Fuzzy Optimal Solution of Fuzzy Transportation Problems with Transshipments

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Abstract. In this paper a new method, named as Mehar's method, is proposed for solving fuzzy transportation problems with transshipments. Also, it is shown that it is better to use Mehar's method as compared to the existing method.

1 Introduction

In conventional transportation and transshipment problems [1] it is assumed that decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real world applications, all these parameters may not be known precisely due to uncontrollable factors. To deal with such situations several authors have represented the different parameters by fuzzy numbers [18] and proposed different methods for solving fuzzy transportation problems [2,9,11,12,13,15,16,17] and fuzzy transshipment problems [3,4,5,6,7,8,10,14].

2 Proposed Method

Kumar et al. [10] proposed fuzzy linear programming approach for finding the fuzzy optimal solution of fuzzy transportation problems with transshipment. In the existing method [10] the fuzzy linear programming formulation of the chosen fuzzy transportation problem with transshipment is converted into four crisp linear programming formulations of crisp transportation problems with transshipment and then all the obtained crisp linear programming problems are solved by Simplex method [1]. But in the literature, it is pointed out that it is better to use modified distribution method [1] for finding the solution of crisp transportation problems as compared to Simplex method.

Due to the same reason, in this section a new method, named as Mehar's method, based on modified distribution method, is proposed for finding the fuzzy optimal solution of same type of problems.

The steps of the proposed method are as follows:

Step 1. Split Table 3 [10] into four crisp transportation tables i.e., Table 1, Table 2, Table 3 and Table 4 respectively.

Step 2. Find the optimal solution a_{ij}; $b_{ij} - a_{ij}$; $c_{ij} - b_{ij}$ and $d_{ij} - c_{ij}$ by solving

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the crisp transportation problems, shown by Table 1; Table 2; Table 3 and Table 4 respectively, by using modified distribution method.
where, $\lambda_{ij} = \frac{a'_{ij}+b'_{ij}+c'_{ij}+d'_{ij}}{4}$, $\rho_{ij} = \frac{b'_{ij}+c'_{ij}+d'_{ij}}{4}$, $\delta_{ij} = \frac{c'_{ij}+d'_{ij}}{4}$, and $\xi_{ij} = \frac{d'_{ij}}{4}$.

Step 3. Find the values of a_{ij}, b_{ij}, c_{ij} and d_{ij} by solving the equations obtained in Step 2 and also find $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$.

Step 4. Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ij}$ in $\sum_{i=1}^{m+n} \sum_{j=1}^{m+n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$.

Table 1. First crisp transportation table

	S_1	S_2	$\cdots$	S_m	D_1	D_2	$\cdots$	D_n	
S_1	0	λ_{12}	$\cdots$	λ_{1m}	$\lambda_{1(m+1)}$	$\lambda_{1(m+2)}$	$\cdots$	$\lambda_{1(m+n)}$	q_1
S_2	λ_{21}	0	$\cdots$	λ_{2m}	$\lambda_{2(m+1)}$	$\lambda_{2(m+2)}$	$\cdots$	$\lambda_{2(m+n)}$	q_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	λ_{m1}	λ_{m2}	$\cdots$	0	$\lambda_{m(m+1)}$	$\lambda_{m(m+2)}$	$\cdots$	$\lambda_{m(m+n)}$	q_m
D_1	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	p_1
D_2	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	p_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
D_n	$\lambda_{(m+n)1}$	$\lambda_{(m+n)2}$	$\cdots$	$\lambda_{(m+n)m}$	$\lambda_{(m+n)(m+1)}$	$\lambda_{(m+n)(m+2)}$	$\cdots$	0	p_1
	p_1	p_1	$\cdots$	p_1	q'_1	q'_2	$\cdots$	q'_n	

Table 2. Second crisp transportation table

	S_1	S_2	$\cdots$	S_m	D_1	D_2	$\cdots$	D_n	
S_1	0	ρ_{12}	$\cdots$	ρ_{1m}	$\rho_{1(m+1)}$	$\rho_{1(m+2)}$	$\cdots$	$\rho_{1(m+n)}$	$r_1 - q_1$
S_2	ρ_{21}	0	$\cdots$	ρ_{2m}	$\rho_{2(m+1)}$	$\rho_{2(m+2)}$	$\cdots$	$\rho_{2(m+n)}$	$r_2 - q_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	ρ_{m1}	ρ_{m2}	$\cdots$	0	$\rho_{m(m+1)}$	$\rho_{m(m+2)}$	$\cdots$	$\rho_{m(m+n)}$	$r_m - q_m$
D_1	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$p_2 - p_1$
D_2	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$p_2 - p_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
D_n	$\rho_{(m+n)1}$	$\rho_{(m+n)2}$	$\cdots$	$\rho_{(m+n)m}$	$\rho_{(m+n)(m+1)}$	$\rho_{(m+n)(m+2)}$	$\cdots$	0	$p_2 - p_1$
	$p_2 - p_1$	$p_2 - p_1$	$\cdots$	$p_2 - p_1$	$r'_1 - q'_1$	$r'_2 - q'_2$	$\cdots$	$r'_n - q'_n$	

Table 3. Third crisp transportation table

	S_1	S_2	$\cdots$	S_m	D_1	D_2	$\cdots$	D_n	
S_1	0	δ_{12}	$\cdots$	δ_{1m}	$\delta_{1(m+1)}$	$\delta_{1(m+2)}$	$\cdots$	$\delta_{1(m+n)}$	$s_1 - r_1$
S_2	δ_{21}	0	$\cdots$	δ_{2m}	$\delta_{2(m+1)}$	$\delta_{2(m+2)}$	$\cdots$	$\delta_{2(m+n)}$	$s_2 - r_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	δ_{m1}	δ_{m2}	$\cdots$	0	$\delta_{m(m+1)}$	$\delta_{m(m+2)}$	$\cdots$	$\delta_{m(m+n)}$	$s_m - r_m$
D_1	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$p_3 - p_2$
D_2	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$p_3 - p_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
D_n	$\delta_{(m+n)1}$	$\delta_{(m+n)2}$	$\cdots$	$\delta_{(m+n)m}$	$\delta_{(m+n)(m+1)}$	$\delta_{(m+n)(m+2)}$	$\cdots$	0	$p_2 - p_1$
	$p_3 - p_2$	$p_3 - p_2$	$\cdots$	$p_3 - p_2$	$s'_1 - r'_1$	$s'_2 - r'_2$	$\cdots$	$s'_n - r'_n$	

Table 4. Fourth crisp transportation table

	S_1	S_2	$\cdots$	S_m	D_1	D_2	$\cdots$	D_n	
S_1	0	ξ_{12}	$\cdots$	ξ_{1m}	$\xi_{1(m+1)}$	$\xi_{1(m+2)}$	$\cdots$	$\xi_{1(m+n)}$	$t_1 - s_1$
S_2	ξ_{21}	0	$\cdots$	ξ_{2m}	$\xi_{2(m+1)}$	$\xi_{2(m+2)}$	$\cdots$	$\xi_{2(m+n)}$	$t_2 - s_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	ξ_{m1}	ξ_{m2}	$\cdots$	0	$\xi_{m(m+1)}$	$\xi_{m(m+2)}$	$\cdots$	$\xi_{m(m+n)}$	$t_m - s_m$
D_1	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$p_4 - p_3$
D_2	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$p_4 - p_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
D_n	$\xi_{(m+n)1}$	$\xi_{(m+n)2}$	$\cdots$	$\xi_{(m+n)m}$	$\xi_{(m+n)(m+1)}$	$\xi_{(m+n)(m+2)}$	$\cdots$	0	$p_4 - p_3$
	$p_4 - p_3$	$p_4 - p_3$	$\cdots$	$p_4 - p_3$	$t'_1 - s'_1$	$t'_2 - s'_2$	$\cdots$	$t'_n - s'_n$	

3 Advantages of the Proposed Method

To show the advantage of the proposed method over existing method [10] a fuzzy transportation problem with transshipment, [Example 5.1, 10], is solved by using the proposed method and it is shown that the obtained results are same while it is easy to apply the proposed method as compared to the existing method [10].

3.1 Results

On solving the fuzzy transshipment problem [10], the obtained fuzzy optimal solution and minimum total fuzzy transportation cost is $\tilde{x}_{11} = (16, 30, 44, 56)$, $\tilde{x}_{13} = (6, 8, 10, 20)$, $\tilde{x}_{14} = (4, 10, 12, 14)$, $\tilde{x}_{16} = (0, 2, 8, 8)$, $\tilde{x}_{21} = (0, 0, 0, 2)$, $\tilde{x}_{22} = (16, 30, 44, 58)$, $\tilde{x}_{26} = (0, 4, 8, 10)$, $\tilde{x}_{33} = (16, 30, 44, 58)$, $\tilde{x}_{44} = (16, 30, 44, 58)$, $\tilde{x}_{54} = (6, 6, 6, 6)$, $\tilde{x}_{55} = (16, 30, 44, 58)$, $\tilde{x}_{66} = (16, 30, 44, 58)$and remaining are $(0, 0, 0, 0)$ and $(8, 38, 90, 166)$.

3.2 Discussion

It can be easily seen that the results of the fuzzy transportation problems with transshipment, obtained by using the existing method [10] and the proposed method are same but as discussed in, Section 2, it is easy to use the proposed method as compared to existing method [10].

4 Conclusion

The shortcoming of an existing method [10] for finding the fuzzy optimal solution of fuzzy transportation problem with transshipment are pointed out and to overcome the shortcoming of the existing method a new method, named as Mehar's method, is proposed for solving the same type of problem.

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