Image Recognition by Affine Tchebichef Moment Invariants

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Abstract. Tchebichef moments are successfully used in the field of image analysis because of their polynomial properties of discrete and orthogonal. In this paper, two new affine invariant sets are introduced for object recognition using discrete orthogonal Tchebichef moments. The current study constructs affine Tchebichef invariants by normalization method. Firstly, image is normalized to a standard form using Tchebichef moments as normalization constraints. Then, the affine invariants can be obtained at the standard form. The experimental results are presented to illustrate the performance of the invariants for affine deformed images.

Keywords: Discrete orthogonal moments, affine transform, moment invariants, Tchebichef moments, pattern recognition.

1 Introduction

Moments invariants were firstly introduced by Hu [1], who proposed a method of deriving moment invariants from algebraic methods. He used geometric moments to generate a set of invariants. However, geometric moments are not derived from a family of orthogonal functions, and are sensitive to noise, especially for higher order moments [2]. Thus, Hu's moment invariants have limit applications. Many literatures have presented novel approaches of applying sequential orthogonal moments to construct moment invariants, such as Zernike moment [3], pseudo-Zernike [4], and Legendre moment [5]. But the accuracy of recognition descends due to the discrete approximation of the continuous integrals [6]. Mukundan proposed the discrete orthogonal Tchebichef moments [7]. The use of discrete orthogonal Tchebichef polynomials as basis function for image moments eliminates the discrete approximation associated with the continuous moments. Our previous work [8] proposed a new approach to derive the translation and scale invariants of Tchebichef moments based on the corresponding polynomials. However the descriptors are only invariant with respect to translation and scale of the pattern. In fact, objects may have other deformation, such as elongation, we still expect them to stay in the same category. The moment invariants proposed in refs. [3-5, 8] do not work well under

H. Deng et al. (Eds.): AICI 2011, Part III, LNAI 7004, pp. 472–480, 2011.

similar transformation. Moment invariants under affine transformation come up consequently to cope with this problem. Reiss [9] and Flusser and Suk [10] independently introduced affine moment invariants and proved their applicability in recognition tasks. These affine moment descriptors are expressed in terms of the central moments of the image data, and are widely used in many applications such as image analysis; pattern recognition and contour shape estimate [11-14]. Rothe et al [15] first proposed the concept of affine normalization. In their work, two different affine decompositions were used. The first called XSR consists of shearing, anisotropic scaling and rotation. The second is the XYS and consists of two shearings and anisotropic scaling. The normalization methods have then been further improved by other researchers [16]. Recently, Zhang et. al [17] proposed affine Legendre moment invariants for watermark detection. The affine moment invariant using continuous orthogonal Legendre moments [17], Zernike moments [14] have been already obtained, no affine moment invariants take the discrete Tchebichef moments into consideration until now. Motivated by their methods [17~18], this paper presents two sets of discrete orthogonal Tchebichef moment invariants using XYS and XSR decomposition. To obtain the proposed affine moment invariants, the study applies the normalization method which is done via decomposition the affine transformation into three successive steps. Then, the normalization is achieved by imposing normalization constraints on some chosen function parameters. The experiment results demonstrate the proposed invariants are effective.

2 Tchebichef Polynomials and Tchebichef Moments

The discrete Tchebichef polynomials are defined as [7]

$$t_n(x) = (1-N)_n \ {}_3F_2(-n,-x,1+n;1,1-N;1) \quad n,x = 0,1,2,...,N-1 \tag{1}$$

where $(a)_k$ is the Pochhammer symbol given by

$$(a)_k = a(a+1)(a+2)...(a+k-1),\ k \geq 1 \text{ and } (a)_0 = 1 \tag{2}$$

and ${}_3F_2(\cdot)$ is the hypergeometric function

$${}_3F_2(a_1,a_2,a_3;b_1,b_2;c) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k}{(b_1)_k (b_2)_k} \frac{c^k}{k!} \tag{3}$$

with the above definitions, Eq.(1) can be rewritten simply as

$$t_n(x) = n! \sum_{k=0}^{n} (-1)^{n-k} \binom{N-1-k}{n-k} \binom{n+k}{n} \binom{x}{k} \tag{4}$$

The discrete Tchebichef polynomials satisfy the following orthogonal property in discrete domain

$$\sum_{x=0}^{N-1} t_n(x) t_m(x) = \rho(n,N) \delta_{nm} \tag{5}$$

where δ_{nm} is the Kronecker symbol and the squared-norm $\rho(n,N)$ given by

$$\rho(n,N)=(2n)!\binom{N+n}{2n+1} \tag{6}$$

The scaled discrete Tchebichef polynomials are defined as

$$\tilde{t}_p(x)=\sum_{k=0}^{p} c_{p,k}^{N} x^k \tag{7}$$

where

$$c_{p,k}^{N}=\sum_{r=k}^{p} S_1(r,k)\frac{(-1)^{p+r}(p+r)!(N-r-1)!}{\sqrt{\rho(p,N)}(p-r)!(r!)^2(N-p-1)!} \tag{8}$$

From (7), one can deduce

$$x^p=\sum_{k=0}^{p} d_{p,k}^{N}\tilde{t}_k(x) \tag{9}$$

where $d_{p,k}^{N}$ ($0 \le k \le p \le N$-1) is the inverse matrix of the lower triangular matrix $c_{p,k}^{N}$ [18], which can be written as:

$$d_{p,k}^{N}=\sum_{m=k}^{p} S_2(p,m)\frac{\sqrt{\rho(p,N)}(2k+1)(m!)^2(N-k-1)!}{(m+k+1)!(m-k)!(N-m-1)!} \tag{10}$$

here $S_1(i, j)$ and $S_2(i, j)$ are the first kind and the second kind of Stiriling numbers[19], respectively.

The discrete Tchebichef moments can be denoted as

$$T_{\mathrm{pq}}=\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{t}_p(x)\tilde{t}_q(y)f(x,y) \tag{11}$$

3 Affine Tchebichef Moments Invariants

The geometric deformations of the pattern can be simplified to affine transformations:

$$\begin{pmatrix} X \\ Y \end{pmatrix}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}+\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \tag{12}$$

where (x, y) and (X, Y) are coordinates in the image plane before and after the transformations, respectively, and (x_0, y_0) is the image centroid coordinates given by [18].

$$x_0 = \frac{c_{00}^N T_{10} - c_{10}^N T_{00}}{c_{11}^N T_{00}}, \quad y_0 = \frac{c_{00}^N T_{01} - c_{10}^N T_{00}}{c_{11}^N T_{00}} \tag{13}$$

Translation invariance can be achieved by locating the origin of the coordinate system to the center of the object. Thus (x_0, y_0) can be ignored and only take the matrix A into consideration. Form above, Zhang et al [18] defined the two-dimensional ($p+q$) order Tchebichef moments of the transformed image $g(X, Y)$ as follows:

$$T_{pq}^{(g)} = A\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{t}_p(a_{11}x + a_{12}y)\tilde{t}_q(a_{21}x + a_{22}y)f(x,y) \quad A = \left|a_{11}a_{22} - a_{12}a_{21}\right| \tag{14}$$

For simplicity, the above equation can be rewritten as

$$T_{pq}^{(g)} = A\sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{s=0}^{m}\sum_{t=0}^{n}\sum_{i=0}^{s+t}\sum_{j=0}^{m+n-s-t}\binom{m}{s}\binom{n}{t}(a_{11})^s(a_{12})^{m-s}(a_{21})^t(a_{22})^{n-t}c_{p,m}^N c_{q,n}^N d_{s+t,i}^N d_{m+n-s-t,j}^N T_{ij}^{(f)} \tag{15}$$

In the following subsection, we will use the normalization method to obtain the affine Tchebichef moment invariants. This study adopts two kinds' decomposition known as XYS and XSR decomposition to reduce the complexity of matrix A, and discusses constraints imposed in each step of XYS and XSR decomposition procedure.

3.1 XYS Decomposition

Using this decomposition method, the transform matrix A can be separated into an x-shearing, a y-shearing and anisotropic scaling matrix, respective.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} \alpha_0 & 0 \\ 0 & \delta_0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \gamma_0 & 1 \end{pmatrix}\begin{pmatrix} 1 & \beta_0 \\ 0 & 1 \end{pmatrix} \tag{16}$$

where the coefficients α_0, δ_0, γ_0 and β_0 are real numbers.

Depend on this decomposition, we can derive a set of Tchebichef moment invariants I_{pq}^{xsh}, I_{pq}^{ysh} and I_{pq}^{as} through the following theorem, and these invariants are invariant to x-shearing, y-shearing and anisotropic scaling, respectively.

Theorem 1. Suppose f be an origin image and g is its x-shearing transformed version such as $g(x, y) = f(x + \beta_0 y, y)$. Then the following $I_{pq}^{xsh(f)}$ are invariant to x-shearing.

$$I_{\mathrm{pq}}^{xsh(f)} = \sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{s=0}^{m}\sum_{i=0}^{s}\sum_{j=0}^{m+n-s}\binom{m}{s}\beta^{m-s}c_{p,m}^N c_{q,n}^N d_{s,i}^N d_{m+n-s,j}^N T_{ij}^{(f)} \tag{17}$$

where β_f is the parameter associated with the origin image f, and the relationship of the origin image parameter and the transformed is $\beta_f = \beta_g + \beta_0$. The parameter β_f can be obtained using method in [15]. Setting $I_{30} = 0$ in (17), one has

$$\begin{aligned}&(c_{33}d_{30}T_{00}+c_{33}d_{31}T_{01}+c_{33}d_{32}T_{02}+T_{03})\times\beta^{3}+[(c_{32}+3c_{33}c_{00}d_{10})(d_{20}T_{00}+d_{21}T_{01}+d_{22}T_{02})\\&+3c_{33}c_{00}d_{11}(d_{20}T_{10}+d_{21}T_{11}+d_{22}T_{12})]\times\beta^{2}+[(c_{31}+2c_{32}c_{00}d_{10}+3c_{33}c_{00}d_{20})\times(d_{10}T_{00}+d_{11}T_{01})\\&+(2c_{32}c_{00}d_{11}+3c_{33}c_{00}d_{21})(d_{10}T_{10}+d_{11}T_{11})+3c_{33}c_{00}d_{22}(d_{10}T_{20}+d_{11}T_{21})]\times\beta\\&+[(c_{30}d_{00}+c_{31}d_{10}+c_{32}d_{20}+c_{33}d_{30})T_{00}+(c_{31}d_{11}+c_{32}d_{21}+c_{33}d_{31})T_{10}+(c_{32}d_{22}+c_{33}d_{32})T_{20}+T_{30}]=0\end{aligned}\tag{18}$$

Theorem 2. Suppose f be an origin image and g is its y-shearing transformed version such as $g(x,y)=f(x,\gamma_0 x+y)$. Then the following $I_{pq}^{ysh(f)}$ are invariant to y-shearing.

$$I_{pq}^{ysh(f)}=\sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{t=0}^{n}\sum_{i=0}^{m+t}\sum_{j=0}^{n-t}\binom{n}{t}\gamma^{t}c_{p,m}^{N}c_{q,n}^{N}d_{m+t,i}^{N}d_{n-t,j}^{N}T_{ij}^{(f)}\tag{19}$$

where γ_f is the parameter associated with the origin image f, and the relationship of the origin image parameter and the transformed is $\gamma_f=\gamma_g+\gamma_0$. Under this composition, the constraint $I_{11}=0$ used to calculate the γ_f. One can have

$$\gamma=-\frac{(c_{10}c_{10}d_{00}d_{00}+2c_{10}c_{11}d_{00}d_{10}+c_{11}c_{11}d_{10}d_{10})T_{00}+(c_{10}d_{00}+c_{11}d_{10})(T_{01}+T_{10})+T_{11}}{(c_{10}c_{11}d_{00}d_{10}+c_{11}c_{11}d_{20}d_{00})T_{00}+(c_{10}d_{00}+c_{11}c_{11}d_{21}d_{00})T_{10}+c_{11}c_{11}d_{22}d_{00}T_{20}}\tag{20}$$

Theorem 3. Suppose f be an origin image and g is its scaling transformed version such as $g(x,y)=f(\alpha_0 x,\delta_0 y)$. Then the following $I_{pq}^{as(f)}$ are invariant to anisotropic scaling

$$I_{pq}^{as(f)}=\sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{i=0}^{m}\sum_{j=0}^{n}\alpha_f^{m+1}\delta_f^{n+1}c_{p,m}^{N}c_{q,n}^{N}d_{m,i}^{N}d_{n,j}^{N}T_{ij}^{(f)}\tag{21}$$

where α_f and δ_f are two parameters associated with the origin image f, such that $\alpha_f=\alpha_0\alpha_g$ and $\delta_f=\delta_0\delta_g$. From above equations, one can receive the invariants, but the problem is how to get the parameters. One way for estimating these parameters is using the constraints $I_{20}=I_{02}=1$ in (21) to computer parameters α_f and δ_f, such as

$$\begin{cases}(c_{22}d_{20}T_{00}+c_{22}d_{21}T_{10}+T_{20})\alpha^{3}\delta+(c_{21}d_{10}T_{00}+c_{21}d_{11}T_{10})\alpha^{2}\delta+c_{20}d_{00}T_{00}\alpha\delta=1\\(c_{22}d_{20}T_{00}+c_{22}d_{21}T_{01}+T_{02})\alpha\delta^{3}+(c_{21}d_{10}T_{00}+c_{21}d_{11}T_{01})\alpha\delta^{2}+c_{20}d_{00}T_{00}\alpha\delta=1\end{cases}\tag{22}$$

3.2 XSR Decomposition

This is another widely used affine decomposition method. Under this decomposition, the matrix A can be written as an x-shearing, an anisotropic scaling and a rotation matrix.

$$\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}\alpha_0&0\\0&\delta_0\end{pmatrix}\begin{pmatrix}1&\beta_0\\0&1\end{pmatrix}\tag{23}$$

where the coefficients α_0, δ_0, β_0 and θ are real numbers.

Using this decomposition, one can derive a set of Tchebichef moment invariants I_{pq}^{xsh}, I_{pq}^{as} and I_{pq}^{rt} through the following theorems, and these invariants are invariant to *x*-shearing, *y*-shearing and anisotropic scaling respectively.

Theorem 4. Suppose *f* be an origin image and *g* is its *x*-shearing transformed version such as $g(x, y) = f(x+\beta_0 y, y)$. Then the following $I_{pq}^{xsh(f)}$ is invariant to *x*-shearing

$$I_{\mathrm{pq}}^{xsh(f)} = \sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{s=0}^{m}\sum_{i=0}^{s}\sum_{j=0}^{m+n-s}\binom{m}{s}\beta^{m-s}c_{p,m}^{N}c_{q,n}^{N}d_{s,i}^{N}d_{m+n-s,j}^{N}T_{ij}^{(f)} \tag{24}$$

where β_f is the parameter associated with the origin image *f*. It can be obtained using the constraint I_{11}=0 in (24).

$$\beta = -\frac{(c_{10}c_{10}d_{00}d_{00} + 2c_{10}c_{11}d_{00}d_{10} + c_{11}c_{11}d_{10}d_{10})T_{00} + (c_{10}d_{00} + c_{11}d_{10})(T_{01}+T_{10})+T_{11}}{(c_{10}c_{11}d_{00}d_{10}+c_{11}c_{11}d_{20}d_{00})T_{00}+(c_{10}d_{00}+c_{11}c_{11}d_{21}d_{00})T_{01}+c_{11}c_{11}d_{22}d_{00}T_{02}} \tag{25}$$

The relationship of the origin image parameter and the transformed is $\beta_f = \beta_g + \beta_0$.

Theorem 5. Suppose *f* be an origin image and *g* is its scaling transformed version such as $g(x, y) = f(\alpha_0 x, \delta_0 y)$. Then the following $I_{pq}^{as(f)}$ is invariant to anisotropic scaling

$$I_{pq}^{as(f)} = \sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{i=0}^{m}\sum_{j=0}^{n}\alpha_f^{m+1}\delta_f^{n+1}c_{p,m}^{N}c_{q,n}^{N}d_{m,i}^{N}d_{n,j}^{N}T_{ij}^{(f)} \tag{26}$$

where α_f and δ_f are two parameters associated with the origin image *f*, such that $\alpha_f = \alpha_0\alpha_g$ and $\delta_f = \delta_0\delta_g$. Setting $I_{20} = I_{02} = 1$ in (26), one has

$$\begin{cases}(c_{22}d_{20}T_{00}+c_{22}d_{21}T_{10} + T_{20})\alpha^3\delta + (c_{21}d_{10}T_{00}+c_{21}d_{11}T_{10})\alpha^2\delta + c_{20}d_{00}T_{00}\alpha\delta = 1\\(c_{22}d_{20}T_{00}+c_{22}d_{21}T_{01} + T_{02})\alpha\delta^3 + (c_{21}d_{10}T_{00}+c_{21}d_{11}T_{01})\alpha\delta^2 + c_{20}d_{00}T_{00}\alpha\delta = 1\end{cases} \tag{27}$$

Theorem 6. Suppose *f* be an origin image and *g* is its rotation transformed version such as $g(x, y) = f(\cos\theta x + \sin\theta y, -\sin\theta x + \cos\theta y)$. Then the following $I_{pq}^{rt(f)}$ is invariant to rotation transform.

$$\begin{aligned}I_{pq}^{rt(f)} = &\sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{s=0}^{m}\sum_{t=0}^{n}\sum_{i=0}^{s+t}\sum_{i=0}^{m+n-s-t}\binom{m}{s}\binom{n}{t}(-1)^t(\cos\theta)^{n+s-t}(\sin\theta)^{m+t-s}\\&\times c_{p,m}^{N}c_{q,n}^{N}d_{s+t,i}^{N}d_{m+n-s-t,j}^{N}T_{ij}^{(f)}\end{aligned} \tag{28}$$

where θ is the parameter associated with the origin image *f*. Setting $I_{30}+I_{12} = 0$ in (28), the parameter θ can be calculated using the following expression.

$$\theta = \frac{1}{2}\arctan(\frac{uT_{11} - vT_{00}}{T_{20} - T_{02}}),\ u = \frac{2c_{22}^{N}c_{00}^{N}}{(c_{11}^{N})^2},\ v = \frac{2c_{22}^{N}(c_{10}^{N})^2}{c_{00}^{N}(c_{11}^{N})^2} \tag{29}$$

Equation (28) is affine invariant to image geometric deformation.

Theorems 1~6 proof is similar to ref.[17] and is omitted here.

Table 1. Invariants values for image butterfly ($N = 120$)

	(p, q)								
XYS	(1, 0)	-0.2389	-0.2389	-0.2389	-0.2389	-0.2389	-0.2389	-0.2389	-0.2389
	(1, 1)	0.3019	0.3019	0.3019	0.3019	0.3020	0.3021	0.3019	0.3019
	(1, 2)	0.5406	0.5406	0.5406	0.5406	0.5407	0.5408	0.5406	0.5406
	(0, 3)	0.1440	0.1440	0.1440	0.1440	0.1438	0.1437	0.1440	0.1440
	(3, 1)	0.6842	0.6842	0.6842	0.6842	0.6842	0.6842	0.6842	0.6842
	(3, 2)	0.9217	0.9217	0.9217	0.9217	0.9219	0.9222	0.9217	0.9217
XSR	(1, 0)	-0.2389	-0.2389	-0.2389	-0.2389	-0.2389	-0.2389	-0.2389	-0.2389
	(1, 1)	0.3020	0.3020	0.3020	0.3020	0.3021	0.3021	0.3020	0.3020
	(1, 2)	0.5412	0.5408	0.5412	0.5408	0.5411	0.5409	0.5408	0.5408
	(0, 3)	0.1449	0.1431	0.1449	0.1431	0.1446	0.1431	0.1431	0.1431
	(3, 1)	0.6845	0.6854	0.6845	0.6854	0.6844	0.6849	0.6854	0.6854
	(3, 2)	0.9236	0.9241	0.9236	0.9241	0.9235	0.9237	0.9241	0.9241

Table 2. Invariants values for number “0” ($N = 30$)

	(p, q)								
XYS	(1, 0)	-0.1899	-0.1899	-0.1899	-0.1899	-0.1899	-0.1895	-0.1913	-0.1906
	(1, 1)	0.3253	0.3252	0.3253	0.3252	0.3252	0.3259	0.3230	0.3241
	(1, 2)	0.5135	0.5134	0.5135	0.5134	0.5134	0.5142	0.5107	0.5121
	(0, 3)	0.0733	0.0734	0.0733	0.0734	0.0734	0.0717	0.0792	0.0763
	(3, 1)	0.5840	0.5841	0.5840	0.5840	0.5841	0.5840	0.5842	0.5841
	(3, 2)	0.7629	0.7627	0.7628	0.7627	0.7627	0.7658	0.7521	0.7574
XSR	(1, 0)	-0.1899	-0.1899	-0.1899	-0.1899	-0.1899	-0.1895	-0.1913	-0.1906
	(1, 1)	0.3261	0.3260	0.3261	0.3261	0.3260	0.3264	0.3247	0.3254
	(1, 2)	0.5172	0.5147	0.5147	0.5172	0.5147	0.5151	0.5134	0.5179
	(0, 3)	0.0799	0.0666	0.0667	0.0801	0.0666	0.0670	0.0653	0.0866
	(3, 1)	0.5860	0.5927	0.5926	0.5860	0.5927	0.5900	0.6021	0.5871
	(3, 2)	0.7771	0.7814	0.7813	0.7772	0.7814	0.7786	0.7907	0.7797

4 Experimental Results

In this section, three test images with different size, as shown in the first row of Tables 1~3, are used to illustrate the invariance properties of the proposed affine invariants to various geometric transformations. These images are shifted up, down, translation, scale, and rotation. Tables 1, illustrates the proposed six invariants with different order *p*, *q* for each image according to XYS and XSR decomposition, respectively. The experiment is performed on another image which is a number "0" and with the size of 30×30. Table 2 is the results of affine Tchebichef moment according to XYS and XSR decomposition. Similarly, Table 3 shows another result of the character *"q"* with 80×80 pixels. From these Tables, one can found both XYS and XSR decomposition preserve invariance for all affine transformation. Experimental results also illustrate that there are more errors exist in scaling invariants, and errors will be increase while the moment order (*p*+*q*) increasing. On the other hand, the experimental results also indicate that the XYS decomposition is better than XSR decomposition.

Table 3. Invariants values for letter "q" ($N = 80$)

	(p, q)								
XYS	(1, 0)	-0.2308	-0.2308	-0.2308	-0.2308	-0.2309	-0.2312	-0.2308	-0.2310
	(1, 1)	0.3057	0.3057	0.3057	0.3057	0.3056	0.3051	0.3058	0.3054
	(1, 2)	0.5360	0.5360	0.5360	0.5360	0.5358	0.5352	0.5361	0.5356
	(0, 3)	0.1324	0.1324	0.1324	0.1324	0.1327	0.1340	0.1322	0.1333
	(3, 1)	0.6675	0.6675	0.6675	0.6675	0.6675	0.6676	0.6675	0.6676
	(3, 2)	0.8950	0.8950	0.8950	0.8950	0.8944	0.8921	0.8953	0.8934
XSR	(1, 0)	-0.2308	-0.2308	-0.2308	-0.2308	-0.2309	-0.2312	-0.2308	-0.2310
	(1, 1)	0.3060	0.3060	0.3060	0.3060	0.3059	0.3056	0.3060	0.3058
	(1, 2)	0.5372	0.5372	0.5372	0.5364	0.5373	0.5360	0.5372	0.5362
	(0, 3)	0.1344	0.1344	0.1345	0.1303	0.1352	0.1299	0.1341	0.1301
	(3, 1)	0.6681	0.6681	0.6682	0.6702	0.6683	0.6729	0.6681	0.6717
	(3, 2)	0.8994	0.8993	0.8994	0.9006	0.8996	0.9033	0.8992	0.9021

5 Conclusions

Considering the orthogonal and discrete characteristics of Tchebichef moments, this study presented two image normalization methods that can give affine Tchebichef moment invariants which were invariant with respect to affine deformation. The numerical experiments were performed with symmetric as well as asymmetric images. The results demonstrated the invariance properties and discriminative capabilities of the proposed descriptors.

Acknowledgments. This work has been supported by National Natural Science Foundation of China under Grant No. 60975004.

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