The Normal Parameter Value Reduction of Soft Sets and Its Algorithm

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Abstract. Some work has been done to such issues concerning parameter reduction of soft sets. However, up to the present, few documents have focused on parameter value reduction of soft sets. In this paper, we introduce the definition of normal parameter value reduction (NPVR) of soft sets which can overcome the problem of suboptimal choice and added parameter values. More specifically, minimal normal parameter value reduction (MNPVR) is defined as a special case of NPVR and a heuristic algorithm is presented. Finally, an illustrative example is employed to show our contribution.

Keywords: Soft sets; Reduction; Parameter value reduction; Normal parameter value reduction.

1 Introduction

In recent years, there has been a rapid growth in interest in soft set theory and its applications. Soft set theory was firstly proposed by a Russian Mathematician Molodtsov [1] in 1999. It is a new mathematical tool for dealing with uncertainties, while a wide variety of theories such as probability theory, fuzzy sets [2], and rough sets [3] so on are applicable to modeling vagueness, each of which has its inherent difficulties given in [4]. In contrast to all these theories, soft set theory is free from the above limitations and has no problem of setting the membership function, which makes it very convenient and easy to apply in practice. Therefore, many applications based on soft set theory have already been demonstrated by Molodtsov [1], such as the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory.

Presently, great progresses of study on soft set theory have been made [5, 6, 7, 8, 9, 10, 11, 12]. And it is worthwhile to mention that some effort has been done to such issues concerning reduction of soft sets. Maji *et al.* [13] employed soft sets to solve the decision-making problem. Later, Chen *et al.* [14] pointed out that the conclusion of soft set reduction offered in [13] was incorrect, and then presented a

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new notion of parameterization reduction in soft sets in comparison with the definition to the related concept of attributes reduction in rough set theory. The concept of normal parameter reduction was introduced in [15], which overcome the problem of suboptimal choice and added parameter set of soft sets. An algorithm for normal parameter reduction was also presented in [15]. However, up to the present, few documents have focused on parameter value reduction of soft sets. So, in this paper, we propose a definition of normal parameter value reduction of soft sets and give a heuristic algorithm to achieve the normal parameter value reduction of soft sets.

The rest of this paper is organized as follows. Section 2 reviews the basic notions of soft set theory. Section 3 gives definitions of normal parameter value reduction and minimal normal parameter value reduction of soft sets. Furthermore we give a heuristic algorithm to achieve them, which are illustrated by an example. Finally Section 4 presents the conclusion from our study.

2 Preliminaries

In this section, we review the definition with regard to soft sets.

Let *U* be a non-empty initial universe of objects, *E* be a set of parameters in relation to objects in *U*, $P(U)$ be the power set of *U*, and $A \subset E$. The definition of soft set is given as follows.

Definition 2.1 (See [4]). *A pair* (F, A) *is called a soft set over U, where F is a mapping given by*

$$F : A \to P(U)$$

That is, a soft set over U is a parameterized family of subsets of the universe U. As an illustration, let us consider the following example, which is quoted directly from [4].

Example 2.1. Let be a soft set (F, E) describe the "attractiveness of houses" that Mr. X is going to purchase. Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$, where there are six houses in the universe *U* and *E* is a set of parameters, e_i $(i = 1,2,3,4,5)$ standing for the parameters "expensive", "beautiful", "wooden", "cheap", and "in the green surroundings" respectively.

Suppose that we have

$F(e_1) = \{h_2, h_4\}$, $F(e_2) = \{h_1, h_3\}$, $F(e_3) = \phi$, $F(e_4) = \{h_1, h_3, h_5\}$, and $F(e_5) = \{h_1\}$, where $F(e_i)$ means a subset of U which elements match the parameter e_i. Then we can view the soft set (F,E) as consisting of the following collection of approximations:

$$(F,E)=\left\{\begin{array}{c}\text{expensive houses}=\{h_2,h_4\},\\ \text{beautiful houses}=\{h_1,h_3\}\\ \text{wooden houses}=\phi\\ \text{cheap houses}=\{h_1,h_3,h_5\}\\ \text{in the green surrounding houses}=\{h_1\}\end{array}\right\}$$

Each approximation has two parts, a predicate p and an approximate value set v. For example, for the approximation "expensive houses $=\{h_2,h_4\}$", we have the predicate name of expensive houses and the approximate value set or value set is $\{h_2,h_4\}$.Thus, a soft set (F,E) can be viewed as a collection of approximations below:

$$(F,E)=\{p_1=v_1,p_2=v_2,p_3=v_3,\cdots,p_n=v_n\}.$$

The soft set is a mapping from parameter to the crisp subset of universe. From such case, we may see the structure of a soft set can classify the objects into two classes (yes/1 or no/0). Thus we can make a one-to-one correspondence between a Boolean-valued information system and a soft set, as stated in Proposition 2.1.

Definition 2.2. *An information system is a 4-tuple (quadruple)* $S=(U,A,V,f)$, *where* $U=\{u_1,u_2,\cdots,u_{|U|}\}$ *is a non-empty finite set of objects,* $A=\{a_1,a_2,\cdots,a_{|A|}\}$ *is a non-empty finite set of attributes,* $V=\bigcup_{a\in A}V_a$, V_a *is the domain (value set) of attribute a,* $f:U\times A\to V$ *is an information function such that* $f(u,a)\in V_a$, *for every* $(u,a)\in U\times A$, *called information (knowledge) function.*

An information system is also called a knowledge representation system or an attribute-valued system and can be intuitively expressed in terms of an information table. In an information system $S=(U,A,V,f)$, if $V_a=\{0,1\}$, for every $a\in A$, then S is called a Boolean-valued information system.

Proposition 2.1. *If* (F,E) *is a soft set over the universe U, then* (F,E) *is a Boolean-valued information system* $S=(U,A,V_{\{0,1\}},f)$.

From Proposition 2.1, a soft set (F,E) as in Example 2.1, it can be represented as a Boolean table as follows:

Table 1. Tabular representation of a soft set in the above example

U	***e₁***	***e₂***	***e₃***	***e₄***	***e₅***
h_1	0	1	0	1	1
h_2	1	0	0	0	0
h_3	0	1	1	1	0
h_4	1	0	1	0	0
h_5	0	0	1	1	0
h_6	0	0	0	0	0

3 The Normal Parameter Value Reduction of Soft Sets

In this section, we depict a definition of normal parameter value reduction of soft sets (NPVR). Furthermore, we introduce the minimal normal parameter value reduction (MNPVR) of soft sets as a special case of NPVR and then present a heuristic algorithm to achieve it. Finally, an illustrative example is given.

3.1 Definition of Normal Parameter Value Reduction of Soft Sets

Suppose $U=\{h_1,h_2,\cdots,h_n\}$, $E=\{e_1,e_2,\cdots,e_m\}$, (F,E) is a soft set with tabular representation. Define $f_E(h_i)=\sum_j h_{ij}$, where h_{ij} are the entries in the table of (F,E).

Definition 3.1 (See [15]). *With every subset of parameters $B\subseteq A$, an indiscernibility relation $IND(B)$ is defined by*

$$IND(B)=\{(h_i,h_j)\in U\times U : f_B(h_i)=f_B(h_j)\}.$$

For soft sets (F,E), $U=\{h_1,h_2,\cdots,h_n\}$, the decision partition is referred to as

$$C_E=\{\{h_1,h_2,\ldots,h_i\}_{f_1},\{h_{i+1},\ldots,h_j\}_{f_2},\ldots,\{h_k,\ldots,h_n\}_{f_s}\},$$

where for subclass $\{h_v,h_{v+1},\ldots,h_{v+w}\}_{f_i}, f_E(h_v)=f_E(h_{v+1})=\ldots=f_E(h_{v+w})=f_i$, and $f_1\geq f_2\geq\ldots\geq f_s$, s is the number of subclasses. In other words, objects in U are classified and ranked according to value of $f_E(.)$ based on the indiscernibility relation.

Definition 3.2. *Given a Boolean-valued information system* $S=\left(U,A,V_{\{0,1\}},f\right)$ *corresponding to a soft set* $\left(F,E\right)$ *, we define:*

$$(1)(e_j,h_{ij})\overset{def}{=}\{h_i \mid (h_i\in U)\wedge(f_{e_j}(h_i)=h_{ij}),i=1,2,...,n;j=1,2,...,m\};$$

$$(2)((e_1,h_{i1}),(e_2,h_{i2}),...,(e_m,h_{im}))\overset{def}{=}\{h_i \mid (h_i\in U)\wedge(f_{e_1}(h_i)=h_{i1,}f_{e_2}(h_i)=h_{i2,}...,$$
$$f_{e_m}(h_i)=h_{im,}i=1,2,...,n\};$$

$$(3)H_E(h_i)=\bigcup_{j\in E}(e_j,h_{ij})=\{(e_1,h_{i1}),(e_2,h_{i2}),...,(e_m,h_{im})\},i=1,2,...,n.$$

Definition 3.3. *Denote* $A_1,A_2,...,A_n\subset E$ *as subsets, if there exist subsets* $A_1,A_2,...,A_n$ *satisfying* $f_{A_1}\left(h_1\right)=f_{A_2}\left(h_2\right)=\cdots=f_{A_n}\left(h_n\right)=t(t\le f_s)$ *, then the parameter values* $H_{A_i}\left(h_i\right)(i=1,...,n)$ *are dispensable, otherwise, they are indispensable.* $H_{B_i}\left(h_i\right)(i=1,...,n)$ *is defined as a normal parameter value reduction, if the two conditions as follows are satisfied*

(1) $H_{B_i}\left(h_i\right)(i=1,...,n)$ *is indispensable*

(2) $f_{E-B_1}\left(h_1\right)=f_{E-B_2}\left(h_2\right)=...=f_{E-B_n}\left(h_n\right)=t$

From the definition of normal parameter value reduction, we know normal parameter value reduction of soft sets keeps the classification ability and rank invariant for decision making. That is, after reducing dispensable parameter value, the decision partition is $C_E'=\left\{\{h_1,h_2,...,h_i\}_{f_1-t},\{h_{i+1},...,h_j\}_{f_2-t},...,\{h_k,...,h_n\}_{f_s-t}\right\}$

Definition 3.4. *Denote* $A_1,A_2,...,A_n\subset E$ *as subsets, if there exist subsets* $A_1,A_2,...,A_n$ *satisfying* $f_{A_1}\left(h_1\right)=f_{A_2}\left(h_2\right)=\cdots=f_{A_n}\left(h_n\right)=t$ *, then the parameter values* $H_{A_i}\left(h_i\right)(i=1,...,n)$ *are dispensable, otherwise, they are indispensable.* $H_{B_i}\left(h_i\right)(i=1,...,n)$ *are defined as a minimal normal parameter value reduction, if the three conditions as follows are satisfied*

(1) $H_{B_i}\left(h_i\right)(i=1,...,n)$ *is indispensable*

(2) $f_{E-B_1}\left(h_1\right)=f_{E-B_2}\left(h_2\right)=...=f_{E-B_n}\left(h_n\right)=t$

(3) $t=f_s$ *(* f_s *is the minimum decision choice value)*

From the above definition it follows that MNPVR does not differ essentially from NPVR. Obviously, MNPVR is a special case of NPVR. Intuitively speaking, MNPVR leads to the final decision partition $C_E' = \{\{h_1, h_2, ..., h_i\}_{f_1 - t}, \{h_{i+1}, ..., h_j\}_{f_2 - t}, ..., \{h_k, ..., h_n\}_{f_s - t = 0}\}$.

3.2 Algorithm of Minimal Normal Parameter Value Reduction of Soft Sets

Here below, we provide an algorithm to illustrate how to achieve the minimal normal parameter value reduction of soft sets.

(1) Input the soft set (F, E) and the parameter set *E*;

(2) Delete the parameter values denoted by 0.

(3) If $f_s = t \neq 0$, reduce the t parameter values denoted by 1 for every $h_i (h_i \in U, 0 \le i \le n)$ until $f_s = 0$.

(4) Put the remainder values as the minimal normal parameter value reduction which satisfies $C_E' = \{\{h_1, h_2, ..., h_i\}_{f_1 - t}, \{h_{i+1}, ..., h_j\}_{f_2 - t}, ..., \{h_k, ..., h_n\}_{f_s - t = 0}\}$.

3.3 Example

Example 3.1. Let (F, E) be a soft set with the tabular representation displayed in Table 2. Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

Table 2. A soft set (F, E)

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	*f(.)*
h_1	1	0	0	0	0	1	0	1	3
h_2	0	1	0	1	1	1	0	1	5
h_3	1	0	1	1	1	1	0	1	6
h_4	1	0	0	1	0	0	0	1	3
h_5	0	1	0	0	1	0	0	1	3
h_6	1	0	0	1	1	0	0	1	4

From table 2, the decision partition is $C_E = \{\{h_3\}_6, \{h_2\}_5, \{h_6\}_4, \{h_1, h_4, h_5\}_3\}$. Clearly $f_E(h_3) = 6$ is the maximum choice value, thus h_3 is the optimal choice object. h_2 is the suboptimal choice object.

Table 3. A normal parameter value reduction table of original table (Table 2)

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	*f(.)*
h_1	-	-	-	-	-	1	-	1	2
h_2	-	-	-	1	1	1	-	1	4
h_3	-	-	1	1	1	1	-	1	5
h_4	-	-	-	1	-	-	-	1	2
h_5	-	-	-	-	1	-	-	1	2
h_6	-	-	-	1	1	-	-	1	3

Given t=1 (namely, $f_{A_1}(h_1)=f_{A_2}(h_2)=\cdots=f_{A_n}(h_n)=t=1$), it is evident that, $A_1=\{e_1,e_2,e_3,e_4,e_5,e_7\}, A_2=\{e_1,e_2,e_3,e_7\}, A_3=\{e_1,e_2,e_7\}, A_4=\{e_1,e_2,e_3,e_5,e_6,e_7\}, A_5=\{e_1,e_2,e_3,e_4,e_6,e_7\}, A_6=\{e_1,e_2,e_3,e_6,e_7\}$ in this example. The normal parameter value reduction of (F,E) is clearly shown in Table 3. And then we get that the decision partition is $C_E=\{\{h_3\}_5,\{h_2\}_4,\{h_6\}_3,\{h_1,h_4,h_5\}_2\}$. h_3 is still the optimal choice object. The normal parameter value reduction keeps the classification ability and rank invariant for decision making.

Table 4. A minimal normal parameter value reduction table of original table (Table 2)

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	*f(.)*
h_1	-	-	-	-	-	-	-	-	0
h_2	-	-	-	-	-	1	-	1	2
h_3	-	-	-	-	1	1	0	1	3
h_4	-	-	-	-	-	-	-	-	0
h_5	-	-	-	-	-	-	-	-	0
h_6	-	-	-	-	-	-	-	1	1

Clearly $f_s = 3$ is the minimum decision choice value in original Table 2. Thus, let $f_{A_1}(h_1) = f_{A_2}(h_2) = \cdots = f_{A_n}(h_n) = f_s = t = 3$. And then we can obtain

$$A_1 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}, A_2 = \{e_1, e_2, e_3, e_4, e_5, e_7\}, A_3 = \{e_1, e_2, e_3, e_4, e_7\},$$
$$A_4 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}, A_5 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}, A_6 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}.$$

The final decision partition is $C_E = \{\{h_3\}_3, \{h_2\}_2, \{h_6\}_1, \{h_1, h_4, h_5\}_0\}$. The results from Table 3 and Table 4 indicate that MNPVR can delete more parameter values in comparison with NPVR, in the case of keeping the classification ability and rank invariant for decision making. Thus MNPVR can be generally interpreted as the minimal degree of NPVR.

4 Conclusion

Some work on parameter reduction of soft sets has been done. They presented a new notion of parameterization reduction in soft sets in comparison with the definition to the related concept of attributes reduction in rough set theory and the concept of normal parameter reduction which overcome the problem of suboptimal choice and added parameter set of soft sets. Unfortunately, the two algorithms are complicated and time-consuming. And up to the present, few documents have focused on parameter value reduction of soft sets. So, in this paper, we propose a definition of normal parameter value reduction (NPVR) of soft sets. More specifically, minimal normal parameter value reduction (MNPVR) is defined as a special case of NPVR and a heuristic algorithm is presented to achieve it, which is very easy to understand and carried out. Finally, an illustrative example is employed to show our contribution.

References

1. Molodtsov, D.: Soft set theory_First results. Computers and Mathematics with Applications 37(4/5), 19–31 (1999)
2. Zadeh, L.A.: Fuzzy sets. Information and Control 8, 338–353 (1965)
3. Pawlak, Z.: Rough sets. International Journal Information Computer Science 11, 341–356 (1982)
4. Molodtsov, D.: The Theory of Soft Sets. URSS Publishers, Moscow (2004) (in Russian)
5. Pei, D., Miao, D.: From soft sets to information systems. In: The proceeding of 2005 IEEE International Conference on Granular Computing, IEEE GrC 2005, pp. 617–621. IEEE Press, Los Alamitos (2005)
6. Herawan, T., Mat Deris, M.: A direct proof of every rough set is a soft set. In: Proceeding of the Third Asia International Conference on Modeling and Simulation, pp. 119–124. AMS, Bali (2009)
7. Feng, F., Li, C., Davvaz, B., Ali, M.I.: Soft sets combined with fuzzy sets and rough sets: A tentative approach. In: Soft Computing - A Fusion of Foundations, Methodologies and Applications, pp. 899–911. Springer, Heidelberg (2009)
8. Feng, F.: Generalized rough fuzzy sets based on soft sets. In: The Proceeding of 2009 International Workshop on Intelligent Systems and Applications, ISA 2009, Wuhan, China, pp. 1–4 (2009)

9. Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabira, M.: On some new operations in soft set theory. Comput. Math. Appl. 57(9), 1547–1553 (2009)
10. Zou, Y., Xiao, Z.: Data analysis approaches of soft sets under incomplete information. Knowl.-Based Syst. 21(8), 941–945 (2008)
11. Xiao, Z., Gong, K., Xia, S., Zou, Y.: Exclusive disjunctive soft sets. Computers and Mathematics with Applications 59(6), 2128–2137 (2010)
12. Herawan, T., Mat Deris, M.: A Soft Set Approach for Association Rules Mining. Knowledge Based Systems (2010), doi:10.1016/j.knosys.2010.08.005
13. Maji, P.K., Roy, A.R.: An application of soft sets in a decision making problem. Computers and Mathematics with Applications 44, 1077–1083 (2002)
14. Chen, D., Tsang, E.C.C., Yeung, D.S., Wang, X.: The parameterization reduction of soft sets and its applications. Computers and Mathematics with Applications 49(5–6), 757–763 (2005)
15. Kong, Z., Gao, L., Wang, L., Li, S.: The normal parameter reduction of soft sets and its algorithm. Computers and Mathematics with Applications 56(12), 3029–3037 (2008)