Finite Precision Extended Alternating Projection Neural Network (FPEAP)

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Abstract. The paper studies finite precision Extended Alternating Projection Neural Network (FPEAP) and its related problems. An improved training method of FPEAP has been present after considering the finite precision influence on the training method of EAP. Then the mathematical relation among the factors influencing the association times has been studied. Finally simulation experiments have been designed and simulation results demonstrate validity of theoretical analyses.

Keywords: Alternating projection, neural network, signal processing, association times, finite precision.

1 Introduction

Alternating Projection Neural Network(APNN)[1] is firstly proposed by Marks II etc. The literature [2] studies APNN thoroughly and proposes a new neural network—Extended Alternating Projection Neural Network (EAP) which functions in the complex domain. The topology architecture and association process of EAP are the same as those of APNN. In the literature [2] the stability of EAP has been studied and strict mathematical proofs to its stability has also been given. The literature [3] obtains the mathematical expression to the steady state value of EAP and gives the sufficient and necessary condition of EAP used for Content Addressable Memory (CAM).

EAP neural network is prone to parallel computation and VLSI design due to its simplicity, consequently has a bright future under the real time processing situations. It has been applied to the signal processing such as band-limited signal extrapolation[4], notch filters[5] and weak signal seperation[6]. In order to expand its application scope and apply it better in other field such as pattern recognition and sensor network, the literature [7] has made further research on the EAP.

While studying EAP the above literatures have made an assumption that each data is of unlimited accuracy and no errors exist during their operations. However, artificial neural networks in practical application are implemented by adopting ASCI or microprocessors. Hence its data is of limited accuracy and error must exist during the operations of data. The paper will study finite precision EAP (FPEAP) and its related problems.

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2 EAP and FPEAP

EAP is full-interconnection neural network and its topology architecture is shown in Figure 1. Suppose EAP is made up of L neurons, the arbitrary neuron i and the neuron j are bidirectional connection. Weight t_{ij} equals weight t_{ji}. Neurons of EAP can be classified into clamped neurons or floating neurons according to their states. The state $s_i(m)$ of arbitrary floating neuron i at time m equals $\sum_{p=1}^{L} t_{pi} s_p(m-1)$, the state $s_j(m)$ of arbitrary clamped neuron j at time m is equal to $s_j(m-1) = \cdots = s_j(0)$. The weight-value of network can be obtained by the training method in the literature [2].

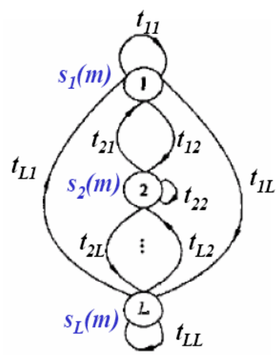

Fig. 1. Illustration of EAP

After EAP has been trained, it is time to decide which neurons are clamped neurons and which neurons are floating neurons. We can assume without loss of generality that neurons 1 through P are clamped and the remaining $Q=L\text{-}P$ neurons are floating. At time m=0 clamped neurons are initialized. If the state vector of EAP is $S(m) = [s_1(m) \quad s_2(m) \quad \cdots \quad s_L(m)]^T$ at time m, the state vector of clamped neurons is $S^P(m)$ and the state vector of floating neurons is $S^Q(m)$, then the state vector of EAP at time m+1 is

$$S(m+1) = \eta T S(m) = \begin{bmatrix} S^P(m+1) \\ S^Q(m+1) \end{bmatrix} = \eta \cdot \begin{bmatrix} T_2 & T_1 \\ T_3 & T_4 \end{bmatrix} \cdot \begin{bmatrix} S^P(m) \\ S^Q(m) \end{bmatrix} = \begin{bmatrix} S^P(m) \\ T_3 S^P(m) + T_4 S^Q(m) \end{bmatrix} \quad (1)$$

Where T is the interconnection matrix, operator η clamps the states of clamped neurons whose states will be altered by T. Thus the whole operating process of EAP is that operator T and operator η function by turns.

The entire operating process of EAP includes two stages, namely training stage and association stage. There exist addition, subtraction, multiplication and division etc at the training stage; there just exist addition and multiplication at the association stage.

If data of FPEAP are all of double precision, then we just discuss relative error. For FPEAP we find that relative error of subtraction of two approximate numbers which are very close or equal is remarkable.

For example, Let L=5, library pattern number N=3, $f_1 = [1 \quad 2 \quad 6 \quad 9 \quad 8]^T$, $f_2 = [2 \quad 4 \quad 9 \quad 8 \quad 7]^T$, $f_3 = [3 \quad 6 \quad 15 \quad 17 \quad 15]^T$, $F = [f_1 \quad f_2 \quad f_3]$. Since $f_3 = f_1 + f_2$, $\|\varepsilon_3\|$ is equal to zero theoretically, but computation errors caused by finite precision make $\|\varepsilon_3\| = 4.6044 \times 10^{-15}$ (Matlab result) to be unequal to zero. Thus the rank of matrix *T* obtained by EAP's weight-learning method is 3, for *P*=2 the spectral radius of matrix T_4 is 1.4192>1, for *P*=3 the spectral radius of T_4 is 1.1498>1. The network is no more stable. Therefore weight-learning method for EAP does not suit FPEAP.

Herein we will provide the following improved weight-learning method for FPEAP:

(a) Let the interconnection matrix *T* equal **0**, $i \leftarrow 1$;

(b) $\varepsilon_i = (I - T) f_i$, where I is $L \times L$ identity matrix, f_i is library pattern;

(c) if $\|\varepsilon_i\| \le \lambda_\alpha$, then f_i is already in the subspace *T* and goto step (d); else $T \leftarrow T + \varepsilon_i \varepsilon_i^H / \varepsilon_i^H \varepsilon_i$;

(d) $i \leftarrow i+1$, if $i > N$ (*N* is number of library pattern) then end, else goto step (b).

In the 3th step of the above method λ_α is threshold value which can be determined by using different method according to practical application.

3 Factors Influencing Association Times

Theoretically the FPEAP will reach the steady state value only after infinite times association. However in practical application it is impossible and unnecessary to make infinite times association since proper error between finite times association value and the steady state value may be permitted. Then what are the factors that influence the association times when the FPEAP is used for CAM?

Suppose FPEAP has *L* neurons, and has learned *N* library patterns f_i (*i=1,2,…,N*). We can assume without loss of generality that neurons 1 through *P* are clamped and the remaining *Q=L-P* neurons are floating.

From the formula (1) in the above section we can deduce the following conclusions:

$$S^P(m) = S^P(m-1) = \cdots = S^P(0) \tag{2}$$

$$S^Q(m) = \sum_{i=0}^{m-1} T_4^i T_3 S^P(0) + T_4^m S^Q(0) \tag{3}$$

Where the matrix T_4 is $Q \times Q$ matrix, and T_4^0 is $Q \times Q$ identity matrix.

According to the formula (3) we can deduce $S^Q(\infty)$ as follows:

$$S^Q(\infty) = \sum_{i=0}^{\infty} T_4^i T_3 S^P(0) + T_4^{\infty} S^Q(0) \tag{4}$$

From the literature [7] we can learn that $T_4^{\infty} = 0$ because the FPEAP is used for CAM. Thus the above formula (4) can be rewritten as follows:

$$S^Q(\infty) = \sum_{i=0}^{\infty} T_4^i T_3 S^P(0) \tag{5}$$

The error between m-times association value $S^Q(m)$ and the steady state value $S^Q(\infty)$ can be defined as follows:

$$Er(m) = \left\| S^Q(m) - S^Q(\infty) \right\| \tag{6}$$

$$= \left\| \sum_{i=0}^{m-1} T_4^i T_3 S^P(0) + T_4^m S^Q(0) - \sum_{i=0}^{\infty} T_4^i T_3 S^P(0) \right\|$$

$$= \left\| T_4^m S^Q(0) - \sum_{i=m}^{\infty} T_4^i T_3 S^P(0) \right\| = \left\| T_4^m \left(S^Q(0) - S^Q(\infty) \right) \right\|$$

While $\left\| S^Q(\infty) \right\| \neq 0$ we can also define the relative error as follows:

$$REr(m) = \left\| S^Q(m) - S^Q(\infty) \right\| / \left\| S^Q(\infty) \right\| \tag{7}$$

$$= \left\| T_4^m \left(S^Q(0) - S^Q(\infty) \right) \right\| / \left\| S^Q(\infty) \right\| \leq \gamma$$

From the formula (7) it is obvious that association times m will be closely related with the permitted relative association error γ, the inital state vector $S^Q(0)$ of all floating neurons and the spectral radius $\rho(T_4)$ of the interconnection matrix T_4 formed by weights of all floating neurons when the FPEAP is used for CAM.

4 Simulation Experiment and Result Analysis

In this section simulation experiments are designed to verify the theoretical analyses in section 3. Suppose the first P neurons are clamped and the remaining $Q=L-P$ neurons are floating. The following simulation experiments will be made on MATLAB R2006a.

Let L=200, N=17, P=50 and Q=150. F=*randn(L,N)*, $S^P(0)$=*randn(P,1)*, $S^Q(0) = 5*$ *randn(Q,1)*, γ=0.1**rand(1,1)*+10^{-4}, where *randn( )* and *rand( )* are both MATLAB function(see MATLAB handbook).

Case 1: we will make experiments for 500 times. For each experiment F, $S^P(0)$ and γ will remain invariant while $S^Q(0)$ varies randomly according to the above given expression. The 500-times variation of the minimum association times $\boldsymbol{m_m}$ satisfying $REr(m_m) \leq \gamma$ with $\left\| S^Q(0) \right\|$ will be illustrated in Figure 2.

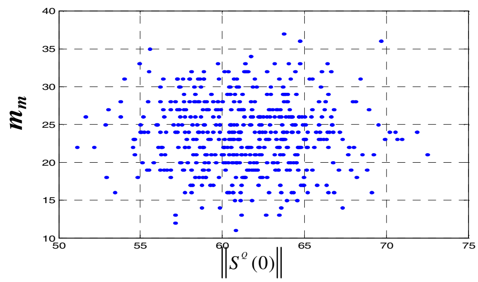

Fig. 2. 500-times experimental results for case 1

Case 2: for each experiment $S^Q(0)$, $S^P(0)$ and γ will remain invariant while F varies randomly according to the above given expression. The 500-times variation of the minimum association times $\boldsymbol{m_m}$ satisfying $REr(m_m) \le \gamma$ with $\rho(T_4)$ will be illustrated in Figure 3.

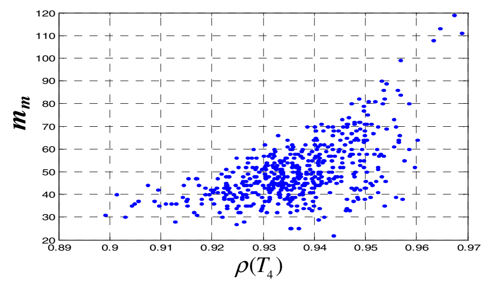

Fig. 3. 500-times experimental results for case 2

Case 3: for each experiment $S^Q(0)$, F and $S^P(0)$ will remain invariant while γ varies randomly according to the above given expression. The variation of the minimum association times $\boldsymbol{m_m}$ satisfying $REr(m_m) \le \gamma$ with $|\ln(\gamma)|$ for 500 times will be illustrated in Figure 4.

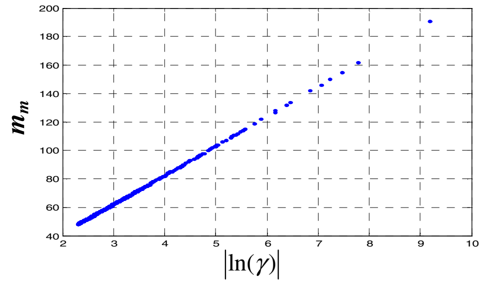

Fig. 4. 500-times experimental results for case 3

Figure 2 shows that $S^Q(0)$ influences the association times $\boldsymbol{m_m}$ and its multi-dimensional property causes the complex variation relation between $\boldsymbol{m_m}$ and $S^Q(0)$.

Figure 3 shows that $\rho(T_4)$ influences the association times $\boldsymbol{m_m}$ and $\boldsymbol{m_m}$ will increase with the increase of $\rho(T_4)$ on the whole.

Figure 4 shows that the association times $\boldsymbol{m_m}$ will almost linearly vary with $|\ln(\gamma)|$. This result can be theoretically analyzed and proved.

5 Peroration

The paper has studied finite precision EAP (FPEAP) and its related problems. Firstly the finite precision influence on the training method of EAP has been considered and an improved training method for FPEAP is present. Then the mathematical relation among association times of FPEAP used for CAM, the permitted relative association error, the inital state vector of all floating neurons and the spectral radius of the interconnection matrix formed by weights of all floating neurons has been studied. Finally simulation experiments have been designed and simulation results demonstrate validity of theoretical analyses.

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