Maintaining Database Integrity with Refinement Types

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Abstract. Taking advantage of recent advances in automated theorem proving, we present a new method for determining whether database transactions preserve integrity constraints. We consider check constraints and referential-integrity constraints—extracted from SQL table declarations—and application-level invariants expressed as formulas of first-order logic. Our motivation is to use static analysis of database transactions at development time, to catch bugs early, or during deployment, to allow only integrity-preserving stored procedures to be accepted. We work in the setting of a functional multi-tier language, where functional code is compiled to SQL that queries and updates a relational database. We use refinement types to track constraints on data and the underlying database. Our analysis uses a refinement-type checker, which relies on recent highly efficient SMT algorithms to check proof obligations. Our method is based on a list-processing semantics for an SQL fragment within the functional language, and is illustrated by a series of examples.

1 Introduction

This paper makes a case for the idea that database integrity should be maintained by static verification of transactional code, rather than by relying on checks at run time. We describe an implementation of this idea for relational databases, where schemas are defined using SQL table descriptions, and updates are written in a functional query language compiled to SQL. Our method relies on a semantics of SQL tables (including constraints) using refinement types, and a semantics of SQL queries in terms of list processing. We describe a series of database schemas, the implementation of transactions in the .NET language F#, and the successful verification of these transactions using the refinement-type checker Stateful F7. Like several recent tools, Stateful F7 relies in part on pushing verification conditions to external SMT solvers, provers whose effectiveness has recently improved at a remarkable rate. Our aim is to initiate the application of modern verification tools for functional languages to the problem of statically-verified database transactions, and to provide some evidence that the idea is at last becoming practical.

1.1 Background: Database Integrity Constraints

SQL table descriptions may include various sorts of constraints, as well as structural information such as base types for columns.

A *check constraint* is an assertion concerning the data within each row of a table, expressed as a Boolean expression.

A *primary key constraint* requires that a particular subset, the *primary key*, of the columns in each row of the table identifies the row uniquely within the table. A key consisting of multiple column labels is called a *composite key*. A *uniqueness* constraint is similar to a primary key constraint but based on a single column.

A *foreign key constraint* requires that a particular subset, a *foreign key*, of the columns in each row of the table refers uniquely to a row in the same or another table. Satisfaction of primary key and foreign key constraints is known as *referential integrity*.

To illustrate these constraints by example consider a table recording marriages between persons, represented by integer IDs. A key idea is that the marriage of A and B is encoded by including both the tuples (A,B) and (B,A) in the table.

An Example Table with Integrity Constraints: Marriage

```
create table [Marriage](
    [Spouse1] [int] not null unique,
    [Spouse2] [int] not null,
  constraint [PK_Marriage] primary key ([Spouse1],[Spouse2]),
  constraint [FK_Marriage] foreign key ([Spouse2], [Spouse1])
      references [Marriage] ([Spouse1], [Spouse2]),
  constraint [CK_Marriage] check (not([Spouse1] = [Spouse2])))
```

The two columns Spouse1 and Spouse2 in the Marriage table store non-null integers. Database integrity in this example amounts to three constraints: marriage is monogamous (you cannot be in two marriages), symmetric (if you marry someone they are married to you), and irreflexive (you cannot marry yourself).

- The uniqueness constraint on the column Spouse1 asserts that nobody is Spouse1 in two different marriages, hence enforcing monogamy.
- The primary key constraint PK_Marriage in conjunction with the self-referential foreign key constraint FK_Marriage asserts that whenever row (A,B) exists in the table, so does the row (B,A), hence enforcing symmetry.
- The check constraint CK_Marriage asks that nobody is married to themselves, hence enforcing irreflexivity.

A buggy transaction on this table may violate its constraints. The sorts of bugs we aim to detect include the following: (1) insertion of null in Spouse1 or Spouse1 (violating the **not null** type annotation); (2) inserting (A,C) when (A,B) already exists (violating the uniqueness constraint); (3) inserting (A,B) but omitting to insert (B,A) (violating the foreign key constraint); and (4) inserting (A,A) (violating the check constraint). We aim to eliminate such integrity violations by static analysis.

1.2 Background: Multi-tier Functional Programming

We consider the common situation where database updates are not written directly in SQL, but instead are generated from a separate programming language via some object-relational mapping. In particular, we consider database transactions expressed in the functional language F# [28], but compiled to SQL for efficient execution in the relational backend. This is an instance of *multi-tier functional programming*, where a single functional program is split across tiers including the web server and the database.

Our mapping is based on three ideas:

(1) We model SQL table definitions as F# types: the whole database is a record type db consisting of named tables, where each table is a list of records, corresponding to the rows of the table.
(2) We provide the user with *standard* functions for create, read, update, and delete operations on each table. We also allow user-supplied *custom* SQL stored procedures, and provide F# functions to call these procedures. Both standard and custom functions are implemented as SQL queries, and can be thought of as imperative actions on a global state of type db.
(3) Users write a transaction as an F# function that interacts with the database by calling a sequence of standard SQL functions and custom stored procedures.

To illustrate point (1), we model our example table definition with the following F# types, where the whole database db is a record with a single field holding the marriages table, which itself is a list of rows.

```
type marriage_row = { m_Spouse1:int; m_Spouse2:int; }
type db = { marriages: marriage_row list; }
```

A row (A,B) is represented by the record:

```
{ m_Spouse1=A; m_Spouse2=B; }
```

The marriage of A and B is represented by the list:

```
[{ m_Spouse1=A; m_Spouse2=B }; { m_Spouse1=B; m_Spouse2=A }]
```

Regarding point (2), we have (among others) the following standard queries as F# functions:

– hasKeyMarriage (A,B) computes whether a row with primary key (A,B) exists in the marriages table.
– deleteMarriage (A,B) deletes the row with primary key (A,B) from the marriages table, if it exists.

We have no user-supplied custom SQL queries for the marriages example, but show such queries in some of our later examples.

Actual transactions (point (3) above) are written as functional code. The following example of a user-written transaction is to dissolve a marriage. Given two spouses A and B, we have to check whether the rows (A,B) and (B,A) exist in the database and remove them both.

An Example Transaction: Divorce

```
let divorce_ref (A,B) =
  if hasKeyMarriage(A, B) then
    deleteMarriage(A, B);
    deleteMarriage(B, A);
    Some(true)
  else Some(false)
```

The body of the function is an expression returning a value of type bool option.

- Some **true** means there was a marriage successfully removed, and we commit;
- Some **false** means there was no marriage to remove, and we commit;
- None would mean that the transaction failed and any updates are to be rolled back (a return value not illustrated by this code).

The code above takes care to check that a marriage between A and B already exists before attempting to delete it, and also to remove both (A,B) and (B,A). Instead, careless code might remove (A,B) but not (B,A). Assuming that the foreign key constraint on the marriage table is checked dynamically, such code would lead to an unexpected failure of the transaction. If dynamic checks are not enabled (for instance since the underlying database engine does not support deferred consistency checking) running invalid code would lead to data corruption, perhaps for a considerable duration. Our aim is to detect such failures statically, by verifying the user written code with a refinement-type checker.

1.3 Databases and Refinement Types

The values of a *refinement type* $x{:}T\{C\}$ are the values x of type T such that the formula C holds. (Since the formula C may contain values, refinement types are a particular form of dependent type.) A range of refinement-type checkers has recently been developed for functional languages, including DML [31], SAGE [14], F7 [4], DSolve [24], Fine [27], and Dminor [6], most of which depend on SMT solvers [23].

A central idea in this paper is that refinement types can represent database integrity constraints, and SQL table constraints, in particular. For example, the following types represent our marriage table.

SQL Table Definitions as Refinement Types:

```
type marriage_row = { m_Spouse1:int; m_Spouse2:int }
type marriage_row_ref = m:marriage_row {CK_Marriage(m)}
type marriage_table_ref = marriages:marriage_row_ref list
  { PKtable_Marriage(marriages) ∧ Unique_Marriage_Spouse1(marriages) }
type State = { marriage:marriage_table_ref }
type State_ref = s:State {FK_Constraints(s)}
```

The refinement types use predicate symbols explained informally below. We give formal details later on.

- CK_Marriage(m) means the record m satisfies the check constraint [CK_Marriage].
- PKtable_Marriage(marriages) means the list of records marriages satisfies the primary key constraint with label [PK_Marriage].
- Unique_Marriage_Spouse1(marriages) means marriages satisfies the uniqueness constraint on column [Spouse1].
- FK_Constraints(s) means the database s satisfies the foreign key constraint with label [FK_Marriage].

1.4 Transactions and the Refined State Monad

The state monad is a programming idiom for embedding imperative actions within functional programs [29]. Pure functions of type $\mathsf{State} \to T * \mathsf{State}$ represent computations that interact with a global state; they map an input state to a result paired with an output state. The *refined state monad* [13,20,8] $[(s_0)C_0]\,x{:}T\,[(s_1)C_1]$ is the enrichment of the state monad with refinement types as follows:

$$[(s_0)C_0]\,x{:}T\,[(s_1)C_1] \triangleq s_0{:}\mathsf{State}\{C_0\} \to x{:}T * s_1{:}\mathsf{State}\{C_1\}$$

The formula C_0 is a precondition on input state s_0, while the formula C_1 is a postcondition on the result x and output state s_1.

A new idea in the paper is to represent SQL queries and transactions as computations in a refined state monad, with the refinement type State being a record with a field for each table in the database, as above. For example, the function divorce_ref has the following type, where the result of the function is a computation in the refined state monad.

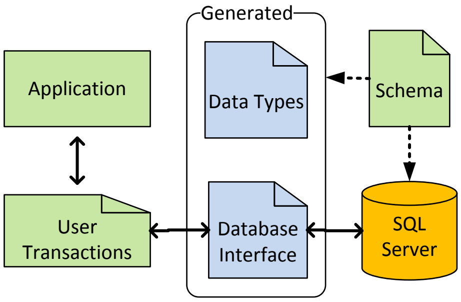
```
val divorce_ref: (int×int) →
  {(s) FK_Constraints(s)} r:bool option {(t) r ≠ None ⇒ FK_Constraints(t)}
```

The return type states that if the function is called in a state s satisfying the foreign key constraints, and it terminates, then it returns a value r of type bool option. Moreover, if r≠None, then the state t after the computation terminates satisfies the foreign key constraints. The type reflects that the code performs sufficient dynamic checks that it never causes a dynamic failure, and that it returns None whenever it leaves the database in an inconsistent state. The case of the function returning None is caught by a transaction wrapper (not shown here) which then aborts the transaction, rolling back the database to its initial state.

By checking with refinement types, we aim to catch buggy code that terminates with Some b, intending to commit and return b, but that does not in fact re-establish the database invariants. In particular, we would catch code that removes say (A,B) but not (B,A), as it does not re-establish the foreign key constraint FK_Constraints(t).

1.5 An Architecture for Verified Database Transactions

We verify a series of example user transactions, according to the diagram below. Each example starts from a database schema in SQL. From the schema our tool generates refinement types to model the database, and also a functional programming interface for a set of pre-packaged stored procedures in SQL (for actions such as querying and deleting items by key, exemplified by the functions lookupMarriage etc mentioned above). Against this interface, the user writes transactional code (exemplified by the function divorce_ref mentioned above) in F#, which is invoked from their application. We verify the code of the user transactions using the typechecker Stateful F7 [8], which implements the refined state monad on top of the typechecker F7 [4]. Additionally, not shown in the diagram, in some examples the schema may also include queries written directly as custom SQL stored procedures; we can also verify these queries by mapping SQL into F#.



Our examples are as follows.

(1) Marriages (see above and Section 3).
We have a single table Marriage(Spouse1, Spouse2) with integrity expressed using SQL table constraints. We describe verifiable transactions to create and delete marriages.
(2) Order processing (see Section 4).
We have tables Orders(OrderID, CustomerID, Name, Address) and Details(OrderID, ProductID, UnitPrice, Quantity) with integrity expressed using primary key, foreign key, and check constraints. We show that an addOrder function, which creates an order with a single detail row, respects all these constraints.
(3) Heap data structure (see Section 5).
We have a table Heap(HeapID, Parent, Content) where each row represents a node in a heap data structure. SQL constraints do not suffice to specify the integrity invariant of this data structure, so we add user-defined constraints written in first-order logic. We verify that integrity is preserved by recursive functions to push and pop elements, which make use of user-defined stored procedures getRoot and getMinChild.

1.6 Contributions of the Paper

Our main contribution is to interpret SQL table descriptions as refinement types, and database updates as functional programs in the refined state monad, so as to verify, by refinement-type checking, that updates preserve database integrity. Hence, verification of the F# and SQL source code proceeds by sending a series of verification conditions in first-order logic to an automatic theorem prover.

Our source code is in the .NET language F#, but our method could be recast for other functional multi-tier languages, such as Links [11], HOP [25], or FLAPJAX [17], and also for object-oriented programming models such as LINQ [19]. We use the type-checker Stateful F7, but we expect our approach to queries and transactions would easily adapt to related verifiers for functional code with state such as Why [13] or YNOT [20], and indeed to verifiers for imperative code, such as those using Boogie [2].

The idea of static verification of database transactions goes back to the 1970s, to work on computing the weakest precondition needed for a transaction to succeed [16,9,26,5]. Theorem proving technology has improved considerably since the idea of static verification of database transactions was first mooted, and an implication of our work is that the idea is at last becoming practical. Moreover, the success of languages with functional features such as F#, Scala [21], and indeed recent versions of C# with closures, is compelling evidence for the significance of functional programming as an object-oriented technology. Hence, our work lays a foundation for statically verifiable database access from mainstream object-oriented platforms.

A technical report includes additional details [1].

2 A Tool to Model SQL with Refinement Types

This section fleshes out the architecture diagram of our system.

Section 2.1 describes the details of the SQL schemas input by our system, including both the data definition part defining the structure of tables, and also the data manipulation part of queries invoked from stored procedures.

Section 2.2 details how our tool generates data types and a database interface from a schema. The database interface consists of a set of F# functions with types, including preconditions and postconditions, specified in the syntax of Stateful F7. When generating the database interface, our tool automatically includes functions to access a set of standard queries, as well as functions to access any custom stored procedures included in the schema.

Section 2.3 gives a symbolic reference implementation for the generated database interface. The symbolic implementation relies on list processing in a similar fashion to Peyton Jones and Wadler [22] and serves as a formal semantics for the interface. We trust, but do not formally verify, that the behaviour of the symbolic implementation corresponds to its interface, as well as to our concrete implementation which works by sending queries to an actual SQL database.

Finally, Section 2.4 extends our schema syntax with the ability to write integrity constraints directly as first-order predicates.

2.1 SQL Schemas: Tables and Stored Procedures

Let c range over constants, x over variables and f,g over table column names. Then, boolean expressions and value expressions occurring within SQL queries are defined by the syntax below. Value expressions include (boolean, integer, and string) constants, binary operations, variables and table field names. Boolean expressions include equations between value expressions, comparisons, conjunction, disjunction, and negation.

Values and Expressions

B	$::= E = E \mid E \odot E \mid B \vee B \mid B \wedge B \mid \neg B$	Boolean expression
$\odot$	$::= < \mid <= \mid > \mid >=$	Comparison operator
E	$::= E \oplus E \mid c \mid x \mid f$	Value expression
$\oplus$	$::= + \mid - \mid * \mid /$	Binary operator

A table declaration defines a table t, and gives a name f_i and type T_i to each of its columns. To indicate uniqueness constraints, each column has a tag u_i, either **unique** or empty. SQL supports several data types; in this work, we only consider Boolean, Int and

String data types which are defined as their counterparts in F#. We may interpret more complex types using appropriate encodings and refinements of the three basic types. We assume that each table has exactly one primary key, which may be composite, exactly one check constraint, and no multiple foreign key references to the same table. (SQL syntax allows multiple check constraints, but these may be conjoined to produce a single check constraint).

Data Definition

DT ::=	Table declaration
table t $(u_i\ f_i : T_i)^{i \in 1..n}$,	name and fields
primary key $\overline{g}$,	primary key
check B,	check constraint
$\kappa_1, \ldots, \kappa_m$	foreign keys
κ ::= **foreign key** $\overline{f}$ **references** $t(\overline{g})$	
T ::= Boolean | Int | String	Type

The syntax of supported SQL queries includes those necessary for selecting, inserting and deleting rows from a table. Let t denote an SQL table name and let $\overline{f}$ be a shorthand for $f_1, \ldots, f_n$ (all the columns of the table) and $\overline{g}$ be a shorthand for $g_1, \ldots, g_m$ (denoting some of the f_i).

Data Manipulation

Q ::= QS | QI | QD | QU	Query
QS ::=	Select query
select [**top** 1] $\overline{g}$	selector
from t	source
where B	criterion
[**order by** f {**asc** | **desc**}]	ordering
QI ::=	Insert query
insert into t	target table
$(f_1, \ldots, f_n)$	table fields
values $(E_1, \ldots, E_n)$	field values
QD ::=	Delete query
delete from t	target table
where B	criterion
QU ::=	Update query
update t **set**	target table
$(g_1, \ldots, g_m)$ =	table fields
$(E_1, \ldots, E_m)$	field values
where B	criterion

A QS query filters all the rows of a table t, based on a boolean criterion B, and projects the selected fields $\overline{g}$. The result of a select query is a table of rows matching the criterion. We consider only **select** queries that contain **top** 1 if and only if they contain an **order by** clause. In this case, the resulting table is ordered in either ascending or descending order based on the single field f, and the first element of the table is returned as the result. A *QI* query adds a row consisting of the values $E_1,\dots,E_n$ in table t. In **insert into** t $(f_1,\dots,f_n)$ **values** $(E_1,\dots,E_n)$, we expect each E_i to be either a variable or a constant. The result of a *QI* query is a number indicating the count of successful insertions. A *QD* query removes from table t all the rows matching the boolean criterion B. Again, the result is a number indicating the count of successful deletions. Finally, a *QU* query modifies fields $\overline{g}$ to contain values $\overline{E}$ for all rows of table t that match the boolean condition B.

The SQL schema syntax includes constructs for databases, tables, procedures, and constraints. A schema is a named tuple of declarations. A declaration can be either a procedure or a table. A procedure abstracts a query Q by giving a name h and making it depend on parameters $a_1,\dots,a_n$. We assume that in a procedure declaration, the query Q only contains variables from $\overline{a}$.

SQL Schema

S	$::=$	**schema** $s(DT_i^{i\in 1..n}, DP_j^{j\in 1..m})$	Schema
DP	$::=$	**procedure** h $(a_i : T_i)^{i\in 1..n} Q$	Procedure declaration

In subsequent sections, we adopt a convenient syntax for advanced queries and assume standard encodings of these syntactic forms in terms of the core query syntax. For example, multi-row insertion is defined in terms of multiple single-row insertions and the star (*) syntax in *QS* queries corresponds to explicitly naming all the columns in the table, in order of their appearance in the table declaration.

2.2 Generating Types and Database Interfaces from Schemas

Our tool maps an SQL schema S to a Stateful F7 module $[\![S]\!]$ by assembling a series of type definitions, predicate definitions, and function signatures. This section describes each of the components in turn.

Translation from S in SQL to $[\![S]\!]$ in Stateful F7:

Let $[\![S]\!]$ be the Stateful F7 module obtained from schema S by concatenating the type and function definitions displayed below: (S1) types from schema declarations; (S2) refinement formulas from constraints; (S3) signatures of standard functions; (S4) signatures of custom functions.

First, in (S1) we fix a type for table declarations and the global type `State` of the refined state monad used by Stateful F7. Second, in (S2) we define logical axioms which correspond to the database constraints. Third, we give types to queries and procedures that manipulate the global state. As discussed earlier, expressions get *computation types* of the form $[(s_0)C_0]\,x{:}T\,[(s_1)C_1]$. Finally, in (S3) and (S4) we give signatures of standard API functions and custom procedures for manipulating the global state. We use **val** `f : T` to give a type to a function in the API.

Below, we assume a fixed schema s defined by S. For every table t in s, assume the definition **table** t $(u_i\ f_i : T_i)^{i\in 1..n}$, **primary key** $\overline{g}$, **check** $B, \kappa_1,\dots,\kappa_l$. Given table t, the translation algorithm works as follows. We generate the type t_key as a tuple of the corresponding types of the primary key fields and let Tf, the type of field f, be given by T_i when $f = f_i$. For each row in the table we create an unrefined record type t_row with the labels corresponding to the column names from the table definition. To associate a **check** constraint with each table row, we refine the row type with the formula CK_t(row) (defined below) and create the refinement type t_row_ref. Values of this type represent rows in the table *t* for which the check constraint holds. The table *t* itself is modelled as a list of refined rows (t_table). Finally, we refine the table type by associating the primary key constraint formula PKtable_t(tab) (defined below in (S2)) with it. Values of this type represent tables for which the primary key constraint holds. Basic types translate directly to their equivalents in Stateful F7. We deal with **not null** and **null** constraints by declaring nullable types as option types.

We can now proceed to the definition of the type corresponding to the database (for a single schema). Without loss of generality, assume that the tables $t_1,\ldots,t_n$ belong to the fixed schema *s*. The database type is a record of refined tables. In the refined state type, the refinement asserts that values of this type will satisfy the foreign key constraints on the database. The normal state type does not have this refinement, denoting that top-level constraints may temporarily be invalidated. A valid transaction may assume that the foreign key constraints hold, and must enforce them on exit, but may internally temporarily violate the constraints.

(S1) Types from Table Declarations

```
type t_key = Tg1× ... ×Tgm
type t_row = {f1:T1; ...; f n:Tn}
type t_row_ref = row:t_row {CK_t(row)}
type t_table_ref = tab:t_row_ref list {PKtable_t(tab) ∧ $\bigwedge_{u_i=unique}$ Unique_t_fi(tab)}
type State = { (t_i : ti_table_ref) $^{i\in 1..n}$ }
```

We now define logical predicates corresponding to SQL table constraints. We assume a translation $\mathrm{L}[\![\cdot]\!]_r$ from SQL boolean and value expressions to logical formulas and terms; the translation is homomorphic except for the base case $\mathrm{L}[\![f]\!]_r \triangleq r.f$.

- CK_t(row) means the check constraint of table t holds of the tuple row.
- PK_t(r,k) means the primary key of row r of table t is k.
- PKtable_t(tab) means the contents tab of table t satisfies its primary key constraint.
- Unique_t_f(tab) means the contents tab of table t satisfies the uniqueness constraint for field f.
- FK_t_u(tab1,tab2) means the contents tab1 of table t satisfies the foreign key constraint with reference to the contents tab2 of table u.
- FK_Constraints(db) means all foreign key constraints in the database db are satisfied.

In the table below, the Stateful F7 keyword **assume** introduces a universally quantified axiom to define each new predicate symbol.

(S2) Refinement Formulas from Constraints

assume ∀row. CK_t(row) $\Leftrightarrow$ $\mathrm{L}[\![B]\!]_{row}$
assume ∀row,$\overline{x}$. PK_t(row,($\overline{x}$)) $\Leftrightarrow \bigwedge_i x_i$ = row.g_i
assume ∀tab. PKtable_t(tab) $\Leftrightarrow$∀row1, row2.
  (Mem(row1,tab) ∧ Mem(row2,tab) ∧ PK_t(row1,(row2.$g_1,\ldots,$ row2.g_m))) ⇒ row1 = row2
assume ∀tab. Unique_t_f(tab) $\Leftrightarrow$∀row1, row2.
  (Mem(row1,tab) ∧ Mem(row2,tab) ∧ row1.f = row2.f) ⇒ row1 = row2
assume ∀tab1,tab2. FK_t_u(tab1,tab2) $\Leftrightarrow$∀x. Mem(x,tab1) ⇒ (∃ y. Mem(y,tab2) ∧
    $\bigwedge_i x.f_i = y.g_i$)
      if $\exists \kappa_i =$ **foreign key** $f_1 \ldots f_m$ **references** $u(g_1 \ldots g_m)$
assume ∀s. FK_Constraints(s) $\Leftrightarrow \bigwedge_{t,u}$ FK_t_u(s.t, s.u)

(A limitation of our semantics for uniqueness constraints and primary keys is that they allow duplicate copies of the same row; the limitation could be lifted by more elaborate semantics but we leave this for future work).

Now that we have translated SQL data declarations, we may proceed to the query and data manipulation languages. The variable s in the translation represents the entire database record, and therefore the expression s.t projects the table t over which the query is performed. A simple select query does not modify the state, and returns a list whose elements are exactly the rows in the table matching the select condition. We write $\mathsf{T}_{\overline{g}}$ for the tuple T1 ×... ×Tn where Ti is the type of field g_i and $n = |\overline{g}|$. A select **top** 1 query also does not modify the state, and returns a list which is either empty or contains one element from the table that matches the select criterion and is less than (or greater than, not shown) any other such element. An insert query may only be called if inserting the row does not invalidate any table constraints, and the new table after running the query is the old table with the inserted row prepended to it. A delete query modifies the corresponding table to contain only those rows not matching the query condition. An update query also modifies the table to contain exactly those rows that do not match the where clause, or the updated version of the rows that do.

Types of SQL Queries

$\mathcal{T}[\![$**select** $\overline{g}$ **from** t **where** B $]\!] \triangleq$
  [(s)] l:$\mathsf{T}_{\overline{g}}$ list [(s') s=s' ∧ (∀ x. Mem(x,l) ⇔ ∃r. $\mathrm{L}[\![B]\!]_r \wedge \overline{r.g} = \overline{x.g}$ ∧ Mem(r, s.t))]
$\mathcal{T}[\![$**select top** 1 $\overline{g}$ **from** t **where** B **order by** f **asc** $]\!] \triangleq$
  [(s)] l:$\mathsf{T}_{\overline{g}}$ list [(s') s=s' ∧ ((l = [] ∧ (∀r. Mem(r,s.t) ⇒ ¬$\mathrm{L}[\![B]\!]_r$)) ∨
    (∃x. $\mathrm{L}[\![B]\!]_x$ ∧ Mem(x,s.t) ∧ l = [{$\overline{g} = \overline{x.g}$}] ∧ (∀r. $\mathrm{L}[\![B]\!]_r$ ∧ Mem(r,s.t) ⇒ r.f >= x.f)))]
$\mathcal{T}[\![$**insert into** t $(f_1,\ldots,f_n)$ **values** $(E_1,\ldots,E_n)$ $]\!] \triangleq$
  [(s) PKtable_t({$\overline{f} = \mathrm{L}[\![\overline{E}]\!]_r$}::(s.t))] unit [(s') s'= {s **with** t = {$\overline{f} = \overline{E}$}::(s.t) }]
$\mathcal{T}[\![$**delete from** t **where** B $]\!] \triangleq$
  [(s)] int [(s') ∃t'. s'= {s **with** t = t'} ∧ (∀r. Mem(r,t') ⇔ $\mathrm{L}[\![\neg B]\!]_r$ ∧ Mem(r,s.t))]
$\mathcal{T}[\![$**update** t **set** $\overline{g} = \overline{E}$ **where** B $]\!] \triangleq$
  [(s)] int [(s') ∃t'. s'= {s' **with** t = t'} ∧ (∀x. Mem(x,t') ⇔ (Mem(x,s.t) ∧ ¬$\mathrm{L}[\![B]\!]_r$) ∨
   (∃r.Mem(r,s.t) ∧ $\mathrm{L}[\![B]\!]_r$ ∧ x= {r **with** $\overline{g} = \mathrm{L}[\![\overline{E}]\!]_r$}))]

The standard API defines functions that look up the existence of keys inside a table, generate fresh keys for a table, checks that an unrefined row satisfies the constraints, inserts a refined row in a table, deletes a row from a table, and updates a row in a table. The function fresh_t is only defined when the primary key is non-composite, that is, the constraint contains a single field f, and indeed is an integer. The lookup_t function takes a numeric key and returns true if it exists inside the table; the fresh_t function generates a new key that does not exist inside the table; the check_t function takes an unrefined row and makes sure that all the check constraints are satisfied for it; the insert_t function takes a refined row to be inserted in a table and starting from a state that satisfies all the primary key and foreign key constraints performs the insertion; the delete_t function takes a key for a table and removes the associated row from the table.

(S3) Signatures of Standard Functions

```
PKfresh_t(tab, (x̄ )) ≜ ∀r. Mem(r, tab) ⇒ ⋁ᵢ(xᵢ ≠ r.gᵢ)
PKexists_t(tab, (x̄ )) ≜ ∃r. Mem(r, tab) ∧ ⋀ᵢ(xᵢ = r.gᵢ)

val hasKey_t: k:t_key →
    [(s) True] b:bool [(s') s=s' ∧(b=false ⇒PKfresh_t(s.t,k)) ∧(b=true ⇒PKexists_t(s.t,k))]
val lookup_t: k:t_key →
     [(s) True] o:t_row_ref option [(s') s=s' ∧(o=None ⇒PKfresh_t(s.t,k)) ∧
          (∀r. o=Some(r) ⇒Mem(r,s.t) ∧PK_t(r,k))]
val fresh_t: unit →[(s) True] k:t_key [(s') s=s' ∧PKfresh_t(s.t,k)]
val check_t: r:t_row →[(s) True] b:bool [(s') s=s' ∧(b=true ⇒CK_t(r))]
val insert_t: r:t_row_ref →[(s) True] b:bool [(s') (b=false ⇒s=s') ∧
                                          (b=true ⇒s'= {s with t = r::(s.t) })]
val update_t: r:t_row_ref →[(s) True] b:bool [(s') (b=false ⇒s=s') ∧
   (b=true ⇒∃t'. s'= {s with t = t'} ∧
          (∀x. Mem(x,t') ⇔(Mem(x,s.t) ∧¬x.ḡ = r.ḡ) ∨(x=r ∧∃y.Mem(y,s.t) ∧y.ḡ = r.ḡ))]
val delete_t: k:t_key →[(s) True] unit [(s') (b=false ⇒s=s') ∧
   (b=true ⇒∃t'. s'= {s with t = t'} ∧(∀x. Mem(x,t') ⇔(Mem(x,s.t) ∧¬x.ḡ = r.ḡ))]
```

To complete the four parts of the definition of $[\![S]\!]$, each custom stored procedure explicitly listed in the schema S is translated to a function signature as follows.

(S4) Signatures of Custom Functions

$$[\![\textbf{procedure}\ \mathsf{h}\ (a_i : T_i)^{i\in 1..n}\ Q\ ]\!] \triangleq$$
$$\textbf{val}\ \mathsf{h} : a_1 : \mathcal{T}[\![T_1\ ]\!] \to \ldots \to a_n : \mathcal{T}[\![T_n\ ]\!] \to \mathcal{T}[\![Q\ ]\!]$$

2.3 Reference Implementation of Database Interface

The dynamic semantics for the subset of SQL that we consider follows Peyton Jones and Wadler [22]. In the following, we assume standard map and filter functions on lists, and also functions max and min that select the maximum and minimum of a list of orderable values. As a convention, we use $\overline{f}$ for the full tuple of columns, and $\overline{g}$ for a subset of the columns. The translation $\mathrm{F}[\![\cdot]\!]_r$, from SQL boolean and value expressions to F# expressions is homomorphic except for the base case $\mathrm{F}[\![f]\!]_r \triangleq r.f$.

Semantics of SQL Queries

$[\![\textbf{select}\ \overline{g}\ \textbf{from}\ t\ \textbf{where}\ B]\!] \triangleq$
$\quad \textbf{let}\ s = \textbf{get}()\ \textbf{in}\ \mathsf{map}\ (\textbf{fun}\ \overline{f} \rightarrow \overline{g})(\mathsf{filter}\ (\textbf{fun}\ r \rightarrow \mathrm{F}[\![B]\!]_r)\ (s.t))$
$[\![\textbf{select top}\ 1\ \overline{g}\ \textbf{from}\ t\ \textbf{where}\ B\ \textbf{order by}\ f\ \textbf{asc}]\!] \triangleq$
$\quad \textbf{match}\ [\![\textbf{select}\ f\ \textbf{from}\ t\ \textbf{where}\ B]\!]\ \textbf{with}\ [] \rightarrow []\ |\ xs \rightarrow$
$\quad \textbf{let}\ m = \mathsf{max}(xs)\ \textbf{in}\ [\mathsf{hd}([\![\textbf{select}\ \overline{g}\ \textbf{from}\ t\ \textbf{where}\ (f = m) \wedge B]\!])]$
$[\![\textbf{insert into}\ t\ (f_1,\ldots,f_n)\ \textbf{values}\ (E_1,\ldots,E_n)]\!] \triangleq$
$\quad \textbf{let}\ s = \textbf{get}()\ \textbf{in set}\ \{s\ \textbf{with}\ t = \{f_1 = E_1,\ldots,f_n = E_n\} :: (s.t)\}$
$[\![\textbf{delete from}\ t\ \textbf{where}\ B]\!] \triangleq$
$\quad \textbf{let}\ s = \textbf{get}()\ \textbf{in set}\ \{s\ \textbf{with}\ t = [\![\textbf{select}\ *\ \textbf{from}\ t\ \textbf{where}\ \neg B]\!]\}$
$[\![\textbf{update}\ t\ \textbf{set}\ \overline{g} = \overline{E}\ \textbf{where}\ B]\!] \triangleq$
$\quad \textbf{let}\ s = \textbf{get}()\ \textbf{in}$
$\quad \textbf{let}\ t1 = [\![\textbf{select}\ *\ \textbf{from}\ t\ \textbf{where}\ \neg B]\!]\ \textbf{in}$
$\quad \textbf{let}\ tB = [\![\textbf{select}\ *\ \textbf{from}\ t\ \textbf{where}\ B]\!]\ \textbf{in}$
$\quad \textbf{let}\ t2 = \mathsf{map}\ (\textbf{fun}\ r \rightarrow \{r\ \textbf{with}\ \overline{\mathrm{F}[\![g]\!]_r = \mathrm{F}[\![E]\!]_r}\})\ tB\ \textbf{in}$
$\quad \textbf{set}\ \{s\ \textbf{with}\ t = t1 @ t2\}$

To translate a simple *QS* query, we first obtain the current database and project the table t we are interested in. We then filter every row r using the translation of the boolean condition B. Finally, we map a projection function, which selects the required subset of columns $\overline{g}$, onto the filtered result. The translation of a *QS* query with **top** and **order by** first narrows the result set using the boolean criterion B. If no rows match the criterion, we simply return the empty list. If multiple rows match the criterion, we find the maximum value of the field f within any row and store that to a temporary variable m. We, finally, use a simple *QS* query to find all the rows that satisfy B for which the field f has the value m and return the head of the list. The translation of a *QI* query involves getting the current database value, and immediately writing it back with the new row being prepended to the existing table. The translation of a *QD* query follows a similar pattern; we get the current database and immediately write back a table consisting of all the rows that do not match the boolean condition. The translation of a *QU* query first saves the initial state, as well as the rows of table t that do not match the criterion B. We then extract the rows that match the criterion B, and map the update over them. Finally, the modified state is written back.

Here is the semantics for the API in F#, with appeal to our semantics of SQL in F#. Below we write $pk(\mathsf{t})$ for the non-empty tuple of field names making up the primary key of table t, and we write $ck(\mathsf{t})$ for the check constraint of table t.

Semantics of API Functions

```
let hasKey_t k = [[select * from t where pk(t)=k]] ≠ []
let lookup_t k =
   match [[select * from t where pk(t)=k]] with
   | [r] → Some r
   | _ → None
let fresh_t () =
   genKey_t [[select top 1 pk(t) from t where true order by pk(t) asc]]
let check_t r = F[[ck(t)]]_r
let insert_t r = let nrows = [[insert into t (f1,...,fn) values (r.f1,...,r.fn)]] in 1 == nrows
let delete_t k = let nrows = [[delete from t where pk(t)=k]] in ()
let update_t k r = (delete_t k; insert_t r)
```

The user code is written in F# and can be executed symbolically against the reference implementation of the database access API above. The same user code is typechecked against the F7 interface and linked against the concrete implementation of the API functions that use a relational database.

2.4 Extension with Application Constraints

We extend the SQL table syntax from Section 2.1 in order to allow user-specified invariants, written in first-order logic. We also replace the definition of refined tables, and the definition of the global database constraint FK_Constraints as follows.

User Constraints

κ	$::= \cdots \mid p$	p is a unary predicate symbol
D	$::= \cdots \mid p$	p is a unary predicate symbol

type t_table_ref = tab:t_table$\{$PKtable_t(tab) $\wedge \bigwedge_{i=1\ldots k} p_i^{\mathsf{t}}(tab)\}$

assume $\forall$db. FK_Constraints(db)$\Leftrightarrow \bigwedge_{t,u}$ FK_t_u(s.t, s.u)$\wedge \bigwedge_i p_i^D$

User constraints $p(x)$, where x will be instantiated either by a table or the entire database, are defined by a user-specified first order logic formula C_p that can contain boolean expressions, quantifiers, and other axiomatized predicates. When defining these formulas, care must be taken to avoid introducing inconsistencies—any program satisfies an inconsistent specification. To help with this, F7 can work in a debug mode which attempts to prove **false** every time an axiom or a refined value is introduced.

3 Completing the Marriages Example

Our goal is to type-check application code that accesses the database and to ensure that it respects the database constraints. To achieve this we need a model of the database, the tables and the constraints inside the host language of the application. Based on the rules from section 2.2 we translate the Marriages table declaration from section 1.1 and generate the appropriate F7 data types with refinements. We now give the complete translation of the marriage example.

3.1 Database Schema

We here repeat the definition of the refined data types corresponding to the marriage table and its rows.

Marriage Data Types

```
type marriage_row = { m_Spouse1:int; m_Spouse2:int }
type marriage_row_ref = m:marriage_row {CK_Marriage(m)}
type marriage_table_ref = marriages:marriage_row_ref list
  { PKtable_Marriage(marriages) ∧ Unique_Marriage_Spouse1(marriages) }
type State = { marriage:marriage_table_ref }
type State_ref = s:State {FK_Constraints(s)}
```

The check constraint, primary key constraint, uniqueness constraint and foreign key constraint are defined below as first-order logical formulas, using the keyword **assume**. We define two auxiliary predicates: PK_Marriage(m, k) states that the primary key of row m is k, and FK_Constraints(s) states that all foreign key constraints (of which there is only one) are satisfied for the database s. The predicate Mem(r,t) checks if row r is present in table t.

SQL Constraints as Formulas

```
assume ∀x,y. CK_Marriage((x, y)) ⇔ x ≠ y
assume ∀m,k. PK_Marriage(m, k) ⇔ k = (m.m_Spouse1, m.m_Spouse2)
assume ∀xs. PKtable_Marriage(xs) ⇔
    ∀x,m. Mem(x, xs) ∧ Mem(m, xs) ∧ PK_Marriage(x, (m.m_Spouse1, m.m_Spouse2))
          ⇒ x = m
assume ∀l. Unique_Marriage_Spouse1(l) ⇔
    ∀x,y. Mem(x, l) ∧ Mem(y, l) ∧ x.m_Spouse1 = y.m_Spouse1 ⇒ x = y
assume ∀d. FK_Constraints(d) ⇔ FK_Marriages_Marriages(d.marriages, d.marriages)
assume ∀marriages’, marriages. FK_Marriages_Marriages(marriages, marriages’) ⇔
    ∀x. Mem(x, marriages’) ⇒
        ∃u. Mem(u, marriages) ∧ PK_Marriage(u, (x.m_Spouse2, x.m_Spouse1))
```

3.2 Access Function API

From the database schema, our tool also generates data manipulation functions which carry preconditions and postconditions corresponding to the database constraints on their arguments. We generate two implementations of these functions: one that works on the abstract model, and one that works on the actual SQL server database via ADO.Net.

The following code fragment contains the type signatures of the automatically generated functions for the marriage example.

Specification of API Functions

```
val checkMarriage: r:marriage_row → [(s)] b:bool [(s')(s = s' ∧ b = true ⇒ CK_Marriage(r))]
val hasKeyMarriage :
  k:(int × int) → [(s)] b:bool [(s')( s = s' ∧
                                      b = false ⇒ PK_Marriages_Fresh(s.marriages, k) ∧
                                      b = true ⇒ PK_Marriages_Exists(s.marriages, k))]
val deleteMarriage :
  k:(int × int) → [(s) PK_Marriages_Exists(s.marriages, k)] unit [(s')
    ContainsiffNotPKMarriage(s, s', k)]
val insertMarriage :
  r:marriage_row_ref → [(s)] b:bool [(s')(
    b = true ⇒ s'.marriages = r :: s.marriages ∧
    b = false ⇒ s = s')]
assume (∀s,t,k. (ContainsiffNotPKMarriage(s, t, k) ⇔
      (∀x. (Mem(x, t.marriages) ⇔ (Mem(x, s.marriages) ∧ not PK_Marriage(x, k))))))
assume (∀marriages,spouse1,spouse2. (PK_Marriages_Fresh(marriages, (spouse1, spouse2))
      ⇔ (∀x. (Mem(x, marriages) ⇒ (spouse1 ≠ x.m_Spouse1 ∨ spouse2 ≠ x.m_Spouse2)))))
assume (∀marriages,spouse1,spouse2. (PK_Marriages_Exists(marriages, (spouse1, spouse2))
      ⇔ (∃x. ((Mem(x, marriages) ∧ spouse1 = x.m_Spouse1) ∧ spouse2 = x.m_Spouse2))))
```

3.3 User-Written Transactions

In addition to the divorce transaction seen in section 1, the user also writes a transaction to marry two people. Note that the foreign key constraint (symmetry) is temporarily invalidated between the two row insertions. The verification will ensure that it is properly reestablished at the end of the transaction.

Marriage Transaction

```
let marry_ref (A,B) =
  if hasKeyMarriage(A,B) then Some(false)
  else if A=B then Some(false)
  else
    insertMarriage {m_Spouse1=A; m_Spouse2=B};
    insertMarriage {m_Spouse1=B; m_Spouse2=A};
    Some(true)

let marry m = doTransact marry_ref m
```

The final line above defines a transaction marry by calling the transaction wrapper doTransact, which ensures that transactions that may violate database integrity are rolled back. The marriage transaction, wrapped and unwrapped, and the transaction wrapper, have the following types.

Wrapping Transactions

```
type α transaction = [(s) FK_Constraints(s)] r:α [(t) FK_Constraints(t)]
type α preTransact =
      [(s) FK_Constraints(s)] r: α option
      [(t) r ≠ None ⇒ FK_Constraints(t)]

val marry_ref: int×int → bool preTransact
val doTransact: (α → β preTransact) → α → (β option) transaction
```

A transaction returning type α is a computation, which if run in a state satisfying the foreign key constraints, if it terminates, returns a value of type α in a state that satisfies the foreign key constraints. Similarly, a pre-transaction returning type α is a computation, which if run in a state satisfying the foreign key constraints, and terminating with a return value of type α option different from None, preserves the foreign key constraints. To go from a pre-transaction, e.g., marry_ref to a transaction, e.g., marry, we use the function doTransact which rolls back the pre-transaction if it returns None.

We verify that the user code above has the types given above by refinement type checking; in particular, we get that the functions marry and toTransact divorce_ref preserve database integrity.

4 Example: A Simple E-Commerce Application

In this section, we illustrate our approach in the context of a typical e-commerce web shopping cart. A user can add products to their cart, update the number of products or remove items from their order. Each operation must leave the database in a consistent state satisfying all database contraints. An operation either successfully completes the database transaction, leaving the database in a new state, or it aborts the transaction and rolls back all the intermediate modifications, leaving the database in its original state.

We store the shopping cart state across two database tables. The first one (Orders) holds order information like customer name and shipping address, while the second one (Details) stores specific details about orders, like the codes of the chosen products, their quantities, and their price. A row in the Details table represents a unique product in an order. The column OrderID is used to associate each order with multiple detail rows.

The following SQL fragment shows the two tables, with their constraints.

SQL Schema

```
create table [Orders](
    [OrderID] [int] not null,
    [CustomerID] [nchar](8) null,
    [ShipName] [nvarchar](40) null,
    [ShipAddress] [nvarchar](60) null,
  constraint [PK_Orders] primary key ([OrderID])
)
create table [Details](
    [OrderID] [int] not null,
    [ProductID] [int] not null,
    [UnitPrice] [money] not null,
    [Quantity] [smallint] not null,
  constraint [PK_Details] primary key ([OrderID], [ProductID]),
  constraint [FK_Details_Orders] foreign key([OrderID])
    references [Orders] ([OrderID]),
  constraint [CK_Quantity] check (([Quantity]>(0))),
  constraint [CK_UnitPrice] check (([UnitPrice]>=(0))))
```

The primary key is the compound key created from the OrderID and the ProductID fields. To ensure referential integrity, we add the constraint that for every row in the Details table, the value of the OrderID field must exist in a row of the Orders table. For data integrity, we ask that for each row in the table, the Quantity is positive and the UnitPrice is non-negative.

E-Commerce Data Types (partial)

```
type State = { orders : orders_ref; details : details_ref}
type State_ref=d:State{ FK_Constraints(d) }
assume ∀d. FK_Constraints(d) ⇔ FK_Details_Orders(d.details, d.orders)
assume ∀ds, os.FK_Details_Orders(ds, os) ⇔
  ∀x. Mem(x,ds) ⇒ ∃u.Mem(u,os) ∧ PK_Orders(u,x.d_OrderID)
```

Given the types corresponding to table definitions (omitted), we represent a database as a record whose labels correspond to the table names. The label types are the refined table types orders_ref and details_ref. A refined state State_ref, is a database for which the foreign key constraint between the tables holds. The foreign key predicate definition says that for every row x of the details table, there exists a row u in the orders table, such that the primary key of u is equal to the d_OrderID field of x.

We verify the user defined pre-transaction addOrder below.

E-Commerce Transaction

```
let addOrder_ref ord =
  let (customerID, shipName, shipAddress, productID, unitPrice, quantity) = ord in
  let oid = freshOrders () in
  let orders : orders_row =
    {o_OrderID = oid; o_CustomerID = Some(customerID);
     o_ShipName = Some(shipName); o_ShipAddress = Some(shipAddress)} in
  let details : details_row =
    {d_OrderID = oid; d_ProductID = productID;
     d_UnitPrice = unitPrice; d_Quantity = quantity} in
  let rowChecks = checkDetails details in
  if rowChecks then let r = insertDetails details in
    if r then let r' = insertOrders orders in
      if r' then Some(true)
      else None
    else None
  else None

let addOrder ord = doTransact addOrder_ref ord
```

In addOrder_ref, since the detail is inserted before the order row, the database passes through a state in which the foreign key constraint is violated. (This code would fail needlessly in some systems, such as SQL Server.) The function addOrder uses the library function doTransact to wrap addOrder_ref with the necessary transaction handling code; type-checking ensures that the transaction is rolled back when necessary to avoid violation of integrity constraints.

5 Example: A Heap-Ordered Tree

This example shows the use of more advanced features of our system, such as user-defined predicates and custom stored procedures. We use a database table to store a heap-ordered tree, where the child nodes store pointers to their parent but not vice versa. We show how to define and typecheck for adding and removing nodes of the heap.

Heap SQL Specification

```
create table [Heap](
    [HeapID] [int] identity (1,1) not null,
    [Parent] [int] not null,
    [Content] [int] not null,
  constraint
    [PK_Heap] primary key CLUSTERED ([HeapID] asc),
  constraint
    [FK_Heap] foreign key ([Parent]) references [Heap] ([HeapID]),
  /×−−− UserConstraint TR_isHeap ×/
  /×−−− UserConstraint TR_uniqueRoot ×/)
```

We add two named application-level invariants (Section 2.4) to the heap table.

– TR_isHeap states that the value stored at every node is greater than that at its parent;
– TR_uniqueRoot states that any two root nodes are equal.

The first-order formulas expressing the application-level invariant predicates are defined in terms of an auxiliary predicate TR_isRoot. This predicate denotes that a given node is the root of a tree, which is defined as the node being its own parent.

User Constraints

```
assume ∀d. TR_isHeap(d) ⇔ (∀x,y. (Mem(x, d) ∧ Mem(y,d) ∧
            x.h_Parent = y.h_HeapID) ⇒ x.h_Content >= y.h_Content)
assume ∀d. TR_uniqueRoot(d) ⇔
              (∀x,y. (TR_isRoot(x,d) ∧ TR_isRoot(y,d)) ⇒ x = y )
assume ∀x,d. TR_isRoot(x,d) ⇔ Mem(x, d) ∧ x.h_Parent = x.h_HeapID
```

We also define two stored procedures: getRoot returns a root node of the tree, while getMinChild returns the smallest child of a given node.

Custom Stored Procedures

```
create procedure getRoot as
    select top 1 * from Heap
    where HeapID = Parent order by Content asc

create procedure getMinChild @rootID [int] as
    select top 1 * from Heap
    where (Parent = @rootID and HeapID ≠ @rootID)
    order by Content asc
```

The form of these stored procedures is very similar, so we detail the translation of only getRoot. Its postcondition is defined as follows. The function can return two different values: the empty list or a list containing one element. If the function returns the empty list, we learn that there is no root element. If one element was returned, the predicate GetRootResult states that it satisfies the where clause, and is from the table, and the predicate GetRootIsMin states that the returned element is the one with the least value of the elements in the table satisfying the where clause.

getRoot

```
val getRoot : unit → [(s)] l:heap_row list
    [(s') s = s' ∧ GetRootResult(l,s) ∧
    ((l = [] ∧ GetRootNotFound(s)) ∨ (∃x. l = [x]))]

assume ∀s,x. GetRootNotFound(s) ∧ Mem(x,s.heaps) ⇒
      not (x.h_Parent = x.h_HeapID)
assume ∀s,l,x. (GetRootResult(l,s) ∧ (l = [x])) ⇒
      (x.h_HeapID = x.h_Parent) ∧ Mem(x, s.heaps) ∧
      PK_Heaps_Exists(s.heaps,x.h_HeapID) ∧ GetRootIsMin(x,s)
assume ∀x,s,r. (GetRootIsMin(x,s) ∧ r.h_HeapID = r.h_Parent
                  ∧ Mem(r, s.heaps)) ⇒ x.h_Content >= r.h_Content
```

In this setting, we define two operations. We can insert a node into the tree, using the function pushAt_int, which adds a node with a given value as a child to the nearest ancestor of a given node that has a value less than the value to insert. With pop_int we can pop the smallest node off the table, causing its smallest child to bubble up the tree, recursively. This recursive procedure is called rebalanceHeap.

Specifications of User Functions

```
val pushAt_int: int×int →bool preTransact
val pop_int: unit →int preTransact
val rebalanceHeap: i:int →
    [(s) FK_Constraints(s) ∧PK_Heaps_Exists(s.heaps,i) ]
    unit [(t) FK_Constraints(t) ]
```

To push an element, we compare it to the root. If it is smaller, it becomes the new root value, otherwise we store it as a child of the root.

Pushing an Element Onto the Heap

```
let rec pushAt_int (i,v) =
  let node = lookupHeap i in
  let newID = freshHeap () in
  match node with
  | None →None
  | (Some(nodeRow)) →
    let {h_Content=c ; h_HeapID=id; h_Parent=par} = nodeRow in
    if v > c then
      let r = {h_Content = v ; h_HeapID = newID; h_Parent = id} in
      if insertHeap r then Some(true) else None
    else
      if hasKeyHeap id then
        if hasKeyHeap par then
          if id = par then
            let nodeRow' = {h_Content=v; h_HeapID=id; h_Parent=par} in
            if updateHeap id nodeRow' then
              let r = {h_Content=c; h_HeapID=newID; h_Parent=id} in
              if insertHeap r then Some(true) else None
            else None
          else pushAt_int (id,v)
        else None
      else None
```

When popping the root, we use rebalanceHeap to let a chain of minimal children "bubble up" one step.

Popping the Root of the Heap

```
let rec rebalanceHeap id =
  let minM = getMinChild(id) in match minM with
  | [] → let res = deleteHeap id in res
  | [minRow] → match minRow with
    | {h_Content=mc; h_HeapID=mid; h_Parent=mpar} →
      if hasKeyHeap mid then
        let r = lookupHeap id in match r with
        | None → ()
        | (Some(u)) → match u with
          | {h_Content=rc ; h_HeapID=rid; h_Parent=rpar} →
            let v = {h_Content = mc; h_HeapID = id ; h_Parent = rpar} in
            updateHeap id v;
            let res = rebalanceHeap mid in res
      else ()

let pop_int () =
  let root = getRoot() in match root with
  | [] → None
  | [rootRow] → match rootRow with
    | {h_Content = c; h_HeapID = id; h_Parent = par} →
      (rebalanceHeap id; Some(c))
```

To verify this more complex example, we needed to add three axioms to the context of the SMT solver. The first axiom states that when updating a row, without changing its primary key, then the same primary keys are present in the database table as before. The second axiom states that if the foreign key constraints hold, and the primary and foreign key fields are unchanged by a single-row update, then the foreign key constraints are not violated. The third axiom states that if a row has no children, then it can be deleted without violating the foreign key constraint.

Axioms

```
assume ∀h,h',k,v,x. UpdateHeap(h',h,k,v) ∧ PK_Heaps_Exists(h,x) ⇒ PK_Heaps_Exists(h',x)
assume ∀h1,h2,x,y. FK_Heaps_Heaps(h1,h1) ∧ Replace(h2,h1,x,y) ∧ x.h_Parent = y.h_Parent
    ∧ x.h_HeapID = y.h_HeapID ⇒ FK_Heaps_Heaps(h2,h2)
assume ∀s,k,s'. FK_Constraints(s) ∧ GetMinChildNotFound(k,s) ∧ DeletedHeap(s,s',k) ⇒
    FK_Constraints(s')
```

Given these axioms, we verify that transactions that add values to or pop values from the tree do not violate the database integrity, including the application-level constraints.

6 Software Architecture and Evaluation

Our implementation consists of a compiler from an SQL schema to a Stateful F7 database interface implementing the translation in Section 2.2, and from an SQL schema to a

Table 1. Lines of user supplied and generated code, and verification information

	User supplied		Generated		Verification	
	transactions	schema	data types	db interface	queries	time
Marriages	38	10	38	48	20	20.890s
E-Commerce	41	23	54	74	16	9.183s
Heap	111	30	54	85	76	80.385s

symbolic implementation of the database in F# implementing the dynamic semantics of Section 2.3. We use the Stateful F7 typechecker to verify the user supplied transactions against the generated interface. Additionally we provide a concrete implementation of the database interface against SQL Server. The core compiler (without Stateful F7) consists of about 3500 lines of F# code split between the SQL parser, the translation rules, and the implementation of the database interface.

We evaluate our approach experimentally by verifying all the examples of this paper; Table 1 summarizes our results. For each example it gives: a) the total number of lines of user supplied code (including the F# transaction code and user-defined predicates, and the SQL schema declaration), b) the number of lines of the automatically generated data types and database interface, and c) the verification information consisting of the number of proof obligations passed to Z3 and the actual verification time. Constraints that affect individual rows or tables like **check**, and **primary key** constraints, unsurprisingly add little time to the verification process. This explains the small verification time of the E-Commerce example, despite having more tables and **check** constraints than the other examples. On the other hand uniqueness, foreign key constraints, and arbitrary user constraints require disproportionately more time to verify.

We express constraints in first-order logic with a theory of uninterpreted function symbols and linear arithmetic. The main challenge when working with first-order solver like Z3 is quantifier instantiation. In certain examples like heap, we found that Z3 was unable to prove the automatically generated predicates. As a result and to assist Z3 with its proof obligations, our compiler implements some additional intermediate predicates. In particular, for universally quantified formulas, we gave predicate names to quantified subformulas such that superfluous instantiations might be avoided. For existentially quantified formulas, Z3 sometimes has problems constructing the appropriate witness, and we instead were forced to add an axiom that referred to predicates that abstracted quantified subformulas. One contributing factor to this problem was that F7 communicates with the SMT solver Z3 using the Simplify format, while the more advanced SMT-Lib format would permit us to add sorting of logical variables, patterns to guide quantifier instantiation, and access to the array theory implemented in Z3 for a more efficient modelling of tables as arrays indexed by the primary key.

Given that our objective is to statically verify that transactional code contains enough checks to preserve the database invariants, we found that applying our approach interactively as we developed the transactional code helped us implement an exhaustive set of checks and made for a pleasant programming experience. Still, our approach leads to verbose code which, when verified, explicitly handles any possible transaction outcome.

7 Related Work

The idea of applying program verification to database updates goes back to pioneering work [16,9] advocating the use of Hoare logic or weakest preconditions to verify transactions.

Sheard and Stemple [26] describe a system for verifying database transactions in a dialect of ADA to ensure that if they are run atomically then they obey database constraints. The system uses higher order logic and an adaptation of the automated techniques of Boyer and Moore.

In the setting of object-oriented databases, Benzaken and Doucet [5] propose that the checking procedures invoked by triggers be automatically generated from high-level constraints, well-typed boolean expressions.

Benedikt, Griffin, and Libkin [3] consider the integrity maintenance problem, and study some theoretical properties of the weakest preconditions for a database transaction to succeed, where transactions and queries are specified directly in first-order logic and extensions. Wadler [30] describes a related practical system, Pdiff, for compiling transactions against a large database used to configure the Lucent 5ESS telephone switch. Consistency constraints on a database with nearly a thousand tables are expressed in C. Transactions in a functional language are input to Pdiff, which computes the weakest precondition which must hold to ensure the transaction preserves database integrity.

To the best of our knowledge, our approach to the problem is the first to be driven by concrete SQL table descriptions, or to be based on an interpretation of SQL queries as list processing and SQL constraints as refinement types, or to rely on SMT solvers.

A recent tool [12] analyzes ADO.NET applications (that is, C# programs that generate SQL commands using the ADO.NET libraries) for SQL injection, performance, and integrity vulnerabilities. The only integrity constraints they consider are check constraints (for instance, that a price is greater than zero); they do not consider primary key and foreign key constraints.

Malecha and others [18] use the Coq system to build a fully verified implementation of an in-memory SQL database, which parses SQL concrete syntax into syntax trees, maps to relational algebra, runs an optimizer and eventually a query. Their main concern is to verify the series of optimization steps needed for efficient execution. In contrast, our concern is with bugs in user transactions rather than in the database implementation. Still, our work in F# is not fully verified or certified, so for higher assurance it could be valuable to port our techniques to this system.

Ur/Web [10] is a web programming language with a rich dependent type system. Like our work, Ur/Web has a dependently typed embedding of SQL tables, and can detect typing errors in embedded queries. On the other hand, static checking that transactions preserve integrity is not an objective of the design; Ur/Web programs may result in a "fatal application error if the command fails, for instance, because a data integrity constraint is violated" (online manual, November 2010).

Refinement-type checkers with state are closely related to systems for Extended Static Checking such as ESC Java [15] and its descendants [2]. To the best of our knowledge, these systems have not previously been applied to verification of transactions, but we expect it would be possible.

8 Conclusion

We built a tool for SQL databases to allow transactions to be written in a functional language, and to be verified using an SMT-based refinement-type checker. On the basis of our implementation experience, we conclude that it is feasible to use static verification to tell whether transactions maintain database integrity.

In the future, we are interested to consider an alternative architecture in which our static analysis of queries is implemented in the style of proof-carrying code on the SQL server itself. Another potential line of work is to model database state within separation logic, and to appeal to its tools for reasoning about updates. Finally, it would be interesting to apply our techniques in the setting of software transactional memory, for example, on top of recent work on semantics for STM Haskell [7].

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