surface, as can be seen from Fig. 15.7. The material parameters used in the finite element model are listed in Table 15.2.

Table 15.2 Material parameters of the deep beam

Parameters	Values
E_c	29240 MPa
E_s	200000 MPa
f_c	38.2 MPa
f_t	3.40 MPa
f_y	754.08 MPa
v	0.2
UST parameter b	0.6
A_t	1.0
B_t=0.6	50000

Figure 15.10 shows the finite element mesh and the loading condition. Two-node truss elements and eight-node brick elements are used to model the steel bars and concrete, respectively. With the two different models (Model I and Model II), the load-deflection curves are obtained and compared with the experimental one in Fig. 15.11.

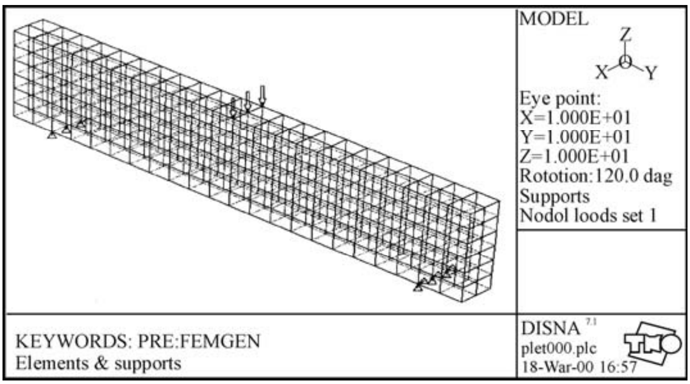


Fig. 15.10 Mesh of the deep beam (Zhou, 2002)

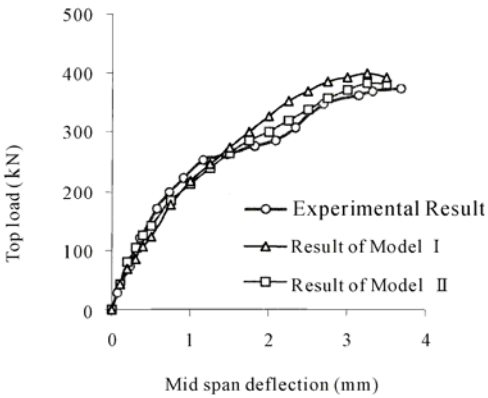


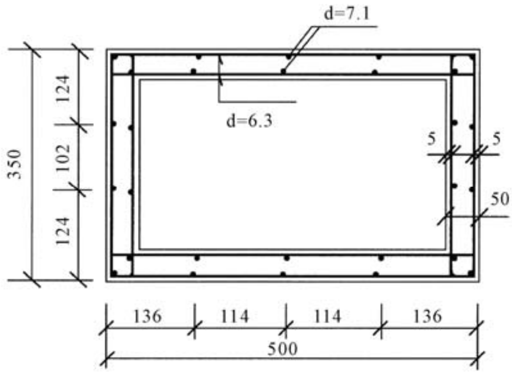
Fig. 15.11 Load-deflection curves of the deep beam (Zhou, 2002)

From this figure, it can be seen clearly that both models give reasonably accurate predictions of the overall response of the beam. Again, the results obtained from Model II are better than those from Model l. In particular for the ultimate capacity of the beam, the experimental value is 375 kN and the analytical values are 399.58 kN for Model I and 384.72 kN for Model II, with an error of 6.55% and 2.59%, respectively.

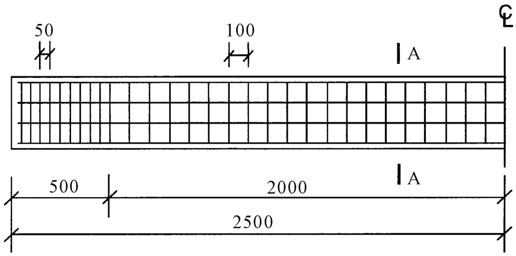
15.6 Elasto-Plastic Analysis of an RC Box Sectional Beam

From the examples in the previous two sections, we can see that Model II can yield better prediction than Model I for the load-deflection responses. In this section, therefore, Model II is employed to analyze the response of an RC box sectional beam under eccentric loading conditions by Zhou. The box girders tested by Rasmussen and Baker (1999) are analyzed here. The dimensions and the reinforcement layout are depicted in Fig. 15.12.

Three different configurations of anti-symmetric loading are studied, as illustrated in Fig. 15.13. Only point loads were applied at the mid-span. Load cases l and 2 invoke bending, torsion and distortion, while load case 3 invokes only torsion and distortion. It is worth noting that, in load case 2, the upward load is half of the downward load. In the test, the specimens were simply supported but torsionally restrained at both ends.

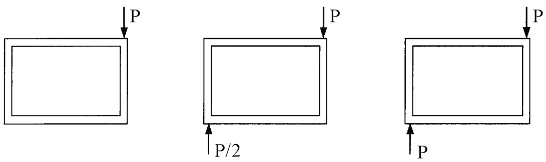


(a) Section A-A



(b) Elevation and reinforcements

Fig. 15.12 Reinforcement layout (dimension in millimeters)



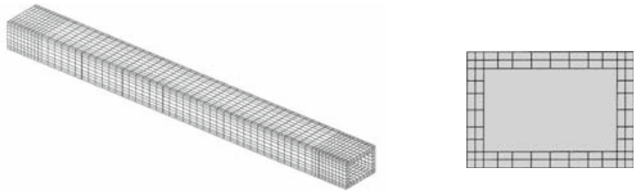
(a) Case 1 (b) Case 2 (c) Case 3

Fig. 15.13 Loading cases in mid-span section (Zhou, 2002)

Figure 15.14(a) shows the finite element mesh used in the calculation. Longitudinally it is divided into 60 segments with respect to the spacing of the stirrups. A two-node truss element and eight-node brick element are used to model the reinforced bars and the concrete, respectively. The concrete cover (5 mm) to the tension steel is ignored, as in the previous analysis. All the steel elements (truss elements) lie on the edges of brick elements (concrete). A perfect bond is assumed in this analysis. The material parameters used in the calculation are shown in Table 15.3.

Table 15.3 Material parameters of the RC box sectional beam

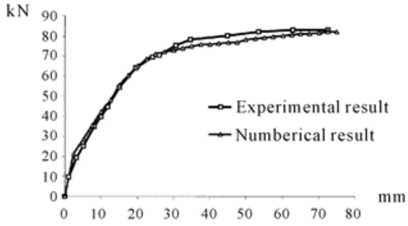
Parameters	Values
E_c	22000 MPa
E_s	200000 MPa
f_c	50.0 Mpa
f_t	4.00 Mpa
f_y	541 Mpa
v	0.2
UST parameter b	0.6
A_t	1.0
B_t=0.6	50000



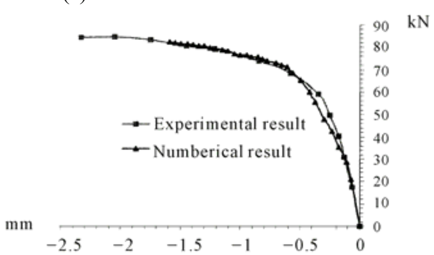
(a) Finite element mesh (b) Cross section

Fig. 15.14 Finite element mesh used in the calculation

Figure 15.15 compares the numerical and the experimental results in load case l. Figure 15.l5(a) plots the load versus the vertical deflection and Fig. 15.15(b) plots the load versus the diagonal distortion. From these figures, it can be seen clearly that the present model is able to yield good prediction of the overall response (vertical deflection and diagonal distortion). In particular for the ultimate capacity of the beam, the experimental value is about 83.0 kN and the analytical value is 82.37 kN with an error of only 0.8%.



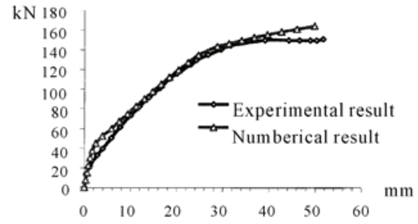
(a) Load versus vertical deflection



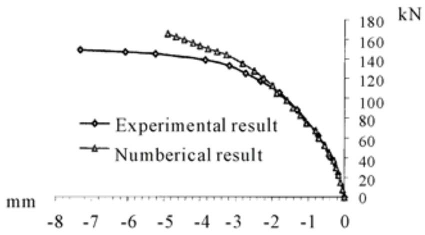
(b) Load versus diagonal distortion

Fig. 15.15 Comparison of numerical results and experimental results for load case 1

Similarly, Fig. 15.16 shows the analytical and experimental results of load versus vertical deflection and load versus diagonal distortion in load case 2.



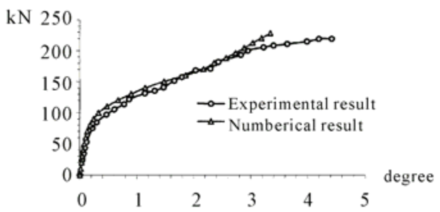
(a) Load versus vertical deflection



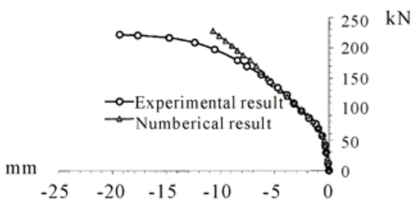
(b) Load versus diagonal distortion

Fig. 15.16 Comparison of numerical results and experimental results for load case 2

Figure 15.17 shows those results in load case 3. From the figures, it can be found that the present model yields accurate predictions of the overall responses except for the near-failure regions, where deviation from experimental results is observed in all load cases, especially in the diagonal distortions. These discrepancies may arise from the differences in the restraint condition at the end supports because, in the test, torsion cannot be strictly restrained at both ends of the beam and end diaphragms were not cast. Nevertheless, the accuracy achieved with the present model can suffice in engineering applications.



(a) Load versus total rotation

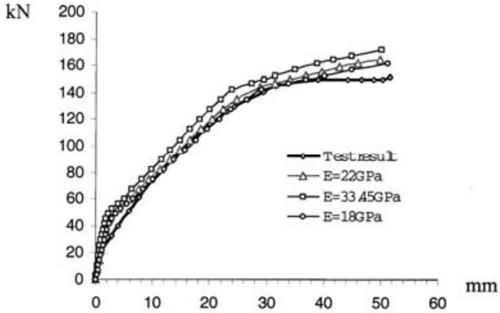


(b) Load versus diagonal distortion

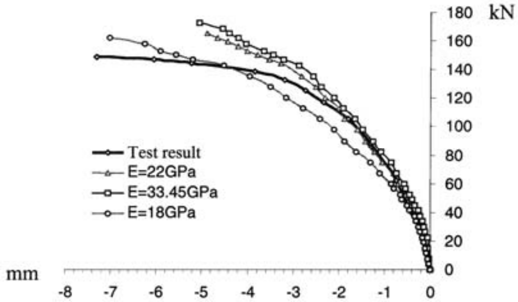
Fig. 15.17 Comparison of numerical results and experimental results for load case 3

On the other hand, the discrepancies may be due to the deviation of the material parameters, in particular the effect of Young's modulus. A sensitivity analysis is performed. Without loss of generality, only load Case 2 is studied here. Figure 15.18 shows the overall response obtained from different values of

Young’s modulus. From the figures, it confirms that the variations in Young’s modulus can affect the results, to some extent.



(a) Load versus vertical deflection



(b) Load versus diagonal distortion

Fig. 15.18 Sensitivity analysis of Young’s modulus for load case 2 (Zhou, 2002)

15.7 Summary

Unified strength theory (UST) is also successfully implemented in several commercial FEM codes and finite difference method codes. The unified strength theory (UST) with different parameter b is also adopted as the failure criterion for the analysis of punching shear failure of beams and slab-column connections by Zhang et al. at Griffith University, Australia. Elasto-plastic analysis for reinforced concrete slabs and high-strength concrete slabs using the unified strength theory has also been successfully studied by Zhou and Wang and Fan at Nangyang Technical University in Singapore.

Through comparison of FE simulation results and the experimental data, conclusions can be drawn that the unified strength theory and its associated flow rule, and a new three-dimensional elasto-plastic-damage constitutive model for concrete, can be successfully implemented into non-linear FEM. The derived load-carrying capacities for all the beams and slabs are in good agreement with the experimental data. Generally, the calculated deflections at different levels of load for all the slabs also reflect the real deformation procedure. The only exception is that the predicted deflections for the high-strength slabs are smaller than for the experimental counterparts, which implies that the high-strength slabs in the simulation are stiffer than the actual slabs. Damage distributions and the reinforcement stress distributions predict well the reinforcement anisotropy of the common concrete slabs and also the failure patterns for the high-strength concrete slabs.

Serial results can be obtained by using the unified strength theory (UST), which can be adopted for more materials. The material models are increasing and forming a series of systematic and effective constitutive relations for practical use. UST and its implementation in computer codes provide us with a very effective base and approach for studying the effect of failure criterion for various problems. It can be used for more materials and more structures. The strength potential of materials may be utilized by using UST with b>0.

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