Chapter 30
Modelling of Water Cycle Processes

Viktor Batyreu and Siarhei Zenchanka

Abstract World drinking-water stocks are limited and this means that water is a limiting factor that defines people's living conditions and ecosystems all over the world. It is clear that modelling the global water cycle is a complex task and can give an approximate estimation only. More exact estimations may be done for separate water cycle processes only when their coefficients and boundary conditions are well defined. The aim of this work is to develop the model of water use as a water cycle process and to define necessary and sufficient conditions of water-using regimes' stability. As a result of modelling, a common criterion of stability of water-using regimes is proposed. This criterion includes the necessary condition based on the radioactive balance criterion, and the sufficient condition that is defined by a pulse migration value. The considered approach permits to define the boundary of stable areas of water system processes and systems themselves. In the area of instability, a crash water issue arises that leads to the disturbance of an assimilated system capacity.

Keywords Modelling · Water cycle · Consumption · Pollution · Concentration · Necessary condition · Sufficient condition

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Introduction

The global water cycle includes water in all physical states (solid state—ice, liquid state—water, gaseous state—water vapour) and considers its spreading in different physical media (solid—soil, liquid—water streams with different physical properties, for example, the Gulf Stream, the El Niño phenomenon in the Pacific Ocean, gas—the atmosphere). Accordingly, different physical states and physical properties have different rates of circulations. The water residence time in different natural objects differs significantly and can change from 1 week for water in biosphere objects to 10,000 years in soil (Vital Water Graphics 2002).

The estimations show that clean water in its different states constitutes 2.5% of total water quantity and available drinking water equals 30% of this quantity.

The limit of drinking-water stocks makes water a limiting factor which defines the living conditions for people and the ecosystems on all the Earth's continents. Lately, the constantly increasing human impact on the environment has led to a significant change in water-use and significant human impact on water systems. Problems connected with the stability and sustainability of such systems are arising. It is clear that modelling of the global water cycle is a complicated problem and can give only approximate estimations. More exact estimations can be made for a separate process of a water cycle with well-defined coefficients.

Milićević et al. (2010) pointed to the fact that "the mathematical models become increasingly important for implementation of Water Framework Directive, particularly in terms of pollution control and management of water resources quality in river basin areas".

Different types of water modelling have been suggested. Among them are the hydrological model suggested by Chang et al. (2009) and the object-oriented model based on the concept of system dynamics suggested by Elshorbagy and Ormsbee (2005).

Saysel (2007) and Fiksel (2006) underlined the complexity, dynamics and nonlinear nature of environmental systems and showed that in systems with multiple feedback loops, it is complicated to predict further system development.

The aim of this paper is to describe the model of water consumption as a process of a water cycle and to define essential and sufficient conditions of water-using regimes.

Terms and Definitions

Any process occurring in nature should be considered in terms of its stability and sustainability.

Consider these terms:

- Stability—quality of being stable (Hornby 1974).

- Stable—having or showing an ability or tendency to maintain, or resist change in, position or form (Surjeet 1988).
- Sustain—keep from falling or sinking (Hornby 1974).

These terms are synonymous but they have some differences.

Fiksel (2006) pointed out that the commonly used notion of "sustainability" as a steady-state equilibrium is not realistic and that "achieving sustainability will arguably require the development of resilient, adaptive industrial and societal systems that mirror the dynamic attributes of ecological systems".

Resilience can be defined as the capacity of a system to tolerate disturbances while retaining its structure and function (Fiksel 2006).

In this work, *stability* is considered as the ability of the system to function without change to its intrinsic structure and to be in a state of equilibrium with the environment.

Sustainability is considered as the ability of the system to return to its initial state after exposure to external impacts. This definition is close to resilience.

For a more exact description of an ecosystem's response to external impacts, it is appropriate to use such characteristics as elasticity and plasticity.

Elasticity is the quality of being elastic, i.e. having the tendency to go back to the normal or previous size or shape after being pulled or pressed (Hornby 1974). At the same time, certain impacts that exceed the threshold value of such a system mean that it is typically destroyed or it transfers to a new quality.

Plasticity is the ability of the system to change its structure under external loading. When loading cuts off the system, it returns to a state close to the initial one.

Concepts such as "elasticity" and "plasticity" are closely connected with such concepts as "potential capacity of the system" ("assimilated system capacity") and "critical load".

Potential capacity of the system is the maximum allowable level of flow into the system during a certain period of time.

Critical load is a maximum allowable level of outflow during continuous time.

These definitions show the importance of the time of the impact on the system, hence all processes must be considered from the point of view of the own system time.

The intrinsic system time is the time that is considered within the period of the system existing or its processes. The intrinsic system time has a different scale, as well as a system dimension.

A Common Model of the Water Cycle Process

The model of water consumption is based on the dynamic model presented in Fig. 1.

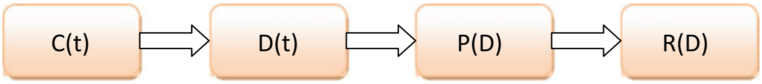

Fig. 1 Dynamic model of estimation of environment quality

Model components:

1. The law of change of concentration with time:

$$C(t) = C0\,\exp(-kt);\ C0 = \text{const};\ C0\,\exp(kt). \tag{1}$$

2. The impact of a pollutant on the system (dose) is defined as an integral of the change of concentration:

$$D(t) = D0(1 - \exp(-kt));\ D0(kt);\ D0(\exp(kt) - 1); D0 = C0/k. \tag{2}$$

3. The probability of system lethal outcome (system destroying) as a function of the dose:

$$P(D) = P0\,\exp(-rD);\ P0 = \text{const};\ P0\,\exp(rD). \tag{3}$$

4. The risk of lethal outcome (system destroying) that is defined as an integral from the law of variation of the probability of the system lethal outcome as a function dose value:

$$R(D) = R0(1 - \exp(-rD));\ R0\,rD;\ R0(\exp(rD) - 1). \tag{4}$$

D0 initial value of concentration;
C0 initial dose;
P0 initial probability (corresponding initial concentration C0);
R0 initial values of the risk of the lethal outcome;
k process constant (rate);
r rate of dose absorption.

Scheme of Water Consumption

Water consumption is an essential part of a global water circle and affects the condition of water resources.

The typical scheme of water consumption is shown in Fig. 2.

The "Diluting" block may be both in the direct chain and be used as a feedback when dumping is realized in a water object near the source.

Consumption regimes can be described as follows:

1. Water economy in the process of production of food products, consumer goods, etc.:

$$C_1 = C_0 e^{-k_1 t};$$

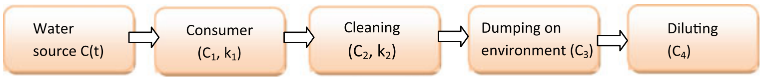

Fig. 2 Scheme of water consumption

2. Water consumption by living organisms:

$$C_1 = C_0;$$

3. Emergency dumping of water from the reservoir, dam breaking, snow avalanche, etc.

$$C_1 = C_0 e^{k_1 t}.$$

C_i, k_i—water concentrations and process rates in the appropriate chain links of water consumption.

The generalized criteria of sustainability of the water consumption regime are suggested, including:

1. the essential condition that was obtained analogously to the criterion for radioactive equilibrium and
2. the sufficient condition determined by the value of the migration pulse $D = \int C(t)dt$.

Migration pulse is a common concept describing a process of transport of energy, matter, etc. from one object to another during a defined time interval.

Such an approach permits us to define the limits of sustainability of the process occurring in water systems and systems in general. In the area of unsustainability, emergency dumping takes place that leads to the disturbance of the potential capacity of the system.

It is possible to consider the system as consisting of N serial links (a finite Markov chain) (Howard 1971):

$$C_1 \to C_2;\ C_1 \to C_2 \to C_3;\ C_1 \to C_2 \to \cdots \to C_N.$$

The value of N for the ecosystems including water systems can be equal to 2–4, for physical and chemical processes—10 and greater, but N is finite (N—number of links in a chain, n—a link number in a chain).

Value C_n is easily calculated at any link of the chain and for any value of N (Kozlov 1991). The obtained formulas for N are very bulky and it is difficult to analyse them. Besides, it is necessary to know the exact ratio between the process rates in different links k_n.

The essential condition of sustainable equilibrium defined as $\frac{C_n}{C_{n-2}} = \text{const}$, when $t \to \infty$ actually defines what part of the concentration in the previous link of the chain passes to the next link and influences its existence.

The sufficient condition (migration pulse) when taking into account the value of the intrinsic time of the system combines the concepts of maximum loading and the potential capacity and defines actually the time of the system's existence in a sustainable state.

Let us consider several examples within the frame of the model shown in Fig. 2.

Linear Regime of Water Consumption

As a linear regime of water consumption, consider the regime when pollutant concentration and water volume are transferred along serial links of the chain and dumping occurs at large distance from the source.

The Two-Element Chain

Consider the two-element chain C1 → C2 (Fig. 3):

The source C_1 is described by $C_1 = C_0 * e^{-kt}$. In this chain, three regimes are possible: (1) $k < 0$, (2) $k = 0$, (3) $k > 0$.

For the first regime:

$$\frac{C_2}{C_1} \to \frac{k_1}{k} * e^{kt};\ D_2 \to \frac{k_1}{k} * C_0 * t.$$

For the second regime:

$$\frac{C_2}{C_1} = k_1 * t;\ D_2 = k_1\, C_0 \frac{t^2}{2}.$$

For the third regime:

$$\frac{C_2}{C_1} = \frac{k_1}{k}\left(1 - e^{-kt}\right);\ D_2 = C_0 * \frac{k_2}{k^2} * e^{kt}.$$

The Three-Element Chain

For such a chain (Fig. 4), the expressions for essential and sufficient conditions are:

When $k_2 < k$:

$$\frac{C_2}{C_1} \cong \frac{k_1}{k_2 - k}\left(1 - e^{-kt}\right);\ D_2 \cong C_0 * \frac{k_1}{k_2 - k}.$$

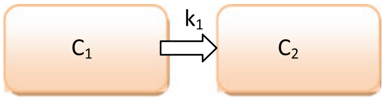

Fig. 3 Model of the two-element chain

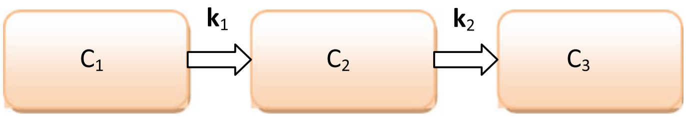

Fig. 4 Model of the three element chain

When $k_2 = k$:

$$\frac{C_2}{C_1} \cong kt = k_2 t, \quad D_2 \cong \frac{k_1}{k^2}.$$

When $k_2 > k$:

$$\frac{C_2}{C_1} \cong \frac{k_1}{k_2 + k}\left(e^{(k_2+k)t} \ 1\right); \ D_2 \sim e^{kt}.$$

The Four-Element Chain

The four-element chain is shown in Fig. 5.

The essential condition:

$$\frac{C_3}{C_2} \cong \frac{k_2}{k_3 - k}.$$

The sufficient condition:

$$D_3 \cong C_0 * \frac{k_2}{k_3 - k}.$$

The Analysis of the Results for Linear Chains

The analysis of the obtained expressions shows that the system consisting of two links (Fig. 3) is unsustainable for any regimes. Adding of the third link (Fig. 4) increases the sustainability of the system for several regimes. When $N \geq 3 \geq 3$ (Fig. 5), the dynamic regime is sustainable in accordance with the first and second conditions. For the chain consisting of N links we can obtain:

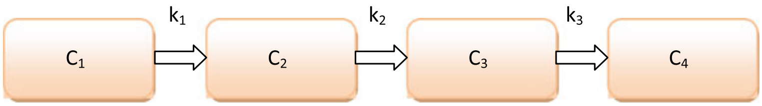

Fig. 5 Model of the four element chain

$$\frac{C_n}{C_{n-1}} \cong \frac{k_{n-1}}{k_n - k}. \quad D_3 \cong C_0 * \frac{k_{n-1}}{k_n - k}.$$

These expressions show that the increase in the number of links leads to system sustainability.

Regime of Water Consumption with Feedback

Consider the influence of feedback on the behaviour of the systems described above.

The Two-Element Chain with Feedback

Let us introduce feedback (k_0) (Fig. 6).

Such a chain is described by a set of equations:

$$\frac{dC_1}{dt} = -k_1C_1 + k_0C_2;$$

$$\frac{dC_2}{dt} = k_1C_1 - k_0C_2.$$

The essential condition for the system existence is:

$$\frac{C_2}{C_1} \sim \frac{A_3}{A_1}$$

and the sufficient condition is:

$$D_1 \sim A_1t;\ D_2 \sim A_3t;$$

where A_1, A_3 are some numerical coefficients.

The Three-Element Chain with Feedback

Let us add the third element to the system described above and consider stability and sustainability conditions for the existence of such a system (Fig. 7).

Consider the case with feedback between elements C_2 and C_1. Such a chain is described by the set of equations:

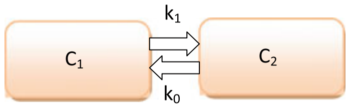

Fig. 6 Model of the two element chain with feedback

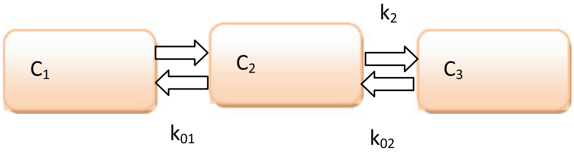

Fig. 7 Model of three-element chain with feedback

$$\frac{dC_1}{dt} = -k_1C_1 + k_{01}C_2 - k_2C_2;$$

$$\frac{dC_2}{dt} = k_1C_1 - k_{01}C_2 - k_2C_2;$$

$$\frac{d\,C_3}{dt} = k_2C_2.$$

When $0 < \lambda_1 < \lambda_2$(roots of the system determinant), the solution of this system gives the following conditions for stability and sustainability:

$$\frac{C_2}{C_1} \sim \frac{A_3}{A_1};\; D_1 \sim \frac{A_1}{\lambda_1} + \frac{A_2}{\lambda_2};\; D_2 \sim \frac{A_3}{\lambda_1} + \frac{A_2}{\lambda_2},$$

where A_1, A_2, A_3 are some numerical coefficients.

When $\lambda_1 = \lambda_2 = \lambda$:

$$\frac{C_2}{C_1} = \frac{A_3 + A_4}{A_1 + A_2};\; D_1 \sim \frac{A_1 + A_2}{\lambda};\; D_2 \sim \frac{A_3 + A_4}{\lambda}.$$

When λ_1 and λ_2 are imaginary, the solution is an oscillating process.

The Four-Element Chain with Feedback

For the four-element chain, consider the case with feedback between link C_2 and C_3 (Fig. 8).

The system behaviour is completely defined by the source regime, i.e. conditions $k < 0$, $k = 0$ or $k > 0$. The results are shown in Table 1. D is a determinant of the set of equations.

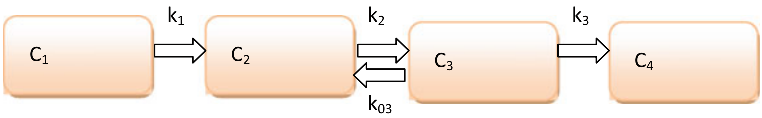

Fig. 8 Model of the four-element chain with feedback

Table 1 Solutions for the four-element chain

	>0D $>$ 0		=0D $=$ 0	
	$\frac{C_3}{C_2}$	D_i	$\frac{C_3}{C_2}$	D_i
$k < 0$	Constant	Constant	Constant	$\sim t$
$k = 0$	Constant	$\sim t$	Constant	$\sim t$
$k > 0$	Constant	e^{kt}	Constant	e^{kt}

The Analysis of the Results for Chains with Feedback

The consideration of the results obtained for the systems with feedback and without it shows that the implementation of feedback significantly influences the system behaviour. In many cases, stabilization of the system functioning takes place. At the same time, there are critical areas where the system collapses or its transition to another state occurs during the time.

Conclusion

An analysis of the functioning of multi-element systems without feedback (linear) and with feedback was made. It was shown that any system has its intrinsic lifetime. At the same time, there are regimes of the system functioning, the lifetimes of which are defined by the lifetime of the source only.

As the water cycle in nature and in industrial systems of water consumption includes elements with the direct link and feedback, the considered approach can be applied to such systems. In many cases, the optimization of the functioning of industrial systems can be reached by means of process rate optimization. In the cases where natural process rates cannot be optimized, it is necessary to take technical and preventive measures to limit them.

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