A Probability Density Function for Energy-Balanced Lifetime-Enhancing Node Deployment in WSN

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Abstract. Energy is one of the scarcest resources in wireless sensor network (WSN). So the issue of preserving energy requires utmost attention. There are many ways to conserve energy in such a network. One primary way of conserving energy is judicious deployment of sensor nodes within the network area so that the energy flow remains balanced throughout the network. This prevents the problem of occurrence of 'energy holes' and ensures prolonged network lifetime. This work first proposes a probability density function (PDF) and derives its intrinsic characteristics. We have shown the PDF's suitability to model the network architecture considered for the work. Next a node deployment algorithm is developed based on this PDF. Further, we have identified necessary constraints involving different network parameters for ensuring energy balance of the entire network. Performance of the deployment scheme is evaluated in terms of energy balance and network lifetime. Finally the scheme is compared with two existing deployment schemes. Simulation results confirm our scheme's supremacy over the two existing schemes in terms of all the two performance metrics.

Keywords: Probability density function, Coverage, Connectivity, Energy balance, Network lifetime.

1 Introduction

A wireless sensor network (WSN) [1] consists of several hundreds of sensor nodes which collect data from their surroundings and send the collected data to their neighbouring nodes in single hop. The neighbouring nodes in turn send the data to the sink either directly or via their one hop neighbouing nodes. The sink processes and transmits the received data to the outside world. Sensor nodes are equipped with battery whose charge cannot be replaced easily after deployment and so the need to conserve energy is a major concern of WSN. The rate of energy depletion in the network primarily depends on the deployment nature of the nodes that further depends on the application environment.

Deployment can be random or pre-determined. In random deployment, nodes are randomly deployed generally in an inaccessible terrain. For example, in the application

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domain of disaster recovery or in forest fire detection, sensors are generally dropped by helicopter in random manner [2]. In pre-determined deployment, number of nodes in a unit area is known apriori and is used in applications where sensors are expensive or their operation is significantly affected by their positions. These applications include placing imaging and video sensors, populating an area with highly precise seismic nodes, monitoring manufacturing plants etc [2].

One important way of conserving energy is through uniform energy or load distribution all over the network. Non-uniform energy dissipation in any part of the network may result in non-functioning of that part leading to the phenomenon of energy hole problem [3] that effects the network lifitime. The non-uniform energy dissipation arises due to uneven data transmissions by certain nodes in the network resulting in extra energy dissipation of those nodes. This problem also causes a substantial amount of energy to remain in the nodes even after network lifetime ends leading to significant wastage of energy [4]. To avoid this, nodes should be deployed in such a manner that the energy dissipation of all nodes takes place uniformly ensuring load balancing throughout the network. A good sensor deployment strategy is one that achieves both energy balance and energy efficiency [5].

Many works reported so far deal with the deployment issue. In [6], [7] authors have proposed a deployment strategy with a target to cover the area of interest. In [8] authors have proposed a deployment scheme to minimize energy consumption in the whole network so that the network lifetime is prolonged. In [9], [10] authors have proposed the deployment scheme for efficient energy usage throughout the network, thereby enhancing the network lifetime. In [11], [12] authors have used standard distribution functions for node deployment showing their capabilities for enhancing the network lifetime. Most of the distribution-function/scheme based deployment strategies have not addressed all the issues of energy balance and network lifetime simultaneously. This motivates us to propose a probability density function based on which the pre-determined node deployment strategy is proposed. The proposed scheme is targeted to achieve energy balance and enhancement of network lifetime.

The rest of the paper is organized as follows. In section 2, literature review is elaborated. The network model considered for the present work is presented in section 3. Section 4 presents the proposed node deployment scheme along with the proposed probability density function based on which the scheme is developed. In section 5, the performance of the scheme is evaluated based on both qualitative and quantitative analysis. Finally in section 6, the paper is concluded with some mention about the future scope of the present work.

2 Related Work

Y. Zou *et al.* [6] have formulated a problem on uncertainty-aware sensor node placement when nodes are dropped from airplanes. They have proposed two pre-determined node placement algorithms- maximum miss probability and minimum miss probability for determining minimum number of required nodes and their locations (mean positions) such that coverage is ensured. However, no attempt has been made for prolonging the network lifetime and balancing energy consumption in the network.

P. K. Agarwal *et al.* in [7] have also proposed a node placement algorithm which requires minimum number of nodes to cover a region. The approach is landmark based where landmarks are the set of finite points in a 2-D space. The algorithm is proposed based on greedy approach to compute the location of sensors. Although authors guarantee coverage of a given region but fail to ensure connectivity amongst the sensors. They are also silent about network lifetime.

D. Ganesan *et al.* [8] have formulated an optimization problem for node placement and transmission structure of data gathering to minimize communication energy. The node placement strategy is first studied in 1-D network and is then extended to 2-D circular network. The algorithm in 2-D network considers the circular area partitioned into wheel like structures where wheels are comprised of a number of spokes. But, the authors have not mentioned the energy balancing criterion and network lifetime of the proposed node placement algorithm.

Wu *et al.* [9] have explored the theoretical aspects of energy hole problem in sensor networks in layered architecture. They have proposed a non-uniform node distribution strategy which ensures maximum energy efficiency in the network. The number of nodes distributed in a layer is determined based on the minimum number of nodes required in the upper adjacent layer. However, the authors are silent about the minimum number of nodes required to be placed in the farthest layer from the sink to maintain connectivity and coverage.

C. Y. Chang *et al.* [10] have proposed two node deployment schemes- distance-based and density-based for balancing power consumption among the sensor nodes. In distance-based scheme, deployment positions of nodes are adjusted such that the nodes' neighbors towards sink are located relatively closer compared to other neighbor nodes. The density-based scheme partitions the network into a number of equal-sized zones, adjusts the density of nodes in each zone by controlling the switching mode as on/off and balances the load of each zone. However, the scheme requires various control mechanisms that are difficult to implement in resource-constrained WSN.

Olariu *et al.* [11] have given a network design guideline for maximizing lifetime and avoiding energy hole with uniform node distribution. They show that uneven energy depletion due to energy hole is unavoidable for free-space model, but can be prevented for multipath model. The authors have provided the design guideline for multipath model with corona architecture in terms of widths and the number of layers. However, they have not explored the potential of non-uniform node deployment.

D. Wang *et al.* [12] propose an analytical model for coverage and network lifetime of sensor networks using two dimensional Gaussian distribution showing more nodes get deployed closer to the sink using this model. They have proposed two deployment algorithms- one for circular and another for elliptical network area using which larger coverage and longer network lifetime are achieved. But the deployment scheme does not ensure energy balancing in the network.

3 Network Model

In this section, we describe the network architecture along with sensing, communication and energy models considered for this work.

3.1 Architecture

We consider a square shaped network area a×a which is covered by a set of annuli. Each such annuli is designated with width r as layer. The sink is considered to be located at the centre of the network area. Nodes are placed in different layers surrounding a single sink. A layer is identified as L_i where i = 1, 2, . . . , N. Here i=1 indicates the layer nearest to the sink and i=N indicates the layer farthest from the sink.

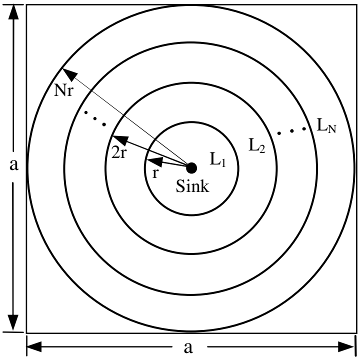


Fig. 1. Layered network area

We assume all the sensor nodes are homogeneous with respect to their initial energy, sensing and communication ranges. The nodes are static and distributed within the network with a given node density. Node density is defined [3] as the ratio of the number of nodes in a layer and the area of the layer. Further we consider that a unit area generates data at ρ bits/sec which is collected by the nodes and sent to the sink after a fixed time-interval q(t). We assume that data is transmitted from layers towards sink following the greedy forwarding routing policy [13] where a node in a layer chooses a neighbor as next-hop when the neighbor is located closest to the destination in comparison with the other neighbors. As a result of this greedy approach, during data transmission from a layer to the sink the data traverses each of the layers only once along the transmission path.

3.2 Sensing and Communication Model

3.2.1 Communication Model

We define a network as connected, if any active node can communicate with the sink node either in single hop or in multiple hops. We assume two nodes can directly exchange messages if their Euclidean distance is not larger than the communication range R_c. The relationship between r (width of a layer) and R_c must satisfy the condition $r \le R_c$ [12] for ensuring connectivity in the network area (Figure 1).

Lemma 1: For a given network area a×a, in order to maintain connectivity of the network, the number of layers (N) stands in relation with R_c as $N \ge \frac{a}{2R_c}$.

Proof: If the radius of each layer in the layered architecture is r, then the distance between the centre of the inner most layer and the farthest edge of any other layer is given by ir (Refer figure 1), where i is the layer number. If the distance between the centre of the inner most layer and the farthest edge of any other layer in the network area is $\frac{a}{2}$, then replacing i by N, we get $Nr \ge \frac{a}{2}$ *or,* $N \ge \frac{a}{2r}$.

Replacing $r \le R_c$ in the above relation, we have $N \ge \frac{a}{2R_c}$.

3.2.2 Sensing Model

We define a unit area to be covered if every point in that area is within the sensing range of at least one active node. The nodes perform observation [14] at an angle of 360°. The maximal circular area centered around a node v, that can be covered by the node is defined as its sensing area S(v). The radius of S(v) is called the v's sensing range [14] R_s. We assume the relationship between r and R_s must satisfy the condition $r \le 2R_s$ [12] for covering the network area (Figure 1) under consideration.

Corollary 1: For a given network area a×a, in order to maintain network coverage, the number of layers (N) stands in relation with R_s as $N \ge \frac{a}{4R_s}$.

Proof: From lemma 1, the relationship between a and N is evaluated as, $N \ge \frac{a}{2r}$. Replacing $r \le 2R_s$ in the relation $N \ge \frac{a}{2r}$, we have $N \ge \frac{a}{4R_s}$.

3.3 Energy Model

We have considered the first order radio model [12] as our energy model where energy consumption of a node is dominated by its wireless transmissions and receptions; so the other energy consumption factors such as for sensing and processing are neglected. According to this radio model, energy consumed by a node for transmission and reception are as follows:

Energy consumption for transmitting (e_{tx}) n-bit data over a distance d is

$$e_{tx}(n,d) = e_{elec}\, n + e_{amp}\, n\, d^2 \tag{1a}$$

Energy consumption for receiving (e_{rx}) n-bit data is

$$e_{rx}(n) = e_{elec}\, n \tag{1b}$$

4 Probability Density Function Based Node Deployment (PDFND)

In this section the node deployment strategy based on probability density function and the implementing algorithm is presented.

4.1 Proposed Probability Density Function (PDF)

The mathematical domain under consideration is divided into a number of concentric circles centered at (0, 0) and having radii increasing arithmetically from R to NR with a difference of R. In the mathematical domain, if (x, y) be a point lying between circle-(i-1) and circle-i, then the probability density at that point is

$$f(x,y;N,i,R)=\frac{k(2i-1)}{N^2i^4},\ \forall (i-1)^2R^2<x^2+y^2\le i^2R^2 \tag{2a}$$

where $i=1,2,\dots,N$ and k is a constant given by $k=\dfrac{N^2}{\pi R^2\left[1+\frac{3^2}{2^4}+\frac{5^2}{3^4}+\cdots+\frac{(2N-1)^2}{N^4}\right]}$.

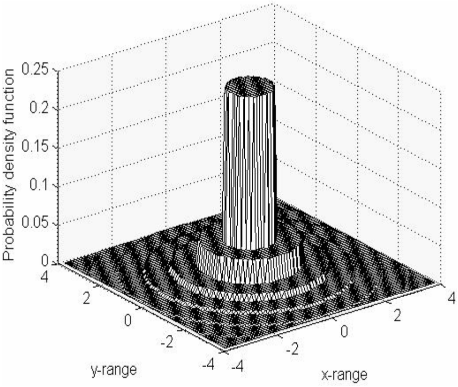


Fig. 2. Surface plot of the PDF

Figure 2 is the 3-D graph of the proposed PDF. The characteristics of the PDF show fall in the functional value with rise in the value of i implying lower probability and vice versa.

Theorem 1: The value of constant k is given by $k=\dfrac{N^2}{\pi R^2\left[1+\frac{3^2}{2^4}+\frac{5^2}{3^4}+\cdots+\frac{(2N-1)^2}{N^4}\right]}$.

Proof: Let p_i denotes the probability at point (x, y) and the point lies between circles (i-1) and i. From the proposed PDF, the probability p_i is

$$p_i=\frac{k(2i-1)}{N^2i^4}\iint f(x,y)\,dx\,dy.$$

In the above relation $\iint f(x,y)\,dx\,dy$ is the domain area. For circular domain, area is $\iint f(x,y)\,dx\,dy = (2i-1)\pi R^2$. The probability of x and y is given as $p_i = \frac{k(2i-1)^2 \pi R^2}{N^2 i^4}$. By fundamental rule of probability, $\sum\sum f(x,y) = 1$

$$\textstyle\sum_{i=1}^{N} p_i = 1 \; or, \; \displaystyle\sum_{i=1}^{N} \frac{k(2i-1)^2 \pi R^2}{N^2 i^4} = 1$$

$$k = \frac{N^2}{\pi R^2 \sum_{i=1}^{N} \frac{(2i-1)^2}{i^4}} = \frac{N^2}{\pi R^2 \left[1 + \frac{3^2}{2^4} + \frac{5^2}{3^4} + \cdots + \frac{(2N-1)^2}{N^4}\right]}.$$

Theorem 2: If two random variables X and Y follow a proposed PDF with parameters N and i, then the cumulative distribution function (CDF) of X and Y is given as

$$F[X \le x, Y \le y] = \frac{k\pi R^2}{N^2}\left[\sum_{j=1}^{i}\left[\frac{(2j-1)^2}{j^4}\right] + \frac{\left(\eta^2 - i^2\right)}{i^4}\right]$$

where (x, y) such that $0 \le x^2 + y^2 \le \eta^2 R^2$, where $i \le \eta \le i+1$.

Proof: The probability of two discrete random variables X and Y for a particular value within a given range of i is considered as

$$\frac{k\pi R^2}{N^2} \sum_{j=1}^{i} \frac{(2j-1)^2}{j^4}. \tag{2b}$$

The probability of the variables X and Y between a given domain area iR and ηR, where $\eta R > iR$ is given as

$$\frac{k}{N^2 i^4}\left[\pi(\eta R)^2 - \pi(iR)^2\right]. \tag{2c}$$

So, the CDF of X and Y using equations (2b) and (2c) is obtained as

$$F[X \le x, Y \le y] = \frac{k\pi R^2}{N^2}\left[\sum_{j=1}^{i}\left[\frac{(2j-1)^2}{j^4}\right] + \frac{\left(\eta^2 - i^2\right)}{i^4}\right].$$

Theorem 3: If the two random variables X and Y follow the proposed PDF with parameters N and i, then the expectation of X and Y is given as

$$E[XY] = \frac{kR^4}{N^2} \sum_{i=1}^{N}\left[\frac{2}{i} + \frac{2}{i^3} - \frac{3}{i^2} - \frac{1}{2i^4}\right].$$

Theorem 4: If the two random variables X and Y follow a proposed PDF with parameter N and i, then the covariance of X and Y is given as

$$\mathrm{Cov}(X,Y)=\frac{kR^4}{N^2}\sum_{i=1}^{N}\left[\frac{2}{i}+\frac{2}{i^3}-\frac{3}{i^2}-\frac{1}{2i^4}\right]-\frac{4kR^3}{3N^2}\left[\sum_{i=1}^{N}\left[\frac{3}{i^2}+\frac{3}{i^3}-\frac{1}{i^4}\right]\right]^2.$$

Due to page limitation, the proofs of Theorems 3 and 4 could not be incorporated.

4.2 Proposed PDF-Based Deployment

The probability density function (PDF) proposed in the previous sub-section is discrete in nature. Our objective is to deploy sensor nodes in the layered network area (Figure 1) with the proposed PDF. The PDF is mapped with the node deployment in a layered network area as follows: the parameter i corresponds to layer number where i=1,2,…,N. Here i=1 indicates the layer nearest to the sink and i=N indicates the layer farthest from the sink. The parameter R corresponds to the width r of the annuli/layer. The density function is designed as a non-uniform one such that higher value of PDF implies a node deployed around the sink and the lower value of PDF is observed as one moves away from the sink. The probability density of deploying a node at point (x, y) for a value of i is

$$\frac{k(2i-1)}{N^2 i^4}\text{, where } i=1,2,\ldots,N. \tag{3a}$$

In equation (3a), k is constant and i is the number of layers in the network. The probability density for nodes deployed within layer-i is

$$\frac{k(2i\text{-}1)A_i}{N^2 i^4}$$

where A_i is the area of layer-i and

$$k=\frac{N^2}{\pi r^2\left[1+\frac{3^2}{2^4}+\frac{5^2}{3^4}+\cdots+\frac{(2N-1)^2}{N^4}\right]}$$

where r is the width of a layer. The area of layer-i is $A_i=\left[(2i-1)\pi r^2\right]$. The probability of deploying nodes at layer-i is $p_i=\frac{k(2i\text{-}1)A_i}{N^2 i^4}$. Replacing the value of A_i, the probability of deployment of nodes at layer-i is

$$p_i=\frac{k(2i-1)^2\pi r^2}{N^2 i^4}. \tag{3b}$$

The number of nodes in layer-i (T_i) is equal to the probability of deploying nodes at layer-i (p_i) multiplied by the total number of nodes (T_{total}) that are to be deployed within the network area i.e.,

$$T_i=p_i\times T_{total}. \tag{3c}$$

The probability expression implies that nodes in a layer are uniformly distributed with equal probability, but this probability varies in different layers.

4.3 Algorithm for Node Deployment

```
1: input a, R(r), $T_{total}$        /* area parameter, width of layer, and total number of
                                        nodes to be deployed */
/* assume r= $R_c$ ; section 3.2.1 */
2: compute $N = \frac{a}{2\times r}$   /* N: no. of layers; section 3.2*/
3: compute k                          /* k: constant; section 4.2 */
4: for i=1; $i \le N$; i + +
5:     compute $p_i$          /* $p_i$ : probability of deploying nodes at layer-i; equation 3(b) */
6:     compute $T_i = p_i \times T_{total}$ /* $T_i$: no. of nodes to be deployed in layer i */
7: end for
```

4.4 Illustrative Example

Let us consider a square area of 200×200 sq unit where 100 nodes are deployed employing the PDF. The area is covered by layers with width (r) of 25 units. The number of layers is $N = \frac{a}{2\times r} = \frac{200}{2\times 25} = 4$ (section 3.2). Replacing the values of N and R (r) in equation (2a), we get k=0.004. The probability of node deployment at layer-i is computed using equation (3b), as $p_i = \frac{0.49\times(2i-1)^2}{i^4}$. Probability of node deployment at layer-1, $p_1 = 0.49$. Probability of node deployment at layer-2, $p_2 = 0.27$. Similarly $p_3 = 0.15$, $p_4 = 0.09$. Using equation (3c), the number of nodes deployed in the 4 layers is: in layer-1 $T_1 = 0.49\times 100 = 49$, in layer-2 $T_2 = 27$, in layer-3 $T_3 = 15$ and in layer-4 $T_4 = 9$.

We observe that the number of nodes deployed in each layer conforms to the non-uniform nature of the PDF. Therefore, it fulfils our objective of deploying more nodes towards sink and lesser nodes at locations away from the sink.

5 Performance Analysis

Performance of the present node deployment strategy is measured based on two parameters such as energy balance and network lifetime. Both qualitative and quantitative analysis is presented here.

5.1 Qualitative Analysis

In this sub-section, parameters involved in maintaining energy balance and enhancing network lifetime are identified.

5.1.1 Energy Balancing

Our objective is balancing energy consumption among all the network layers so that network lifetime is maximized. A WSN with layered architecture is said to be energy balanced when all nodes of the network use up their energy at the same time [9].

Nodes of all the layers except those belonging to the farthest layer from the sink, spend their energy for transmitting their own data, receiving data from nodes of adjacent layers farther from the sink and forwarding the received data. Nodes of the farthest layer from the sink spend their energy only for transmitting their own data.

Energy required for a node to transmit (e_{tx}) n-bit of data over the distance R_c is- $e_{tx}(n, R_c) = e_{elec}\, n + e_{amp}\, n\, R_c^2 = e_t n$, where $e_t = e_{elec} + e_{amp}\, R_c^2$ [using equation (1a)].

Similarly, energy required for a node to receive (e_{rx}) n-bit data is- $e_{rx}(n) = e_{elec}\, n = e_r n$, where $e_r = e_{elec}$ [using equation (1b)].

Let ECR_i denotes energy consumption rate i.e., energy consumption per unit time-interval q(t) (section 3.1) of layer-i. As the last layer consumes energy for transmitting its own sensed data, for i=N,

$$ECR_N = e_t \times \rho \times A_N \tag{4a}$$

where $\rho \times A_N$ is the number of bits transmitted for the layer N per second. As the rest of the layers consume energy both for transmitting its data and for receiving and forwarding the other outer layers data, for i=1,2,…,(N-1)

$$ECR_i = e_t \times \rho \times A_i + \left[(e_t + e_r) \sum_{j=i+1}^{N} (\rho \times A_j) \right] \tag{4b}$$

where $e_t \times \rho \times A_i$ for transmitting its own data and $(e_t + e_r) \sum_{j=i+1}^{N} (\rho \times A_j)$ for receiving & forwarding the other outer layers data.

All the nodes of the network use up their energy at the same time [9], means that the ratio of total initial energy content of a layer and ECR of that layer is same for all layers in the network. So for energy balancing, the following condition must be satisfied-

$$\frac{T_1 \times E_{Initial}}{ECR_1} = \frac{T_2 \times E_{Initial}}{ECR_2} = \cdots = \frac{T_i \times E_{Initial}}{ECR_i} \tag{4c}$$

where T_i is the number of nodes in layer-i and $E_{Initial}$ is the initial energy in each node.

From equation (4c), the condition required for balancing energy throughout the network is-

$$\frac{ECR_i}{T_i} = \frac{ECR_{i+1}}{T_{i+1}}$$

$$\frac{T_i}{T_{i+1}} = \frac{e_t \times \rho \times A_i + \left[(e_t + e_r) \sum_{j=i+1}^{N} (\rho \times A_j) \right]}{e_t \times \rho \times A_{i+1} + \left[(e_t + e_r) \sum_{j=i+2}^{N} (\rho \times A_j) \right]} \quad \text{[using equation (4b)].}$$

Simplifying the above relation by dropping ρ and replacing A_i by $(2i-1)\pi r^2$ in sequence results in-

$$\frac{T_i}{T_{i+1}} = \frac{e_t \times (2i-1)\pi r^2 + \left[(e_t + e_r) \sum_{j=i+1}^{N} (2j-1)\pi r^2 \right]}{e_t \times (2i+1)\pi r^2 + \left[(e_t + e_r) \sum_{j=i+2}^{N} (2j-1)\pi r^2 \right]}$$

$$\frac{T_i}{T_{i+1}}=\frac{e_t\times(2i-1)+(e_t+e_r)\sum_{j=i+1}^{N}(2j-1)}{e_t\times(2i+1)+(e_t+e_r)\sum_{j=i+2}^{N}(2j-1)}. \tag{4d}$$

From the proposed PDF (section 4 & Appendix) LHS of equation (4d) is evaluated as

$$\frac{T_i}{T_{i+1}}=\frac{(2i-1)^2(i+1)^4}{(2i+1)^2 i^4}.$$

The RHS of equation (4d)

$$\frac{e_t\times(2i-1)+(e_t+e_r)\sum_{j=i+1}^{N}(2j-1)}{e_t\times(2i+1)+(e_t+e_r)\sum_{j=i+2}^{N}(2j-1)}.$$

Both the LHS and RHS are ratios where denominator and numerator have same power of i and therefore, these two terms are approximately equal. So if the nodes are deployed employing the proposed PDF, it fulfils the objective of energy balancing.

5.1.2 Network Lifetime

Network lifetime is defined in terms of network coverage. It is the time till the proportion of dead nodes exceeds a certain threshold, which may result in loss of coverage of a certain region, and/or network partitioning [4]. Energy consumption rate by the nodes of a layer can be calculated using equations (4a) and (4b). Energy consumption per unit q(t) by each node in layer-i (ER_i) is given as

$$ER_i=\frac{ECR_i}{T_i\times q(t)} \quad \text{for i=1, 2, …, N} \quad \text{[from equation (4c)].}$$

As our scheme is energy balanced, the lifetime of a node is same as lifetime of a layer or network lifetime. The lifetime of each node in layer-i is

$$LT_i=\frac{E_{Initial}}{ER_i}.$$

Putting the value of ER_i in the above relation, we have

$$LT_i=\frac{E_{Initial}\times T_i}{ECR_i}q(t)$$

$$LT_i=\frac{E_{Initial}\times T_i}{e_t\times\rho\times A_i+\left[(e_t+e_r)\sum_{j=i+1}^{N}(\rho\times A_j)\right]}q(t). \tag{5}$$

The parameters- $E_{Initial}$, T_i, q(t), e_t, e_r, ρ, A_i affect the lifetime of a node or layer. As the values of e_t, e_r, $E_{Initial}$, A_i are constant, we have concentrated on rest of the three parameters as mentioned below.

5.1.2.1 Number of Nodes in Each Layer (T_i). From equation (5) it is observed that with the increase in number of nodes in each layer, keeping the other parameters fixed, the network lifetime increases. Therefore, network lifetime is directly proportional to the number of nodes in each layer.

5.1.2.2 Interval of Periodic Data Collection $\left(q(t)\right)$. From equation (5) it is inferred that as the interval of periodic data collection rate increases, the lifetime of the network also increases keeping the other parameters unchanged. Increase of interval of periodic data collection refers to less data collection, thereby resulting in reduced energy consumption.

5.1.2.3 Information Generation rate(ρ). In equation (5) lifetime of the network is inversely proportional to the information generation rate which means that with increase in information generation rate, node has to sense more data. So, more energy consumption takes place leading to shortening of network lifetime.

5.2 Quantitative Analysis

The effectiveness of the proposed node deployment scheme, reported in section 4.2 is evaluated through simulation. Moreover all the theoretical claims made through qualitative analysis presented in section 5.1 are justified by simulation results.

5.2.1 Simulation Environment

The simulation is performed using MATLAB (version 7.1). Simulation results of PDFND are compared with two existing schemes namely node deployment with Gaussian distribution (NDGD) [12] and node deployment with Uniform distribution (NDUD) [15]. We assume perfect MAC layer issues while considering this work. Extensive simulation has been performed and average results of 2000 independent runs have been taken while plotting the simulation graphs.

5.2.2 Simulation Metrics

To evaluate the performance of PDFND, energy balance and network lifetime as defined in sections 5.1.1 and 5.1.2 respectively have been considered as performance metrics. The number of deployed nodes is varied from 120 to 700. We define two more parameters-energy consumption rate per node in a layer and average residual energy of each layer for evaluating the extent of energy balance in the network**.** Though the concept of the parameter energy consumption rate per node is used in section 5.1, the same is formally defined here.

Energy consumption rate per node (ER): It is defined as energy consumption by a node per unit time. It is evaluated as $\text{ER}=\dfrac{\text{Energy consumption of a layer}}{\text{Number of nodes in the layer}}$

Average residual energy per node (Avg RE per node): It is defined as the residual energy of a node in a layer after network lifetime ends. It is evaluated as

$$\text{Avg RE per node}=\frac{\text{Sum of residual energy of nodes in a layer}}{\text{Number of nodes in the layer}}$$

Two sets of experiments are conducted for evaluating the performance of the present scheme and two other competitor schemes. One set of experiment measures energy balancing in the network and the last set verifies the enhancement of network lifetime. For each set, experiments have been conducted for two different network sizes viz. network with 3 and 7 layers deploying 120 and 700 nodes respectively. The parameters and their corresponding values used for simulation are listed in Table 1.

Table 1. Simulation Parameters

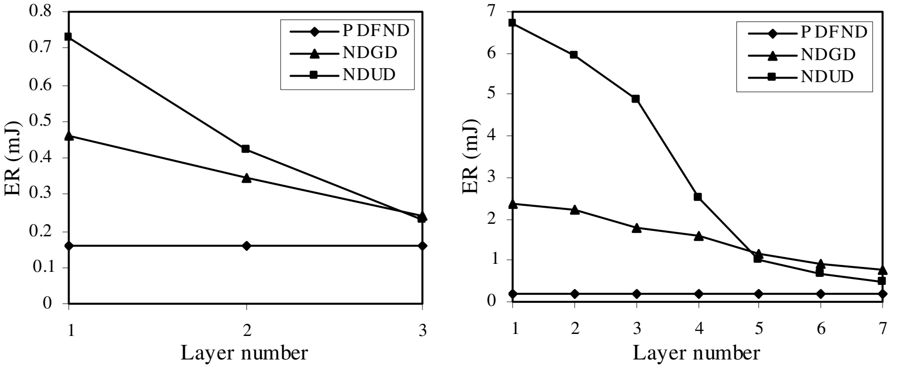
Parameters	Value
Initial energy $(E_{Initial})$	50 J
e_{elec}	50 nJ/bit
e_{amp}	10 pJ/bit/m^2
Communication range of a node (R_c)	160 m
Sensing range of a node (R_s)	80 m
Information generation rate (ρ)	0.1 bits/sec
Interval of periodic data collection (q(t))	1 sec
Network area	1 km^2 ~ 5 km^2

5.2.3 Energy Balancing

In this sub-section energy balancing of the scheme is evaluated in terms of the following two parameters.

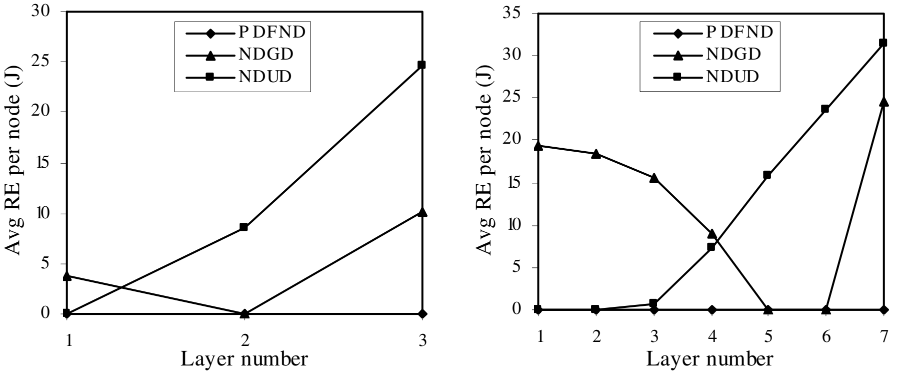
5.2.3.1 ER. Figure 3 shows ER for different network sizes. We observe that for PDFND the ER for a particular network size is constant for all layers and this rate varies with network sizes. For example, for network with 3 and 7 layers the ER is 0.16 mJ and 0.21 mJ respectively. These results imply that the ER for PDFND increases with increase in network area. In NDGD, ER for different layers varies considering a particular network size. So for NDGD, irrespective of network size, node in layer-1 has the maximum ER and node in farthest layer has the lowest ER. Therefore, nodes deployed in layers nearer to the sink drain out their energy much more quickly in comparison to nodes deployed in layers farther away from the sink. Similar observation holds for NDUD. This justifies our claim that PDFND is energy balanced whereas this is not true for both NDGD and NDUD.

5.2.3.2 Avg RE Per Node. Figure 4 illustrates the comparison of PDFND with NDGD and NDUD considering avg RE per node as a performance metric. Node deployment using NDGD or NDUD, results in an abrupt change in avg RE per node in each layer, independent of network size. In NDGD, energy of nodes in layer-2 (Figure 4(a)) and nodes in each of the layers-5 & 6 (Figure 4(b)) is drained out completely, though the nodes of other layers in the network retain sufficient energy for carrying out normal network operation that causes the phenomenon of energy hole. Similarly for NDUD, energy of nodes in layer-1 (Figure 4(a)) and nodes in each of layers-1, 2 (Figure 4(b)) is drained out completely though the nodes of other layers in the network have adequate energy for normal network operation. So NDUD also suffers from energy hole problem.



(a) 3-layer network (b) 7-layer network

Fig. 3. Energy consumption rate per node for various network sizes

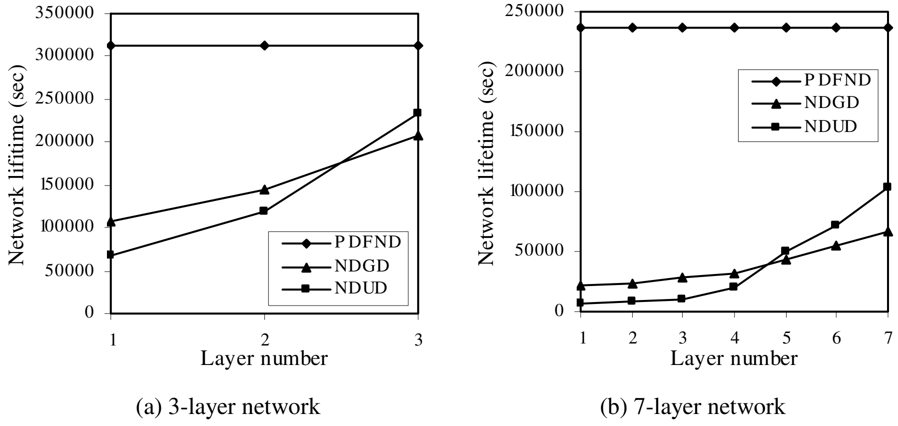


(a) 3-layer network (b) 7-layer network

Fig. 4. Avg RE per node for various network sizes

5.2.4 Network Lifetime

We consider data collection interval (q(t)) as 1 sec. The graphs in Figures 5(a) and 5(b) represent the network lifetime for two different network sizes. The network lifetime of PDFND is 50.65% and 55.02% more than that of NDGD and NDUD respectively for 3-layer network. For 7-layer it is 83.91% and 83.61% more than that of NDGD and NDUD respectively. Moreover, in PDFND the flat nature of the plot ensures that network lifetime terminates in more or less same time in all the layers as compared to NDGD and NDUD. This ensures energy in PDFND is balanced to a greater extent than both the competent schemes.



(a) 3-layer network (b) 7-layer network

Fig. 5. Network lifetime for various network sizes

6 Conclusion

In this work we have proposed a pre-determined node deployment scheme in wireless sensor network using a probability density function defined by us. The target of the scheme is to achieve energy balancing and enhance network lifetime while maintaining coverage and connectivity. We have provided theoretical formulation of energy balancing and network lifetime. Based on this analysis we have derived certain constraints, involving network parameters, to be satisfied to achieve the target. An algorithm is also developed to implement the scheme. We claim that our scheme successfully achieves the target. The claim is substantiated by performing both qualitative and quantitative analysis. Finally the results of quantitative analysis are compared with two existing works [12] [15] of node deployment that clearly demonstrates our scheme's dominance over the existing works.

As a future extension of our work, the deployment strategy may be made more realistic by considering 3-D environment. Moreover, the scheme may be analyzed for further improvement considering various QoS parameters.

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Appendix

From equations 3(b) and 3(c), the number of nodes deployed in layer-i and layer-(i+1) is given as

$$T_i = p_i \times T_{total} = \frac{k(2i-1)A_i}{N^2 i^4} \times T_{total} \text{ and } T_{i+1} = p_{i+1} \times T_{total} = \frac{k(2i-1)A_{i+1}}{N^2 i^4} \times T_{total}$$

respectively.

Dividing T_i by T_{i+1} and canceling k, T_{total}, and N^2 from denominator and numerator in sequence, we have

$$\frac{T_i}{T_{i+1}} = \frac{p_i}{p_{i+1}} = \frac{(2i-1)(i+1)^4 A_i}{(2i+1)i^4 A_{i+1}}.$$

The above relation is simplified by replacing the value of $A_i = (2i-1)\pi r^2$ and $A_{i+1} = (2i+1)\pi r^2$ as

$$\frac{T_i}{T_{i+1}} = \frac{(2i-1)^2 (i+1)^4}{(2i+1)^2 i^4}.$$