

Image Feature to Take the Edge of the Research Methods by Anisotropic Diffusion

Qi Wang¹ and Shaobin Ren²

¹ College of Mechanical Engineering, Taiyuan University of Technology,
No. 79 West Yingze Street, Taiyuan 030024, Shanxi, China
wqmngp@sina.com

² College of Science, Taiyuan University of Technology, No. 79 West Yingze Street,
Taiyuan 030024, Shanxi, China
rsb_long@126.com

Abstract. Like to raise the inspection as the quality of the picture, noise and enhance the image of the test is the image of the main objective. Its main objective is to reinforce the edge of specific characteristics for the measurement. This article against partial differential equations(PDE) in image processing the application of the study and learn and form the operator combines to form, based on the gradient and the edge of the detection methods, and simulations validate the test.

Keywords: component, anisotropic diffusion, partial differential equations, mathematical morphological, Visual Basic.

1 Introduction

The PDE (partial differential equations), research has been 300 years of history. However, it is only used in image processing research focus in recent years. Treatment of partial differential equations has the advantage of computational mathematics there are many numerical methods can be used. For example, the image of a model to be able to effectively change their image; hot proliferation of partial differential equations, which can simulate image from the physical to the process to degenerate; reverse the spread of partial differential equations can be successfully conducted image of the standards set; function structure of the equation can be better implementation of step-wise tracking, and was used in the image of the division [1].

2 Partial Differential Equations Theory

Partial differential equations arising in mechanics, geometry, physics and theoretical study of subjects. Mainly used in image processing is denoising content [2]. Gabor First, in 1960 the system of partial differential equations are considered, the original image and the blurred image is proportional to the difference between the approximation of its Laplace transform them to the relevant classification.

Let the original image is u_0 , The blur process can use the following linear heat equation: (Equation 1)

$$\frac{\partial u}{\partial t} = \varepsilon \Delta u, u(0) = u_0 \quad (1)$$

The Δ is the Laplace operator. It can be expressed as discrete: (Equation 2)

$$u_{n+1} = u_n + \varepsilon \Delta u_n \quad (2)$$

On the contrary, the process of image ambiguity can use the following formula: (Equation 3)

$$\frac{\partial u}{\partial t} = -\varepsilon \Delta u \quad (3)$$

Its input is the inspection images u_0 , Discrete form of the type: (Equation 4)

$$u_{n+1} = u_n - \varepsilon \Delta u_n \quad (4)$$

Discrete form of these two different representatives of the two completely opposite physical meaning, in which formula (1) represents the heat transfer process, commonly known as the heat equation; the formula (3) indicates that the heat transfer of the anti-process, often called the reverse heat equation. It is based on two different equations, PDE method is divided into two large branches. Linear heat equation for anisotropic diffusion for edge detection and image denoising; and the nonlinear inverse diffusion processing for image restoration.

3 Anisotropic Diffusion Filtering Theory

Image restoration filtering results in the output signal to noise ratio increased at the same time, inevitably leads to blurred edges. So Perona and Malik proposed a method of nonlinear image smoothing - Anisotropic Filtering [3]: (Equation 10, Equation 5)

$$\frac{\partial u}{\partial t} = \text{div}(c \cdot \nabla u) \quad (5)$$

In the above formula, u is the input signal, div is the gradient operator, said the gradient, c is the diffusion coefficient. Usually c is the image gradient function, which increases monotonically with the gradient decreased. The range $[0,1]$. The diffusion system determines the direction of diffusion, provides a proliferation of local adaptive control strategies, the spread of the location in the noise as much as possible, and stop at the edges of the image. Diffusion coefficient is commonly used: (Equation 6)

$$c_1 = e^{-\frac{\|\nabla u\|^2}{\lambda}} \quad (6)$$

In the above formula, ∇_i is the gradient in the direction of i , λ is the gradient threshold, which determines the edge to be retained is. Another commonly used diffusion operator is: (Equation 7)

$$c_2 = \frac{1}{1 + (\|\nabla_i u_i\| / \lambda)^{1+\alpha}} \quad (\alpha > 0) \quad (7)$$

The traditional idea is to detect the image low-pass filtering, and then calculated on the basis of gradient changes, as the image edge detection. Many scholars proposed a variety of different situations the diffusion coefficient of the program [4][5][6][7], their common feature is the first image with different pre-treatment, but this approach often leads to image edge of the drift . Morphological gradient operator is calculated corrosion is calculated directly on the original first difference, and then use the edge enhancement morphological gradient operator corrosion and reduce noise. Morphological operations will not bring the edge because of the location of the offset, so using this method can accurately locate the edge of the image, but also has strong anti-noise ability.

Corrosion in the morphology of a feature than the structural elements can be eroded by a small image details. This is combined into the erosion and dilation morphological opening operation to remove the root causes of image noise. However, computing the minimum criteria for corrosion of operation makes the image details are often not retained. PDE is the use of corrosion morphology after the extraction of image edge gradient image, it can get a good noise immunity and precise positioning, but also has strong anti-noise ability.

Morphological transformation is the most basic form of expansion and corrosion. Suppose a function $f : R^d \rightarrow R$ represents a d-dimensional signal (when $d = 2$ that an image). $g : B \rightarrow R$ That defined in the compactly supported function on a structure. The input f function is the structure function g multi-scale expansion $f \oplus g$, and corrosion $f \ominus g$ can be defined as: (Equation 8, Equation 9)

$$(f \oplus g_s)(x) = \sup_{v \in sB} \{ f(x-v) + sg\left(\frac{v}{s}\right) \} \quad (8)$$

$$(f \ominus g_s)(x) = \inf_{v \in sB} \{ f(x-v) - sg\left(\frac{v}{s}\right) \} \quad (9)$$

$g_s : B \rightarrow R$ is the structure function of the multi-scale representation, $sB = \{sb : b \in B\}$, $s \geq 0$, $g_s(x) = sg(x/s)$, $s > 0$. When the structure function is a constant equal to zero, it is usually referred to as flat structure function g , Multi-scale dilation and erosion at this time reduced to the following formula: (Equation 10, Equation 11)

$$(f \oplus g_s)(x) = \sup_{v \in sB} (x-v) \quad (10)$$

$$(f \ominus gs)(x) = \inf_{v \in sB} (x + v) \tag{11}$$

The operator is applied to solve partial differential equations. Which use flat structuring elements: $B = \{(x, y) : \sqrt{x^2 + y^2} \leq 1\}$, The image $f(x, y)$ expansion of (corrosion) of the result: (Equation 12)

$$\begin{cases} \partial_t u = \pm |\nabla u| \\ u_0 = f(x, y) \end{cases} \tag{12}$$

Osher and Sethian's description of the unilateral use of upwind difference for corrosion PDE algorithm is one relatively simple formula (Equation 13).

$$\begin{aligned} \frac{u_{ij}^{n+1} - u_{ij}^n}{\nabla t} = & -((\min(\frac{u_{i-ij}^n - u_{ij}^n}{h_1}, 0))^2 + (\min(\frac{u_{i+ij}^n - u_{ij}^n}{h_2}, 0))^2 \\ & + (\min(\frac{u_{ij-i}^n - u_{ij}^n}{h_3}, 0))^2 + (\min(\frac{u_{ij+i}^n - u_{ij}^n}{h_4}, 0))^2)^{1/2} \end{aligned} \tag{13}$$

Morphological erosion operator defined as equation 14:

$$u_{ij}^{n+1} = E(u_{ij}^n) = c \bullet (u_{ij}^n + \Delta t \mathcal{E}_{ij}^n) \tag{14}$$

4 New PDE-Based Morphological Erosion Operator

Gradient edge detection method based on the calculation usually original edge gradient was smoothed before, the purpose is to reduce the noise impact of the edge gradient. Usually the first low-pass filter, and then the gradient calculations. But it actually only consider if the signal is noisy, directly calculate the gradient will generate a lot of nothing to do with the great value of the edge, the first low-pass filter can be the maximum inhibition. The actual result is the original image from fine to coarse scale space reduced scale space, it often will affect the exact location of the edge. Many solutions have been proposed for this program. Optimization of the frequency domain, such as edge detection, wavelet edge detection methods.



Fig. 1. Lenna adding noise2%



Fig. 2. Canny operator results

Morphological erosion operator based on the gradient image can not directly change the image edge denoising. Therefore, after considering the corrosion noise directly applied to the edge gradient image feature extraction. This means there is more than the original method of positioning the edge of capacity and robustness. New approaches in the calculation of the original image smoothing difference do before, but the application of PDE morphological erosion operator to do noise reduction on the difference image.



Fig. 3. LOG operator results



Fig. 4. Morphological results

4.1 Morphological Erosion Operator Edge Detection Algorithm

Define horizontal and vertical multi-scale operators as equation 15 and 16.

$$d_{\sigma}^h(f) = \frac{f(x + \sigma, y) - f(x - \sigma, y)}{2\sigma} \quad (15)$$

$$d_{\sigma}^w(f) = \frac{f(x, y + \sigma) - f(x, y - \sigma)}{2\sigma} \quad (16)$$

Where σ is the scale factor, input image in the σ scale gradient vector can be expressed as equation 17:

$$M_{\sigma} = \sqrt{(d_{\sigma}^h)^2 + (d_{\sigma}^w)^2} \quad (17)$$

Gradient direction angle is expressed as equation 18:

$$A_{\sigma} = \arg(d_{\sigma}^h + id_{\sigma}^w) \quad (18)$$

4.2 Algorithm Steps

- at a certain scale to solve the input image vertical and horizontal gradient images.
- respectively, horizontal and vertical gradient image using the formula (11), the morphological erosion operation.
- the use of formula (17) Calculate the edge of the image.

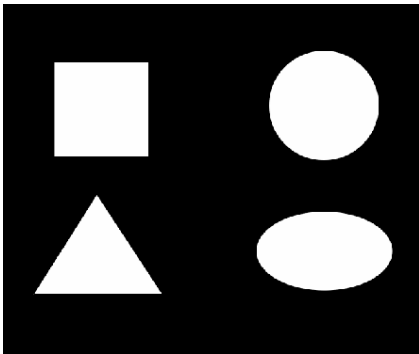
Calculation process in the experiment, for an image of the morphological operation can be repeated, so as to constitute an iterative process. Determined according to the image signal to noise ratio the number of iterations, the number of multi-effect, but also the corresponding computation increases linearly. Therefore, in the process of calculating the image analysis needs to select the appropriate number of iterations.



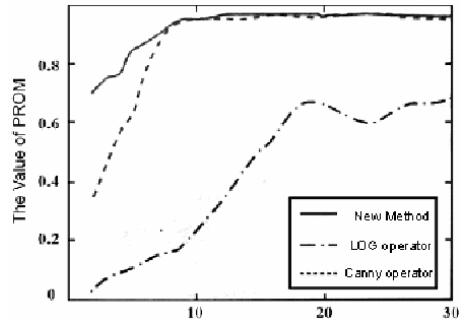
Fig. 5. Software interface



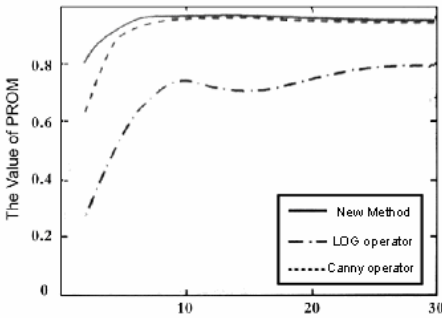
Fig. 6. Results



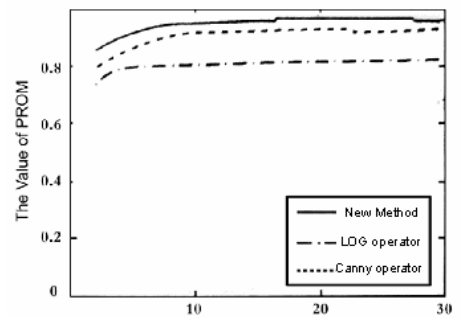
(a) Synthetic image



(b) Scale to take $\sigma = 0.5$ comparison of three ways



(c) Scale to take $\sigma = 1$ comparison of three ways



(d) Scale to take $\sigma = 2$ comparison of three ways

Fig. 7. Image edge detection methods with different accuracy compared

Experiment calculated according to each generated image to determine whether to continue the calculation, a process similar to the detection of the calibration process, comparing the new image with the original image, the edge of the key position to determine whether the circumstances meet the requirements.

4.3 Experimental Comparison

PDE-based edge to verify the accuracy of detection methods, refer to Pratt's evaluation curve [8] criteria, programming, using the Canny operator, respectively, LOG operator on Lena image after treatment compared with the algorithm. The results shown in Fig.7. From the results, Canny operator and LOG operator is very sensitive to salt and pepper noise, when the noise ratio greater than 2%, these two operators in the detection process has basically been unable to confirm the exact location of the edge. PDE algorithm is of such noise has a strong anti-jamming capability.

5 Conclusion

Using morphological edge detection algorithm of corrosion is the biggest characteristic of the original image without noise reduction but directly in the original image to obtain the finest scale gradient image. Using the gradient of the image edge detection, image smoothing can effectively avoid the edge caused by drift. Therefore, this method can effectively solve the detection process, the noise caused by sensor detection mechanism of difficult problems, in terms of automatic identification will have broad application prospects.

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