Chapter 8
Summary

The flow of water through porous media in partially saturated conditions is an important, and on the other hand complex problem, relevant to several disciplines of science and engineering. The present book is a contribution to this field of research, which particularly focuses on the problem of numerical solution of the flow equations and on capturing the influence of Darcy-scale material heterogeneities in the field-scale models of water flow in unsaturated porous media.

A general background to the mathematical modeling of flow in porous media was given in Chap. 2. The two-phase flow of water and air in the unsaturated zone was discussed, as well as the simplified, and more widely used Richards approach, which neglects the air flow, assuming instantaneous equilibration of the air pressure with the atmosphere. In each case the flow is described by nonlinear partial differential equations of parabolic type. The nonlinearity results from complex relationships between the capillary pressure, saturations and permeabilities of the two fluid phases, which can be represented by several types of analytical functions.

In practical applications, the unsaturated flow equations usually must be solved using numerical methods. This is performed in several stages. Spatial discretization using finite difference, finite element or finite volume approach transforms a partial differential equation into a system of ordinary or algebraic differential equations with respect to time. Next, implicit temporal discretization results in a system of nonlinear algebraic equations for each time step. The system must be solved iteratively, and the subsequent corrections to the vector of unknowns are obtained from the solution of a system of linear algebraic equations, arising for each iterations. In Chap. 3 basic numerical techniques relevant to each of those stages were outlined and the similarities and differences between various available approaches were discussed. The numerical scheme based on vertex-centred finite volume discretization, used in the following chapters, was presented in more detail.

One aspect of the numerical solution of the Richards equation, which received particular attention in Chap. 4, was the problem of approximation of the relative conductivity between adjacent nodes of the numerical grid. A number of averaging schemes available in the literature were compared and evaluated. The performance of

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simple methods, like arithmetic, geometric or upstream averaging is highly dependent on the form of relative permeability function, boundary conditions of the problem and node spacing. The relationship between the value of the average permeability and the interplay of capillary and gravity forces was explored in the framework of the so-called Darcian approach. The Darcian averaging is based on consideration of steady-state flow between adjacent nodes. A method of this type, proposed by the author in an earlier paper, was comprehensively evaluated. It was shown to be a viable alternative to other existing methods, offering accurate approximation for a wide range of relative permeability functions and grid sizes.

Another important issue related to the modeling of water flow in the unsaturated zone is the upscaling, i.e. the development of models which describe in the average terms large-scale behaviour of media showing small-scale heterogeneity. This work focused on upscaling from Darcy scale to field scale and a brief overview of the problem was presented in Chap. 5. The importance of factors such as local heterogeneity pattern, separation of scales, existence of the representative elementary volume and balance of capillary, viscous and gravity forces was discussed. The following Chaps. 6 and 7 focused more specifically on upscaling of flow in a porous medium composed of disconnected inclusions embedded in a continuous background material. The upscaled models obtained with the periodic homogenization approach were presented. The main point was to show that even if a relatively simple flow model is assumed at the Darcy scale, the large-scale flow in a heterogeneous medium may have non-standard characteristics.

In Chap. 6 the influence of inclusion–background diffusivity ratio was analyzed. The results of a numerical experiment indicated qualitative differences between the media with weakly permeable and highly permeable inclusions. In the former case, the increase of inclusions' diffusivity led to a monotonic acceleration of the infiltration process. In contrast, for weakly diffusive inclusions the large scale response of the system was strongly related to the actual value of the diffusivity ratio, measured with regard to the scale parameter ε. Depending on this value, the outflow can be either retarded or accelerated in comparison to the homogeneous medium. Theoretical analysis showed that for the diffusivity ratio of ε^2 the flow is dominated by local non-equilibrium of the capillary pressure at the scale of a single representative elementary volume, leading to prolonged water transfer between inclusions and background. This process is represented by an additional "memory" term arising in the large scale equation. A generalized upscaled equation was discussed, which is suitable for the whole range of the inclusion–background diffusivity ratio. A preliminary experimental verification of this model was also described.

Chapter 7 deals with the heterogeneity in the air entry pressure. This important parameter represents the value of the capillary pressure below which the air phase loses its connectivity and cannot flow. If the entry pressure in inclusions is lower than in the background material, two types of phenomena can be expected. First, during infiltration some amount of air becomes trapped in the inclusions, as it cannot overcome the entry pressure to move through the water-saturated background. Second, if the medium is initially fully water-saturated the drainage of inclusions cannot start earlier than the drainage of the background material, because only then the air phase can form connected paths through the medium. These two cases cannot be represented by the standard Richards equation, and significant differences were observed in the Darcy-scale numerical solutions of the Richards equation and the reference two-phase model. Such differences do not occur for a homogeneous medium. The performance of the Richards equation can be greatly improved by introducing upscaled capillary and permeability functions which account for the entry pressure effects and show quasi-hysteresis in the range of capillary pressures below the entry pressure of the background material. It is important to stress that the hysteresis results purely from the heterogeneous structure of the medium at Darcy scale, and not from the pore scale effects, which are neglected in the analysis.

In the author's view, several issues considered in this book offer perspectives for further research. For instance, the performance of the improved method for inter-nodal permeability averaging should be evaluated more thoroughly for multidimensional problems, involving unstructured grids and anisotropic media. Moreover, an attempt can be made to extend the presented approach to the case of two-phase flow, by adding viscous forces to the capillary–gravity balance. There are also numerous possibilities for further development of the presented upscaling approach. They include a more general formulation, which would account for both local non-equilibrium of the fluid pressures, discussed in Chap. 6, and air-entry effects, considered in Chap. 7. Another direction would be to introduce a more realistic local-scale description, encompassing hysteresis in the hydraulic functions of each material, inter-phase mass transfer and deformations of the solid skeleton. Finally, the need for a comprehensive experimental verification of the upscaling approaches must be emphasized.