Symbolic-Numeric Investigation of the Aerodynamic Forces Influence on Satellite Dynamics

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Abstract. An approach for symbolic-numeric stability analysis of equilibrium orientations of a satellite in a circular orbit under the influence of gravitational and aerodynamic forces is considered. The stationary motions of a satellite are governed by a system of nonlinear algebraic equations. A computer algebra method based on an algorithm for the construction of a Groebner basis and the resultant concept is proposed for determining all equilibrium orientations of a satellite with a given aerodynamic torque and given principal central moments of inertia. It is shown that equilibrium orientations are determined by real solutions of algebraic equation of the twelfth degree. Evolution of domains with a fixed number of equilibria is investigated in detail. The stability analysis of equilibria is performed on the basis of Lyapunov theorem. The equilibrium orientations and their stability are analyzed numerically.

1 Introduction

Celestial mechanics is one of the most popular fields where symbolic computations are necessary to do very bulky calculations for solving many significant problems. In astrodynamics, successful application of computer algebra methods is a rare occasion in scientific papers. In this work, an example of symbolic–numeric investigation of satellite's dynamics under the influence of gravitational and aerodynamic torques is presented. It is a well known result that a satellite with different moments of inertia in the central Newtonian force field in a circular orbit has 24 equilibrium orientations [1]. However, at altitudes from 250 up to 500 km, rotational motion of a satellite is subjected to aerodynamic torque too. Therefore, it is necessary to study the joint action of gravitational and aerodynamic torques and, in particular, to analyze all possible satellite's equilibria in a circular orbit. Such solutions are used in practical space technology in the design of passive control systems of satellites.

This problem is considered in many papers. The basic problems of satellite's dynamics with an aerodynamic attitude control system have been presented in [1]. In [2], [3], and [4] all equilibrium orientations were found in some special cases, when the center of pressure is located on a satellite's principal central axis

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of inertia and on a satellite's principal central plane of inertia. The effect of the atmosphere on a satellite is reduced to the drag force applied to the center of pressure and directed against velocity of the satellite's center of mass relative to the air. The center of pressure is assumed to be at a fixed point in the satellite body.

In the present work, the problem of determining the classes of equilibrium orientations for general values of aerodynamic torque is considered. The equilibrium orientations are determined by real roots of the system of nonlinear algebraic equations. The investigation of equilibria was possible due to application of Computer Algebra Groebner basis and resultant methods. Evolution of domains with a fixed number of equilibria is investigated numerically in dependence of four dimensionless system parameters. Sufficient conditions for stability of all equilibrium orientations are obtained using generalized integral of energy.

2 Equations of Motion

Consider the motion of a satellite subjected to gravitational and aerodynamic torques in a circular orbit. We assume that 1) the gravity field of the Earth is central and Newtonian, 2) the satellite is a triaxial rigid body, 3) the effect of atmosphere on a satellite is reduced to the drag force applied at the center of pressure and directed against the velocity of the satellite's center of mass relative to the air, and the center of pressure is fixed in the satellite body. To write the equations of motion we introduce two right-handed Cartesian coordinate systems with origin in the satellite's center of mass O. $OXYZ$ is the orbital reference frame. The axis OZ is directed along the radius vector from the Earth center of mass to the satellite's center of mass, the axis OX is in the direction of a satellite's orbital motion. $Oxyz$ is the satellite's body reference frame; Ox, Oy, Oz are the principal central axes of inertia of a satellite. The orientation of the satellite's body reference frame $Oxyz$ with respect to the orbital reference frame is determined by means of the Euler angles ψ (precession), ϑ (nutation), and φ (spin). The direction cosines in transformation matrix between the frames $OXYZ$ and $Oxyz$ have the form:

$$\begin{aligned}
a_{11} &= \cos(x, X) = \cos\psi\cos\varphi - \sin\psi\cos\vartheta\sin\varphi, \\
a_{12} &= \cos(y, X) = -\cos\psi\sin\varphi - \sin\psi\cos\vartheta\cos\varphi, \\
a_{13} &= \cos(z, X) = \sin\psi\sin\vartheta, \\
a_{21} &= \cos(x, Y) = \sin\psi\cos\varphi + \cos\psi\cos\vartheta\sin\varphi, \\
a_{22} &= \cos(y, Y) = -\sin\psi\sin\varphi + \cos\psi\cos\vartheta\cos\varphi, \\
a_{23} &= \cos(z, Y) = -\cos\psi\sin\vartheta, \\
a_{31} &= \cos(x, Z) = \sin\vartheta\sin\varphi, \\
a_{32} &= \cos(y, Z) = \sin\vartheta\cos\varphi, \\
a_{33} &= \cos(z, Z) = \cos\vartheta.
\end{aligned} \tag{1}$$

Then equations of the satellite's attitude motion can be written in the Euler form [1], [2]:

$$
\begin{aligned}
&A\dot{p} + (C - B)qr - 3\omega_0^2(C - B)a_{32}a_{33} = \tilde{h}_2 a_{13} - \tilde{h}_3 a_{12},\\
&B\dot{q} + (A - C)rp - 3\omega_0^2(A - C)a_{31}a_{33} = \tilde{h}_3 a_{11} - \tilde{h}_1 a_{13},\\
&C\dot{r} + (B - A)pq - 3\omega_0^2(B - A)a_{31}a_{32} = \tilde{h}_1 a_{13} - \tilde{h}_3 a_{11},
\end{aligned}
\qquad (2)
$$

$$
\begin{aligned}
p &= \dot{\psi} a_{31} + \dot{\vartheta}\cos\varphi + \omega_0 a_{21},\\
q &= \dot{\psi} a_{32} + \dot{\vartheta}\sin\varphi + \omega_0 a_{22},\\
r &= \dot{\psi} a_{33} + \dot{\vartheta} + \omega_0 a_{23}.
\end{aligned}
\qquad (3)
$$

Here p, q, r are the projections of the satellite's angular velocity onto the axes Ox, Oy, Oz; A, B, C are the principal central moments of inertia of the satellite; ω_0 is the angular velocity of the orbital motion of the satellite's center of mass. $\tilde{h}_1 = -a_p Q$, $\tilde{h}_2 = -b_p Q$, $\tilde{h}_3 = -c_p Q$, Q is the atmospheric drug force acting on a satellite; a_p, b_p, c_p are the coordinates of the center of pressure of a satellite in the reference frame $Oxyz$. The dot designates differentiation with respect to time t.

Equations (2) along with (3) form a closed system of equations of motion of the satellite, for which the Jacobi Integral is valid

$$
\begin{aligned}
H = {} & \frac{1}{2}(A\bar{p}^2 + B\bar{q}^2 + C\bar{r}^2) + \frac{3}{2}\omega_0^2[(A - C)a_{31}^2 + (B - C)a_{32}^2] + \\
& + \frac{1}{2}\omega_0^2[(B - A)a_{21}^2 + (B - C)a_{23}^2] - (\tilde{h}_1 a_{11} + \tilde{h}_2 a_{12} + \tilde{h}_3 a_{13}),
\end{aligned}
\qquad (4)
$$

where $\bar{p} = p - \omega_0 a_{21}$, $\bar{q} = q - \omega_0 a_{22}$, $\bar{r} = r - \omega_0 a_{23}$.

3 Equilibrium Orientations of a Satellite

Putting in (2) and (3) $\psi = \text{const}$, $\vartheta = \text{const}$, $\varphi = \text{const}$ and introducing the notation $\tilde{h}_i = \omega_0^2 \bar{h}_i (i = 1, 2, 3)$, we obtain the equations

$$
\begin{aligned}
(C - B)(a_{22}a_{23} - 3a_{32}a_{33}) &= \bar{h}_2 a_{13} - \bar{h}_3 a_{12},\\
(A - C)(a_{21}a_{23} - 3a_{31}a_{33}) &= \bar{h}_3 a_{11} - \bar{h}_1 a_{13},\\
(B - A)(a_{21}a_{22} - 3a_{31}a_{32}) &= \bar{h}_1 a_{12} - \bar{h}_2 a_{11},
\end{aligned}
\qquad (5)
$$

which allow us to determine the satellite's equilibria in the orbital reference frame.

Let $A \neq B \neq C$. Substituting the expressions for the direction cosines from (1) in terms of Euler angels into Eqs. (5), we obtain three equations with three unknowns ψ, ϑ, φ. The second procedure for closing Eqs. (5) is to add the following six orthogonality conditions for the direction cosines:

$$
a_{i1}a_{j1} + a_{i2}a_{j2} + a_{i3}a_{j3} = \delta_{ij} \qquad (6)
$$

where δ_{ij} is the Kronecker delta and $(i, j = 1, 2, 3)$. Equations (5) and (6) form a closed system with respect to the direction cosines, which also specifies the equilibrium solutions of a satellite. We state the following problem for the system of equations (5), (6): determine all nine direction cosines, i.e., to find all the equilibrium orientations of the satellite when A, B, C, $\bar{h}_1$, $\bar{h}_2$, and $\bar{h}_3$ are given. The problem has been solved only for some specific cases when the center of pressure is located on a satellite's principal central axis of inertia Ox, when $\bar{h}_1 \neq 0, \bar{h}_2 = \bar{h}_3 = 0$ [2], [3] and when the pressure center locates in the satellite's principal central plane of inertia Oxz of the frame $Oxyz$ and $\bar{h}_1 \neq 0$, $\bar{h}_2 = 0$, $\bar{h}_3 \neq 0$ [4]. In the case $\bar{h}_1 = \bar{h}_2 = \bar{h}_3 = 0$, it has been proved that the system (5), (6) has 24 solutions describing the equilibrium orientations of a satellite-rigid body [1].

Here we consider the general case of the problem of defining the equilibria of a satellite when $\bar{h}_1 \neq 0$, $\bar{h}_2 \neq 0$, $\bar{h}_3 \neq 0$. A Computer Algebra approach to define all the equilibrium orientations of a satellite will be used. Projecting Eqs. (5) onto the axis of the orbiting frame $OXYZ$, we get the algebraic system, using the method given in [5]

$$\begin{aligned} Aa_{21}a_{31} + Ba_{22}a_{32} + Ca_{23}a_{33} = 0, \\ Aa_{11}a_{21} + Ba_{12}a_{22} + Ca_{13}a_{23} - (\bar{h}_1 a_{21} + \bar{h}_2 a_{22} + \bar{h}_3 a_{23}) = 0, \\ 3(Aa_{11}a_{31} + Ba_{12}a_{32} + Ca_{13}a_{33}) + \bar{h}_1 a_{31} + \bar{h}_2 a_{32} + \bar{h}_3 a_{33} = 0. \end{aligned} \tag{7}$$

A solution of the system (6), (7) can be obtained using an algorithm for the construction of Groebner bases [6]. The method of Groebner bases is used to solve systems of nonlinear algebraic equations. It comprises an algorithmic procedure for reducing the problem involving polynomials of several variables to investigation of a polynomial of one variable. Using the computer algebra system Maple [7] Groebner[gbasis] package with *tdeg* option, we calculate the Groebner basis of the system (6), (7) of nine polynomials with nine variables a_{ij} $(i, j = 1, 2, 3)$ under the ordering on the total power of the variables. In the list of variables in the Maple Groebner package we use nine direction cosines, and in the list of polynomials, we include the polynomials from the left-hand sides f_i $(i = 1, 2, ...9)$ of the algebraic equations (6), (7):

map(factor,Groebner[gbasis]([f1,f2,f3, ... f9],tdeg(a11, a12, a13, ... a33))). Here we write down the polynomials in the Groebner basis that depend only on the variables a_{31}, a_{32}, a_{33}

$$\begin{aligned} 9[(B-C)^2 a_{32}^2 a_{33}^2 + (C-A)^2 a_{31}^2 a_{33}^2 + (A-B)^2 a_{31}^2 a_{32}^2] = \\ = (\bar{h}_1 a_{31} + \bar{h}_2 a_{32} + \bar{h}_3 a_{33})^2 (a_{31}^2 + a_{32}^2 + a_{33}^2), \\ 3(B-C)(C-A)(A-B) a_{31} a_{32} a_{33} - [\bar{h}_1 (B-C) a_{32} a_{33} + \\ + \bar{h}_2 (C-A) a_{31} a_{33} + \bar{h}_3 (A-B) a_{31} a_{32}](\bar{h}_1 a_{31} + \bar{h}_2 a_{32} + \bar{h}_3 a_{33}) = 0, \\ a_{31}^2 + a_{32}^2 + a_{33}^2 = 1. \end{aligned} \tag{8}$$

Introducing the new variables $x = a_{31}/a_{32}$, $y = a_{33}/a_{32}$, $h_i = \bar{h}_i/(B - C)$, $\nu = (B - A)/(B - C)$, we deduce two equations for determining of x and y.

$$a_0 y^2 + a_1 y + a_2 = 0,$$
$$b_0 y^4 + b_1 y^3 + b_2 y^2 + b_3 y + b_4 = 0, \tag{9}$$

where

$$\begin{aligned}
a_0 &= h_3(h_2(1-\nu)x - h_1),\\
a_1 &= \nu(3(1-\nu) + h_3^2)x + (h_1 x + h_2)(h_2(1-\nu)x - h_1),\\
a_2 &= \nu h_3(h_1 x + h_2)x,\\
b_0 &= h_3^2,\\
b_1 &= 2h_3(h_1 x + h_2),\\
b_2 &= (h_1 x + h_2)^2 + h_3^2(1 + x^2) - 9 - 9(1 - v)^2 x^2,\\
b_3 &= 2h_3(h_1 x + h_2)(1 + x^2),\\
b_4 &= (h_1 x + h_2)^2(1 + x^2) - 9\nu^2 x^2.
\end{aligned}$$

Invoking the resultant concept we eliminate the variable y from the equations (9). Expanding the determinant of the resultant matrix of Eqs.(9), with the help of Maple symbolic matrix function, we obtain a twelfth degree algebraic equation in x:

$$p_0 x^{12} + p_1 x^{11} + p_2 x^{10} + p_3 x^9 + p_4 x^8 + p_5 x^7 + \\ + p_6 x^6 + p_7 x^5 + p_8 x^4 + p_9 x^3 + p_{10} x^2 + p_{11} x + p_{12} = 0, \tag{10}$$

the coefficients of which depend in a rather complicated way on the parameters ν, h_1, h_2, h_3

$$p_0 = (1-\nu)^6 p_{12}, \qquad p_1 = -(1-\nu)^5 p_{11}, \qquad \dots \tag{11}$$
$$p_{11} = 2h_1^3 h_2^3(2(1-\nu)h_2^2 - 2h_1^2 - \nu h_3^2 - 3\nu(1-\nu)), \quad p_{12} = -h_1^4 h_2^4.$$

By the definition of the resultant, to every root x of Eq.(10) there corresponds a common root y of the system (9). It can easily be shown that to every real root x of Eq.(10) there correspond 2 solutions for (5), (6). Since the number of real roots of Eq.(10) does not exceed 12, the satellite in a circular orbit can have at most 24 equilibria in the orbiting reference frame.

Using Eq.(10), (11) we can determine numerically all the relative equilibrium orientations of the satellite and analyze their stability. We have analyzed numerically dependence of the number of real solutions of Eq.(10) on the parameters, using factorization method. For a fixed values of ν and h_3, the number of real roots was determined at the nodes of a uniform grid in the plane (h_1, h_2). We have used the values of $\nu = 0.2$, $\nu = 0.4$, $\nu = 0.6$, $\nu = 0.8$ $(|\nu| < 1)$.

In the present work, we have implemented the bifurcation values of the parameters h_1 and h_3, corresponding to the qualitative change of domains with a fixed number of equilibria, which were defined in [4] for the special case when

$\bar{h}_1 \neq 0$, $\bar{h}_2 = 0$, $\bar{h}_3 \neq 0$. In [4] all the equilibrium solutions are determined by real roots of the algebraic equations of fourth degree and bifurcation values of parameters h_1 and h_3 when the number of real roots changes were found analytically: $|h_1| = 1$, $|h_3| = 1$, $|h_1| = 3$, $|h_3| = 3$, $|h_1| = 6$, $|h_3| = 6$. For this special case in the intervals $|h_1| < 1$, $|h_3| < 1$ – 24, 20, and 16 equilibria exist; in the next intervals $1 < |h_1| < 3$, $1 < |h_3| < 3$ – 16, 12, and 8 equilibria exist and in the intervals $|h_1| > 6$, $|h_3| > 6$ only 8 equilibria exist. We have used these bifurcation values of h_1 and h_3 when $h_2 = 0$ for our numerical calculations. For the first interval when $|h_3| < 1$ we define numerically the evolution of domains with 24, 20, and 16 equilibria. We have used a small step of the parameter h_3 ($h_3 = 0.1, 0.15, 0.25, 0.35, 0.5, 0.75, 0.8, 0.9$) because for $|h_3| < 1$ there are small domains with a fixed number of real roots of Eq.(10). For example, at $h_3 = 0.1$ ($\nu = 0.2$) when the parameter values $|h_1| < 0.2$ and $|h_2| < 0.2$ there is domain of existence of 24 equlibria (12 real roots). For the intervals $0.2 < |h_1| < 0.5$ and $0.2 < |h_2| < 0.6$ – 20 equlibria exist (10 real roots). For the next intervals $0.5 < |h_1| < 0.7$ and $0.6 < |h_2| < 0.7$, there is domain of existence of 16 equlibria (8 real roots). For $0.7 < |h_1| < 2$ and $0.7 < |h_2| < 2.5$ – 12 equlibria exist (6 real roots), and for $|h_1| > 2$ and $|h_2| > 2.5$ only 8 equlibria exist (4 real roots). At $h_3 = 0.25$ ($\nu = 0.2$) when the parameter values $|h_1| < 0.2$ and $|h_2| < 0.2$ there is domain of existence of 24 equlibria. For the intervals $0.2 < |h_1| < 0.3$ and $0.2 < |h_2| < 0.45$ – 20 equlibria exist. For the next intervals $0.3 < |h_1| < 0.8$ and $0.45 < |h_2| < 0.6$ there is domain of existence of 16 equlibria. For $0.8 < |h_1| < 1.6$ and $0.6 < |h_2| < 2.2$ – 12 equlibria exist, and for $|h_1| > 1.6$ and $|h_2| > 2.2$ only 8 equlibria exist. Analysis of the numerical results for $|h_3| < 1$ shows that five domains with the 24, 20, 16, 12, and 8 equilibria exist in the plane (h_1, h_2) for the intervals $0 < |h_3| \leq 0.8$. When we cross the bifurcation value $h_3 = 0.8$ domain with the 24 equilibria vanish and in the intervals $0.8 < |h_3| < 1$ only four domains with 20, 16, 12, and 8 equilibria exist. The value $|h_3| = 1$ is also bifurcation as in the special case. When we cross the bifurcation value $h_3 = 1$ the domain with 20 equilibria vanishes. In the interval $1 < |h_3| < 3$, only three domains with the 16, 12, and 8 equilibria exist. The value $|h_3| = 3$ is bifurcation, as in the special case. When we cross the bifurcation value $h_3 = 3$ the domain with 16 equilibria vanishes. In the interval $3 < |h_3| < 6$, only two domains with 12 and 8 equilibria exist. When the values of parameter $|h_3|$ of the aerodynamic torque are more than 6, the satellite has only 8 equilibrium orientations, which correspond to four real roots of Eq.(10).

4 Stability Analysis of Equilibria

To investigate the stability of equilibrium solutions $\psi = \psi_0 =$ const, $\vartheta = \vartheta_0 =$ const, $\varphi = \varphi_0 =$ const satisfying Equations (5), we can use the the Jacobi Integral of energy (4) as the Lyapunov function. After replacement $\psi \to \psi + \psi_0$, $\vartheta \to \vartheta + \vartheta_0$, $\varphi \to \varphi + \varphi_0$ where ψ, ϑ, φ are small deviations from the satellite's equilibrium ψ_0, ϑ_0, φ_0, the energy integral takes the form

$$H = \frac{1}{2}(A\bar{p}^2 + B\bar{q}^2 + C\bar{r}^2) + \frac{1}{2}(B-C)(A_{11}\psi^2 + A_{22}\vartheta^2 + A_{33}\varphi^2 + \\ + 2A_{12}\psi\vartheta + 2A_{13}\psi\varphi + 2A_{23}\vartheta\varphi) + O_3(\psi,\vartheta,\varphi) = \text{const}, \quad (12)$$

where coefficients A_{ij} depend on the parameters $\nu, h_1, h_2, h_3, \psi, \vartheta, \varphi$ in the form

$$\begin{aligned}
A_{11} &= \nu(a_{11}^2 - a_{21}^2) + (a_{13}^2 - a_{23}^2) + h_1 a_{11} + h_2 a_{12} + h_3 a_{13},\\
A_{22} &= (3+\cos^2\psi_0)(1-\nu\sin^2\varphi_0)\cos 2\vartheta_0 - \frac{1}{4}\nu\sin 2\psi_0\cos\vartheta_0\sin 2\varphi_0 -\\
&\quad - (h_1\cos\vartheta_0\sin\varphi_0 + h_2\cos\vartheta_0\cos\varphi_0 - h_3\sin\vartheta_0)\sin\psi_0,\\
A_{33} &= \nu((a_{22}^2 - a_{21}^2) - 3(a_{32}^2 - a_{31}^2)) + h_1 a_{11} + h_2 a_{12}, \qquad (13)\\
A_{12} &= -\frac{1}{2}\sin 2\psi_0\sin 2\vartheta_0 + \nu(a_{11}a_{23} + a_{13}a_{21})\sin\varphi_0 -\\
&\quad - (h_1 a_{31} + h_2 a_{32} + h_3 a_{33})\cos\psi_0,\\
A_{13} &= \nu(a_{12}a_{22} + a_{12}a_{21}) + h_1 a_{22} - h_2 a_{21},\\
A_{23} &= -\frac{3}{2}\nu\sin 2\vartheta_0\sin 2\varphi_0 + \nu(a_{21}\cos\varphi_0 + a_{22}\sin\varphi_0)a_{23} -\\
&\quad - (h_1\cos\varphi_0 - h_2\sin\varphi_0)a_{13}.
\end{aligned}$$

It follows from Lyapunov theorem that the equilibrium solution is stable if the quadratic form (12),(13) is positive definite, i.e., the following inequalities take place:

$$\begin{aligned}
&A_{11} > 0, \qquad B > C,\\
&A_{11}\, A_{22} - A_{12}^2 > 0, \qquad (14)\\
&A_{11}\, A_{22}A_{33} + 2A_{12}A_{23}A_{13} - A_{11}A_{23}^2 - A_{22}A_{13}^2 - A_{33}A_{12}^2 > 0.
\end{aligned}$$

Substituting the expressions for A_{ij} from (13) for the corresponding equilibrium solution into (14), we obtain the conditions for stability of this solution. Using integral (12),(13), we have analyzed numerically stability conditions (14) for the equilibrium solutions. Analysis of the numerical results shows that stable equilibrium orientations of a satellite exist even for large aerodynamic torque when $|h_i| \geq 6.0$ $(i = 1, 2, 3)$. For such values of aerodynamic torque only eight equilibria exist, and two of them are stable. At $0 < |h_i| < 6.0$ $(i = 1, 2, 3)$ both four and two stable equilibria exist.

5 Conclusion

In this work, the attitude motion of a satellite under the action of gravitational and aerodynamic torques in a circular orbit has been investigated. The main attention was given to determination of a satellite equilibrium orientation in the orbital reference frame and to analysis of their stability. The symbolic-numeric method of determination of all the satellite equilibria is suggested in general case ($\bar{h}_1 \neq 0$, $\bar{h}_2 \neq 0$, $\bar{h}_3 \neq 0$). The symbolic computation system Maple is applied to reduce the satellite stationary motion system of nine algebraic equations with nine variables to a single algebraic equation of the twelfth degree with one variable, using the Groebner package for the construction of a Groebner basis and the resultant approach. It was shown that the equilibrium orientations are determined by real roots of single algebraic equation of the twelfth degree. Using this result of symbolic calculations we conclude that the satellite subjected to gravitational and aerodynamic torques can have no more than 24 equilibrium orientations in a circular orbit. The evolution of domains with a fixed number of equilibrium orientations was investigated numerically in the plane of two parameters (h_1, h_2) at a different values of parameters ν and h_3. Some general bifurcation values of h_3 corresponding to the qualitative change of domains with a fixed number of equilibria were determined. On the basis of numerical calculation, we can conclude that the number of satellite's isolated equilibria is no less than 8. Using the Lyapunov theorem, the sufficient conditions of stability of the equilibrium orientations are investigated numerically at different values of aerodynamic parameters. Analysis of the numerical simulation shows that the number of stable equilibria is no less than two. All the calculations considered here were implemented with the computer algebra system Maple. The results of the study can be used at the stage of preliminary design of the satellite with aerodynamic control system.

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