A Variable Precision Covering Generalized Rough Set Model

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Abstract. The covering generalized rough sets are an improvement of traditional rough set model to deal with more complex practical problems which the traditional one cannot handle. A variable precision extension of a covering generalized rough set model is proposed in this paper. Some properties are investigated.

Keywords: Rough sets, covering, variable precision rough set model.

1 Introduction

Rough set theory was introduced by Pawlak [18] to account for the definability of a concept with an approximation in an approximation space. It captures and formalizes the basic phenomenon of information granulation. The finer the granulation is, the more concepts are definable in it. For those concepts not definable in an approximation space, their lower and upper approximations can be defined.

Rough set theory has found practical applications in many areas such as data mining and data analysis. Successful applications of the rough set theory depend on the understanding of its basic notions, various views, interpretations and formulations of the theory, and potentially useful generalizations of the basic theory [22, 23]. There have been extensive theoretical research on rough set theory [3–13, 15].

However, partition or equivalence relation, as the indiscernibility relation in the traditional rough set theory, is still restrictive and it may limit the applications of the rough set models. Therefore, many generalized rough set models are proposed. In this paper, we focus on covering-based rough sets. Extensive research on this subject can be found in [1, 2, 14, 16, 19, 20, 24, 25, 27].

Zhang et al. combined covering rough set model with variable precision rough set model and proposed covering rough set model based on variable precision by using the intersection of all the minimum description set of an element (essentially the concept of neighborhood) in [26]. Liu et al. gave more properties of Zhang's model in [17]. Sun et al. extended Zhang's model using the concept neighborhood by granular approach in [21]. The elementary granules in the

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model are the neighborhoods. In this paper, we look the elements of a covering as elementary granules and construct a new variable precision covering generalized rough set model by granular approach.

2 Preliminaries

Let (U, R) be an approximation space, where U is the universe and R is an equivalence relation on U. With each approximation space (U, R), two operators on $\mathcal{P}(U)$ can be defined. For any $X \subseteq U$, then the lower approximation of X and the upper approximation of X are defined as:

$$R_{-}(X) = \bigcup\{[x]_R | [x]_R \subseteq X\} \tag{1}$$

$$R^{-}(X) = \bigcup\{[x]_R | [x]_R \cap X \neq \emptyset\} \tag{2}$$

The pair $< R_{-}(X), R^{-}(X) >$ is called a rough set. X is termed definable set(also termed exact set) in approximation space (U, R) if and only if $R_{-}(X) = R^{-}(X)$. For the sake of simplicity, the lower approximation and upper approximation are also denoted as $\underline{X}$ and $\overline{X}$ respectively.

Let U be a finite and nonempty set called the universe, and $\mathcal{C}$ a finite family of nonempty subsets of U. $\mathcal{C}$ is called a covering of U if it satisfies $\bigcup_{C \in \mathcal{C}} = U$, then the pair $(U, \mathcal{C})$ is called a covering approximation space.

The covering approximation operators is an extension of Pawlak approximation operators. It can be obtained by replacing the equivalence classes with the elements of a covering in granule-oriented definition of Pawlak approximation operators.Consequently, the lower and the upper approximation operators are not necessarily dual operators. Given a covering approximation space $(U, \mathcal{C})$, for any $X \subseteq U$, then the lower approximation of X and the upper approximation of X are defined as [19, 20]:

$$\underline{X} = \bigcup\{K | K \in \mathcal{C}, K \subseteq X\} \tag{3}$$

$$\overline{X} = \bigcup\{K | K \in \mathcal{C}, K \cap X \neq \emptyset\} \tag{4}$$

Example 1. Let $U = \{u1, u2, u3, u4, u5, u6\}$, $\mathcal{C} = \{\{u1, u2\}, \{u1, u2, u3\}, \{u5, u6\}, \{u1, u3, u4\}, \{u3, u4, u5\}\}$. Let $X = \{u1, u2\}$, then we have

$\underline{\underline{X}} = \bigcup\{K | K \in \mathcal{C}, K \subseteq X\}$=$\{u1, u2\}$

$\overline{X} = \bigcup\{K | K \in \mathcal{C}, K \cap X \neq \emptyset\} = \{u1, u2\} \cup \{u1, u2, u3\} \cup \{u1, u3, u4\} = \{u1, u2, u3, u4\}$.

Let $Y = \{u4, u5\}$, we get

$\underline{Y} = \bigcup\{K | K \in \mathcal{C}, K \subseteq X\}$=$\emptyset$

$\overline{Y} = \bigcup\{K | K \in \mathcal{C}, K \cap X \neq \emptyset\} = \{u5, u6\} \cup \{u1, u3, u4\} \cup \{u3, u4, u5\} = \{u1, u3, u4, u5, u6\}$.

3 Variable Precision Extension for Covering Generalized Rough Set Model

Essentially, the idea of variable precision rough set model is based on the generalization of the notion of the standard set inclusion relation [28]. Given any nonempty subsets A and B of the universe, We say that the set A is included in the set B with an inclusion error β:

$$A \subseteq_\beta B \Longleftrightarrow e(A,B) \leq \beta$$

$$e(A,B) = \begin{cases} 1 - \frac{card(A \cap B)}{card(A)} & A \neq \emptyset \\ 0 & A = \emptyset \end{cases}$$

The quantity $e(A,B)$ is called the inclusion error of A in B. The value of β should be limited in [0 ,0.5).

Now, we will give a variable precision extension of covering generalized rough set model.

Definition 1. *Let $(U, \mathcal{C})$ be a covering approximation space. For $X \subseteq U$, the β-lower approximation of of X, the β-upper approximation of X, the β-boundary region of X and the β-negative region of of X are defined as*
$\underline{apr}^\beta(X) = \bigcup\{K | K \in \mathcal{C}, e(K,X) \leq \beta\}$
$\overline{apr}^\beta(X) = \bigcup\{K | K \in \mathcal{C}, e(K,X) < 1-\beta\}$
$bnd^\beta(X) = \bigcup\{K | K \in \mathcal{C}, \beta \leq e(K,X) \leq 1-\beta\}$
$neg^\beta(X) = \bigcup\{K | K \in \mathcal{C}, e(K,X) \geq 1-\beta\}$

The lower approximation of the set X can be interpreted as the collection of the subset blocks from the cover $\mathcal{C}$ that can be classified into X with the classification error not greater than β. Similarly, the β-negative region of X is the collection the subset blocks from the cover $\mathcal{C}$ that can be classified into the complement of X with the classification error not greater than β.

Example 2. Let $U = \{u1, u2, u3, u4, u5, u6\}$, $\mathcal{C} = \{\{u1, u2\}, \{u1, u2, u3\}, \{u5, u6\}, \{u1, u3, u4\}, \{u3, u4, u5\}\}$. Let $X = \{u1, u2\}, \beta = 0.3$, then we have
$\underline{apr}^\beta(X) = \bigcup\{K | K \in \mathcal{C}, e(K,X) \leq \beta\} = \{u1, u2\}$
$\overline{apr}^\beta(X) = \bigcup\{K | K \in \mathcal{C}, e(K,X) < 1-\beta\} = \{u1, u2\} \cup \{u1, u2, u3\} \cup \{u1, u3, u4\} = \{u1, u2, u3, u4\}$.

Example 3. Let $U = \{u1, u2, u3, u4, u5, u6\}$, $\mathcal{C} = \{\{u1, u2\}, \{u1, u2, u3\}, \{u5, u6\}, \{u1, u3, u4\}, \{u3, u4, u5\}\}$. Let $Y = \{u4, u5\}, \beta = 0.4$, then we have
$\underline{apr}^\beta(Y) = \bigcup\{K | K \in \mathcal{C}, e(K,X) \leq \beta\} = \{u3, u4, u5\}$
$\overline{apr}^\beta(Y) = \bigcup\{K | K \in \mathcal{C}, e(K,X) < 1-\beta\} = \{u5, u6\} \cup \{u3, u4, u5\} = \{u3, u4, u5, u6\}$.

Proposition 1. *If $\beta = 0$, we have*
$\underline{apr}^\beta(X) = \underline{X}$
$\overline{apr}^\beta(X) = \overline{X}$.

Proof. $\underline{apr}^{\beta}(X) = \underline{apr}^{0}(X) = \bigcup\{K|K \in \mathcal{C}, e(K,X) \leq 0\} = \bigcup\{K|K \in \mathcal{C}, 1 - \frac{card(K\cap X)}{card(K)} \leq 0\} = \bigcup\{K|K \in \mathcal{C}, \frac{card(K\cap X)}{card(K)} \geq 1\} = \bigcup\{K|K \in \mathcal{C}, K \subseteq X\} = \underline{X}$.

On the other hand, $\overline{apr}^{\beta}(X) = \overline{apr}^{0}(X) = \bigcup\{K|K \in \mathcal{C}, e(K,X) < 1 - 0\} = \bigcup\{K|K \in \mathcal{C}, 1 - \frac{card(K\cap X)}{card(K)} < 1\} = \bigcup\{K|K \in \mathcal{C}, \frac{card(K\cap X)}{card(K)} \geq 0\} = \bigcup\{K|K \in \mathcal{C}, K \cap X \neq \emptyset\} = \overline{X}$.

Proposition 2. *Let* $(U, \mathcal{C})$ *be a covering approximation space. We have*
$\underline{apr}^{\beta}(X) = neg^{\beta}(\sim X)$
where, $\sim X = U - X$.

Proof. $\underline{apr}^{\beta}(X) = \bigcup\{K|K \in \mathcal{C}, e(K,X) \leq \beta\} = \bigcup\{K|K \in \mathcal{C}, 1 - \frac{card(K\cap X)}{card(K)} \leq \beta\} = \bigcup\{K|K \in \mathcal{C}, \frac{card(K\cap X)}{card(K)} \geq 1 - \beta\} = \bigcup\{K|K \in \mathcal{C}, \frac{card(K\cap \sim X)}{card(K)} \leq \beta\} = \bigcup\{K|K \in \mathcal{C}, e(K,X) \geq 1 - \beta\} = neg^{\beta}(\sim X)$.

Proposition 3. *Let* $(U, \mathcal{C})$ *be a covering approximation space. Let* $X, X_1, X_2 \subseteq U$*. We have*
(1)$\underline{apr}^{\beta}(\emptyset) = \emptyset$;
(2)$\overline{apr}^{\beta}(\emptyset) = \emptyset$;
(3)$\underline{apr}^{\beta}(U) = U$;
(4)$\overline{apr}^{\beta}(U) = U$;
(5)$X_1 \subseteq X_2 \Rightarrow \underline{apr}^{\beta}(X_1) \subseteq \underline{apr}^{\beta}(X_2), \overline{apr}^{\beta}(X_1) \subseteq \overline{apr}^{\beta}(X_2)$;
(6)$\underline{apr}^{\beta}(X_1 \cap X_2) \subseteq \underline{apr}^{\beta}(X_1) \cap \underline{apr}^{\beta}(X_2)$;
(7)$\overline{apr}^{\beta}(X_1 \cap X_2) \subseteq \overline{apr}^{\beta}(X_1) \cap \overline{apr}^{\beta}(X_2)$;
(8)$\underline{apr}^{\beta}(X_1) \cup \underline{apr}^{\beta}(X_2) \subseteq \underline{apr}^{\beta}(X_1 \cup X_2)$;
(9)$\underline{apr}^{\beta}(X_1) \cup \underline{apr}^{\beta}(X_2) \subseteq \underline{apr}^{\beta}(X_1 \cup X_2)$.

Proof. (1) $\underline{apr}^{\beta}(\emptyset) = \bigcup\{K|K \in \mathcal{C}, e(K,\emptyset) \leq \beta\} = \bigcup\{K|K \in \mathcal{C}, 1 - \frac{card(K\cap\emptyset)}{card(K)} \leq \beta\} = \bigcup\{K|K \in \mathcal{C}, 1 \leq \beta\}$. Since $0 \leq \beta < 0.5$, we have $\underline{apr}^{\beta}(\emptyset) = \emptyset$.

(2) $\overline{apr}^{\beta}(\emptyset) = \bigcup\{K|K \in \mathcal{C}, e(K,\emptyset) \leq 1 - \beta\} = \bigcup\{K|K \in \mathcal{C}, 1 - \frac{card(K\cap\emptyset)}{card(K)} \leq 1 - \beta\} = \bigcup\{K|K \in \mathcal{C}, 1 \leq 1 - \beta\}$. Since $0 \leq \beta < 0.5$, we have $\overline{apr}^{\beta}(\emptyset) = \emptyset$.

(3) $\underline{apr}^{\beta}(U) = \bigcup\{K|K \in \mathcal{C}, e(K,U) \leq \beta\} = \bigcup\{K|K \in \mathcal{C}, 1 - \frac{card(K\cap U)}{card(K)} \leq \beta\} = \bigcup\{K|K \in \mathcal{C}, 0 \leq \beta\}$. Since $\mathcal{C}$ is a covering, we have $\underline{apr}^{\beta}(U) = U$.

(4) $\overline{apr}^{\beta}(U) = \bigcup\{K|K \in \mathcal{C}, e(K,U) \leq 1 - \beta\} = \bigcup\{K|K \in \mathcal{C}, 1 - \frac{card(K\cap U)}{card(K)} \leq 1 - \beta\} = \bigcup\{K|K \in \mathcal{C}, 0 \leq 1 - \beta\}$. Since $\mathcal{C}$ is a covering, we have $\overline{apr}^{\beta}(U) = U$.

(5) For any $x \in \underline{apr}^{\beta}(X_1)$, $\exists K \subseteq \mathcal{C}$ such that $x \in K, e(K, X_1) \leq \beta$. By definition, we get $e(K, X_1) = 1 - \frac{card(K\cap X_1)}{card(K)}$, $e(K, X_2) = 1 - \frac{card(K\cap X_2)}{card(K)}$. If $X_1 \subseteq X_2$,we get $\frac{card(K\cap X_2)}{card(K)} \geq \frac{card(K\cap X_1)}{card(K)}$, i.e. $1 - \frac{card(K\cap X_2)}{card(K)} \leq 1 - \frac{card(K\cap X_1)}{card(K)}$. It follows $e(K, X_2) \leq e(K, X_1)$. Consequently, we have $x \in \underline{apr}^{\beta}(X_2)$. Hence, $\underline{apr}^{\beta}(X_1) \subseteq \underline{apr}^{\beta}(X_2)$.

Similarly, we can get $\overline{apr}^{\beta}(X_1) \subseteq \overline{apr}^{\beta}(X_2)$.

(6)-(9)are easy to get by (5).

Proposition 4. *Let* $(U, \mathcal{C})$ *be a covering approximation space. For* $X \subseteq U, 0 \leq \beta_1 \leq \beta_2 < 0.5$*, we have*
$\underline{apr}^{\beta_1}(X) \subseteq \underline{apr}^{\beta_2}(X)$;
$\overline{apr}^{\beta_2}(X) \subseteq \overline{apr}^{\beta_1}(X)$.

Proof. For any $x \in \underline{apr}^{\beta_1}(X)$, we know $\exists K \in \mathcal{C}$, such that $e(K, X) \leq \beta_1$. Since $0 \leq \beta_1 \leq \beta_2 < 0.5$, we know $e(K, X) \leq \beta_2$, i.e. $x \in \underline{apr}^{\beta_2}(X)$. It follows that $\underline{apr}^{\beta_1}(X) \subseteq \underline{apr}^{\beta_2}(X)$.

For any $x \in \overline{apr}^{\beta_2}(X)$, we know $\exists K \in \mathcal{C}$, such that $e(K, X) \leq 1 - \beta_2$. Since $0 \leq \beta_1 \leq \beta_2 < 0.5$, we know $e(K, X) \leq 1 - \beta_2 \leq 1 - \beta_1$, i.e. $x \in \overline{apr}^{\beta_1}(X)$. It follows that $\overline{apr}^{\beta_2}(X) \subseteq \overline{apr}^{\beta_1}(X)$.

Proposition 5. *Let* $(U, \mathcal{C})$ *be a covering approximation space. For* $X \subseteq U, 0 \leq \beta < 0.5$*, we have*
$\underline{X} \subseteq \underline{apr^{\beta}(X)}$;
$\overline{apr}^{\beta}\overline{(X)} \subseteq \overline{X}$.

Proof. It is straightforward from last proposition.

4 Conclusion

In this paper, a new variable precision covering based rough set model is proposed. Some properties are investigated.

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