Chapter 1
Inverse Analysis: Introduction

Numerical simulations have been established as a powerful tool used in practically all fields of engineering and science. A large number of commercial codes is developed to solve the, so-called *direct problems (or forward problems)*, which consist of finding the solution in terms of response fields when a complete set of input data defining uniquely the solution is known. Since these codes require the knowledge of some parameters on which the solution depends, sometime in engineering practice it is required to solve an *inverse problem*, defined as the one where some of the "effects" (responses) are known but not some of the "causes" leading to them, namely parameters on which the system depends. These problems are tackled within, relatively young and still growing scientific branch which in modern literature (e.g. [1–3]) is found under the name of *Inverse Analyses*.

The name "inverse" comes from the assumption that the logical path in reasoning (or solving the problems) goes from the causes to the effects. Therefore, solving the problem in opposite direction can be identified as inverse (Fig. 1.1).

In the scientific and engineering practice, some observed phenomena are usually represented by models. These models are further used for, hopefully successful, prediction of responses, for a given input parameters. In this context, an inverse problem can be defined as the one in which some model parameters need to be obtained from the given observed data. This definition already anticipates, what in general holds, that to solve an inverse problem is more difficult than to solve the forward one. These difficulties arise from the fact that inverse problems are typically ill-posed, meaning that usually some of the conditions of the uniqueness of the solution do not exist. Certain types of ill-posedness will be discussed in more details later in this chapter.

This book deals with inverse problems that emerge in the structural engineering field. The main idea is to present just the small portion of theory needed to be understood for a successful implementation of a fully working inverse analyses procedure in the present context. This chapter should serve to make the reader familiar with the main concepts and all the necessary parts that one needs to put together in order to solve an inverse problem.

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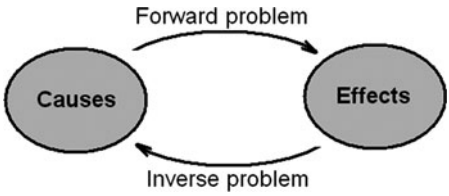

Fig. 1.1 Scheme of solving forward/inverse problems

1.1 Inverse Problems in Science and Engineering

We already anticipated that the term *inverse problem* refers to any general situation in which it is required to solve the problem in the opposite direction, starting from "effects" as known and identifying "causes" that lead to them. Inverse problems are inherently connected to direct problems. This connection is established through the model used to simulate a given phenomenon. To solve a direct problem, in scientific or engineering context, means to find analytical or numerical solution for ordinary or partial differential equations that are describing it. On the other hand, an inverse problem is the one in which the solution is known and the objective is to determine the complete forward problem for which that solution is possible.

In order to solve the direct problem there should be known a minimum set of information describing it, referred to as *the condition of uniqueness*. Therefore, when we talk about the solution of differential equations, the uniqueness condition should include the following information:

- The description of geometry under consideration
- Boundary conditions
- Initial conditions
- Properties of all involved materials (i.e. constants entering into differential equations)

Inverse problem is the one in which *some* of these data are missing and they should be identified from the known solution of the problem. Already at this stage some of the problems connected with the inverse analyses can be foreseen. For example it may be possible to face the situation where not only just one set of missing parameters makes the solution possible. It practically means that the inverse problems may not satisfy the uniqueness condition.

This can be illustrated on the following example. Let us consider a simple case in which the model is represented with an analytical equation given by $y = ax^2 + bx + c$. Let the forward solution be the one that uses the value of x to compute y. It is obvious that when the problem is solved as forward, to one value of x corresponds only one value of y (Fig. 1.2). On the other hand the associated inverse problem is not unique, since the solution y_1 can be produced by a given model for two different values of x. This lack of uniqueness is common for some inverse problems and as such represents a significant drawback. Further in the book it will be shown how these types of problems are tackled within the inverse analyses in the present context.

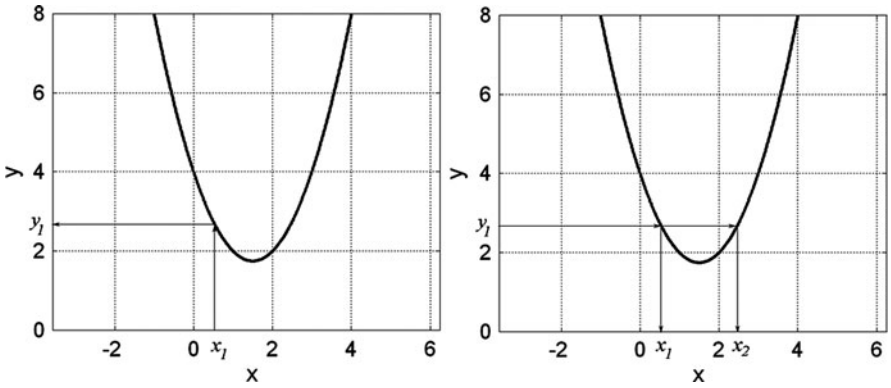

Fig. 1.2 Uniqueness of the solution: existing for the direct problem (*left*) and missing for the associated inverse problem (*right*)

Inverse analyses presented in this book deal with the numerical models. What makes this group of problems particularly interesting and challenging from the engineering and scientific point of view is that they represent a synergic combination between experimental mechanics, numerical simulations and mathematical programming. This combination may be anticipated from the above definition of inverse problems. As mentioned before, the inverse analysis procedure is in general designed in order to assess some of the missing information about the system. The system of interest needs to be modeled by the use of the appropriate technique (in the context presented in this book usually numerical). This model depends on some information (e.g. geometry, boundary condition, parameters, etc.) some of which are known, others should be identified from the known solution. The solution is known as a result of the previously preformed experiment. Finally, a discrepancy function is constructed that quantifies the difference between measured quantities and their computed counter-parts, which is subsequently minimized by the use of optimization algorithms coming from mathematical programming.

1.2 Classification of Inverse Problems and Their Application

There are many ways to classify inverse problems. Going back to their definition, as the problems in which the goal is to identify some missing information, a logical classification could follow based on the type of the missing data. Therefore we can distinguish:

- **Backward problems:** represent problems where the initial conditions are unknown;
- **Boundary inverse problems:** in which the goal is to determine boundary conditions;

- **Shape design:** problems where the shape and the size of the domain need to be determined;
- **Force determination:** problems in which the external action, i.e. forces are unknown;
- **Parameter identification problems:** where constants that are entering into governing equation, primarily into the constitutive models, are to be found.

There are many successful applications of inverse analyses of a different kind. The first two groups of inverse problems cited here are very frequently applied in thermo problems. Using the inverse analyses procedure it is possible to determine both temperatures and heat fluxes on inaccessible surfaces by performing measurements in more appropriate zones (see e.g. [4, 5]). Within Computational Fluid Mechanics (CFD) there are a lot of examples of aerodynamic shape optimization by the use of inverse analyses approach (see e.g. [6, 7]). Force identification problems, are another example of successful application of inverse analyses theory in order to assess, for instance unknown impact or contact forces (e.g. [8]) usually by the measurements of displacements.

In general, inverse analyses are applied in all those fields of science and engineering where the quantities of interest are not directly measurable, and they are therefore assessed through other measured values. Very well established examples of this type can be found in geotechnical engineering considering measurements based on electrical resistivity tomography. This technique provides the measure of conductivity of specimen that is further, through inverse analysis procedure, related to various parameters of soils such as porosity, degree of saturation and hydraulic conductivity (see e.g. [9, 10]).

The last example leads us to, probably the most dispersed group of inverse analyses problems, namely the parameter identification problems. This book will give a particular focus to the parameter identification problems, even though most of the material presented here is applicable also to other groups of inverse problems.

In the last years a big effort was made in the direction of developing numerical models that can reproduce the mechanical behavior of different materials and structures. Nowadays it is possible to use rather complicated material models that are usually already implemented in the commercial codes. These models are capable to capture different physical phenomena (e.g. plasticity, viscoplasticity, damage, fracturing) some times even on different scales. Assuming that they can correctly describe the behavior of the real phenomenon, their accuracy still depends very much on correct identification of the constants (parameters) entering into the governing equation. Sometimes, these parameters are not directly measurable in the laboratories, and so their quantification becomes an important issue. For some simple cases the transition between measured responses in the experiment and required model parameter values can be established relatively easy (like for example construction of a stress–strain curve from measured force and elongation in the uniaxial tensile test). For more complicated material models the link between experiments and computational models is not that trivial. This gap between

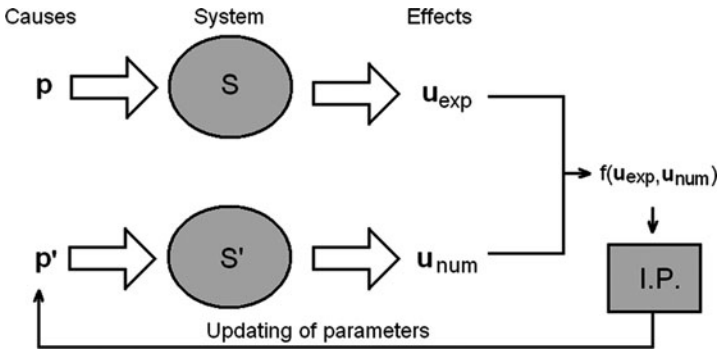

Fig. 1.3 Schematic representation of the inverse analysis procedure

measured material and structural responses and required constitutive model parameters by the computational models is bridged by the inverse analyses theory.

Applying the general concept previously discussed to the parameter identification problems let us denote by S (Fig. 1.3) a certain system, that actually represents an experiment performed on some structure or material specimen. This is the first phase of the inverse analysis procedure that should provide some meaningful measurable quantities. The same process is further modeled (traditionally in the structural context by finite element method) in order to produce a numerical counter part of the system, denoted by S'.

If a perturbation is applied on a real system S (i.e. the system is subjected to a certain load) it will react by giving a response which may be represented by a number of measurable quantities collected in vector u_{exp}. For structural mechanics problems, these can be forces, displacements, crack openings etc. On the other hand, the numerical model is designed to compute the response of the system subjected to the same perturbation, for any given model parameter vector p', collecting the sought parameters within inverse analysis procedure. The response of numerical model of the system represents the calculated values of exactly the same quantities as those measured in the experiment, which are stored in vector u_{num}. The results from the experiment are put together with their computed counterpart in order to form a discrepancy function that should quantify the difference between the two responses (Eq. 1.1)

$$f = \left\| \mathbf{u}_{\text{exp}} - \mathbf{u}_{\text{num}} \right\| \tag{1.1}$$

This function is further minimized with respect to sought parameters. For the minimization a well established algorithms from mathematical programming are usually used, that within some iterations are capable to find parameter values that are minimizing the objective function. In other words the solution of the inverse problem should result with the parameter values for which the adopted numerical model is giving the closest results to those that were measured in the experiment. If the numerical model is done very well, and if the experimental measurements are

performed with precise tool with a low level of noise, then at the end of this minimization procedure the numerical result should match exactly the experimental one. In more realistic cases, there will always be a small discrepancy between the two responses, but in any case for well posed inverse problems a result of the minimization should provide parameter values that produce the minimum possible difference for the given measurements and used numerical model.

In order to better understand the main problems one is facing within parameter identification problems let us give a closer look on how an inverse analysis problem is set within the scope of structural mechanics.

1.3 Parameter Identification Problems in Structural Analyses: Setting Up the Problem

Parameter identification problems in material and structural mechanics are used usually in order to characterize the material properties. The approach of inverse analyses gives the possibility to keep the experiment relatively complicated making possible to capture more complex material models described with large number of parameters.

A popular experiment that is used to characterize materials is an instrumented indentation test that originates in the traditional hardness test. It represents the process in which the tip of indenter, usually conical, spherical or pyramidal, is forced into the surface of testing specimen in order to leave a permanent deformation. Unlike the traditional hardness test, in the instrumented one, there is a constant monitoring of applied force and obtained penetration, and so the result of the test is a so-called indentation curve like the one visualized in Fig. 1.4.

The information of material response taken from the indentation curve proved to be rather useful to characterize both elastic and plastic properties. Several authors (see e.g. [11, 12]) have proposed some semi-empirical approaches to correlate the data provided by indentation curves with the Young modulus and yield strength of the material. Using this test within inverse analyses framework turned out to be very much useful making possible to characterize more complicated material models.

As the first improvement with respect to semi-empirical formulae, an inverse analyses procedure can characterize more accurately material parameters just from the indentation curve itself. Since the indentation test can be simulated, with nowadays available commercial FE software, rather accurately, it gives a possibility to compare computed indentation curve with the measured one, without a need to measure contact area and therefore introduce additional potential error of experimental measurements (like in the semi-empirical approach proposed by Oliver and Pharr [11]). A number of successful inverse analyses procedures based on instrumented indentation can be found in the literature: application in characterization of functionally graded materials [13], characterization of thin films [14] assessment of quasi-brittle fracture properties [15], and so on.

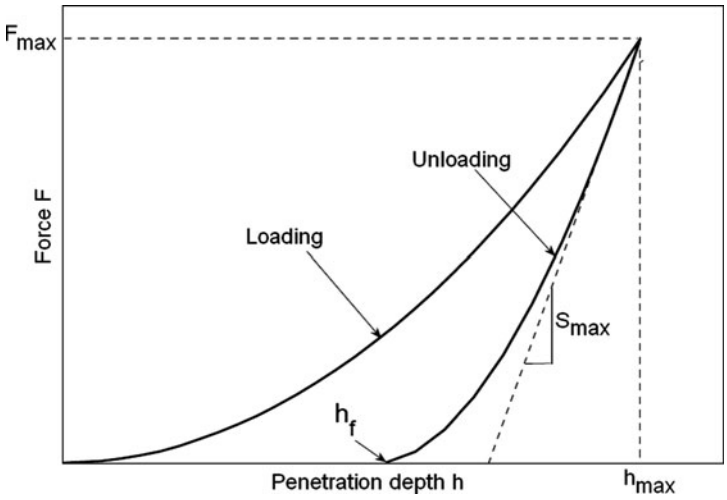

Fig. 1.4 Indentation curve resulting from an instrumented indentation test

Another interesting field of the application of parameter identification procedures is in the assessment of damages in structures that are in service. Since the main concept of inverse analyses is to combine simulation with the experiment, it is evident that the experiment can be rather complicated and performed even in situ directly on the structure. One of the typical examples of this is the assessment of possible deterioration of existing concrete dams due to alkali silica reaction or, in general, due to extreme loadings such as earthquakes and floods. For this purpose a so-called "flat-jack" test is used that consists of performing cuts in the structure (which, considering the size of the dam, are considered as practically non-destructive test); in created slots a flat-jack is inserted that is further pressurized and the resulting displacements are measured. Combining the experiment with numerical modeling of the test a practical inverse analyses procedure can be designed in order to assess the elastic properties of the concrete (see e.g. [16, 17]).

From previous discussion it is obvious that the very first phase in setting the inverse analyses problem for the parameter characterization is to choose the experiment. The experiment should be selected to be as simple as possible but at the same time it needs to activate all the parameters that needs to be assessed. This part is verified usually in a numerical simulation of the experiment.

The parameter identification problems in the context of structural analyses are starting form the assumption that the material model is known and they should result in a calibration of the constants entering into it (e.g. assessment of Young modulus, yield limit, exponent of hardening, etc.). Using the numerical simulation with the adopted material model, it is possible to perform an optimization of experiment in order to find the most appropriate perturbation (i.e. loading pattern of the specimen or structure) and to select those measurements that are the most sensitive to changes of sought parameters.

After the configuration of experiment is decided, the numerical phase is taken a step further in order to perform a so-called sensitivity analyses. Sensitivity analysis is usually carried out over some range of parameter values (the range is adopted based on the expected values of sought parameters for a given problem), and it should result with quantitative information (namely the derivatives of measurable quantities with respect to each of south parameters) about how sensitive measurements are to the perturbations on each of the sought parameters.

Once that the previous numerical study is performed, resulting in the experimental setup from which the measurable quantities with satisfying sensitivity to sought parameters are selected, a subsequent phase proceeds, in which the discrepancy function is formed that represents some norm of a difference between experimental and computed results. It should be mentioned that it is very important to include the uncertainty of the measurements into the objective function. In the form given by Eq. 1.1 if we denote by **R** residual vector that represents the difference between the measured quantities and computed ones, namely

$$\mathbf{R} = \mathbf{u}_{EXP} - \mathbf{u}_{COM}(\mathbf{p}) \tag{1.2}$$

then the objective function (to be minimized) denoted by ω should have the following form

$$\omega(\mathbf{p}) = \mathbf{R}^T \cdot \mathbf{M} \cdot \mathbf{R} \tag{1.3}$$

where **M** is a weight matrix that should somehow take into account uncertainties of the measurements. For example, matrix **M** can be a diagonal matrix assigning to each component of the residual vector **R** a weight inversely proportional to the corresponding measurement scattering. Another possibility could be to represent matrix **M** as the inverse of the (symmetric and positive definite) covariance matrix (diagonal in case of uncorrelated quantities) when each experimental data is provided together with a standard deviation, expressing the uncertainty typical of the measurement device.

To illustrate the importance of weighting the experimental data in the presence of numerical noise, let us consider the following example. Let us assume that the results of some experimental measurements of a certain state variable (e.g. temperature, velocity, etc.) over a space coordinate has the form like the one presented in Fig. 1.5. Let us assume that we would like to predict the distribution of this quantity with an analytical model given by

$$q = p_1 \cdot \sin(p_2 \cdot x) \tag{1.4}$$

The analytical model is governed by two parameters: p_1 and p_2 while x is a spatial coordinate, and q is a quantity of interest. Now we would like to calibrate this model by solving an inverse problem in order to find values of parameters that minimize the difference between experimental data visualized in Fig. 1.5 and those computed by Eq. 1.4. The inverse analyses procedure gives the following values of

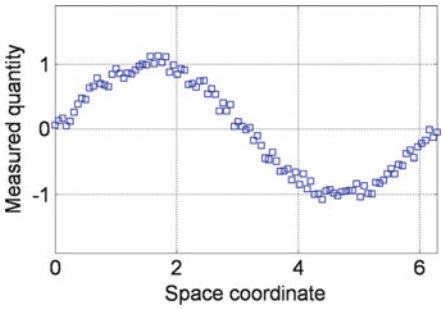

Fig. 1.5 Example of experimental measurements of one quantity in one-dimensional space

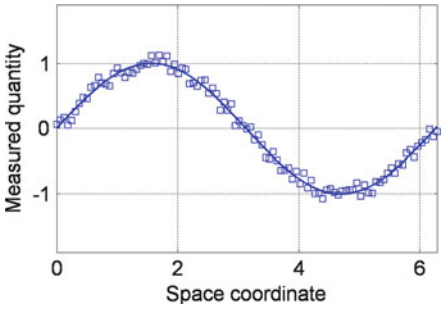

Fig. 1.6 Experimental data and analytical curve that minimizes the objective function

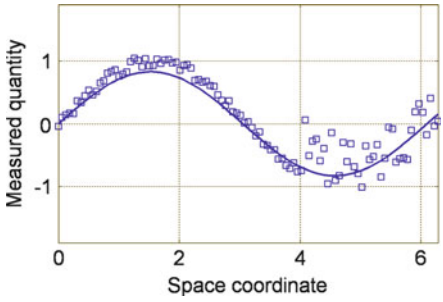

Fig. 1.7 Experimental data with different noise level in different zones, and analytical curve that minimizes objective function without weighting coefficients

parameters that minimize the objective function: $p_1 = 1.013$ and $p_2 = 1.003$ (Fig. 1.6).

Let us assume now that we carry out the same experiment, but this time in the zone where space coordinate is larger than 4 the measurements are performed with larger mistake. Using the objective function that doesn't weight differently the data (say the one defined by Eq. 1.4 with **M** being identity matrix), the procedure converges to different set of parameters, namely $p_1 = 0.828$ and $p_2 = 1.037$ (Fig. 1.7).

As it can be noticed from the figure, the inverse analyses procedure converged to analytical curve that tries to fit both the zone with more accurate measurements and the one with the larger noise. If both of these zones are of equal importance (i.e. no weighting is introduced) resulting parameters that minimize the function are assessed

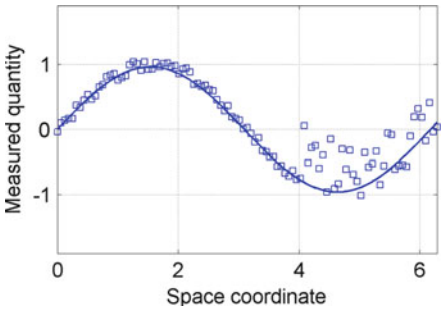

Fig. 1.8 Experimental data with different noise level in different zones, and analytical curve that minimizes objective function with weighting coefficients associated to experimental scattering

with larger error. If we solve now the same problem but this time giving different weighting coefficients to the measurements in the zone $x > 4$ (say three times less weight due to approximately three times larger scattering in that zone) the procedure converges to the following parameter values: $p_1 = 0.960$ and $p_2 = 1.016$ (Fig. 1.8).

Comparing the results it can be noticed that when the experimental data are not weighted, the assessment procedure results in larger errors in estimates, since the function minimizes the difference in an average, least square sense, which can lead to relatively large errors in sought parameters.

Taking into account also the measurement uncertainties is an important issue in all those situations in which the experimental data are not measured with the same accuracy in all zones. This is frequently encountered in the inverse analyses based on instrumented indentation, when the residual imprint is used as experimental data, which is mapped after the removal of the indenter. These measurements are affected with different errors in diverse zones, usually due to optical interferences or other obstacles. In such cases, defining the objective function as a simple difference between measured and computed quantities could result in significant errors on parameter estimates.

Once that the discrepancy function is formulated (namely the measurable quantities are selected and appropriate weighting coefficients are employed) in a subsequent phase, the goal is to minimize this function with respect to sought parameters. In the problems considered in this book the optimization algorithms used for this purpose are numerical, and are not providing the solution in closed form. These algorithms are traditionally iterative and they seek the minimum of the objective function by a successive approaching within a sequence of calculations.

The minimization problems dealt within the context discussed here are always constrained, since the parameters are representing some physical quantities (i.e. Young modulus, yield limit, etc.) and therefore cannot take any arbitrary values. What makes the optimization problem even more complicated is that the objective function, in most of the cases is not convex, and sometimes even not continuous.

There are many different algorithms that one may employ to solve the constrained minimization problem. The choice of appropriate algorithm for the given problem may have a crucial effect on the overall effectiveness of the whole inverse analyses procedure. Without entering into a detailed survey of all possible

choices with their advantages and drawbacks at this stage let us just point out some more important features that need to be verified.

Among many possible classifications of optimization algorithms let us for a moment focus our attention to the classification based on the information on the objective function required by the algorithm. Taking this criterion into account all the algorithms can be classified in the following groups: **zero order algorithms**, which are involving only calculations of the objective functions (e.g. direct search by Nelder-Mead algorithm, Genetic Algorithms etc.); **first order algorithms** that need first derivatives of the objective function (e.g. steepest descent method, trust region method) and **second order alogrithms** requiring information on the Hessian, namely second derivatives of the objective function (e.g. Newton method).

Considering that the function to be minimized in our case is not analytically defined, it means that the computation of the derivatives is performed numerically, usually by finite differences. It implies that to compute the first derivatives of the objective function that depends on two parameters, it is required to perform three computations of system responses (namely three simulations): one for the reference values of the parameters, and remaining two for the separate perturbations of the two parameters. The fact that the derivatives are computed numerically already penalizes the second order methods, since using finite differences to computed full Hessian matrix for the two parameter case will involve six computations (with respect to three needed for first derivatives). Due to the increased computing times connected with second derivatives usually the second order methods are not used, since their potentially better behavior is penalized by the increased computing times.

In general, first order methods are less costly computationally than zero order methods (namely they involve fewer simulations). These algorithms start from some initial guess for the parameters, and then in each iteration they compute the first derivatives of the objective function. Based on these computations an evaluation of the parameter values for the next iteration is performed. The general scheme for first order algorithms is presented in Fig. 1.9.

Relaying on first derivatives only, these methods are incapable of distinguishing between the local and global minima. In other words the first order methods will converge to any mathematical minimum of the function (namely the one for which the first derivatives are equal to zero). The fact that the minimum is not a global one can be easily verified since the computed response for the converged parameters will differ from the expected one. In such cases the optimization procedure should be repeated starting from different initialization (i.e. different guess parameters). The fact that algorithm converged to the local minimum doesn't mean that the parameter estimate procedure doesn't work. Since it can be easily verified it practically represents only a loss of computational time. However, if the objective function for the given problem turns out to have a large number of local minima, then the approach with the use of first order algorithms will result as ineffective since the solution will depend on the initialization point. Even though the local minima can be verified as such, and therefore it may be confirmed that the set of parameters is not the solution we were looking for, still the presence of large

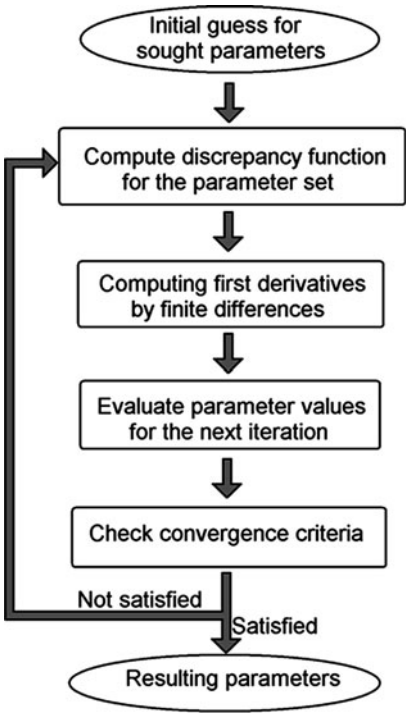

Fig. 1.9 General scheme of first order optimization algorithms

number of local minima makes the procedure not very effective since it may require a large number of initializations until it finally finds the global minimum.

If it turns out that the objective function to be minimized has a large number of local minima, then it is more appropriate to use a zero order optimization algorithms, like for example Genetic Algorithms (GA). These algorithms belong to a so-called soft computing group and will be discussed in more details in the following Chapter. At the moment let us just anticipate that GA, in general, avoid stacking in the local minima and are therefore much more effective if the objective function has a multiple local minima. This nice feature however, comes for the price of increased computing times and therefore the use of GA is justifiable only in the optimization problems with a large number of local minima. In other situations it is more convenient to use first or second order algorithms.

More serious problem which can be encountered within the inverse analysis procedure is the presence of more global minima in terms of two or more parameter sets that produce the same response of the system. In the shape optimization problems this doesn't represent a serious malfunction of the procedure since the goal is to find the shape that optimizes given requirements. An example of this can be a shape optimization problem with the goal of overall weight minimization under constrain of prescribed admissible stresses. If the objective function turns out to have two or more global minima in this case it means that the same weight can be

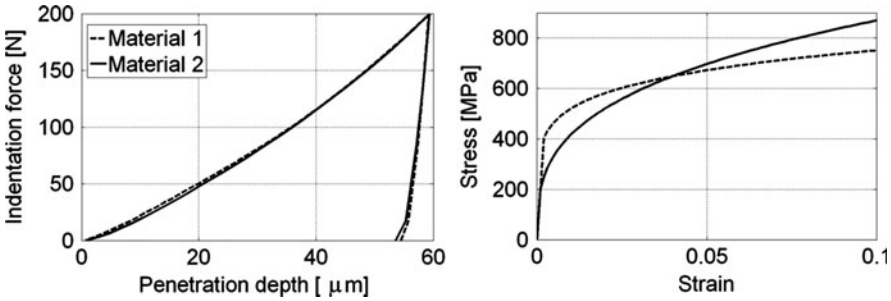

Fig. 1.10 The same response to the indentation test (*left*) of two different materials with different stress–strain curves (*right*)

obtained by different shapes and then it is on the designer to choose the one, so the optimization problem is considered to be solved.

In the material characterization problems the presence of two or more global minima represents a serious drawback of the procedure since it practically means that there are two possible material parameter sets which can produce the same response to the selected experiment. Typical example of this is a response to the indentation test in terms of the indentation curve. Figure 1.10 visualizes the simulation of the indentation test of two different materials with stress–strain curves visualized on the right-hand side of the figure. The left-hand side graph shows that the indentation curves are practically the same. This unpleasant feature of the indentation test is evidenced in the literature (see e.g. [18]) where the authors addressed this problem, and labeled materials as "mystical", since they appear to be undistinguishable.

From the inverse analyses point of view this represents a typical problem of ill-posedness with the discrepancy function having two (or more) global minima. Unlike the case with local minima, that can be solved by changing the optimization algorithm, in the case of more global minima nothing can be improved by changing the optimization algorithm, since, from mathematical point of view both parameter sets minimizes the function and it's not possible to distinguish which one is a solution for the inverse problem. The core of the problem is in the objective function itself, or to be more precise, in the selection of measurable quantities used to construct it. The only way to change it is to take into account additional measurements (or even additional experiment) in order to enrich the experimental information. This usually helps since in general, it is unlikely that two different materials will have the same response in terms of all the measurable field variables, or to two different experiments.

In the context of previously discussed undistinguishable materials in terms of response to the indentation test, Bolzon et.al. in [19] showed that this problem is regularized if also the residual imprint is measured and added to the objective function. Physically this means that even though the two materials (like those in Fig. 1.10) can produce the same indentation curve, there will be a difference in

terms of residual imprint geometry. This intervention changed the objective function which became more regular and doesn't have anymore multiple global minima.

The problem of more than one global minimum is produced by the compensation of influences between some of the parameters (like in the example of indentation test between yield stress and exponent of hardening). As such, it cannot be identified in the sensitivity analyses phase. The sensitivity analysis is done in order to verify that the selected measurements are influenced by the changes on the parameters. However, even if they are influenced by the parameter perturbations it may occur that some of the effects can be compensated producing therefore more than one set of parameters that minimize to the same level the objective function.

This suggests that, after all of the previously described steps are performed, a final check of the whole procedure should be performed. This part is done using the so-called pseudo-experimental data.

The only way to confirm that the inverse problem is set well and that it is capable of finding the sough parameters, is by solving it for the case where the solution is known. Therefore we should provide the system response for which the parameters are know. Since for this purpose truly experimental data with known parameters may not be available, it is more convenient to use pseudo-experimental data. Pseudo-experimental data represent computer generated data, resulting from the simulation with known parameters. When such data are used as inputs to the inverse analyses, the solution is known, and if the procedure is set well it should result with the same parameters as those used to generate pseudo-experimental data.

Furthermore, such verification should be performed for various parameter combinations, spread around the zone of interest, to confirm that the problem is well-posed not only just in one zone. It is important to do so since, the previously discussed problems of compensation between parameters may occur for some parameter combinations and not for others. As a result of these checks usually graphs of the type presented in Fig. 1.11 are designed for each of the parameters to evidence the error on estimates.

The result visualized in Fig. 1.11 represents an output of well set inverse problem. It shows that the parameter 1 is identified with relatively uniform error, for 50 different parameter combinations (presented in the abscissa). The error is uniform practically in all of the zones and it doesn't exceed the level of 1% (ordinate).

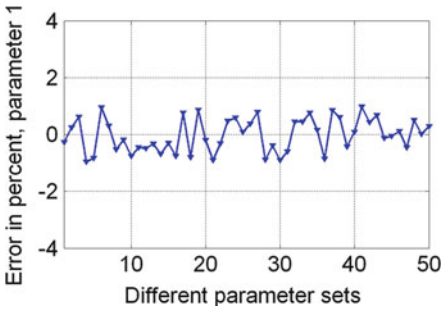

Fig. 1.11 Typical representation resulting from pseudo-experimental testing

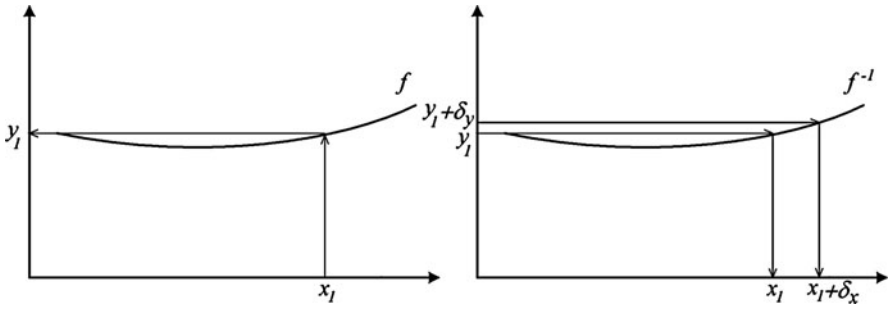

Fig. 1.12 Stabile solution for the direct problem (*left*) and not stable for the associated inverse problem (*right*)

This first pseudo-experimental check was dealing with "clean" numerical data used in the form resulting from the numerical simulations. Since the real experimental data are always subjected to a certain measurement error, it is important to check how the inverse procedure will behave in the presence of noise. Some times inverse problems don't have the same stability of the solution as direct problems (namely to the small perturbation of inputs there is a corresponding small variation of outputs, see Fig. 1.12). Since these perturbations are expected to occur due to errors in measurements it is important to check how the procedure reacts to them. This check should be performed with pseudo-experimental data, by subjecting the inputs, resulting from previously performed simulations, to a random "noise" that should imitate the excepted measurement error. After this intervention, the same study is performed which results in the graphs of the type given in Fig. 1.11, constructed for the different noise levels. From this test, it should be confirmed that the perturbation of the inputs of the certain level should result with more or less the same level or error of the estimated parameters. This study should provide valuable information on the required level of accuracy of the measured quantities.

Another possible resource of error in the identification procedure can come from the numerical simulation. Even when all the quantities are known for given experiment (namely, geometry, loads, constants entering into material models etc.) there will always be an error in modeling resulting in slightly different simulated response from the one measured in the experiment. This modeling error is rather systematic and within inverse analysis procedure can produce the same, systematic, type of error on the estimates.

To illustrate the influence of modeling error let us consider a simple case in which an experiment is performed on a cantilever, of the unknown elastic properties, with the concentrated load at the end. The discrepancy function is formed using, say ten measured displacements along the upper surface. Let us assume that the goal is to design an inverse analysis procedure that should minimize this function in order to assess the Young modulus. If the cantilever is modeled with the coarse FE mesh with a small number of elements over the thickness (see Fig. 1.13), the numerical model will be more rigid than the actual one, resulting

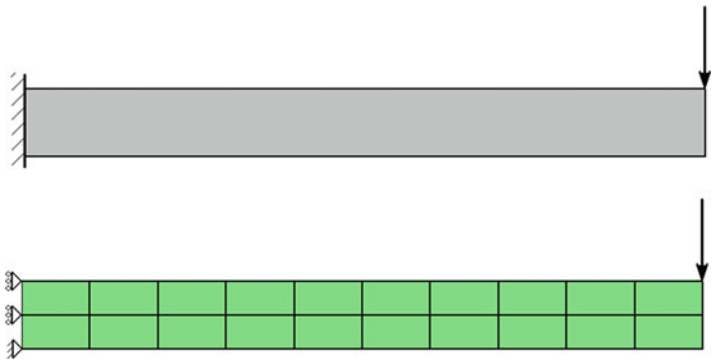

Fig. 1.13 Scheme of cantilever experiment, and numerical model with the coarse mesh

in smaller displacements. Using such model within inverse analyses procedure will produce a systematic error on estimated Young modulus resulting in smaller value than it should be, since the procedure searches the parameter for which the result will match the measured one. An over-stiff behavior is therefore compensated by a softer material. This brings a systematic error to the identification procedure resulting in underestimation of the parameters even when the experimental measurements are performed precisely.

The only way to check how good numerical model is would be to perform the experiment on the specimen with known material parameters and to compare it with the simulation. In the cases when such experimental data are not available, similar check, at least in qualitative terms, can be performed using pseudo-experimental data. It is done by perturbing the simulated data with a certain percentage, but this time with systematic error (i.e. on one side, like increase or decrease computed values resulting from the simulations). Using such data puts in evidence how the modeling error of given magnitude will affect the estimates of the parameters. If they remain within acceptable range it means that no further improvements are necessary on the numerical model. In opposite case it means that there is no significant stability of the given inverse problem and so the accuracy of the numerical model plays an important role. In such circumstance the modeling error should be verified before using the model in the context of inverse analyses.

1.4 Summary

In this Chapter a brief introduction to the inverse problems was given. A general classification and some of the applications in engineering and science are mentioned. A bit more detailed description of the parameter identification problems, as they are the main topic of this book, is then presented.

In the context of structural problems, it was shown which main ingredients need to be put together in order to design a fully operative identification procedure. For this purpose, a sequence of steps and procedures one needs to perform, together with the main possible problems which can be faced, is presented. This sequence can be summarized as follows:

1. The first step of each parameter identification procedure is the selection of the experiment. The experiment should be simple enough in order to be easily executed, but at the same time complicated enough to activate all the sought parameters for which the procedure is designed;
2. In the second step a numerical model of the experiment is build. Traditionally the FEM is used in the structural context. Numerical model should represent a compromise between the required level of accuracy, and the computing time having in mind the repetitive simulations required by the optimization algorithms;
3. Once that a reliable numerical model is constructed, it should be used to perform sensitivity analyses to verify that the selected measurable quantities are sensitive to the variation of parameters. If they turn out to be not sensitive enough they should be changed, or the experiment should be replaced with another one;
4. After the experiment setup is fixed, the objective function is designed that should quantify the discrepancy between the measured quantities and their computed counter-parts. It is important to apply the correct weighting coefficients that should account for measurable uncertainties. Further, an adequate optimization algorithm should be selected for the given problem with primer focus on the possible presence of local minima.
5. Finally, the whole procedure should be tested using pseudo-experimental, computer generated data. This step should put in evidence a possible ill posedness of the inverse problem in terms of the existence of more than one parameter set that minimize to the same extent the objective function (more global minima). If this turns out to be the case the experimental measurements should be enriched, or even an additional experiment should be considered. The pseudo-experimental testing should also be performed using different levels of "noise" (both randomly distributed, to simulate the expected error of measurements, and systematic one, to account for possible modeling errors). The test with noisy data should show how stabile the solution is with respect to the small perturbations of the inputs.

From this chapter it should be clear that one of the crucial parts of any inverse analysis procedure that determines to a large extent the overall success of the whole process is the optimization algorithm. Therefore, subsequent chapter will give a more detailed survey of different optimization algorithms which can be used in the present context.

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