Chapter 2
Chaotic Signals and Their Use in Secure Communications

This chapter introduces nonlinear dynamical systems known as chaotic systems and describes their suitability for application to secure communications. A nonlinear or chaotic signal is characterised by its high sensitivity to parameter and initial condition perturbations, the random like nature and broadband spectrum [1]. From a nonlinear dynamical perspective, chaotic motion is a motion which possesses at least one positive Lyapunov exponent. Furthermore, for a given set of parameters and initial conditions chaotic motion is highly deterministic. Among other applications, these properties make chaotic systems suitable for the application in secure communications [2-9]. One of the main reasons for the increased security of communication provided by the chaotic signals is their broadband nature. In many cases the broadband nature of a chaotic system allows for the effective spectral cover up of the message by the chaotic carrier. In addition, the high sensitivity of chaotic signals to parameter and initial condition perturbations often can act as the encryption keys. In this chapter, the distinguishing features of chaotic systems are first presented and some approaches, used to identify chaotic behavior, are introduced. Furthermore, the approaches and the suitability of chaotic systems to the implementation within secure communication systems are examined. Finally, some of the noise reduction techniques, used to filter chaotic communication systems, are introduced.

2.1 Chaotic Systems

One of the earliest observations of nonlinear behaviour was made in 1961 by the Japanese electrical engineer, Yoshisuke Ueda. The observation occurred when Ueda conducted analog computer simulations of the Duffing/Van der Pol mixed type equation:

$$\frac{d^2\upsilon}{dt^2} - \mu(1-\gamma\upsilon^2)\frac{d\upsilon}{dt} + \upsilon^3 = B\cos(\nu t) \qquad (2.1.1)$$

where: $\mu = 0.2$, $\gamma = 8$, $B = 0.35$ and $\nu = 1.02$.

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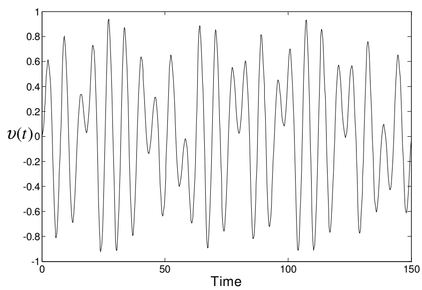
The phenomenon output by the computer subsequently became known as chaos [10]. At around the same time American meteorologist, Edward Lorenz, independently discovered chaos in a third order autonomous system. Since then, a large number of chaotic systems have appeared in the literature [1].

Chaotic systems can be divided into those described by differential equations, known as flows, and those described by difference equations, known as maps [1,11]. The dynamics of a chaotic system can be represented in the time domain as time series or in phase space as a strange attractor [1,11]. The time series and the corresponding "broken-egg" strange attractor, obtained by numerically integrating equation 2.1.1, are shown in Figure 2.1a and Figure 2.1b, respectively.



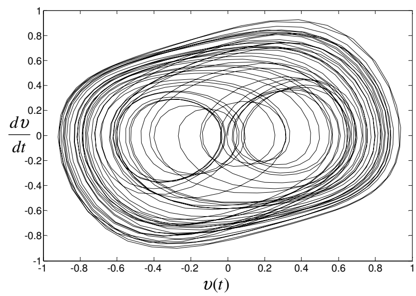


Fig. 2.1a The broken egg chaotic time series, $v(t)$ **Fig. 2.1b** The broken egg strange attractor

The time series graph of Figure 2.1a is obtained by simply plotting the amplitude of the signal against time. On the other hand, the strange attractor is obtained by plotting two or more of the state variables of the system against each other. The state variables of the system are most often defined as the first or the second derivative of the time series, or a combination of those. It is readily verifiable that the system of equation 2.1.1 can also be represented in the state-space form of equation 2.1.2:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= \mu y - \mu \gamma x^2 y - x^3 + B\cos(vz) \\ \dot{z} &= 1 \end{aligned} \tag{2.1.2}$$

where $x = v$, $y = \dot{v}$ and $z = t$ are the state variables of the system.

2.1.1 Chaotic Flows

The chaotic system of equation 2.1.1 (2.1.2) is an example of a chaotic flow. The Lorenz chaotic flow, which is an example of another well known flow, is now presented and its broadband nature and high sensitivity to parameter perturbations demonstrated. Further examples of some of the well known flows, such as the Rossler [12] and the Rucklidge flow [13], can be found in [1,12,13].

The dynamics of the Lorenz chaotic system, described by equation 2.1.3:

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= rx - y - xz\,, \\ \dot{z} &= -b\,z + xy \end{aligned} \tag{2.1.3}$$

are shown in Figure 2.2 when the parameter $\sigma = 10$, $r = 28$ and $b = 8/3$.

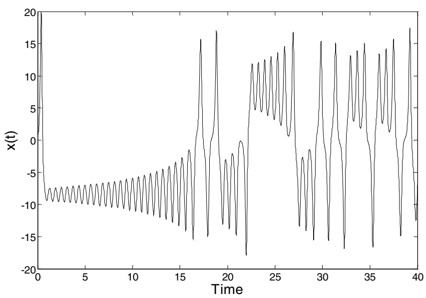


Fig. 2.2a The Lorenz chaotic time series, $x(t)$

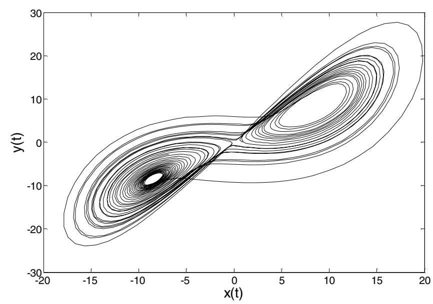


Fig. 2.2b The Lorenz strange attractor

The dynamics of the strange attractor of a chaotic flow are referred to as a trajectory [1]. The trajectory of a chaotic flow is characterised by a smooth, continuous nature. An example of a chaotic flow is a turbulent flow of water from a pipe [1].

The broadband nature of the Lorenz chaotic flow can be observed from Figure 2.3 where the power spectral density of the Lorenz x signal has been plotted against the normalized frequency. Furthermore, the high sensitivity of the Lorenz chaotic flow to parameter perturbations is demonstrated in Figure 2.4. It can be observed from Figure 2.4 that a small alteration to a parameter of the system causes the system to generate an entirely different chaotic signal. It is shown in chapter 6 how this property of chaotic signals can be used in the design of secure chaotic communication systems.

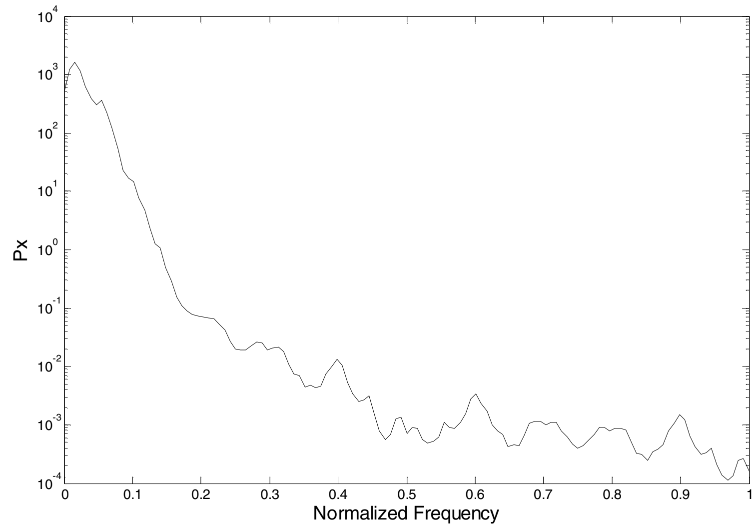


Fig. 2.3 The power spectral density (Px) of the Lorenz x signal versus the normalized frequency

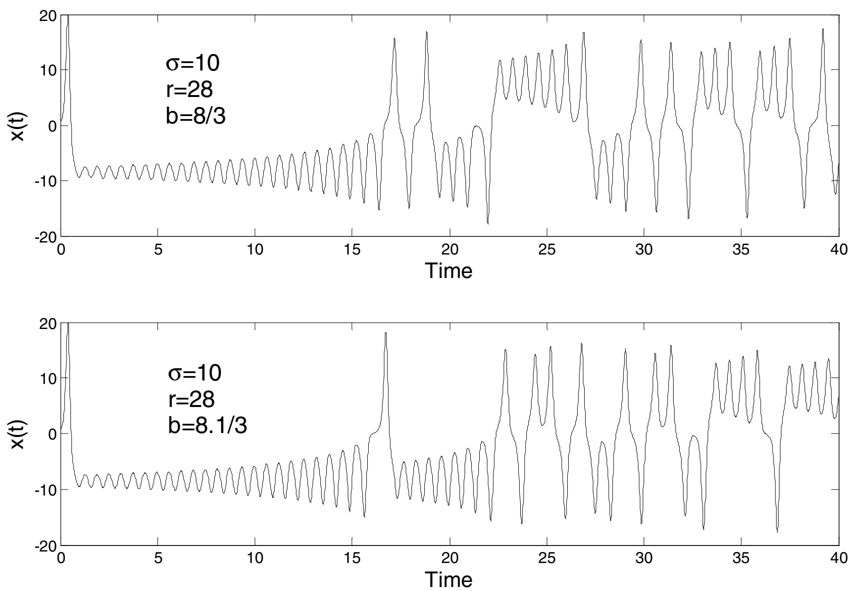


Fig. 2.4 Sensitive dependence on the parameter perturbations within the Lorenz chaotic flow

2.1.2 Chaotic Maps

The dynamics of one of the most well known chaotic maps, the Hénon map:

$$X_{n+1} = 1 - aX_n^2 + bY_n$$
$$Y_{n+1} = X_n \tag{2.1.4}$$

are shown in Figure 2.5 when the parameter $a = 1.4$ and $b = 0.3$.

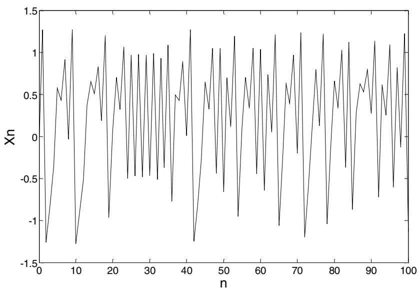


Fig. 2.5a The Hénon chaotic time series, X_n

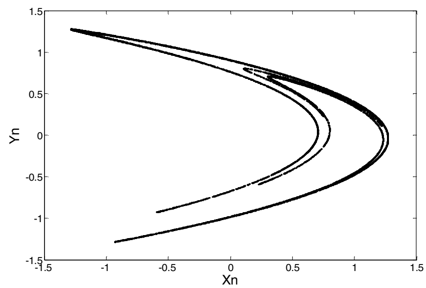


Fig. 2.5b The Hénon map

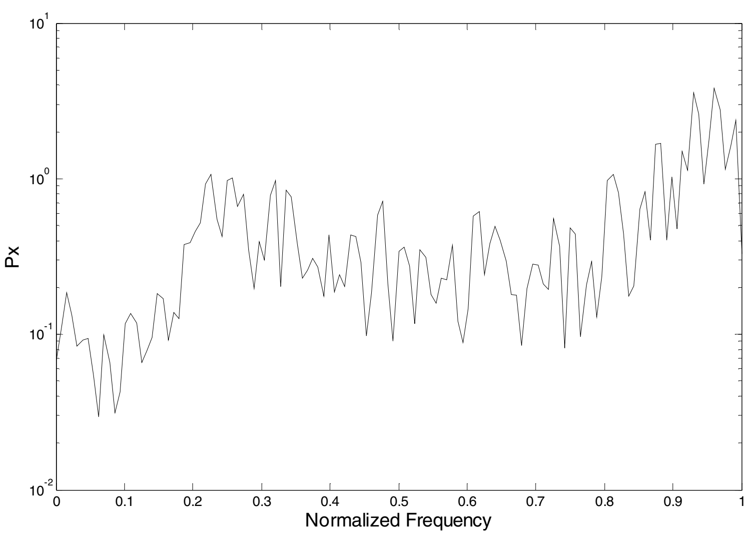


Fig. 2.6 The power spectral density (Px) of the Hénon X_n signal versus normalized frequency.

The broadband nature of the Hénon chaotic map can be observed from Figure 2.6 where the power spectral density of the Hénon X_n signal has been plotted. Furthermore, the high sensitivity of the Hénon chaotic map to parameter perturbations is demonstrated in Figure 2.7. As for the Lorenz chaotic flow, it can be observed from Figure 2.7 that a small alteration to the parameter of the Hénon map causes the system to generate an entirely different chaotic signal.

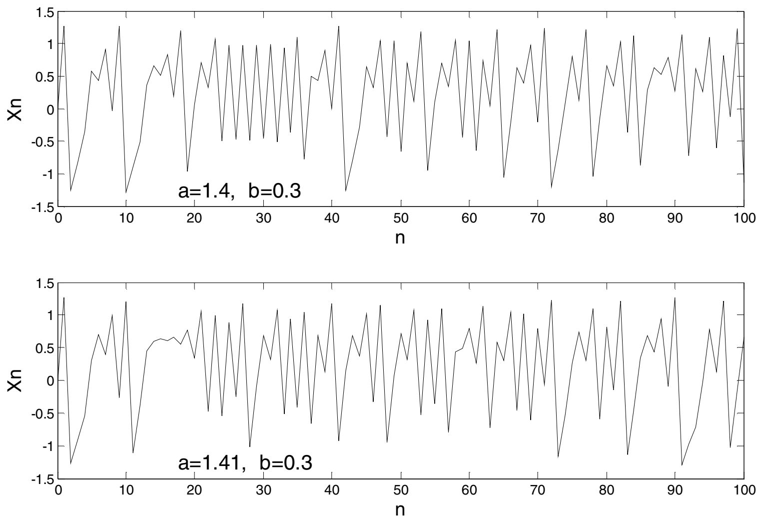


Fig. 2.7 Sensitive dependence on the parameter perturbations within the Hénon chaotic map

The logistic map is an example of another well known chaotic map. The logistic map time series are generated using equation 2.1.5 [14].

$$X_{n+1} = 1 - 2X_n^2 \tag{2.1.5}$$

The dynamics of the logistic map are shown in Figure 2.8 [15]. Furthermore, the dynamics of some of the other well known maps, such as the cusp, Lozi and Chirikov chaotic map, can be found in [1].

The dynamics of the chaotic map are referred to as an orbit [1]. In contrast to the trajectory of chaotic flows, the orbit of a chaotic map is characterised by a non-smooth, discontinuous motion. It can be observed from Figures 2.5 and 2.8, that each chaotic system has its own signature in phase space, that is, a unique attractor characterising it. An example of a chaotic map is the non-periodic dropping of water from a pipe [1].

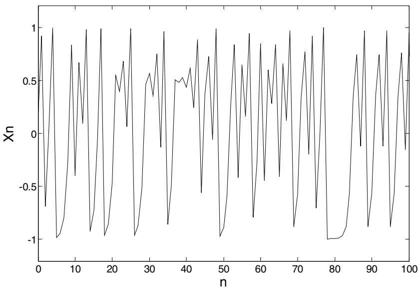


Fig. 2.8a The logistic chaotic time series, X_n

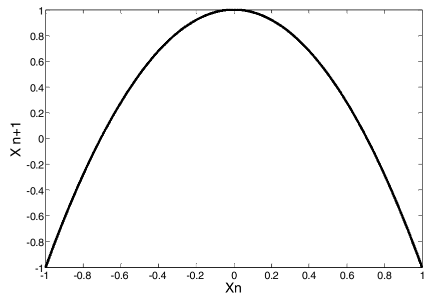


Fig. 2.8b The logistic map

2.2 Lyapunov Exponents

One of the main characteristics of chaotic systems is that they are deterministic, but extremely sensitive to the starting points, that is, their initial conditions. By high sensitivity to the initial conditions it is meant that the two trajectories (orbits), starting from infinitesimally close initial conditions, quickly diverge in phase space. This phenomenon is illustrated in Figures 2.9 and 2.10 on the Lorenz chaotic flow and the Hénon chaotic map time series, respectively. However, given the knowledge of the exact initial conditions, chaotic systems are predictable. It is shown in the next section how this property of chaotic signals can be used to hide (encrypt) messages within a communication system.

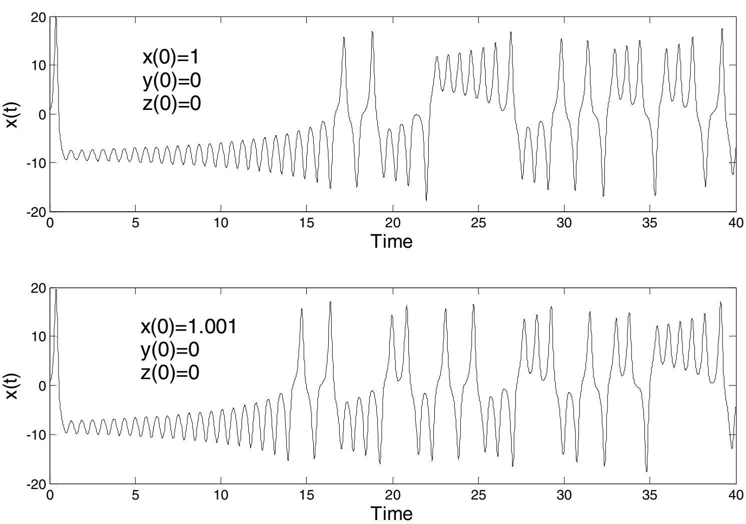


Fig. 2.9 Sensitive dependence on the initial conditions, denoted by $x(0)$, $y(0)$ and $z(0)$, within the Lorenz chaotic flow

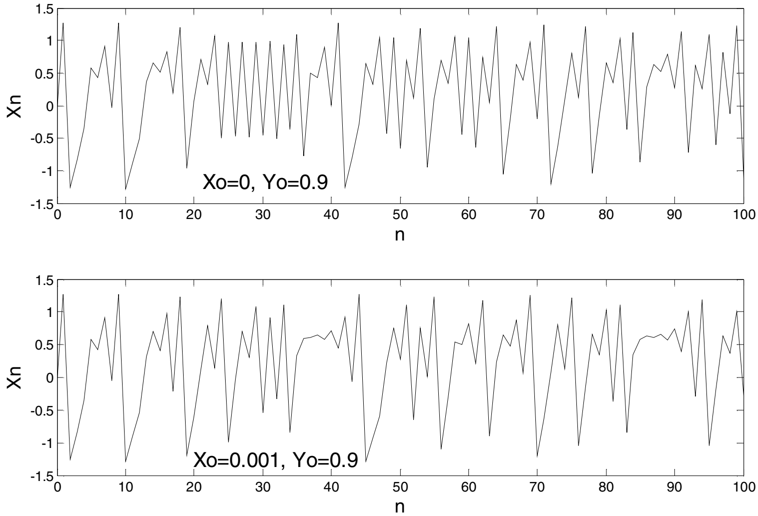


Fig. 2.10 Sensitive dependence on the initial conditions, denoted by X_0 and Y_0, within the Hénon chaotic map

The Lyapunov exponents of a system under consideration characterise the nature of that particular system. They are perhaps the most powerful diagnostic in determining whether the system is chaotic or not. Furthermore, Lyapunov exponents are not only used to determine whether the system is chaotic or not, but also to determine how chaotic it is. They are named after the Russian mathematician, Aleksandr Mikhailovich Lyapunov, who introduced the idea around the turn of the 19th to the 20th century [16,1]. The Lyapunov exponents characterise the system in the following manner. Suppose that d_0 is a measure of the distance among two initial conditions of the two structurally identical chaotic systems. Then, after some small amount of time the new distance is:

$$d(t) = d_0 2^{\lambda t}, \tag{2.2.1}$$

where λ denotes the Lyapunov exponent.

For chaotic maps, equation 2.2.1, is rewritten in the form of equation 2.2.2:

$$d_n = d_0 2^{\Lambda t}, \tag{2.2.2}$$

where Λ denotes the Lyapunov exponent and n a single iteration of a map.

The choice of base 2 in equations 2.2.1 and 2.2.2 is arbitrary [16]. The Lyapunov exponents of equations 2.2.1 and 2.2.2 are known as local Lyapunov exponents as they measure the divergence at one point on a trajectory (orbit). In order to obtain a global Lyapunov exponent the exponential growth at many points along a trajectory (orbit) must be measured and averaged [16]. Therefore, the global, or the largest, Lyapunov exponent is represented by equation 2.2.3:

$$\lambda = \frac{1}{t_N - t_0} \sum_{k=1}^{N} \log_2 \frac{d(t_k)}{d_0(t_{k-1})} \tag{2.2.3}$$

Similarly, for chaotic maps, the global Lyapunov exponent is defined by equation 2.2.4:

$$\Lambda = \lim_{n\to\infty} \frac{1}{N} \sum_{k\to\infty}^{N} \log_2 \left| \frac{d\, f(X_n)}{dX} \right| \tag{2.2.4}$$

where $f(X_n) = X_{n+1}$.

A motion is said to be chaotic if the global Lyapunov exponent is greater than zero [16,1]. A motion with a negative global Lyapunov exponent implies a fixed point or a periodic cycle [1]. In certain cases it is possible to analytically evaluate Lyapunov exponents of the system [1]. If, however, analytical evaluation is not possible, one must resort to the numerical evaluation [16,1].

A chaotic system has as many Lyapunov exponents as it has dimensions. However, the global (largest) Lyapunov exponent is the most important one as its evaluation determines whether the system is chaotic or not. For instance, the one-dimensional logistic map of equation 2.1.5 (Figure 2.8) has a single positive Lyapunov exponent. The two-dimensional Henon map of equation 2.1.4 (Figure 2.5) has two Lyapunov exponents, one negative and the other positive. Furthermore, the Lorenz chaotic flow of equation 2.1.3 (Figure 2.2) has three Lyapunov exponents, one positive, one negative and one equal to zero.

Beside Lyapunov exponents, there are other techniques used to determine whether a system under consideration is chaotic or not, such as the correlation dimension [1] and the Kaplan-Yorke (or Lyapunov) dimension [1]. Unlike the Lyapunov exponent, which measures the attractor's average predictability, the dimension of an attractor measures its complexity. The attractor dimension is less than but not equal to the number of variables of a chaotic system. Furthermore, it is not an integer, but a fraction. Thus the attractor dimension is also called the fractal dimension.

2.3 Application of Chaos to Communications

Unlike pseudo random signals, which are limited in number and are periodic, chaotic systems can theoretically produce infinite numbers of chaotic signals which are non-periodic. This property and the broadband nature of chaotic signals make them of particular interest in secure communications. In this book, two approaches to chaotic communication systems are investigated. The first approach, investigated in the sixth chapter, is that based on the principles of chaotic synchronization [17]. The second approach, investigated in the seventh chapter, is that based on the classical synchronization techniques used within DS-CDMA systems.

2.3.1 Chaotic Communication Systems Based on the Principles of Chaotic Synchronization

The general block diagram which demonstrates the principles of chaotic synchronization is presented in Figure 2.11. In Figure 2.11, the master chaotic system transmits one or more of its signals to the slave system. The slave system is another chaotic system, which in general, can be entirely different from the master system. Depending on the nature of the master signal supplied to the slave system, the slave system may or may not synchronize to the master system. If the master-slave system does not synchronize for a given master signal(s), it is possible to design a controller at the slave side which enforces synchronization. The principles of chaotic synchronization are thoroughly discussed in the next chapter.

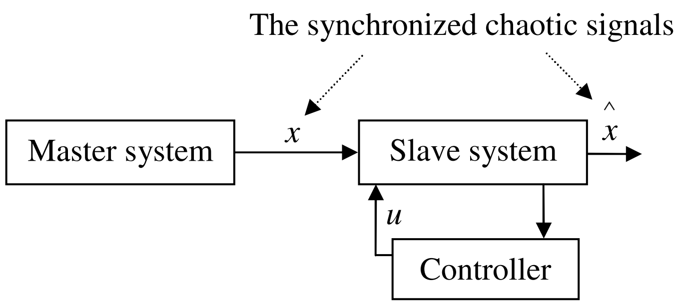


Fig. 2.11 General block diagram demonstrating the principles of chaotic synchronization, where x denotes the master and $\hat{x}$ the slave signal

Once the master-slave synchronization has been achieved, it is possible to design a communication system based on the principles of chaotic synchronization. The general block diagram of such a communication system is illustrated in Figure 2.12. The communication system of Figure 2.12 is therefore entirely based on the principles of chaotic synchronization and an ideal synchronization within it cannot be assumed. This is in contrast to DS-CDMA based systems where one can assume perfect synchronization in order to evaluate the benchmark performance, as was explained in the first chapter. In Figure 2.12, the sequence synchronization unit and the linear operator have been specifically highlighted to clarify the relation of this chaotic synchronization based system to the general system of Figure 1.22. As will be shown in chapter 6, the message m of Figure 2.12 can be encrypted within the chaotic carrier x via the parameter or the initial condition perturbations, or by simply adding it directly onto the chaotic carrier.

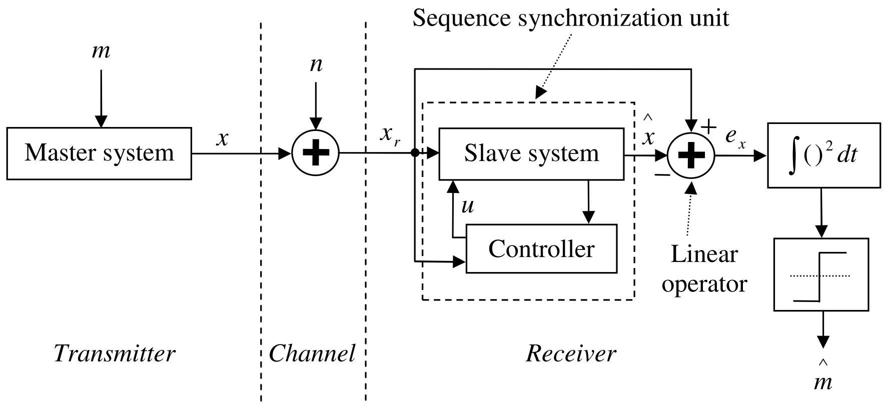


Fig. 2.12 General block diagram of the chaotic communication system based on the concept of chaotic synchronization

2.3.2 Chaotic Communication Systems Based on the DS-CDMA Principle

The implementation of chaotic signals within chaos based DS-CDMA systems is possible due to the fact that the chaotic signals are approximately mutually orthogonal. In particular, this property is more dominant within signals generated by chaotic maps than chaotic flows.

For instance, consider the logistic map, whose time series is generated using equation 2.1.5 [14] and whose dynamics are shown in Figure 2.8 [15]. The two different chaotic time series generated by the same logistic map, but with different initial conditions, are highly orthogonal as is demonstrated in Figure 2.13a by the cross-correlation function with no dominant peaks. The autocorrelation function of the logistic map time series is presented in Figure 2.13b showing the dominant peak. The length of the logistic map time series used to produce Figures 2.13a and 2.13b is equal to 511 points (chips). In Figures 2.13a and 2.13b t denotes the time delay. Also, note that the correlation functions have been normalized to the peak of the autocorrelation function.

As opposed to the logistic map of equation 2.1.5, the Lorenz chaotic flow of equation 2.1.3, for instance, has the correlation properties illustrated in Figures 2.14a and 2.14b. The length of the Lorenz flow time series used to produce Figures 2.14a and 2.14b is equal to 2001 points (chips). In contrast to Figure 2.13a, it can be observed from Figure 2.14a that the cross correlation function of the Lorenz flow contains dominant peaks which are strongly pronounced. Furthermore, whereas the autocorrelation function of the logistic map resembles an impulse function, with a single dominant peak at $t = 0$, the autocorrelation function of the Lorenz flow does not. This can in particular be observed by comparing the close ups of Figures 2.13b and 2.14b and observing that the logistic map autocorrelation function has a sharp falloff from 1 to 0 at $t = 0$ and $t = \pm 1$, whereas the Lorenz flow does not. This indicates that the logistic map time series is more orthogonal to itself than the Lorenz flow time series. Therefore, in this book, only the logistic map time series will be used within a DS-CDMA system for spreading.

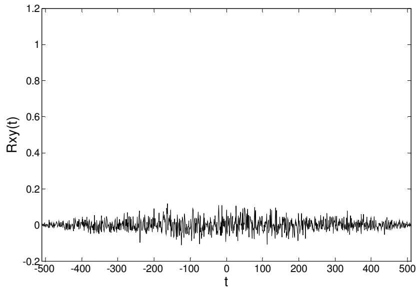


Fig. 2.13a Cross-correlation of logistic map time series

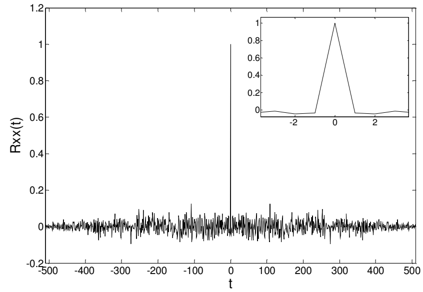


Fig. 2.13b Autocorrelation of logistic map time series. The close up is shown in the top right hand corner

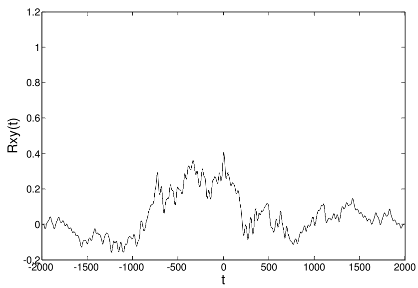


Fig. 2.14a Cross-correlation of Lorenz flow time series

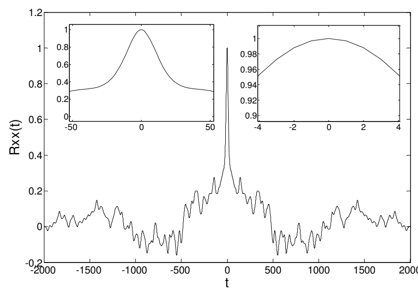


Fig. 2.14b Autocorrelation of Lorenz flow time series. The close ups are shown in the top right and left hand corners

A DS-CDMA system where chaotic signals are used to spread data is termed chaos based DS-CDMA system. A chaos based DS-CDMA communication system with perfect sequence synchronization assumed is shown in Figure 2.15 [18].

In Figure 2.15, $x(t)$ denotes the chaotic spreading signals which are multiplied by the binary message signals $m(t)$. The products are then summed to produce the signal $c(t)$ which is transmitted through the channel:

$$c(t) = \sum_{i=1}^{M} m_i(t) A_i \, x_i(t) \tag{2.3.1}$$

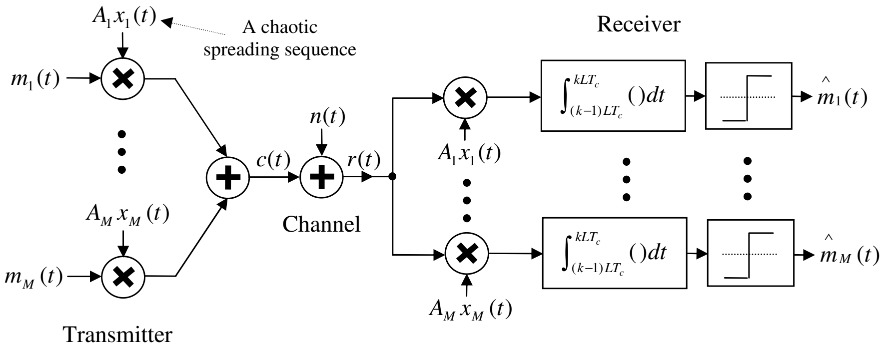


Fig. 2.15 The chaos based DS-CDMA system with perfect sequence synchronization assumed

The received signal $r(t)=c(t)+n(t)$ is despread and correlated with the punctual despreading codes to recover the message $\hat{m}_q(t)$ of each of the M users in the system:

$$\begin{aligned}\hat{m}_q(t) &= \mathrm{T_h}\left[\int_{(k-1)LT_c}^{kLT_c} r(t)\,A_q x_q(t)\,dt\right]\\ &= \mathrm{T_h}\left[\int_{(k-1)LT_c}^{kLT_c}\left\{\sum_{i=1}^{M} m_i(t)\,A_i\,x_i(t)+n(t)\right\}\cdot A_q x_q(t)\,dt\right]\\ &= \mathrm{T_h}\left[\begin{array}{l}\int_{(k-1)LT_c}^{kLT_c} m_q(t)\,A_q^2 x_q^2(t)\,dt\\ +\int_{(k-1)LT_c}^{kLT_c}\sum_{\substack{i=1\\ i\neq q}}^{M} m_i(t)\,A_i A_q\,x_i(t)\,x_q(t)\,dt+\int_{(k-1)LT_c}^{kLT_c} n(t)A_q x_q(t)\,dt\end{array}\right]\end{aligned} \tag{2.3.2}$$

where, $\mathrm{T_h}[\;]$ is the signum function which denotes the thresholding operation and assigns either a -1 or a 1 depending on whether the value in the brackets is negative or positive, respectively [19]. It is assumed that all the received signals have the same average power.

Due to the mutually orthogonal properties of the chaotic spreading sequences produced by the logistic map with different initial conditions, as demonstrated in Figures 2.13a and 2.13b, equations 2.3.3a and 2.3.3b are expected to hold:

$$\int_{(k-1)LT_c}^{kLT_c} m_q(t)\,A_q^2 x_q^2(t)\,dt > 0 \qquad \text{if} \quad m_q(t)=1 \tag{2.3.3a}$$

$$\int_{(k-1)LT_c}^{kLT_c} m_q(t)\, A_q^2 x_q^2(t)\, dt < 0 \qquad if \quad m_q(t) = -1 \tag{2.3.3b}$$

Provided that the power of noise in the system and the interferences among different users are comparatively low to the power of the signal, the noise and the interferences terms of equation 2.3.2 are expected to be approximately equal to zero, that is: $\int_{(k-1)LT_c}^{kLT_c} n(t) A_q x_q(t)\, dt \approx 0$ and $\int_{(k-1)LT_c}^{kLT_c} \sum_{\substack{i=1 \\ i\neq q}}^{M} m_i(t)\, A_i A_q\, x_i(t)\, x_q(t)\, dt \approx 0$, so that equation 2.3.2 takes on the form of equations 2.3.4a and 2.3.4b:

$$\hat{m}_q(t) = 1 \qquad if \quad m_q(t) = 1 \tag{2.3.4a}$$

$$\hat{m}_q(t) = -1 \qquad if \quad m_q(t) = -1 \tag{2.3.4b}$$

Therefore, by assigning the unique initial conditions to each of the M users provides for the increased security of transmission as only the users with the same initial conditions can decrypt the message at the receiver.

2.4 Noise Reduction within Chaotic Communication Systems

In the previous section, the two main approaches to the implementation of chaotic systems to secure communications have been described. In this section, the techniques of noise reduction by means of de-noising (or filtering) are now briefly introduced.

Noise removal from chaotic time series has been attempted by a number of researchers [20-27], among others, and is still an active area of research. Filtering methods include linear filters [20,22] and different wavelet techniques [20,21,23-25], among other. A potential application of chaotic filtering techniques lies in chaotic communication systems [20,26,27]. In this book, the linear and wavelet techniques have been developed and used to filter a newly proposed chaotic communication system based on the principles of chaotic synchronization [20]. While the general block diagram of a chaotic communication system with the filter embedded inside the receiver is shown in Figure 2.16, the complete results are presented in the appendix. The appendix should be read only after reading chapters 3-6.

In Figure 2.16, the filter unit processes the received signal x_r and produces its filtered version x_f. The filtered signal x_f is then fed into the slave system. In this book, three different kinds of filtering techniques have been developed and

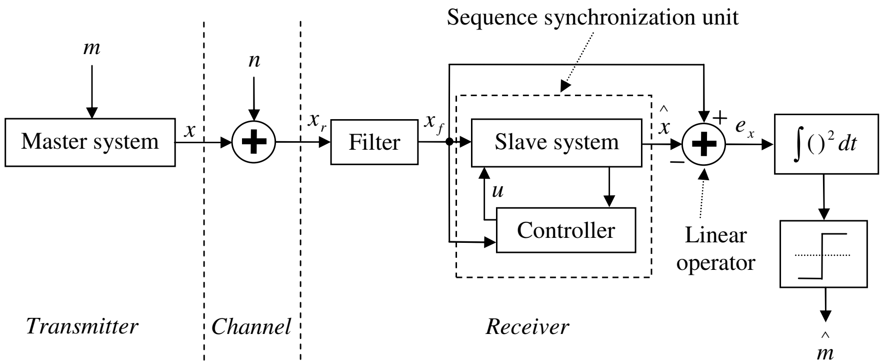


Fig. 2.16 General block diagram of a chaotic communication system based on the concept of chaotic synchronization with the filter unit incorporated

implemented within the chaotic communication system. The filtering techniques include those based on the running average finite impulse response (FIR) filter [1] and those within the Haar wavelet [28] and the Daubechies wavelet domain [28]. It has been shown, in terms of the bit error rate, that both linear and wavelet filters significantly improve the noise performance of the system [20].

2.5 Conclusion

In this chapter, nonlinear dynamical systems, known as chaotic systems, have been introduced and their suitability to the application to secure communication systems outlined. Chaotic behaviour was recognised by the scientific community in the early sixties. It was characterized by apparent random behaviour, high sensitivity to parameter and initial condition perturbations and broadband spectrum. These were some of the properties that led to the belief that chaotic signals could be used within secure communications. Here, two different types of chaotic systems, known as flows and maps, have first been introduced and their broadband nature and high sensitivity to parameter and initial condition perturbations demonstrated. The Lyapunov exponents which are used to diagnose and characterize the system have then been presented. Furthermore, the two different approaches of implementing chaotic systems within secure communication systems have been outlined. These include chaotic communication systems based on the principles of chaotic synchronization and those based on the DS-CDMA principle. Finally, some of the filtering techniques that can be used within chaotic communication systems have been briefly introduced.

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