From “Reading" Math to “Doing" Math: A New Direction in Non-visual Math Accessibility

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Abstract. The ability to understand, apply, and manipulate mathematical concepts is a cornerstone of any scientific discipline; as such it is an irreplaceable component of the training and education of any students. While the advent of online and e-learning technologies has enabled the breaking of several barriers and promoted wider access to educational opportunities, it has also furthered the disenfranchisement of visually impaired students. Despite the several efforts in the field of enhancing accessibility for those with visual impairments especially in the educational discipline, it is obvious that there is still a lack of contributions in making mathematics manipulation processes accessible. This paper presents a framework that facilitates doing and manipulating mathematical algebraic content in a way that is convenient, accessible, and usable as well for students with visual impairments.

Keywords: Accessibility, Math manipulation, Visual impairments.

1 Introduction

Mathematics is a key conceptual framework that is essential for everybody, even for those who are not actively pursuing educational objectives. The learning of mathematics commences in the very early grades of school, which is an evidence of the significance of this science. The level and difficulty of mathematics learning goes up sharply when advancing from one grade to another in the educational system; and that is clear in contents like algebra and geometry [10]. Proficiency in mathematics is a requirement to be enrolled into a variety of scientific majors, and consequently, it is a gateway to find a good place in the world of many critical professions, i.e. scientific, academic, technological, industrial, etc.

Mathematical notations rely on a predominantly visual language, which creates serious challenges for individuals with any type of visual impairments. Mathematics in nature is a complex knowledge that challenges learners in all its levels: basic, intermediate, and advanced. Complexity of mathematics spans also to involve the way it is represented and encoded. The complexity of mathematical notation comes from its two-dimensionality, spatial nature, non linearity [11], and being context sensitive. What adds to its complexity and makes it more challenging for both sighted and unsighted individuals, [10] mathematics is a language that highly

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necessitates exactness, definiteness, totality and comprehensibility of presentation. To summarize:

- Mathematical formulae are inherently two-dimensional entities, and the multidimensionality is critical to interpret the meaning of the formulae e.g.

$$\frac{\sqrt{b^2 - 4ac}}{2a} \tag{1}$$

- The interpretation of mathematical content is highly *contextual* - thus focusing the presentation on individual formulae could lead to ambiguous or incorrect interpretations; for example, a formula like $g(x + y)$ could be alternatively interpreted as the product between g and $x+y$ or as the application of the function g to $x + y$.

All of these issues lead to the conclusion that visually impaired individuals are passively affected by barriers that limit their chances from getting fair advantage of education. To better estimate the size of the problem, according to the Braille Institute [3], about 5 million or one out of 20 preschool-aged children and around 12.1 million children ages 6-17 have visual impairments. 80% of what students learn in school is visually presented information [13].

In mathematical problem solving process, the visually impaired students are challenged in three ways: (1) how to access the information that addresses the mathematical problem in hand; (2) the act of mapping information extracted from the given problem to the proper representation style; (3) how to provide the arrived answer in a readable way for both sighted and unsighted individuals [2,11]. Reading and writing mathematical expressions are among the most challenges visually impaired individuals encounter. The literature has highlighted the difficulties associated with access to mathematical content by individuals with visual impairments [14]. Several researchers have identified the problem of math accessibility for individuals with visual disabilities as a bottleneck in the access to training and careers in science, technology and mathematics (STEM) [12].

An extensive literature has been developed to overcome barriers and problems associated to non-visual accessibility of mathematics. The literature has provided a plethora of approaches aimed at promoting accessibility of mathematical formulae (see [14] for a recent survey). However, there have been relatively limited efforts to enhance *mathematics manipulation technologies* for people who are visually impaired. Focusing exclusively on the reading of mathematical formulae - which has represented the main focus of the current literature - provides only limited help to visually impaired students. In particular:

- The focus on formulae detracts from the opportunity of enhancing understanding by placing the formulae in their context and using the context to develop more effective presentation modalities;
- The greatest challenge for most students is not on reading mathematics, but on understanding and using/applying its concepts.

The process of learning mathematical content is inseparable from the ability to practice it - by solving exercises or by manually developing the sequence of logical or algebraic steps that are often implicit in the arguments present in the studied text. The literature has, to date, provided very limited efforts aimed at enabling visually impaired students to writing and practicing mathematics (e.g., [19]).

Let us consider a simple example: we wish to simplify (a + b) (2a - b):

1. The first step is to develop the product, by multiplying each term in the parenthetical expression by each term in the second parenthetical expression; specifically,

   (a) Extract the first term from the first expression, a, and multiply this by each term in the second expression: $2a^2 - ab$
   (b) Extract the second term from the first expression, b, and repeat the process: $2ab - b^2$
   (c) Combine the two expressions: $2a^2 - ab + 2ab - b^2$

2. The second step is to identify occurrences of terms that can be combined and replace them with their combination: $2a^2 + ab - b^2$.

This example illustrates a process that is challenging without any visual aid - selecting sub-expressions, scanning expressions, create copies of expressions. A purely textual presentation of expressions would require repeated scans and jumps.

In this project, we propose a framework to facilitate manipulation of mathematical concepts by individuals with visual disabilities, starting at the level of basic algebra. The framework is to enable visually impaired individuals to have a complete control over navigation functions through a full history of manipulation steps they carry out. In particular, the framework allows the visually impaired students studying middle school level algebra to manipulate algebraic expressions in recorded accessible consecutive steps that at the end lead them to a final answer to the initial problem. The framework also enables visually impaired students to access each component in any step, review their manipulated expressions, correct mistakes done at any point of time in case of having any, work on particular components separately, save interesting results for later usage, and use a scratch paper like environment.

The central concept of the proposed framework is the idea of workspace:

- The workspace is composed of blank pages, where the student can conduct his/her exercises; pages can be named and directly accessed;
- The workspace provides the ability to place arbitrary bookmarks, within distinct formulae, and position navigation on any requested bookmark (i.e., the student can keep their “fingers" pointing to different locations of the formulae); bookmarks are hierarchically organized to facilitate retrieval of different positions within exercises and formulae;
- The workspace allows the student to select formulae (represented by roots of MathML sub-trees), copy them, or delete them; each formula can be manipulated through selection of sub-formulae, insertion of new terms or removal of terms;
- The workspace provides distinct domain-specific layers that combine the basic term manipulations (selection, copying, removal, replacement) with domain-specific transformations (e.g., multiplication of terms, reversal of fractions);
- The workspace makes it possible to temporarily hide parts of formulae.

The design of workspaces relies on aural presentation of formulae and bookmarks, and the use of hot access keys to control scanning and manipulation of terms. The implementation relies on the representation of expressions in MathML (or its realization in the MSOffice OOXML format); workspaces are implemented as a web tool that provides a virtual notebook with the described features.

2 Prior Research

Among the commonly used approaches in accessibility, screen readers, e.g., JAWS [9] and Microsoft narrator, are considerable software tools that help visually impaired individuals to have an access to many on screen presented content, i.e., text and image or video descriptions (alternative text). However, this is not truly applicable for all kinds of content, especially the mathematical one, for the earlier mentioned characteristics of mathematics, in addition to the fact that some mathematical content is represented as images. Software magnifying tools on the other hand, are very useful aids in case of low vision, while unfortunately, they are not of any advantage in a situation of complete blindness. The limitations of these approaches strongly encouraged the development of other tools.

The field of promoting mathematics accessibility for the visually impaired individuals has attracted researchers and developers that dramatically changed the way of handling mathematics. Research projects influenced the gradual development of other approaches. As a result of introducing MathML, a W3C standard [12], mathematics became available online in a convenient format that can be neatly and easily rendered. However, availability is not everything towards usability, in a sense that many user categories like those of visual impairments were not able to make use of mathematics and take advantage of the availability of mathematical content - even after introducing MathML. Static approaches that use Braille to render mathematics, and Dynamic approaches in which audio is used in rendering are the main categories of accessibility approaches [14].

Math2Braille for converting MathML to Braille standard [4], LAMBDA which uses linear mathematical encoding for mathematics representation [6], and MAVIS to overcome the problem of backtranslation between Braille and LaTeX [12] are examples of the static approaches in which several new Braille notations were created for mathematics. Examples of dynamic presentation approaches include AsTeR for reading mathematics through parsing LaTeX documents to produce a tree structure which in turn facilitates navigation and tagging [15], MathGenie for reading and navigating equations, also for conversions between different formats i.e., Nemeth to LaTeX [18], and the Microsoft Internet Explorer plug-in MathPlayer for reading mathematical content that is represented in MathML [5, 17]. InftyEditor [20], Infty converters for conversion between different standards [20], and WinTriangle for reading and editing documents [8] are examples of tools that enable the non-visual creation and editing of mathematical content. A different direction is represented by tools to interactively translate MathML into voiceXML, render pages by audio, and facilitate voice interaction from end users [16]. [1] proposed a framework to allow both the linear and hierarchical navigation of mathematical expressions.

3 The New Framework Development

Recent technological advances changed the way students receive education, moving from complete dependency on classroom and materialistic tools to more of virtual communication mediums and software based curriculums. For educational purposes, visually impaired students are increasingly required to deal with online available mathematical content. Visually impaired students, while navigating online materials, might be exposed to mathematical content, which has to be properly detected. Once detected, mathematical expressions should be rendered separately from the original navigated page. Next comes the need to provide the ability to reason about mathematical content and perform algebraic manipulations. The visually impaired student should be able to manipulate and reach to an answer to the original algebraic expression. Through all these stages, accessibility is an issue that must be present for the visually impaired. Fig. 2 shows the overall structure of our system.

The ultimate goal of this new framework is to enable visually impaired students to access online mathematical content and to perform algebraic manipulations for solving algebraic problems. Unlike LiveMath and other online creating and editing tools, this new direction of educational tools is not meant to directly or indirectly give answers or carry out manipulations for end-users; instead, it allows visually impaired individuals to practice algebra and solving techniques without any guidance and results given from the system. Moreover, it keeps a history of steps of all manipulations carried out so far by the visually impaired students, so that they can easily go back and forth to review what has been so far made, or simply saying, navigation between different components in the hierarchy of steps is made possible without tedious efforts from the end-user side, which facilitates building a full and clear image of what they have been doing.

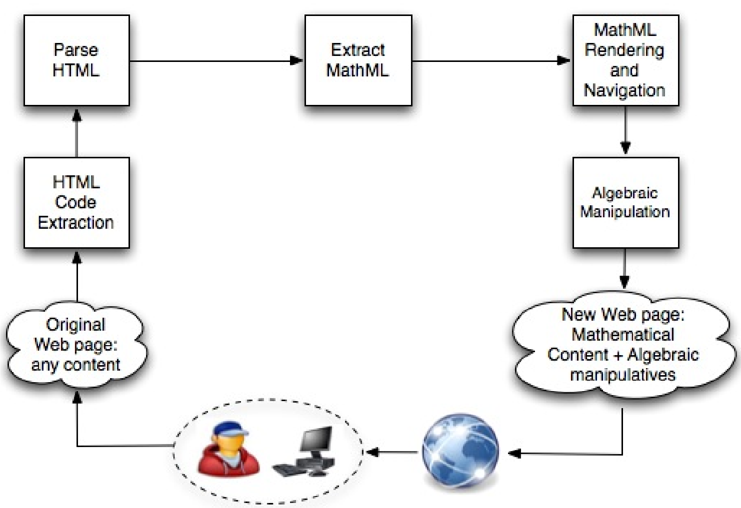

Fig. 1. The Framework structure

The easy to access history of manipulations gives the visually impaired students the capability to make sure what they are making, how so far they handled the expression under consecutive manipulation steps they made, and what expression they got at the end of each step. This approach reduces the amount of visually impaired individual's dependency on sighted people help, decreases the amount of time the visually impaired students would invest in manipulation when using other traditional tools, lowers the need to memorize temporary results, and facilitates practicing of mathematics skills.

4 Functional Requirements and Design

To come up with an effective solution, the following functional requirements were taken into consideration:

- Accessibility support
- It entails understandable representation, clear rendering, user friendly interface, and easily manageable functionalities. The use of audio rendering and hot access keys are among the features the framework has.
- Self controlled environment
- The visually impaired student controls what functions to use, either right or wrong like pencil and paper.
- Hierarchical recording of manipulation steps
- Every step carried out should be recorded for later revision and making sure that things are going in the right direction.
- Error correction and backtracking
- The ability to backtrack, correct errors, and cancel mistaken steps. This helps avoid redoing things from the very beginning.
- Easy navigation
- Going back and forth between steps should be an easy mission that visually impaired students should not bother.
- Bookmarking and working on certain sub-expressions
- Bookmarking prevents the recreation of interesting sub-expressions, also helps reducing the amount of information to be memorized. Working on certain sub-expressions helps flexible dealing with smaller parts that collectively form the original expression.
- The ability to save work
- The work can be saved for later revisions, study, and training. Teachers also can benefit from this option so as they see how their students practice algebra.

The basic operations one needs in solving an equation are taken into consideration. No solution hint or guidance will be given either implicitly or explicitly, since the approach acts the same as giving a piece of paper and pencil for a student and asking them to solve an equation. Non-visual accessibility is supported through the deployment of audio feedback, hot access keys, and interaction with the MathPlayer plug-in [5,17].

The system includes a plug-in that works on parsing MathML source code to capture any mathematical content on the navigated page. After the mathematical content detection step, and with a control from the visually impaired student, a new space opens to enable the visually impaired student to manipulate mathematical expressions freely. Moreover, TTS and audio feedback approaches are used to guide the visually impaired student to the content of the page. In addition, hot access keys will be deployed for the ease of access to help visually impaired in carrying out manipulations. Mouse and keyboard navigations are supported, and for later, voice commands and haptic device navigation and control might be supported.

Initially, the following operations (distinguished in bold) which are mainly derived from the mathematical algebraic perspective, are considered:

- Eliminate - To delete a term (get rid of it when no longer needed) from the whole expression being manipulated. It is necessary to mention that when talking about a term; it can be a factor, variable, number, operator, etc.
- Combine like terms - To collect terms that fall in same category.
- Cancel - To cancel terms together when the effect of one cancels the effect of the other.
- Enclose - To make a set of consecutive terms look like one unit.
- Add - To add an existent term to the expression. This is clear and straightforward in algebraic manipulations.
- Subtract - Like the way the previous function works, this function is to subtract an existent term from the expression.
- Add new - To add a nonexistent term to the expression.
- Subtract new - To subtract a nonexistent term from the expression.
- Insert - To insert a new term, this operation is commonly used in the context of manipulation although the terminology is not formal in mathematical definition of operations for manipulating mathematics.
- Replace - To replace an existent term by a new one, like what said about the previous function, common but not formal.
- Multiply - To multiply the expression by an existent term. This is clear and straightforward in algebraic manipulations.
- Divide by - To divide the expression by an existent term. This is clear and straightforward in algebraic manipulations.
- Factor out - To handle the process of extracting factors.
- Expand - To carry out the distribution process in case of multiplications to what is inside brackets.
- Apply roots - To apply any degree root to the selected term.
- Apply powers - To apply any power to the selected term.
- Work on selected term - To separately manipulate the selected term away of the whole original expression, facilitating more manageable manipulations.
- Open a scratch paper - To work on an empty space to carry out intermediate operations and manipulations.
- Open Calculator - To open an accessible basic calculator that helps doing calculations.

- Save for later - To keep interesting components or results for later use, in other words to bookmark desired components.
- Undo - To go back one step from the currently under focus step, to correct mistakes made in any step prior to the active one.

5 Future Work

It has been found that, while there exists extensive work contributed in accessibility, availability, and usability of mathematics for people who are visually impaired, there is still an obvious absence of attention paid for promoting and driving mathematics manipulation accessibility for students with visual impairments. The contribution presented in this paper is considered a new direction and a starting point for an extendable research work that aims at handling the problem of lacking accessible mathematics manipulation frameworks to help students who are visually impaired overcome their print disabilities. Going beyond what is addressed in this framework, the next natural step is to provide coverage for a larger body of mathematical content, beyond elementary algebra, e.g., by including support for all content covered in college algebra and extend to basic geometry.

The current framework has been designed and developed to provide the basic functionalities required to support extraction of mathematical content and execution of algebraic transformations. The next step involves creation of layers that implement mathematical transformations that are relevant to different mathematical content areas. The design of these layers should be informed by the modalities of mathematical manipulation used by visually impaired students. We have initiated a series of interviews and focus groups with visually impaired students from the New Mexico School for the Blind and Visually Impaired. These user studies will be used to develop the set of transformations and to guide the way they are presented to the users.

6 Conclusions

Visually impaired individuals are productive and are of equal importance for the societies they are involved in like sighted people. Especially in the context of mathematics, the students who are visually impaired are taught exactly the same content and curricula sighted students receive at school. This indicates the need to provide visually impaired students with tools that enable them to reason with mathematical content in a manner comparable to that of sighted students. While the existing literature has offered tools to enable the ``reading'' of mathematical content, there are no tools offering the equivalent of ``pen-and-paper'' to solve mathematical problems.

In this paper, we present the preliminary design of a framework, currently under development, for enhancing math manipulation accessibility for middle school students who are visually impaired. Unlike previous contributions, which are exclusively aimed at enhancing rendering and reading of mathematical content, this framework provides tools to support the processes of mathematical manipulation available. The framework is accessible and offers a convenient solution to allow students who are visually impaired to practice with basic algebra and perform algebraic manipulations of simple formulae. The framework currently support transformations for elementary algebra and it is being informed and validated by focus groups, in cooperation with the local school for the Blind and Visually Impaired.

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