Spatial Econometric OD-Flow Models

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Christine Thomas-Agnan and James P. LeSage

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Abstract

Spatial interaction or gravity models have been used in regional science to model flows that take many forms, for example, population migration, commodity flows, traffic flows, and knowledge flows, all of which reflect movements between origin and destination regions. This chapter focuses on spatial autoregressive extensions to the conventional least-squares gravity models that relax the assumption of independence between flows. These models, proposed by LeSage and Pace (2008, Spatial econometric modeling of origin-destination flows. J Reg Sci 48(5):941–967, 2009), define spatial dependence in this type of setting to mean that larger observed flows from an origin region *A* to

C. Thomas-Agnan (✉)
G.R.E.M.A.Q., Toulouse School of Economics, Toulouse, France
e-mail: christine.thomas@tse.fr.en

J.P. LeSage
Department of Finance and Economics, Texas State University – San Marcos, San Marcos, TX, USA
e-mail: jlesage@spatial-econometrics.com

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a destination region Z are accompanied by (i) larger flows from regions nearby the origin A to the destination Z, say regions B and C that are neighbors to region A, which they label origin dependence; (ii) larger flows from the origin region A to regions neighboring the destination region Z, say regions X and Y, which they label destination dependence; and (iii) larger flows from regions that are neighbors to the origin (B and C) to regions that are neighbors to the destination (X and Y), which they label origin-destination dependence. Spatial spillovers in these models can take the form of spillovers to both regions/observations neighboring the origin or destination in the dyadic relationships that characterize origin-destination flows as well as network effects that impact all other regions in the network. We set forth a simulation approach for these models that can be used to produce scalar expressions for the various types of spillover impacts that arise from changes in the explanatory variables of the model.

83.1 Introduction to Gravity or Spatial Interaction Models

Gravity models have often been used to explain origin-destination (OD) flows that arise in regional science such as trade, transportation, and migration. In the regional science literature, the gravity model has been labeled a spatial interaction model (Sen and Smith 1995) because the regional interaction is directly proportional to the product of regional size measures. In the case of interregional commodity flows, the measure of regional size is typically gross regional product or regional income. The model predicts more interaction in the form of commodity flows between regions of similar (economic) size than regions dissimilar in size. For the case of migration flows, population would be a logical measure of regional size, and in other contexts such as knowledge flows between regions, LeSage, Fischer, and Scherngell (2007) use regional knowledge stocks measured by patents to reflect size.

Theoretical motivations for spatial interaction modeling are numerous, for example, Wilson (1967) and Roy (2004) provide a macroeconomic statistical equilibrium development, Smith (1975) and Sen and Smith (1995) rely on a microeconomic choice-theoretic approach, and Fischer (2002) and Fischer and Reismann (2002) take a neural network approach that treats spatial interaction models as universal function approximations.

Historically, motivations for these models took the view that spatial interaction implies movement of entities, and that this has little to do with spatial association (Getis 1991). These models attempt to explain variation in observed flows between origin and destination regions using (i) *origin-specific* attributes that characterize the ability of the origins to generate outflows, (ii) destination-specific attributes that attract inflows, and (iii) variables reflecting the spatial separation of origin and destination regions. The traditional assumption was that including *separation variables* (such as distance, borders, language, and cultural differences between regions) should fully account for observed spatial dependence in flows. Curry (1972) was an earlier dissenter from this view, advancing a theoretical motivation for the presence of spatial dependence in flows after conditioning on conventional variables, and Griffith and Jones (1980) reported spatial correlation in residuals of conventional spatial interaction models. The notion that use of distance functions in conventional spatial interaction models effectively captures spatial dependence in the interregional flows being analyzed was further challenged by Porojan (2001) for the case of international trade flows, and Lee and Pace (2005) for retail sales. Both studies reported residuals from conventional models that exhibited spatial dependence. Despite these findings, most applied work continued to assume independence between flow observations relying on conventional least-squares models to explain observed variation in flows. One exception is Bolduc, Laferriere, and Santarossa (1992) who explicitly model the disturbances using a spatial autoregressive process.

LeSage and Pace (2008) define spatial dependence in a spatial interaction setting to mean that larger observed flows from an origin region A to a destination region Z are accompanied by (i) larger flows from regions nearby the origin A to the destination Z, say regions B and C that are neighbors to region A, which they label *origin dependence*; (ii) larger flows from the origin region A to regions neighboring the destination region Z, say regions X and Y, which they label *destination dependence*; and (iii) larger flows from regions that are neighbors to the origin (B and C) to regions that are neighbors to the destination (X and Y), which they label *origin-destination dependence*. Using this definition of spatial dependence, modeling of spatial dependence in regional flows requires a spatial autoregressive specification.

LeSage and Pace (2008) show how to produce maximum likelihood estimates for a spatial autoregressive specification of the spatial interaction model. This model includes spatial lags of the dependent variable similar to conventional spatial autoregressive models in an effort to directly model spatial dependence in flows. Fischer and Griffith (2008) use the approach introduced by LeSage and Pace (2008) to include spatial lags for the model disturbances. LeSage and Pace (2009) show how to produce Bayesian Markov Chain Monte Carlo estimates for their spatial econometric variant of the spatial interaction model. While the motivation provided by LeSage and Pace (2008) for the spatial econometric approach to spatial interaction modeling is purely econometric, Behrens, Ertur, and Koch (2012) provide a theoretical justification for such models.

Section 83.2 introduces the conventional spatial interaction model that assumes independence between observed flows and relies on ordinary least-squares estimation methods.

In Sect. 83.3, we introduce the spatial autoregressive extension of LeSage and Pace (2008). Section 83.4 discusses a number of problems that arise in applied modeling of regional flows that can invalidate use of maximum likelihood (or Bayesian) estimation of the model from LeSage and Pace (2008). These problems provide fertile ground for future research in spatial interaction modeling.

While the focus in LeSage and Pace (2008, 2009) was on maximum likelihood and Bayesian estimation of spatial autoregressive interaction models, there is also a need to consider how estimates from these models should be properly interpreted. The subject of interpreting estimates from independent and spatial autoregressive spatial interaction models is taken up in Sect. 83.5. We set forth a simulation approach for these models that can be used to produce scalar expressions for the various types of spillover impacts that arise from changes in the explanatory variables of the model. Spatial spillovers in these models can take the form of spillovers to both regions/observations neighboring the origin or destination in the dyadic flow relationships that characterize origin-destination flows as well as network effects that impact all other regions in the network. We also make the point that interpretation of estimates from conventional independent spatial interaction models may be improved using this approach.

83.2 Gravity or Spatial Interaction Models Based on Independence

Regression models attempt to explain variation in the n^2 flows between the n regions in a closed network of regional flows. The $n \times n$ flow matrix Y is converted to an $n^2 \times 1$ vector by stacking columns. The flow matrix might be arranged so the i, jth element reflects a flow from region j to region i, which has been labeled an origin-centric flow arrangement by LeSage and Pace (2008). Many trade models rely on the convention that the i, jth element of the flow matrix represents a flow from region i to j, which would be a destination-centric arrangement of the flows. If we let y^o denote the origin-centric vector of flows and y^d a vector created by stacking columns from a destination-centric arrangement, there is a vec-permutation matrix P that can be used to relate these two different orderings. Specifically, $Py^o = y^d$, and using properties of permutation matrices, $y^o = P^{-1}y^d = P'y^d$.

A regression model that has been labeled a gravity model captures the notion that the size of the two regions and the distance between them are important factors that determine the magnitude of flows between regions. For example, if one starts with the standard gravity model (c.f., Eq. (6.4) in Sen and Smith 1995) shown in Eq. (83.1) and applies a log transformation, the regression in Eq. (83.2) arises.

$$\mu(i,j) = CX_o(i)X_d(j)H(i,j) \tag{83.1}$$

In Eq. (83.1), $\mu(i,j)$ represents the expected flows from region i to region j (assuming a destination-centric flow matrix), while $X_d(i)$, $X_o(j)$ denote sizes of the destination and origin and $G(i,j)$ represents resistance or deterrence to flows between the origin and destination, typically modeled using some function of distance between regions i and j. To facilitate the log transformation, $X_o(i)$ can be specified using $X_o(i)^{\beta_o}$ and similarly, $X_d(j) = X_d(j)^{\beta_d}$, while $H(i,j)$ is some function of distance between regions i and j, for which we might use a power function, $D(i,j)^\gamma$, where $D(i,j)$ is the distance between regions i and j.

A point made by LeSage and Pace (2009) is that conventional work with these models has relied on mathematics emphasizing dyads i, j which has severe limitations for thinking about flows in the context of a network. Spatial dependence reflects relationships between observations, and is typically modeled using vectors and spatial weight matrices to express relations between observations. LeSage and Pace (2008) use the matrix/vector representation of the log-transformed dyad expression in Eq. (83.1) shown in Eq. (83.2), which more closely mirrors notation from conventional regression modeling. It should also be noted that another population formulation directly models flows as: $F(i,j) = \exp\left(\sum_{k=1}^{R} \beta_k \log X_{kij}\right) + \varepsilon_{ij}$, where the disturbance term is additive. This produces a Poisson model suitable for flows taking the form of counts and requires maximum likelihood estimation (Gourieroux et al. 1984).

$$y = \alpha \iota_{n^2} + X_o \beta_o + X_d \beta_d + \gamma g + \varepsilon \tag{83.2}$$

In Eq. (83.2), y is an $n^2 \times 1$ vector of (logged) flows constructed by stacking the columns of the $n \times n$ flow matrix Y, where we will assume a destination-centric organization throughout this chapter. Similarly, applying the log transformation to the $n \times n$ matrix of distances $D(i,j)$ between the n destination and origin regions and stacking the columns results in a vector of logged distances g, with associated coefficient γ. LeSage and Pace (2008) show that $X_o = \iota_n \otimes X$, where X is an $n \times R$ matrix of characteristics for the n regions, $\bigotimes$ represents a Kronecker product, and ι_n is an $n \times 1$ vector of ones. In the simplest case, X might represent a vector with the appropriate size measure for each region, but without loss of generality this may be a matrix containing R characteristics of the regions that are thought to explain variation in flows. We note that this represents a general case where the same set of explanatory variables is used for both origins and destinations. A special case might involve selection of a subset of variables in the matrix X for use as origin characteristics, and another subset of variables for the destination characteristics. However, the general case maybe the preferred approach to specification, since inclusion of additional unimportant explanatory variables does not bias least-squares estimates, whereas exclusion of important explanatory variables can result in omitted variables bias.

The Kronecker product repeats the same values of the n regions in a strategic fashion to create a vector (or matrix) of sizes associated with each origin region, hence use of the notation X_o to represent these explanatory variables reflecting origin characteristics. Ultimately, use of Kronecker products in conjunction with matrix algebra allowed LeSage and Pace (2008) to express simple estimators that avoid storing multiple copies of the same numerical values, which is computationally inefficient. The matrix/vector $X_d = X \otimes \iota_n$ arranges the n regional characteristics to match the vector y, producing explanatory variables associated with each destination region. The vectors β_o and β_d are $R \times 1$ parameter vectors associated with the origin and destination region characteristics, respectively. The scalar parameter γ reflects the effect of the vector of logged distances g on flows, which is traditionally thought to be negative. The parameter α denotes the constant term parameter, and the $n^2 \times 1$ vector ε represents zero mean, constant variance, zero covariance disturbances, consistent with the Gauss-Markov least-squares assumptions. We note that the assumption of normally distributed disturbances consistent with least-squares implies that the dependent variable flows are also normally distributed. This is not consistent with some flows which represent count data, for example, counts of persons migrating or commuters traveling from one region to another. However, the log transformation may help to produce more normally distributed flows. We will have more to say about this issue in Sect. 83.4, where problems that affect maximum likelihood estimation of spatial interaction models are discussed.

LeSage and Pace (2008) note that the algebra of Kronecker products can be used to avoid the need to form $n^2 \times R$ matrices X_o, X_d which require a great deal of computer storage involving repeated numerical values. This can be seen by examining the $(2R + 2) \times (2R + 2)$ moment matrix formed using: $Z = (\,\iota_{n^2} \quad X_o \quad X_d \quad g)$ shown in Eq. (83.3), where we use G to represent the $n \times n$ matrix of logged distances.

$$Z'Z = \begin{pmatrix} n^2 & 0_k & 0_k & \iota_n' G \iota_n \\ 0_k' & nX'X & 0_k' 0_k & X' G \iota_n \\ 0_k' & 0_k' 0_k & nX'X & X' G \iota_n \\ \iota_n' G \iota_n & \iota_n' G' X & \iota_n' G' X & tr(G^2) \end{pmatrix} \tag{83.3}$$

Similarly, the matrix product $Z'y$ involving the matrix Z' of dimension $(2R + 2) \times n^2$ and the $n^2 \times 1$ vector of flows can be formed as shown in Eq. (83.4), where tr denotes the trace operator.

$$Z'\mathrm{vec}(Y) = \begin{pmatrix} \iota_n^2 \\ X_o' \\ X_d' \\ g' \end{pmatrix} y = \begin{pmatrix} \iota_n' \mathrm{Y} \iota_n \\ \mathrm{X}' \mathrm{Y} \iota_n \\ \mathrm{X}' \mathrm{Y}' \iota_n \\ \mathrm{tr}(GY) \end{pmatrix} \tag{83.4}$$

This allows calculation of the parameter estimates $\delta = (\alpha\, \beta_o\, \beta_d\, \gamma)'$ using only the $n \times R$ matrix X, the $n \times n$ flow matrix Y, and the $n \times n$ matrix of logged distances G that appear in $Z'Z$ and $Z'y$, as shown in Eq. (83.5).

$$\hat{\delta} = \left[(1/n^2)Z'Z\right]^{-1}(1/n^2)Z'y \tag{83.5}$$

Interpretation of the estimates β_o, β_d has followed that used in typical regression, where these parameters reflect the influence (positive or negative) of changes in origin and destination characteristics on the magnitude of flows. Since the model has been log-transformed, these estimates can be interpreted as elasticities. A negative estimate for the rth destination characteristic indicates that this reduces flows to the destination, whereas a positive coefficient points to a factor that increases flows to the destination. A similar interpretation applies to the coefficients β_o, which measure the positive or negative influence of origin characteristics on flows. We will have more to say about this approach to interpreting the coefficients β_o, β_d later. The coefficient γ should be negative, consistent with the notion that (logged) distance acts as a friction to reduce flows.

83.3 Spatial Autoregressive Interaction Models

Intuitively, changes to the characteristics of a single region i will impact both inflows and outflows to all other regions engaged or connected with region i as either an origin or destination. For example, a (ceteris paribus) decrease in taxes in region i would lead to inflows of population to this region from (potentially) all other regions and a decrease in outflows of population to (potentially) all other regions.

LeSage and Pace (2008) suggest that flows across networks involving origins and destinations are likely to exhibit spatial dependence. They define spatial dependence in this type of setting to mean that larger observed flows from an origin region A to a destination region Z are accompanied by (i) larger flows from regions nearby the origin A to the destination Z, say regions B and C that are neighbors to region A, which they label origin dependence; (ii) larger flows from the origin region A to regions neighboring the destination region Z, say regions X and Y, which they label destination dependence; and (iii) larger flows from regions that are neighbors to the origin (B and C) to regions that are neighbors to the destination (X and Y), which they label origin-destination dependence.

Casual observation of migration flows in a network of counties is consistent with this type of observation. If there are a large number of migrants moving away from a county A (say a county near the Detroit metropolitan area), we would expect to see migrants also moving away from other counties B and C near Detroit (presumably due to unfavorable labor market conditions). Similarly, if a large number of migrants are moving into a county Z (say a county in the Austin metropolitan area), we would expect to see migrants also moving into other counties X and Y in the Austin metropolitan area (presumably because of favorable labor market conditions).

LeSage and Pace (2008, 2009) propose a spatial regression extension of the independent empirical gravity model from Eq. (83.2) shown in Eq. (83.6).

$$\begin{aligned} Ay &= \alpha\iota_{n^2} + X_o\beta_o + X_d\beta_d + g\gamma + \varepsilon \\ A &= (I_{n^2} - \rho_o W_o)(I_{n^2} - \rho_d W_d) \\ &= (I_{n^2} - \rho_o W_o - \rho_d W_d + \rho_w W_w) \\ W_o &= I_n \otimes W \\ W_d &= W \otimes I_n \\ W_w &= W_o \otimes W_d = W_d \otimes W_o = W \otimes W \end{aligned} \tag{83.6}$$

The term A can be viewed as a spatial filter that captures origin-based dependence, destination-based dependence, and origin-destination-based dependence. (The filter implies a restriction that $\rho_w = -\rho_o\rho_d$. This restriction need not be imposed during estimation, so we address the more general case here and allow for an unrestricted parameter ρ_w.) The model and associated data generating process (DGP) for the spatial autoregressive interaction model take the forms shown in Eqs. (83.7) and (83.8), respectively, where we rely on the earlier definitions of Z and δ.

$$y = \rho_o W_o y + \rho_d W_d y + \rho_w W_w y + Z\delta + \varepsilon \tag{83.7}$$

$$y = (I_{n^2} - \rho_o W_o - \rho_d W_d + \rho_w W_w)^{-1}(Z\delta + \varepsilon) \tag{83.8}$$

The spatial lag formed by the matrix product $W_d y$ extracts flows from neighbors to each destination region in the vector of origin-destination flow dyads to form a linear combination of flows from neighboring destinations. In the case where the $n \times n$ spatial weight matrix W represents a fixed number, say m, of equally weighted nearest neighbors, the spatial lag vector would contain an average of flows from the m neighboring destinations. The matrix W is a conventional (row-normalized) spatial weight matrix of the type used in cross-sectional regressions involving n regions. This spatial lag captures destination-based dependence, with the parameter ρ_d measuring the strength of destination-based dependence.

A similar interpretation applies to the spatial lag formed by the product $W_o y$, which reflects a linear combination of flows from regions neighboring the origin, again for each origin-destination dyad in the flow vector. The scalar parameter ρ_o reflects the strength of origin-based dependence. The spatial lag $W_w y$ forms a linear combination of flows from neighbors to the origin and flows from neighbors to the destination, and the parameter ρ_w represents the magnitude of this type of dependence.

The stability restrictions for the spatial dependence parameters require that $1/\lambda_{\min} < \rho_o + \rho_d + \rho_w < 1$, where $\lambda_{\min}$ is the minimum eigenvalue of the matrix W. In practice, values of -1 are often used to replace $1/\lambda_{\min}$, since this avoids the need to calculate the minimum eigenvalue of the matrix W.

LeSage and Pace (2008) provide details concerning maximum likelihood estimation for the spatial autoregressive interaction model, and LeSage and Pace (2009) set forth a Bayesian MCMC estimation scheme. Both of these exploit the computationally efficient moment matrices involving the sample data expressed using the smaller dimension matrices.

83.4 Problems That Arise in Applied Modeling of Flows

Maximum likelihood estimation methods require that the disturbances in the model follow a normal distribution, which implies that the dependent variable flows are also normally distributed. As already noted, many flows are more properly viewed as count data magnitudes, for example, flows of population or commuters migrating or traveling between regions. There are limitations to the ability of the log transformation to convert count data to a form consistent with a normal distribution, especially when a large number of flows between regions take on zero values. For a flow matrix involving small regions, there are likely to be a large number of zero flow magnitudes. For example, migration flows between US counties in the contiguous states over a 5-year period exhibit zeros for over 90 % of the county-to-county flows. A solution for problems involving large numbers of zero flows as well as small flows would be development of Poisson variants of the spatial autoregressive spatial interaction model. Some work has been done in this area. Lambert, Brown, and Florax (2010) set forth a two-stage estimation procedure for a spatial autoregressive Poisson model, that is one not representing a spatial interaction model. LeSage, Fischer, and Scherngell (2007) introduce spatially structured origin and destination effects in a Poisson model involving counts of interregional patent citations, but their model does not involve spatial lags of the dependent variable. LeSage and Llano (2006) introduce a Tobit variant of a model that contains spatially structured origin and destination effects parameters, which can address cases involving smaller numbers of zero flows. Ranjan and Tobias (2007) also use a Tobit approach but rely on semi-parametric origin and destination effects parameters. The use of Tobit models is an attempt to address a common practice where practitioners modify the dependent variable vector using: $\ln(1 + y)$ to accommodate the log transformation. Since this transformation ignores the mixed discrete/continuous nature of the flow distribution, it should lead to downward bias in the coefficient estimates for the model. An appropriate approach to addressing the problem of a large number of zero flows as well as small flows and the count nature of many flows remains an area for future research.

Another factor contributing to non-normality in flow magnitudes is the presence of large flows within regions, those located on the main diagonal of the flow matrix, relative to smaller flows between regions, those on the off-diagonal elements. Since the objective of spatial interaction modeling is typically a model that explains variation in interregional flows, practitioners often view intraregional flows as a nuisance. Some common practices are (i) to simply set observed intraregional flows to zero values (Tiefelsdorf 2003; Fischer et al. 2006) and (ii) introduce dummy variables for these observations (Behrens et al. 2012). For the case of the independence model, these approaches are fine, but they can have adverse impacts on spatial autoregressive interaction models. Inclusion of zero magnitudes for intraregional flows in a model that includes spatial lags such as W_{oy}, W_{dy} will produce aberrant observations when these flows become part of the linear combination of neighboring values to the origin or destination.

LeSage and Pace (2008) propose using a separate set of explanatory variables in the spatial autoregressive interaction specification to deal with large flow magnitudes on the main diagonal of the flow matrix. This separate model is embedded into the specification by adjusting the explanatory variables matrices X_o, X_d and the intercept vector ι_n to have zero values for the n observations associated with the main diagonal elements (intraregional flows) of the flow matrix. They then introduce an additional explanatory variables matrix containing only n nonzero observations, those associated with intraregional flows that were set to zero in the matrices X_o, X_d. In addition, new intercept vectors are introduced: one that contains zeros for observations associated with intraregional flows and ones for all others, and a second that contains ones for only the intraregional flow observations.

This approach allows nonzero intraregional flows to be included in the dependent variable vector y which is used to form the spatial lags W_{oy}, W_{dy}, W_{wy}. The part of flow variation associated with the large diagonal elements is explained by the embedded model variables allowing the coefficient estimates associated with the adjusted explanatory variables to more accurately characterize variation in interregional flows. LeSage and Fischer (2010) provide an example of the improvement that arises from this approach.

Assuming the problems of zero flows and the count nature of some flow magnitudes can be solved for the case of the spatial autoregressive spatial interaction model, there is still the issue of how to properly interpret estimates from this model.

83.5 Interpreting Spatial Interaction Models

A first point to note is that we should not interpret the coefficient estimates β_d, β_o and γ as if they were least-squares estimates that reflect partial derivative changes in the dependent variable associated with changes in the explanatory variables. LeSage and Pace (2009) point out that this mistaken approach to spatial autoregressive (SAR) models has been used in much of the past spatial econometrics literature.

We present a method that can be used to relate changes in characteristics of a single region i to flows across the $n \times n$ network of flows between the n regions for the case of the spatial autoregressive interaction model. This issue has not been tackled in the literature, yet it is essential for interpreting the coefficient estimates β_o, β_d in the spatial autoregressive interaction model.

83.5.1 A Numerical Illustration for the Nonspatial Gravity Model

Prior to setting forth our method for quantifying how changes in the rth characteristic of region i impact flows, we provide a simple numerical illustration to fix ideas. Using the DGP in Eq. (83.8), we generated a set of flows using $n = 8$ regions with $\beta_d = 1$, $\beta_{\mathrm{o}} = 0.5$, $\delta = -0.5$, $\rho_d = 0.4$, $\rho_o = 0.4$, and $\rho_w = -p_o \times \rho_d = -0.16$. No disturbance term was used, and the single vector $x' = (40\ 30\ 20\ 10\ 7\ 10\ 15\ 25)$ was used, so we have the case where $R = 1$. A set of n latitude and longitude coordinates (both equal to 1, 2, ..., 8) were used to produce an n^2 vector of (logged) distances g and the associated spatial weight matrix W based on two nearest (distanced) neighbors. The systematic order of the latitude-longitude coordinates produces regions configured to lie on a line, with a simplified spatial weight matrix configuration. For example, region 3 has regions 2 and 4 as the two nearest neighbors, region 4 has regions 3 and 5 as the two neighbors, and so on. This greatly simplifies things relative to real-world data. The weight matrix for our example is shown in Eq. (83.9).

$$W = \begin{pmatrix} 0.0 & 0.5 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.0 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.0 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.5 & 0.0 & 0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.0 & 0.5 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.5 & 0.0 \end{pmatrix} \tag{83.9}$$

A discrete change in each element/region $i = 1, \ldots, 8$ of the vector x of one unit was made and the discrete changes arising in the $n \times n$ flow matrix as a result of these perturbations were recorded. For each change in the value x_i for a single region i, a new flow matrix was generated and subtracted from the original flow matrix to illustrate how changes in the characteristics of a single region impact the matrix of flows.

An important point to note here is that unlike the conventional spatial autoregressive model where the matrix X contains characteristics for each of the n regions, the matrices X_o, X_d in the spatial autoregressive interaction model strategically repeats values of the $n \times R$ matrix X to form $n^2 \times R$ matrices $X_o = \iota_n \otimes X$, $X_d = X \otimes \iota_n$. An implication is that when we change the characteristic/element of a single region i (which we denote using x_i), it produces a set of changes in n elements of the matrix X_o and changes in n elements of the matrix X_d. Together, this set of $2n$ altered values in the matrices X_o, X_d produce the change in flows that results from changing characteristics of the ith region, that is $x_i + 1$. This has computational implications for how we calculate the effects arising from changes in the explanatory variables of this model. Unlike the conventional SAR model, we do not need to calculate changes in each of the n^2 elements of the vectors X_o and X_d to produce scalar summary measures of the impact of these changes on the flows. Although this approach is valid, it requires more computational effort. Instead we can consider only n changes in each observation i of the matrix/vector X as producing a total derivative response. There will be a vector of $n^2 \times 1$ responses in the flows (which can be viewed as a change in the $n \times n$ flows matrix Y) arising from a change in a single characteristic of the ith region, x_i. This single element total derivative change works through a series of $2n$ associated changes that arise in the $n^2 \times R$ model explanatory variables X_o, X_d.

Intuitively, increasing a single region i's characteristic (say the size of region x_i) means this region will (i) attract more inflows as a destination from all n regions (including itself which takes the form of more intraregional flows within region i) and (ii) produce increased outflows to all n regions (including itself). This facet of changes in the characteristic of a single region is what accounts for the model repeating the same altered value of x_i (the new size for region i) n times in the vector/matrix X_o, and n times in X_d. Given this, it is computationally inefficient to consider conventional partial derivatives that would independently change each of the n^2 elements in X_o or X_d and examine their impact on the flow matrix. Changes to individual elements of X_o and X_d need not be considered given the structure of the model (and associated data generating process).

There may be applied modeling situations where different explanatory variables are used to model the origin and destination characteristics of the regions that are thought to be important for explaining variation in flows. In these situations, the argument used above regarding changes to a single explanatory variable x_i for each region will not be valid. The more computationally inefficient approach of using conventional partial derivatives that would independently change each of the n^2 elements in X_o and X_d would need to be used in order to examine the impact of these changes on the flow matrix. We discuss this type of situation when providing a numerical illustration.

Results showing the changes in the $n \times n$ flow matrix associated with a change in the third region's characteristic, x_3, by one unit for the case of the independent (nonspatial) gravity model in Eq. (83.2) are shown in Eq. (83.10). These were produced by setting $\rho_o = \rho_d = \rho_w = 0$ in the spatial gravity model from Eq. (83.8), which results in the independent gravity model from Eq. (83.2).

$$\Delta Y/\Delta x_3 = \begin{pmatrix} 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.50 & 0.50 & 1.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{pmatrix} \quad (83.10)$$

The role of the independence assumption is clear in Eq. (83.10), where we see from row 3 that the change of outflows from region 3 to all other regions equals 0.5, which is the value of the coefficient β_o in our example. Similarly, column 3 exhibits identical changes in inflows to region 3, taking the value 1 of the coefficient β_d in our example. The diagonal (3,3) element reflects a response equal to $\beta_o + \beta_d$, the sum of the changes in flows into and out of region 3, reflecting the change in intraregional flows arising from the change in x_3. We have only $2n$ nonzero changes in flows by virtue of the independence assumption. All changes involving flows into and out of regions other than region 3 are zero.

Our method for producing scalar summary measures of the impacts arising from changes in characteristics of the regions involves averaging over the cumulative flow impacts associated with changes in all regions, $i = 1, \ldots, n$, analogous to the approach taken by LeSage and Pace (2009) for the conventional SAR model. Doing this produces $\sum_{i=1}^{n} (\Delta Y/\Delta x_i)$, a cumulative total effects (TE) matrix shown in Eq. (83.11), which is the sum of $n = 8$ different changed flow matrices of the type shown in Eq. (83.10) for the case where $i = 3$. This matrix (TE) can be decomposed into flow matrices reflecting origin effects (OE), destination effects (DE), network effects (NE), and intraregional effects (IE) arising from changing a single characteristic in all regions by one unit.

$$TE = \begin{pmatrix} 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 \end{pmatrix} \tag{83.11}$$

The matrix of cumulative intraregional effects can be constructed using the main diagonal elements of the TE matrix, $IE(i,i) = \sum_{i=1}^{n} (\Delta Y_{(i,i)}/\Delta x_i)$. The matrix of (cumulative) intraregional effects is shown in Eq. (83.12), where we see that these are identical and equal to the value of the coefficients $\beta_o + \beta_d$ from our example. These are located on the main diagonal which reflects changes in intraregional flows.

$$IE = \begin{pmatrix} 1.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.5 \end{pmatrix} \tag{83.12}$$

The matrix of (cumulative) origin effects can be constructed using the ith row of the flow changes matrix excluding the intraregional effect from the diagonal element. Specifically, $OE(i,.) = \sum_{i=1}^{n} (\Delta Y_{(i,.)}/\Delta x_i) - IE(i,i)$, where we use $OE(i, .)$ and $\Delta Y_{(i,.)}$ to denote the ith row of the OE matrix and flow changes matrix ΔY. The result is shown in Eq. (83.13), where we see that these are identical and equal to the value of the coefficient β_o from our example. The main diagonal is zero since this reflects changes in intraregional flows which we have excluded from our definition of origin effects.

$$OE = \begin{pmatrix} 0.0 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.0 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.0 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.0 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.0 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.0 \end{pmatrix} \tag{83.13}$$

The matrix DE of (cumulative) destination effects is based on using the ith column of the flow changes matrix, excluding the intraregional effect from the diagonal element. Of course, the OE and DE definitions would reverse if we were relying on an origin-centric flow matrix instead of the destination-centric one. Specifically, $DE(.,i) = \sum_{i=1}^{n} (\Delta Y_{(.,i)}/\Delta x_i) - IE(i,i)$, where we use $DE(., i)$ and $\Delta Y_{(.,i)}$ to denote the ith column of the DE matrix and flow changes matrix ΔY. The result is shown in Eq. (83.14), where we see that these are identical and equal to the value of the coefficient β_d from our example. Again, the main diagonal is zero since this reflects changes in intraregional flows which we have excluded from our definition of destination effects.

$$DE = \begin{pmatrix} 0.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 0.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 0.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.0 \end{pmatrix} \quad (83.14)$$

The matrix of cumulative network effects represents all flow changes that spill-over on regions other than the origin and destination region whose characteristics were changed. This can be constructed by subtraction: $TE - IE - OE - DE = NE$. For the nonspatial gravity model, the cumulative network effects matrix NE contains all zeros, since this model does not allow for spillovers to regions not involving origin and destination regions by virtue of the independence assumption. Scalar summary measures of the total effects as well as the decomposition into origin, destination, intraregional, and network effects can be constructed using averages of the (matrices) of cumulated changes in flows. This is accomplished by averaging over row-sums and then column-sums, which follows the approach taken by LeSage and Pace (2009) for the SAR model. This produces the results shown in the first column of Table 83.1. (One can also average over column-sums and then row-sums to produce identical results as noted by LeSage and Pace (2009).)

Applying our decomposition with this computationally inefficient approach would lead to scalar summary measures for the impact of changing all elements in the vector X_o presented in the second column of Table 83.1. Similarly, our decomposition with this approach would lead to scalar summary measures for the impact of (independently) changing each element in the vector X_d shown in the third column of Table 83.1. The sum of these two sets of scalar summary effects estimates constructed using independent changes in all elements of X_o and X_d shown in the fourth column of Table 83.1 equals the result shown in the first column. We will see that this is also the case for the spatial autoregressive variant of the gravity model.

An important point to note is that this approach differs from the conventional interpretation of nonspatial gravity models where the coefficient β_o is interpreted as a partial derivative reflecting the impact of changes in origin characteristics and β_d that is associated with changing destination characteristics. Although the conventional approach that used the coefficient sum $\beta_o + \beta_d$ as a measure of the total effect on flows arising from changes in origin and destination characteristics would

Table 83.1 Scalar summary measures of effects for the nonspatial model from a change in the (single) characteristic x averaged over all regions $i = 1, \ldots, 8$

	Δx_i	$\Delta X_{o,i}$	$\Delta X_{d,i}$	$\Delta X_{o,i} + \Delta X_{d,i}$
Origin effects	0.4375	0.4375	0.0000	0.4375
Destination effects	0.8750	0.0000	0.8750	0.8750
Intraregional effects	0.1875	0.0625	0.1250	0.1875
Network effects	0.0000	0.0000	0.0000	0.0000
Total effects	1.5000	0.5000	1.0000	1.5000

produce a correct inference, the appropriate decomposition into origin, destination, and intraregional effects has been missing from this literature.

Another point is that one can use changes in each element of the $n^2 \times 1$ vectors X_o and X_d to arrive at the same scalar summary measures as shown in Table 83.1. However, this would require that we sequence through changes in n^2 individual elements of X_o and also n^2 elements of X_d, recording the change in the $n \times n$ matrix of flows that arise from this sequence of $2n^2$ changes, which is computationally much more difficult. We would also need to aggregate the changes in flows arising from changes in both X_o and X_d to produce final results. To avoid this, we can exploit the special structure of the $n^2 \times R$ matrices X_o, X_d as they relate to the underlying $n \times R$ matrix X.

As noted above, there could be applied modeling situations where practitioners choose to include a specific characteristic only in the X_o or X_d vector, but not in both. As an example, consider a model for commuting-to-work flows. The number of residents might be used as a size measure for origin regions whereas the number of business establishments might be used as a size measure for the destination regions. In this case, it might be more appropriate for interpretative purposes to report separately scalar effects summaries arising from the calculations involving changing all elements in the vector X_o and X_d. We will have more to say about this in the next section.

83.5.2 A Numerical Illustration for the Spatial Gravity Model

Using the same numerical values set forth in the previous section, but setting $\rho_o = \rho_d = 0.4$ and $\rho_w = -\rho_o\rho_d = -0.16$, we carried out the same experiment where each value of x_i, $i = 1, \ldots, 8$ was changed by one unit. The resulting changes in the flow matrix were recorded, with the total flow effects arising from the change in x_3 shown in Eq. (83.15).

$$\Delta Y/\Delta x_3 = \begin{pmatrix} 0.688 & 0.688 & 2.064 & 0.612 & 0.309 & 0.246 & 0.233 & 0.233 \\ 0.688 & 0.688 & 2.064 & 0.612 & 0.309 & 0.246 & 0.233 & 0.233 \\ 1.376 & 1.376 & 2.752 & 1.300 & 0.997 & 0.934 & 0.921 & 0.921 \\ 0.650 & 0.650 & 2.026 & 0.574 & 0.271 & 0.208 & 0.195 & 0.195 \\ 0.498 & 0.498 & 1.875 & 0.423 & 0.119 & 0.056 & 0.044 & 0.044 \\ 0.467 & 0.467 & 1.843 & 0.391 & 0.088 & 0.025 & 0.012 & 0.012 \\ 0.460 & 0.460 & 1.837 & 0.385 & 0.082 & 0.018 & 0.006 & 0.006 \\ 0.460 & 0.460 & 1.837 & 0.385 & 0.082 & 0.018 & 0.006 & 0.006 \end{pmatrix} \qquad (83.15)$$

One difference between this spatial model result and the nonspatial model is the presence of network effects, shown by the nonzero elements in rows and columns other than 3. This means that a change in say the attractiveness of region 3 impacts flows throughout the network. Of course, the largest impacts reside in the 3rd row and column, since the change in attractiveness of region 3 has the largest impact on flows into and out of region 3 from all other regions. The magnitude of impact declines as we move further from the (3,3) element in the up/down or left/right direction in column and row 3. This arises due to decay with higher-order neighbors typical of spatial autoregressive processes. We see a similar pattern for elements not in the third row and column, where the change in flows decline in magnitude for elements further away from the (3,3) element. This reflects a decline in the magnitude of network spillovers with an increase in the number of paths through which the flows must pass.

It is also important to note that the interpretation of partial derivatives in cross-sectional spatial models such as this is that these reflect a long-run, steady-state equilibrium. The estimated changes in flows would be those that arise in response to the increased attractiveness of region 3 as we move to a new steady-state equilibrium. For example, we would conclude that changes in the attractiveness of region 3 would produce these changes in flows throughout the network, reflecting the level of flows we would expect to see in a new steady-state equilibrium.

Applying our approach for calculating scalar summary measures of the impacts arising from changes in characteristics of the regions described in the previous section we arrive at $TE = \sum_{i=1}^{n} (\Delta Y/\Delta x_i)$, shown in Eq. (83.16). The most obvious facet of the cumulative TE matrix is that the effects are much larger than in the case of the nonspatial gravity model. An examination of the components' (IE, OE, DE, NE) decomposition shows the source of these differences in effects on flows arising from changes in regional characteristics. A similarity with the nonspatial model TE matrix is that total effects are identical for all observations/regions, which is always the case for SAR models. This is because the spatial weight matrix W has row-sums of unity (see Elhorst 2010).

$$TE = \begin{pmatrix} 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 \\ 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 \\ 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 \\ 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 \\ 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 \\ 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 \\ 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 \\ 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 & 4.166 \end{pmatrix} \quad (83.16)$$

The cumulative intraregional effects matrix is shown in Eq. (83.17), where we see that the values are not equal to $\beta_o + \beta_d$ as in the nonspatial model. They are also not equal to the diagonal elements from the cumulative TE matrix. This is because there are feedback loops that arise in spatial models, where impacts on neighbors work their way back to the own region. To see this, consider that spatial autoregressive models rely on a data generating process: $y = (I_n - \rho W)^{-1}(X\beta + \varepsilon)$, where the matrix inverse can be expressed as an infinite series: $I_n + \rho W + \rho^2 W^2 + \rho^3 W^3 + \ldots$. The matrix W has zeros on the main diagonal, but the matrices $W^2, W^3, \ldots$ do not. This is because by virtue of the definition of a second-order neighbor reflected by the matrix W^2, region i is a second-order neighbor to itself, a neighbor to a neighboring region. The feedback effects on intraregional flows account for some of the difference between the value of 4.166 for the main diagonal of the TE matrix in the spatial model and the nonspatial model, where we found a value of 1.5.

$$IE = \begin{pmatrix} 2.632 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 2.747 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 2.752 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 2.728 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 2.728 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 2.752 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 2.747 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 2.632 \end{pmatrix} \quad (83.17)$$

The nonzero network effects (NE) account for the remaining differences, as can be seen from the diagonal of the matrix for these cumulative effects, shown in Eq. (83.18). (The values from the diagonal of NE and IE do not exactly equal those of TE because digits were truncated when forming the matrices for presentation.) Nonzero network effects also feedback onto intraregional flows, and these account for a large part of the difference between the spatial and nonspatial model effects estimates. An important point to keep in mind is that variation in all of the effects estimates over the regions might be greater than in our simple example, where the spatial configuration of the regions represents one of the simplest. The magnitudes of network effects will depend on the spatial configuration of the regions involved, with regions that have more links to other regions experiencing larger network effects relative to regions that are relatively more isolated with less links to other regions.

$$NE = \begin{pmatrix} 1.533 & 0.869 & 1.148 & 1.406 & 1.456 & 1.451 & 1.456 & 1.533 \\ 0.869 & 1.419 & 0.804 & 1.303 & 1.404 & 1.410 & 1.417 & 1.494 \\ 0.996 & 0.767 & 1.414 & 0.855 & 1.310 & 1.388 & 1.411 & 1.489 \\ 1.398 & 1.289 & 0.847 & 1.437 & 0.868 & 1.302 & 1.397 & 1.480 \\ 1.480 & 1.397 & 1.302 & 0.868 & 1.437 & 0.847 & 1.289 & 1.398 \\ 1.489 & 1.411 & 1.388 & 1.310 & 0.855 & 1.414 & 0.767 & 0.996 \\ 1.494 & 1.417 & 1.410 & 1.404 & 1.303 & 0.804 & 1.419 & 0.869 \\ 1.533 & 1.456 & 1.451 & 1.456 & 1.406 & 1.148 & 0.869 & 1.533 \end{pmatrix} \quad (83.18)$$

The cumulative OE and DE matrices for the spatial model are shown in Eqs. (83.19) and (83.20), where we also see values that differ over the regions and exhibit magnitudes greater than the 0.5 and 1.0 values representing coefficients β_o, β_d from the nonspatial model. As in the other cases, these reflect changes in flows arising from interactions modeled by origin-, destination-, and origin-destination dependence in the spatial gravity model. Intuitively, changing the characteristics of a single region will impact inflows to and outflows from that region, but also impact flows to regions neighboring the origin and impact flows to regions neighboring destination regions, and impact flows from regions neighboring the origin to regions neighboring the destination.

$$OE = \begin{pmatrix} 0.000 & 1.243 & 0.954 & 0.893 & 0.880 & 0.878 & 0.877 & 0.877 \\ 1.358 & 0.000 & 1.298 & 0.995 & 0.932 & 0.919 & 0.916 & 0.916 \\ 1.376 & 1.376 & 0.000 & 1.300 & 0.997 & 0.934 & 0.921 & 0.921 \\ 1.005 & 1.005 & 1.292 & 0.000 & 1.289 & 0.989 & 0.929 & 0.929 \\ 0.929 & 0.929 & 0.989 & 1.289 & 0.000 & 1.292 & 1.005 & 1.005 \\ 0.921 & 0.921 & 0.934 & 0.997 & 1.300 & 0.000 & 1.376 & 1.376 \\ 0.916 & 0.916 & 0.919 & 0.932 & 0.995 & 1.298 & 0.000 & 1.358 \\ 0.877 & 0.877 & 0.878 & 0.880 & 0.893 & 0.954 & 1.243 & 0.000 \end{pmatrix} \tag{83.19}$$

$$DE = \begin{pmatrix} 0.000 & 2.053 & 2.064 & 1.867 & 1.829 & 1.837 & 1.832 & 1.755 \\ 1.938 & 0.000 & 2.064 & 1.867 & 1.829 & 1.837 & 1.832 & 1.755 \\ 1.793 & 2.022 & 0.000 & 2.010 & 1.859 & 1.843 & 1.833 & 1.755 \\ 1.763 & 1.871 & 2.026 & 0.000 & 2.009 & 1.875 & 1.840 & 1.756 \\ 1.756 & 1.840 & 1.875 & 2.009 & 0.000 & 2.026 & 1.871 & 1.763 \\ 1.755 & 1.833 & 1.843 & 1.859 & 2.010 & 0.000 & 2.022 & 1.793 \\ 1.755 & 1.832 & 1.837 & 1.829 & 1.867 & 2.064 & 0.000 & 1.938 \\ 1.755 & 1.832 & 1.837 & 1.829 & 1.867 & 2.064 & 2.053 & 0.000 \end{pmatrix} \tag{83.20}$$

Using the same approach set forth in the previous section to produce scalar summary measures of the total effects, as well as the decomposition into origin, destination, intraregional, and network effects by averaging the matrices produced the results given in the second column of Table 83.2.

The third and fourth columns show results based on calculating flow matrix responses to changes in each element of the $n^2 \times 1$ vectors X_o, X_d, which were added to produce the fifth column. In this case where a single characteristics vector x was used to form X_o and X_d, these equal the scalar summary effects produced by considering only n changes in elements of x_i.

Consider again the example involving commuting-to-work flows, where the number of residents is used as a size measure for origin regions and the number of business establishments as a size measure for the destination regions, so X_o and X_d are distinct. Interpreting results for this type of model would require reporting both columns three and four from Table 83.2. Summing these two different scalar summary measures would make less sense in this situation, since changes in X_o do not imply changes in X_d and vice versa. This would lead to a slight change in interpretation, where changes in X_o (residents at the origin) lead to an origin, destination, intraregional, network, and total effects on flows, as do changes in X_d

Table 83.2 Scalar summary measures of effects for the spatial interaction model arising from a change in a single characteristic x averaged over all regions $i = 1, \ldots, 8$

	Δx_i	$\Delta X_{o,i}$	$\Delta X_{d,i}$	$\Delta X_{o,i} + \Delta X_{d,i}$
Origin effects	0.9129	0.7920	0.1209	0.9129
Destination effects	1.6445	0.0605	1.5840	1.6445
Intraregional effects	0.3394	0.1131	0.2263	0.3394
Network effects	1.2698	0.4233	0.8466	1.2698
Total effects	4.1667	1.3889	2.7778	4.1667

(business establishments at the destination). This type of model specification could be viewed as an a priori zero restriction on the coefficient for the characteristic residents at the destination as well as a zero restriction on the coefficient for business establishments at the origin. It should be possible to include the full set of explanatory variables (residents and business establishments) in the set of model characteristics for both origins and destinations and then test the validity of the a priori zero restrictions. This would involve a test for significant differences between the full and nested model scalar summary effects estimates. If there are no differences in conclusions regarding the size and significance of the scalar summaries, then the restrictions are consistent with the sample data.

83.6 Conclusion

Recently introduced spatial autoregressive extensions of the spatial interaction model hold a great deal of promise for regional modeling of flows. However, there are still a great many obstacles to the wide use of these models in applied situations. First, these models require flow magnitudes that can be transformed to reflect a normal distribution. This is not the case for flow matrices containing a large number of zero values, large diagonal elements reflecting intraregional flows, or count magnitudes. There is a need for future research regarding implementation of a spatial autoregressive Poisson interaction model.

Beyond the issue of estimating model parameters, there is also a need to carefully consider how these parameters are interpreted. In the case of the independent spatial interaction model, changes in characteristics of a single region can exert impacts on inflows from all other regions, outflows to all other regions, as well as intraregional flows. These impacts can be measured by considering rows and columns of the flow matrix. We set forth a proposal for calculating scalar measures of impact that average over changes applied to a single explanatory variable (regional characteristic) for all regions. The approach allows separation of row/column and diagonal element impacts arising in the flow matrix, which we label origin, destination, and intraregional effects. Past applications of regression-based spatial interaction models that assume flows are spatially independent seem to have overlooked this aspect of the partial derivative impacts associated with changes in characteristics of regions.

For the case of the spatial autoregressive interaction model, interpretation of the model estimates in terms of their partial derivative impacts on flows is more complicated.

Changes to a single region's characteristics can impact not only inflows from all other regions, outflows to all other regions, and intraregional flows, but also all other flows in the flow matrix. These impacts can be measured using changes taking place in rows, columns, and the diagonal and off-diagonal elements of the flow matrix as a result of a change to a single region's characteristic. We propose a scheme for calculating scalar summary measures for these impacts that we label origin, destination, intraregional, and network effects.

Specifics regarding simulation of the partial derivative impact estimates based on the estimated distribution for the model parameters were not discussed here. This would require using the estimated variance-covariance matrix for the model parameters to generate draws for each model parameter. These could be used in conjunction with the approach proposed here to produce a distribution of the scalar estimates for the various types of impacts. These empirically derived distributions could serve as the basis for inference regarding significance of the various types of impacts.

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