A Study of Dynamic Decoupling Fuzzy Control Systems by Means of Generalized Inverse Matrix

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Abstract. In the dynamic decoupling fuzzy control system, if singular or rectangle matrices are encountered in the state equation, the only way is to consider the decoupling control over partial states of the system. This paper presents a new conclusion with fewer parameters by virtue of generalized inverse matrix and realized decoupling fuzzy control over the system with multiple variables, which demonstrates good control effect.

Keywords: Nonlinear dynamic inverse, Decoupling control, Generalized inverse matrix, Fuzzy controller.

1 Introduction

In the study of nonlinear dynamic inverse, we can introduce nonlinear input to counteract the influence of the nonlinear factors in the system and obtain the expected linear dynamic model in the decoupling way. If the method of dynamic inverse and that of fuzzy control are combined, then the joint method will possess advantages of both. For convenience and without loss of generality, we take the linear multiple variable system into account, but the conclusions obtained can also be applicable in the nonlinear multiple variable system.

Let

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

be the state equation of the system.

If B can be invertible, then the decoupling control of the system can be realized. But in most circumstances, B is a rectangle matrix, and thus what we can do is to reduce (1) to a form of block, and then realize decoupling control over partial states of the system through dynamic inverse method [1～3]. In this paper, we employ the concept of generalized inverse of a singular or rectangle matrix to realize the decoupling fuzzy control over the system (1) with multiple variables.

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2 Preliminaries

Let C^n be the complex n-dimension vector space and $C^{m\times n}$ be the set of $m\times n$ matrices and $C_r^{m\times n}=\{A\in C^{m\times n}$, rank A=r}. Assume R(A)={y∈C^m: y=Ax, x ∈C^n } is the range of A and N(A)={x∈C^n : Ax=0} is the null space of A. Let A^H denote the conjugate transpose matrix of a complex matrix A.

Definition 1[4]

Let $A\in C^{m\times n}$, if $X\in C^{n\times m}$ satisfies

$$AXA=A,$$
$$XAX=X,$$
$$(AX)^H=AX,$$
$$(XA)^H=XA.$$

then X is called Moore-Penrose generalized inverse of A, denoted by A^+.

The four equations in definition 1 are called Moore-Penrose equations. Each equation has some meaning and can be conveniently applied in practice[5,6]. For different purpose, one usually considers X that partially satisfies the equations, which is referred to as weak inverse.

Definition 2

Let $A\in C^{m\times n}$ and $A\{i, j, \ldots, l\}$denote the set of all matrices $X\in C^{n\times m}$ which satisfy the i-th, j-th, … and l-th Moore-Penrose equation. A matrix $X\in A\{i, j,\ldots,l\}$ is denoted by $X = A^{(i,j,\ldots,l)}$ and is referred to as the $\{i, j,\cdots,l\}$ -inverse of A.

In this paper, we present a new result expressing the set $A\{1\}$ ($A^{(1)}\in C^{n\times m}$), which involves the fewest arbitrary parameters. With the expression, the decoupling fuzzy control is more effective.

Lemma 1[7]

Let $A\in C^{m\times n}$, $b\in C^m$. Then the system of linear equations

$$Ax = b \tag{2}$$

is consistent if $AA^{(1)}b = b$ for all $A^{(1)}$.

In this case, the general solution of (2) is

$$x = A^{(1)}b + (I - A^{(1)}A)y$$

where $y\in C^n$ is arbitrary .

Lemma 2[7]

Let $A\in C_r^{m\times n}$, $r>0$. Then there exists a column full-rank matrix $F\in C_r^{m\times r}$ and a row full-rank matrix $G\in C_r^{r\times n}$ such that $A=FG$.

Lemma 3[7]

If $A^{(1)} \in A\{1\}$, then $R(AA^{(1)}) = R(A)$, $N(A^{(1)}A)=N(A)$and $R((A^{(1)}A)^H) = R(A^H)$.

3 Applying Generalized Inverse of Matrix to Realize Decoupling Fuzzy Control over the System

Theorem 1

Let $E \in C^{n\times n}$ be an idempotent matrix .Then

1) E^H and $I - E$ are idempotent;
2) The eigenvalues of E are 0 are 1, and the multiplicity of 1 is rank E;
3) rank E = trace E;
4) $E(I - E) = (I - E)E = O$;
5) $Ex = x$, iff $x \in R(E)$;
6) $N(E) = R(I - E)$.

Proof

The proof of 1)~5) is straightforward by the definition of idempotency.

By 2) and the fact that the trace of every square matrix equals the sum of all its characteristic values as a repeated root is calculated as many times as its multiplicity, we can easily obtain 3).

6) is verified by applying Lemma 1 to the equation $Ex = 0$.

Theorem 2

Let the square matrix E has the full-rank decomposition $E = FG$. Then E is idempotent iff $GF = I$.

Proof

If $GF = I$, then we have

$$(FG)^2=FGFG=FG \,. \tag{3}$$

Conversely, since F is a column full-rank matrix, its {1}-inverse is its left inverse. Similarly, G is a row full-rank matrix implies that its {1}-inverse is its right inverse. Hence

$$F^{(1)}F = GG^{(1)} = I \,.$$

Now It follows from (3) that $GF=I$ by multiplying $F^{(1)}$ on the left and $G^{(1)}$ on the right.

Theorem 3

Let $A \in C_r^{m\times n}$, $A^{(1)} \in A\{1\}$ be any fixed element and let $F \in C_{n-r}^{n\times(n-r)}$, $\mathrm{M}^H \in \mathrm{C}_{m-r}^{m\times(m-r)}$, $P \in C_r^{n\times r}$ be any given matrices whose columns are a basis of $N(A)$, $N(A^H)$ and $R(A^{(1)}A)$ respectively. Then the general solution of Moore-Penrose equation

$$AXA = A \tag{4}$$

is

$$X = A^{(1)} + FY + PZM \tag{5}$$

where $Y \in C^{(n-r)\times m}$, $Z \in C^{r\times(m-r)}$ are arbitrary, i.e.

$$A\{1\} = \{A^{(1)} + FY + PZM\colon Y \in C^{(n-r)\times m}, Z \in C^{r\times(m-r)}\}. \tag{6}$$

Proof

It is obvious that $AF = O$, $MA = O$, and thus the right side of (5) satisfies (4), i.e. $X \in A\{1\}$. It follows from Lemma 3 and 6) in Theorem 1 that

$$R(I_n - A^{(1)}A) = N(A) \text{ and } R((I_m - AA^{(1)})^H) = N(A^H).$$

By Lemma 2, there must exist unique matrices G, H, D such that $FG = I_n - A^{(1)}A$, $HM = I_m - AA^{(1)}$, $PD = A^{(1)}A$.

Since these multiplications are idempotent, by Theorem 2,

$$GF = I, \ \ DP = I, \ \ MH = I. \tag{7}$$

and

$$GP = O, \ \ \ DF = O \tag{8}$$

are obviously valid.

From (5), (7) and (8), it follows that

$$Y = G(X - A^{(1)}), \ \ Z = D(X - A^{(1)})H. \tag{9}$$

Now let X be an arbitrary element in $A\{1\}$. Substituting Y and Z in (9) into (5), we can see that (5) is satisfied. Therefore, (5) is the general solution of (4).

In the system (1), if B is rectangle matrix, let

$$u(t) = B^{(1)}[V(t) - Ax(t)],$$

where $B^{(1)} \in B\{1\}$, $V(t) = \dot{x}(t)$ is the state equation of the system. Choose $V(t)$ as

$$V(t) = K[x_c(t) - x(t)] \tag{10}$$

where K is a diagonal matrix and c stands for instruction.

By (10), we have

$$\dot{x}(t) = K[x_c(t) - x(t)] \tag{11}$$

Hence the decoupling control is realized in the system with multiple variables. By choosing the diagonal elements of the matrix K, the closed loop pole of the system can be arbitrarily set.

By means of self-tuning method of fuzzy control rule with correct factors, a well-behaved fuzzy controller can be designed by optimizing the correct factors, proportion and integral factor. The dynamic performance of the system output will be significantly improved by replacing the matrix K in (11) and adjusting fuzzy control rules and optimizing the parameters involved in the fuzzy controller.

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