Knowledge, Skills, Competencies: A Model for Mathematics E-Learning

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Abstract. This work concerns modelling competence in mathematics in e-learning environment. Competence is something complex, which goes further the cognitive level, but involves meta-cognitive and non-cognitive factors. Anyway it requires the students to master knowledge and skills, at first, and some measurable abilities, defined as competencies. We present a model that, exploiting the innovative technological features of the platform IWT, aims at defining a personalised learning experience allowing the students to build up their competence in mathematics.

Keywords: mathematics learning, knowledge, skill, competence, competency, e-learning.

1 Introduction

This paper is framed in the areas of e-learning and mathematics education. We assume that integrating research outcomes in both areas is of paramount importance. The effort of the author has been devoted to exploit the technological potential of e-learning based on the domain-specific results from research in mathematics education, in order to model competence in mathematics in an e-learning environment. Competence is something complex, which goes beyond the cognitive level, but involves meta-cognitive and non-cognitive factors. Anyway it requires the students to master knowledge and skills, at first, and some measurable abilities we are going to see in details in section 2.2.

There is a general agreement on the importance of mastering mathematics: everyone needs to use mathematics in his or her personal life, in the workplace, and in further learning. In the same way , it is well known that effectiveness of the teaching/learning process requires personalization, as different students have different styles, needs, preferences, attitudes.

Through the chapter we focus on the definition of a model apt to generate personalized learning experiences allowing the students to master not only pieces of knowledge or skills, but to handle competencies in mathematics. To this aim, first of all we are going to define what are knowledge, skills and competencies in mathematics. From the technological point of view, we refer to the e-learning platforms we have used, which is named IWT (Intelligent Web Teacher). It is

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equipped with features of LCMS (Learning Content Management System), adaptive learning system and allows the definition of personalized and collaborative teaching/learning experiences by means of the explicit representation of the knowledge and the use of techniques and tools of Web 2.0.

To begin with, in sections 2 and 3, we give an overview of the theoretical framework as well as the technological one.

Section 4 describes a model for mathematics learning. It is based on a multi-level graph representation of the domain, distinguishing the knowledge, the skill and the competence levels. Then the description of the work-flow needed o generate a tailored learning experience is given.

Finally, section 5 wraps up and explores some opportunities for future research.

2 Theoretical Framework

Mathematical literacy is concerned with “the capacity of students to analyse, reason and communicate effectively as they pose, solve and interpret mathematical problems in a variety of situations” [1]. Commonly speaking, this requires the students to become *competent* in mathematics. From the educational viewpoint, what does it means? And which are the teaching implications?

Many authors [2], [3], [4], [5], [6] have tried to explain what competence in mathematics is. All of them agree that it is something complex and dynamic, which is based on basic notions and procedures (knowledge and skills), but it goes beyond cognitive factors. Meta-cognitive and non-cognitive factors are strongly involved, such as awareness of own cognitive resources and of the thinking processes, self-regulation capabilities, for the former [7], and attitude, acceptance of the stimulus to use own knowing, desire and will to integrate it *in itinere* when necessary, for the latter [3].

2.1 Knowledge and Skills

Mathematical competence is not something “to be taught”, rather it is a long-term goal for the teaching/learning process. Anyway it requires, as pre-requisites, mathematics domain’s knowledge of declarative-propositional type, that is *knowledge* (“to know”), and of procedural type, that is *skill* (“to know how”). Note that both of them are equally important. Given a problem, first of all knowledge allows us to understand and analyze it, then it provides us the required theory, rationale and background to formulate and validate theoretical approaches in order to tackle the problem. Anyway, knowledge alone is not sufficient to solve the problem without procedural skills. The latter produce practical results and consequences, they make concrete our theoretical thoughts and reasoning. At the same time, skills without knowledge may lead to perform correct procedures starting from wrong data, thus producing senseless results.

According to the above distinction, we can try to list some basic requirements as shown in the following table:

Table 1. Classification of the main types of mathematical content

Content's type	***Knowledge***	***Skill***
Definition	Statement	Procedural/computational practice, if applicable
Theorem	Statement	Procedural/computational practice
Theorem	Proof	Procedural/computational practice
Algorithm		Performance of the algorithm
Example/counterexample	Description	
Exercise		Computational skills
Problems		Solving standard problems

Each content may have just theoretical interest, thus it represents just *knowledge*, or be a procedure, that is just *skill*, or have both theoretical and practical counterparts.

For instance, consider the concept of "rank" of a matrix and its definition as the greatest order of any non-zero minor in the matrix (the order of a minor being the size of the square sub-matrix of which it is the determinant). We can apply this as a procedure to compute the rank. The Kronecker theorem gives a more effective way of computing it, , which corresponds to a procedure, but knowing the proof of the theorem adds nothing to skills. On the other hand, the proof of the Gram-Schmidt theorem, stating the existence of an orthonormal basis for an Euclidian vector space of finite dimension, gives a constructive method to compute such a basis. Similarly the definition of the echelon form of a matrix is completely distinct from the techniques able to reduce a matrix in echelon form (Gauss' algorithm).

We can say that knowledge can be seen as factual information or content about a given topic or area, which can be stored, retrieved and autonomously elaborated by the learner during the learning process. We can say that knowledge represents "factual mathematics". On the other hand skills can be associated to what Skemp [8] called "instrumental mathematics" which is characterised by formulas, to keep in mind, exercises, products, and which corresponds to "instrumental comprehension", which means to be able to practice rules, without knowing why.

2.2 Competence and Competencies

According to Niss [5], "*possessing mathematical competence means having knowledge of, understanding, doing and using mathematics and having a well-founded opinion about it, in a variety of situations and contexts where mathematics plays or can play a role*". In order to make more factual the notion of mathematical competence, we can consider a mathematical competency as a clearly recognizable distinct major constituent in mathematical competence [5]. Niss has distinguished eight characteristic cognitive mathematical competencies, which also PISA 2009 [8] refers to. The following table lists them, grouped in two cluster [5]:

Table 2. Clusters related to cognitive mathematical competencies

The ability to ask and answer questions in and with mathematics	***The ability to deal with mathematical language and tools***
Mathematical thinking competency	Representation competency
Problem handling competency	Symbols and formalism competency
Modelling competency	Communication competency
Reasoning competency	Tools and aids competency

Let us see them in more details.

Mathematical thinking competency includes understanding and handling of scope and limitations of a given concepts, abstracting and generalizing results, posing questions and knowing the kinds of answers, distinguishing between different sorts of mathematical statements (theorems, conjectures, definitions, conditional and quantified statements, etc.).

Problem handling competency includes investigating and formulating pure/applied and closed/open mathematical problems and solving such kinds of problems, in various ways.

Modelling competency includes analyzing, validating, performing in given contexts, monitoring existing models.

Reasoning competency includes understanding the logic of a proof or of a counterexample, proving statements, following and assessing others' reasoning.

Representation competency includes distinguishing different kinds of representations of mathematical entities, understanding the relations between different representation of the same object, transforming representations – treatment and conversion [10].

Symbols and formalism competency includes understanding and handling symbolic and formal language, translating back and forth between symbolic language and verbal language.

Communication competency includes understanding others' mathematical texts and expressing oneself about mathematical contents, in different semiotic representation systems and registers[1] [11].

Tools and aids competency includes knowing and reflectively using different tools and aids for mathematical activity.

As seen, competencies correspond to relational mathematics [8], which consists in reasoning, thinking, problems, processes; which is reflected by "relational comprehension", which is to be aware of connections and reasons.

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[1] A register is defined as a linguistic variety according to use, i.e., the linguistic means developed to express meanings related to some context and goals.

Note that to acquire competence as well as competencies requires the students to be involved in didactical and adidactical situations[2] [12] where the students accept to be active actor building their *knowing* instead of passive repeater of what have learnt.

3 Technological Framework

From the technological point of view, in this paper we refer to the platform IWT (Intelligent Web Teacher)), we have used in our practices. It is a distance learning platform, realized at the Italian Pole of Excellence on Learning & Knowledge, equipped with features of LCMS, adaptive learning system and allowing the definition of personalized and collaborative teaching/learning experiences by means of the explicit representation of knowledge and the use of techniques and tools of Web 2.0. Due to the presence of three models (Didactic, Student, Knowledge), IWT allows the student to reach the learning objectives defined by delivering a personalised course which takes into account his/her specific needs, previous knowledge, preferred learning styles, didactical model more suitable to the knowledge at stake and to the mental model (then engagement) of the learner. Let us briefly describe the three models.

The Knowledge Model (KM) is able to represent in an intelligible manner for the computer the information associated to the available didactic material. It makes use of: 1) ontologies, that allow to formalise cognitive domains through the definition of concepts and relations between the concepts, 2) learning objects (LOs), that are defined as “any digital resource that can be reused to support learning” [13], 3) metadata, that are descriptive information and allow to tag each LO in order to associate it to one or more concepts defined in an ontology.

The Learner Model (LM) is able to catch (automatically) the previous knowledge and the knowledge the learners little by little acquire during his/her learning and the learning preferences (seen as cognitive abilities and perceptive capabilities) shown with regard to important pedagogical parameters such as: media, didactic approach, interaction level, semantic density, etc. The model is composed of three elements: a Cognitive State, a set of Learning Preferences and a set of Evolution Rules.

The Didactic Model (DM) defines the optimal modalities of knowledge transfer to the students on the basis of the domain (formalised in the KM) and to the characteristics of the involved student (formalised in the LM). Through this model

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[2] In an environment which has been organized for the purpose of learning a special subject, we can talk about an adidactical situation, when the didactical intention is no longer explicit. The teacher suggests an activity without declaring the purpose of it; the student is well-aware that all activities in the classroom are meant to build up new knowledge, but in this case she/he does not know exactly what she/he is going to learn. If she/he decides to participate, accepting to get implicated, then she/he frees her/himself from “contract” constraints and participates in an adidactical activity. In this case, the teacher is just a spectator, that is to say, she/he is not explicitly implicated in the knowledge management. We talk about didactical situation when the specific didactic objective of the teacher is explicit since the beginning: the teacher openly informs her/his students about the knowledge content that is at issue in that moment.

IWT can personalise the didactic experience on the basis of the previous knowledge of the single learners and of their learning preferences.

Let us now briefly described how IWT is able to generate a tailored unit of learning, taking advantages of the previous models [14], [15], [16]. When student access to the course the first time, IWT is able to automatically generate for each student the best possible learning path according to the information available in the student model, to the course specifications and to the LOs available in the repository. At first the ontology is used to create the list of the concepts needed to reach the target concept of the course. Then the information of the student model is used to update this list according to the cognitive state and to choose the more suitable LOs according to the learner preferences. The choice is made possible taking the LOs whose metadata better matches with the learner preferences data. Moreover the platform is able to dynamically update the learning path according to the outcomes of the intermediate tests.

The described approach to personalisation, which is more ‘teacher-centered’ rather than ‘learner-centred’ as the flow of the LOs is predefined and determined by the platform (that is the teacher/instructor), is combined with a non-linear approach, which allows the learners to navigate among the various alternative resources available and to select and also to create their own preferred ones.

4 Modelling Mathematics Learning

In the following we are going to extend the IWT Knowledge Model in order to distinguish mathematical knowledge, skills and competencies and then take them into consideration for defining and setting up tailored learning experiences.

4.1 Domain Representation

A rough domain formalisation could consist in teaching according to “fundamental nodes”. With this term we refer to “those fundamental concepts which occur in various places of a discipline and then have structural and knowledge procreative value” [17]. Teaching by fundamental nodes means “to weave a conceptual map, strategic and logic, fine and smart”, where each concept actually is the goal of a complex system with mesh and anyway no concept stands completely alone and each of them is part of a relations’ web rather than being single “conceptual object” [3].

Let us consider an example choosing the topic “Matrices”. We can take the following as fundamental nodes: Matrix definition, Operations with matrices, Determinant, Rank, Echelon matrix, Inverse matrix. A conceptual map could be the one shown in the Fig. 1, where the edges describe the links existing among various nodes, which can be of different kind: from those purely relational – for instance, the need of the concept of determinant in order to define the rank – to the instrumental ones – such as the use of the echelon matrices to compute the inverse of a given matrix.

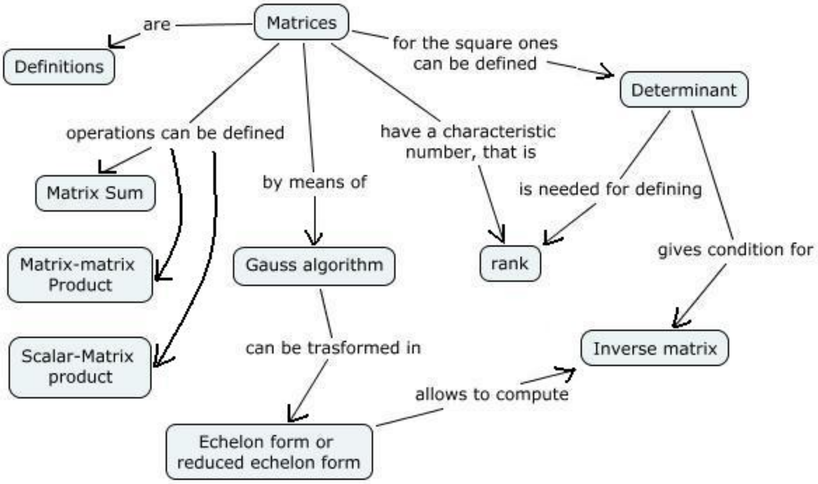

Fig. 1. This shows an example of conceptual map for the topic "Matrices"

Thus the relations appeared in the map of Fig. 1 have intersection with both knowledge and skill levels, as well as with relational and instrumental mathematics.

We wish to manage the previous map using the ontologies in IWT and the corresponding implemented relations, that are "HasPart", "IsRequiredBy" and "SuggestedOrder". We want to consider a multi-level graph representation: an ontology related to the fundamental nodes (corresponding to the use of ontology at the present in IWT, as shown in Fig. 2), an ontology related to the knowledge, an ontology related to the skills. All these should help students in mastering knowledge and skills which are pre-requisites to be involved in learning activities aimed at acquiring competencies (corresponding to the conceptual map in Fig. 1).

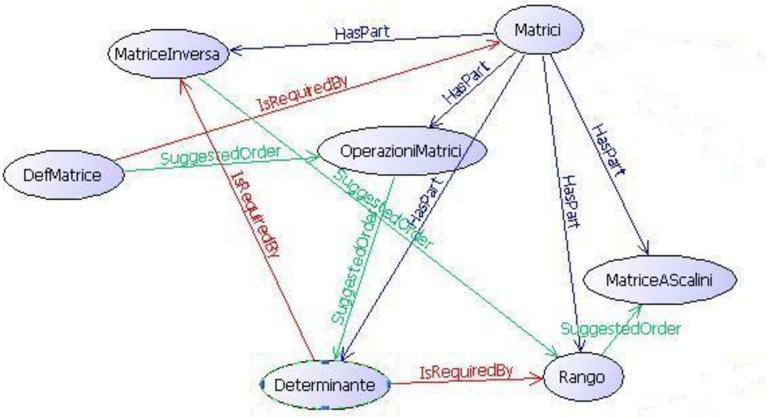

Fig. 2. This shows an example of "fundamental nodes" for the topic "Matrices"

The idea that the fundamental nodes actually are the goals of a relations' web let us to consider each of them as "root" of a further graph (ontology), where the levels of knowledge, skill and competency are made explicit.

- Knowledge level: here we consider nodes corresponding to definitions, theorems, examples, and so on (see table 1); Fig. 3 schematises the possible typologies of nodes and the connecting edges represent possible relations among various typologies, some of them mandatory, in some sense (designed as continuous lines) and some others optional (designed as discontinuous lines): for instance, given a definition of a mathematical entity, some characterizing properties can be proved, thus a link with one or more theorems is foreseen, and at the same time some examples or counterexamples can or cannot be generated.

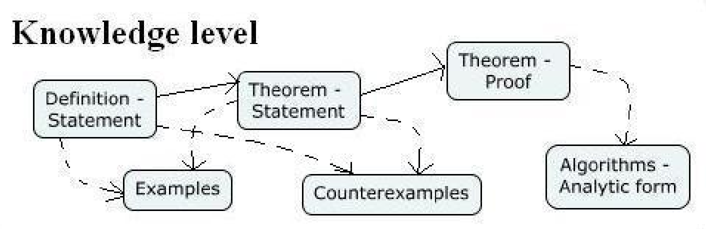

Fig. 3. Generic ontology at knowledge level

- Skill level: here we consider nodes corresponding basically to computational methods and standard solving problem capabilities (see Table 1); Fig. 4 outlines the possible typologies of nodes and the connecting edges representing possible relations among various typologies are designed with discontinuous lines to indicate that they can or cannot concur to the reached skill: for instance solving standard procedure requires one or more "elementary" calculation capabilities.

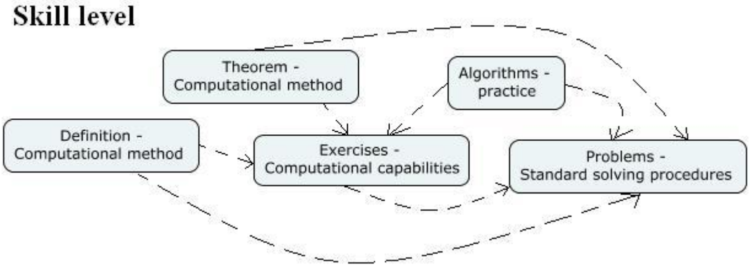

Fig. 4. Generic ontology at skill level

- Competency level: here we take into account the eight clusters described in section 2.2 and the possible links among them. Note that the given classification does not correspond to the separation of the clusters: it is evident that they are closely connected and some differences are actually very fine. This means that it is quite impossible to focus on just one competency. Fig. 5 schematises the eight clusters. The links can concern the whole cluster or some specific items, depending on the domain topic. The figure shows just some examples of linked items.

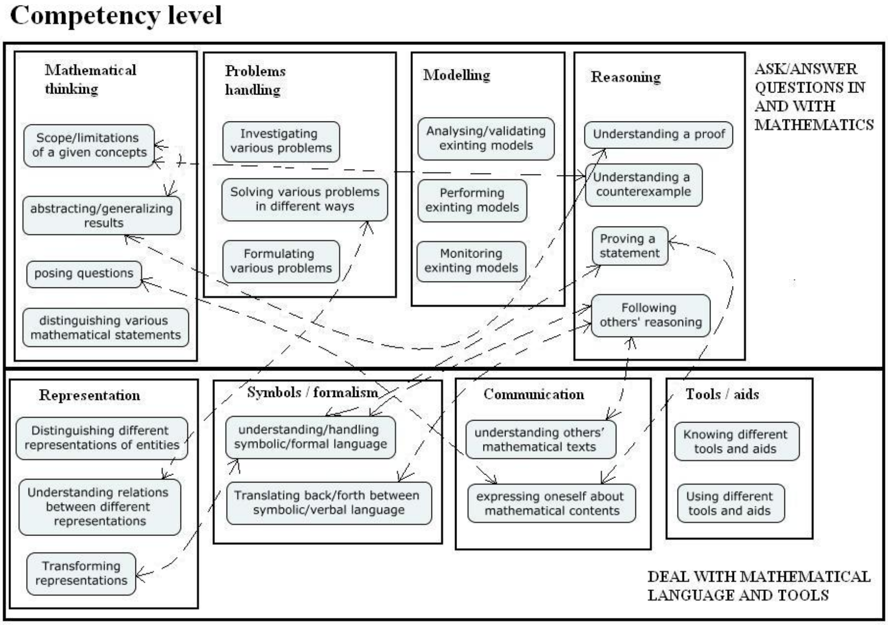

Fig. 5. Generic ontology at competency level

4.2 The Creation of a Personalised Learning Experience

First of all, the domain expert has to represent the domain by means of ontologies where fundamental nodes are roots of corresponding knowledge nodes as well as skills nodes. Further he has to specialize the competencies related to those nodes.

Now let us see which are the steps needed to define a personalised learning experience. The teacher has to define the learning goals of the learning experience. This is made by various choices:

a) The target concepts among the fundamental nodes;
b) The typologies at Knowledge level, if desired;
c) The typologies at Skill level, if desired;
d) The typologies at Competency level, if desired.

Each choice corresponds to the generation of a personalised learning experience, differing one from one other with respect to the global learning goal, as we are going to describe:

1) Aim at making students able to master knowledge and/or skills:
given the target concepts, the ontology of the fundamental nodes allows to create the list of the concepts needed to reach them. For each node of this list, a corresponding list at Knowledge and/or Skill levels is generated, according to the chosen typologies. Then a further list comes from the join of the latter ones, according to the fundamental concepts order. Now the information stored in the Student Model are used to update the list according to the student cognitive state and to choice the best LOs according to the student preferences, as described in section 3.
2) Aim at making students able to master competencies:
as seen in section 2.2, competencies stand cross wisely along all the mathematical domain, so they are not linked to specific knowledge or skill. Anyway, it has been proved [14] that students' success in performing activities related to competencies are strongly influenced by the level of their mastering of knowledge and skills involved in the activities. Thus, given competency typologies (at macro or micro level) some learning activities can be setting up, according to some templates. These latter specify the methodology and the tools or services required. Then the concepts from Knowledge and/or Skill levels to be involved have to be fixed. The concepts will be assumed as and target concepts of a personalised course, generated according to the procedure described in 1) and it will be the pre-requisite for the learning activity. The templates can be equipped with material related to the implementation of the general methodology referred to specific knowledge and skills, that can be re-used.

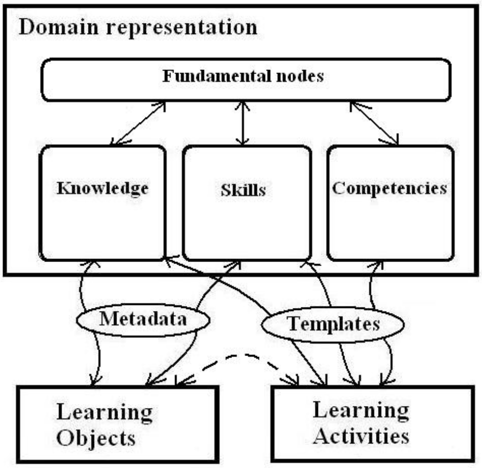

Fig. 6. Model of learning experience generation

5 Future Trends

In this paper we have proposed a model for tailored learning experience generation, taking into consideration knowledge, skill and competency levels in mathematics learning and taking advantages from the Knowledge Model of the platform IWT. The model foresees to design ontologies for the cited three levels concerned the chosen domain. The ones related to the first two levels should be used according to the current generation of cognitive-based personalised unit of learning, consisting in the delivery of suitable LOs. The third one should be used to generate learning activities aimed at students' mastering of some competencies.

We plan to go further on this way, along two main directions:

- definition of details for implementation of learning activities and interconnection with the unit of learning needed as pre-requisites;
- definition of assessment procedures for competencies according to the requirements of PISA [9], both in closed and open form [14], [18].

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