Image Decompositions and Transformations as Peaks and Wells

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Abstract. An image may be decomposed as a difference between an image of peaks and an image of wells. This decomposition depends upon the point of view, an arbitrary set from where the image is considered: a peak appears as a peak if it is impossible to reach it starting from any position in the point of view without climbing. A well cannot be reached without descending. To any particular point of view corresponds a different decomposition. The decomposition is reversible. If one applies a morphological operator to the peaks and wells component before applying the inverse transform, one gets a new, transformed image.

1 Introduction

A binary image is made of particles and holes. Each particle may contain one or several holes and each hole one or several particles. These structures may be deeply nested. For describing this structure, J.Serra [8] introduced the homotopy tree, H.Heijmans [2] called it the adjacency tree. R. Keshet [3] and C. Ballester [1] studied it in depth and gave algorithms for constructing it. They then extended this tree construction to grey tone images, each in a different way, resulting in the so called tree of shapes.

In the present paper we propose a decomposition which decomposes any image into a peak and a well component, given a particular set, called point of view, an arbitrary set from where the image is considered: a peak appears as a peak if it is impossible to reach it starting from any position in the point of view without climbing. A well cannot be reached without descending. If the point of view intersect the minimum of a well, then this particular well will not be considered as well, since it is possible to reach any of its nodes without descending. To any particular point of view corresponds a different decomposition. Serra, Keshet of Ballester base their decomposition on a point in the background from which it is possible to apply hole-filling. Starting from an image X, a hole filling algorithm will produce a peak image P_1, then define the residue $R_1 = P_1 - X$, on which we can perform again a hole filling with a peak and residue, and so on until no residue is left. Summing up all peaks on one hand and all holes on the other hand produces the same result as our method if we adopt as view point. However, our method can use any set as view point and is also applicable to grey tone images.

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Given a reference set X, called view-point set, we decompose an image in a difference between two components, one representing its peaks, the other its wells. They represent respectively the sums of positive and negative variations of the image if one follows a path starting in X. Ch.Ronse proposed an identical decomposition for functions of bounded variation on a poset P, with the final aim to find a sound way for constructing a flat operator on gray level image from a non-increasing operator on binary images [6].

The decomposition is reversible. If one applies a morphological operator to the peaks and wells component before applying the inverse transform, one gets a new, transformed image. In order to identify the admissible transforms on the peak and well components we study their algebraic structure, showing that they form a complete lattice. Openings, closings and morphological filters may then be derived from an adjunction defined on this lattice. Reconstruction openings, also called razings are also allowed opertors on this lattice. The last part of the paper is devoted to illustrations, showing how chosing an optimal point of view for the decomposition leads to interesting results. We conclude with a final discussion.

2 Decomposing an Image into Peaks and Wells

2.1 A Hiking Metaphor

A grey tone image may be considered as a topographic surface. Consider a hiker going from position x to position y on a montaineous landscape along a given route. Its tiredness will depend upon the total amount of climbing he has to do on his trip: no matter if he goes up and down, he only sums up the difference of level when he climbs. He may then chose the route along which this sum is minimal. Obviously, this minimal amount depends upon the starting point. If all possible starting points belong to a set X, then he may chose both the starting point x within X and the route between x and y which requires the minimum of climbing ; this minimal sum of climbing between X and y is a measure of the difficulty to reach y on the topographic surface if one starts from X.

Summing up only the differences of levels on the descending portions of the path would similarly yield a measure linked to the preceding measure: for a route between x and y, the altitude of y is equal to the sum of the altitude of x plus the total amount of climbing minus the total amount of descending. The next section gives a precise meaning to this hiking metaphor.

2.2 The Decompostion into Peaks and Wells

Let f be an image defined on a grid. This grid may be considered as a graph, where the pixels are the nodes and where two neighboring pixels i and j are linked by two arcs, one from i to j and the other from j to i. The weights of node i is f_i, the value of f at i. We consider three graphs e^+, e^- and $e^\#$ characterized by the following distribution of weights on the edges:

– $e^+ : f^+_{ij} = \vee(f_j - f_i, 0)$ positive for upwards transitions and null otherwise.
– $e^- : f^-_{ij} = \vee(f_i - f_j, 0)$ positive for downwards transitions and null otherwise.
– $e^\# : |f_{ij}| = |f_j - f_i|$

Consider now an arbitrary path $\pi = (x_1, x_2, ..., x_n)$ between two nodes x_1 and x_n. We may decompose $f(x)$ along the path π as follows:

$f(x_n) = f(x_n) - f(x_{n-1}) + f(x_{n-1}) - f(x_{n-2}) + ... + f(x_2) - f(x_1) + f(x_1)$

Since $f_j - f_i = f^+_{ij} - f^-_{ij}$ for neighboring pixels i, j in the path π, we get $f(x_n) = f(x_1) + \sum_{ij \in \pi} f^+_{ij} - \sum_{ij \in \pi} f^-_{ij}$.

Among all possible paths between x_1 and x_n, there is a path $\widehat{\pi}$ for which $\sum_{ij \in \pi} f^-_{ij}$ is minimal. As $f(x_1) + \sum_{ij \in \pi} f^+_{ij} - \sum_{ij \in \pi} f^-_{ij}$ has a constant value, the expression $f(x_1) + \sum_{ij \in \pi} f^+_{ij}$ is also minimal on $\widehat{\pi}$ and so is their sum $f(x_1) + \sum_{ij \in \pi} f^+_{ij} + \sum_{ij \in \pi} f^-_{ij} = f(x_1) + \sum_{ij \in \pi} |f_{ij}|$.

The quantities $f(x_1) + \sum_{ij \in \widehat{\pi}} f^+_{ij}, \sum_{ij \in \widehat{\pi}} f^-_{ij}$ and $f(x_1) + \sum_{ij \in \widehat{\pi}} |f_{ij}|$ respectively represent the positive, negative and total variation along $\widehat{\pi}$ in G.

The length of the shortest path $\widehat{\pi}(x_1, y)$ on e^+ between x_1 and any node y of G is a function $\Theta_{x_1}(e^+, f)$ which depends only upon x_1 and f and which takes the value $f(x_1) + \sum_{ij \in \widehat{\pi}(x_1, y)} f^+_{ij}$ on the node y. Consider now multiple starting points of paths belonging to a set X. The minimum $\Theta_X(e^+, f) = \bigwedge_{z \in X} \Theta_{x_1}(e^+, f)$ represents the peak component of each node ; for the node y it represents the minimal value taken by $f(z) + \sum_{ij \in \widehat{\pi}(z, y)} f^+_{ij}$ for all paths starting at a node z in X and joining y.

We have seen that if $f(x_1) + \sum_{ij \in \widehat{\pi}(x_1, y)} f^+_{ij}$ is minimal on the path $\widehat{\pi}(x_1, y)$, then $\sum_{ij \in \widehat{\pi}(x_1, y)} f^-_{ij}$ and $f(x_1) + \sum_{ij \in \pi} |f_{ij}|$ also are minimal on this same path. We obtain like that a function $\Theta_{x_1}(e^+, f)$ depending only upon x_1 and f and taking the value $\sum_{ij \in \widehat{\pi}(x_1, y)} f^-_{ij}$ on the node y, representing the well component of y and a function $\Theta_{x_1}(e^\#, f)$ taking the value $f(x_1) + \sum_{ij \in \widehat{\pi}(x_1, y)} |f_{ij}|$ representing the total variation at y.

As before we consider the minimum of these functions on all paths starting in X and joining y and define $\Theta_X(e^-, f) = \bigwedge_{z \in X} \Theta_{x_1}(e^-, f)$ and $\Theta_X(e^\#, f) = \bigwedge_{z \in X} \Theta_{x_1}(e^\#, f)$.

Finally, to each view point X corresponds a decomposition of the function f into a difference between a function representing the peaks and another representing the valleys: $f = \Theta_X(e^+, f) - \Theta_X(e^-, f)$. This relation also shows how f may be reconstructed from ist peak and well components.

2.3 Setting the Scenario of Trains Circulating on Graphs

This section explains how to construct these functions $\Theta_X(e^+, f)$ and $\Theta_X(e^-, f)$. $\Theta_X(e^+, f)(y)$ represents the minimal value taken by $f(z) + \sum_{ij \in \widehat{\pi}(z,y)} f_{ij}^+$ for all paths starting at a node z in X and joining y. This is an unconventional shortest path problem, as the length of the path $\pi = (x_1, x_2, ..., x_n)$ is equal to the sum of the arcs leading from x_1 to x_n, plus the initial weight f_{x_1}. We again use a metaphor. The graph may be considered as a railway network, where the nodes are railway stations and the arrows are connections between them. Trains may follow all possible paths on G. However, they may only start from a subset $X \subset N$ of railway stations.

Consider a particular train. It starts at station $s \in X$ at time $\tau(s)$, follows a path $\theta = (x_0 = s, x_1, ..., x_n = t)$ where x_i and x_{i+1} are two railway stations linked by an arrow $(i, i+1)$ of E, weighted by the time $e_{i,i+1}$ needed for following it. The arrival time at destination is then $\tau(s) + \sum_{i,i+1 \in \theta} e_{i,i+1}$.

We now consider all trains starting at all possible nodes and taking all possible routes, and observe the earliest time when each node is reached by a train; if a train arrives at i before $\tau(i)$, we replace $\tau(i)$ by this first arrival time. For some nodes no train ever arrives ; for such a node i, all trains coming from another node arrive at i after $\tau(i)$. For others, no train ever departs: if $\tau(i)$ is too late, no train starting from i has a chance to be the first to reach another node ; this is in particular the case if $\tau(i) = \infty$. Some nodes cumulate both situations, and no train departs or arrives.

The resulting shedules $\widehat{\tau} = \Theta_X(e, \tau)$ depend on the distribution of initial departure times and crossing times of each edge. Defining τ_X^∞ by $\tau_X^\infty = \tau$ on X and $\tau_X^\infty = \infty$ elsewhere, it is obvious that $\Theta_X(e, \tau) = \Theta_X(e, \tau_X^\infty)$, since no train with an infinite departure time has the chance to reach another node first. $\Theta_X(e, \tau_X^\infty)$ is clearly an opening on the initial distribution of departure times on all nodes; it is obviously anti-extensive and increasing. It is also idempotent, as a second scheduling would not change the distribution $\tau(i)$ any further. We also remark that if $\tau(s) = 0$ for $s \in X$, the resulting schedules simply are the shortest path between X and all other nodes.

2.4 Harmonizing the Schedules Is a Shortest Path Problem in a Completed Graph

The preceding unconventional shortest path problem on the graph G can be transformed into a conventional one on an augmented graph G_X, obtained by adding to G a dummy node Ω with weight $\tau(\Omega) = 0$ and dummy edges (Ω, i) between Ω and each node i of X, with $e_{\Omega i} = \tau_i$. Since the travelling time along the edge (Ω, i) is τ_i, it is equivalent for a train to start from i at time τ_i or from Ω at time 0 and follow the edge (Ω, i) for reaching i.The earliest time for a train to arrive at any node k of G is the total duration of the shortest path between Ω and k. It follows that scheduling the graph G amounts to constructing the shortest path between Ω and all other nodes in the graph G_X, which is a classical problem in graph theory for which many algorithms exist. In "Scheduling trains with delayed departures" (http://hal.archives-ouvertes.fr/hal-00547261/fr/), we have presented the algorithms of Moore Dijkstra and of Berge and shown that, if the introduction of a dummy node Ω and dummy edges (Ω, i) is a useful support for thinking, it is not necessary in practice.

3 The Lattice of Floodings with a Unique Regional Minimum

We are now able to decompose a function f into its peak and wells components, given the point of view set X. The function f may be reconstructed by $f = \Theta_X(e^+, f) - \Theta_X(e^-, f)$. Applying an operator ψ to each component leads to a transform $\widehat{\psi}(f) = \psi\Theta_X(e^+, f) - \psi\Theta_X(e^-, f)$. This section analyses the algebraic structure of the peak and wells components in order to identify which operators ψ are admissible and transform a peak or wells component into an other grey tone image with the same characteristics.

The schedule $\widehat{\tau_X}(x) = \Theta_X(e, \tau)$ is the length of the shortest path between Ω and the node x. The edge weights being non negative, there exists a non ascending path between any node and Ω for the valuations $\widehat{\tau_X}$. Hence Ω is the only regional minimum of $\widehat{\tau_X}$.

The following formulations are equivalent:

- Ω is the only regional minimum of $\widehat{\tau_X}$.
- the only regional minima of τ belong to X.
- $\widehat{\tau_X}$ is invariant by the swamping which imposes Ω as only regional minimum ; that is the reconstruction closing of $\widehat{\tau_X}$ with a marker function 0_Ω^∞ equal to 0 at Ω and ∞ elsewhere. This operation, also called flooding, is an anti-extensive leveling [5], written $\Lambda^+(\widehat{\tau_X}, 0_\Omega^\infty)$.
- $\widehat{\tau}$ is invariant by the swamping which imposes the minima of X as only regional minima ; that is the flooding $\Lambda^+(\widehat{\tau}, \tau_X^\infty)$.
- considering the threshold at valuation λ, the subgraph spanning all nodes with a valuation $\tau < \lambda$ (we call it the background at level λ) has only one connected component, and this component contains Ω.

We call F_X the lattice of all functions verifying the previous equivalent criteria.

3.1 Infimum and Supremum in the Lattice F_X

F_X is a complete lattice, with the ordinary order relations for functions, 0 as minimal element, ∞ as maximal element.

If h_1 and h_2 are two functions of F_X, their infimum in F_X is simply $h_1 \wedge h_2$. As a matter of fact, the bakgrounds of h_1 and h_2 at level λ comprise each one connected component ; as they both contain Ω, their union, which is the background of $h_1 \wedge h_2$ also consists in one connected component containing Ω.

Now, $h_1 \vee h_2$ may have regional minima outside X, which have to be suppressed by swamping. For this reason the supremum $h_1 \sqcup h_2$ is equal to $\Lambda^+(h_1 \vee h_2;\ (h_1 \vee h_2)_X^\infty)$.

Remark: These operators are not distributive one with another.

3.2 An Adjunction in the Lattice F_X

If for each node x we define a neighborhood V_x, the erosion of a function f with this variable structuring element is $\varepsilon_V f(x) = \bigwedge\limits_{y \in V_x} f_y$. This operator is an erosion both in the ordinary lattice of functions as in the lattice F_X. Its adjunct dilation in the ordinary lattice is $\delta_V f(y) = \bigvee\limits_{x \in V_y^t} f_x$, where $V_y^t = \{x \mid y \in V_x\}$. Being an adjunction, they verify for each couple (f, g) of functions the relation: $f < \varepsilon_V g \Leftrightarrow \delta_V f < g$.
As $\delta_V f(y) = \bigvee\limits_{x \in V_y^t} f_x$ does generally not belong to F_X, we construct $\widetilde{\delta_V} f(y) = \bigsqcup\limits_{x \in V_y^-} f_x = \Lambda^+(\delta_V f, (\delta_V f)_X^\infty)$. To establish that $\widetilde{\delta_V}$ is the ajunct of ε_V in F_X, we have to prove that $\delta_V f < g \Leftrightarrow \Lambda^+(\delta_V f, (\delta_V f)_X^\infty) < g$.

On one hand, floodings being extensive we have $\Lambda^+(\delta_V f, (\delta_V f)_X) < g \Rightarrow \delta_V f < g$.
On the other hand, if f and g belong to $F_{X.}$, then $\delta_V f < g \Rightarrow \Lambda^+(\delta_V f, (\delta_V f)_X) < \Lambda^+(g, g_X)$. But $\Lambda^+(g, g_X) = g$, as g is invariant by the flooding $\Lambda^+(g, g_X)$.
Concatenating all equivalences, we get for any couple of functions in F_X: $f < \varepsilon_V g \Leftrightarrow \delta_V f < g \Leftrightarrow \widetilde{\delta_V} f = \Lambda^+(\delta_V f, (\delta_V f)_X) < g$, showing that $(\varepsilon_V f, \widetilde{\delta_V} f) = (\bigwedge\limits_{y \in V_x} f_y, \bigsqcup\limits_{x \in V_y^-} f_x)$ is indeed an adjunction on F_X.

Classically $\varepsilon_V \widetilde{\delta_V}$ is then a closing and $\widetilde{\delta_V} f \varepsilon_V$ an opening. Based on these openings one may construct all classical morphological filters based on openings and closings [4],[9].

Razings clip peaks and do not create new holes. Thus the image of F_X by any type of razing or reconstruction opening is F_X. Finally, we obtain a large family of operators which operate on and in F_X.

4 Min, Max and Morphological Operators Through the Decomposition in Peaks and Wells

Suming up, given a set X, serving as view-point, we are now able to decompose any image f into a cumulative image of its peaks f_X^+ and a cumulative image of its wells f_X^-, such that $f = f_X^+ - f_X^-$. Both functions belong to F_X , a lattice where we defined a supremum, infimum and an adjunction. Furthermore this lattice is stable by any type of razing.

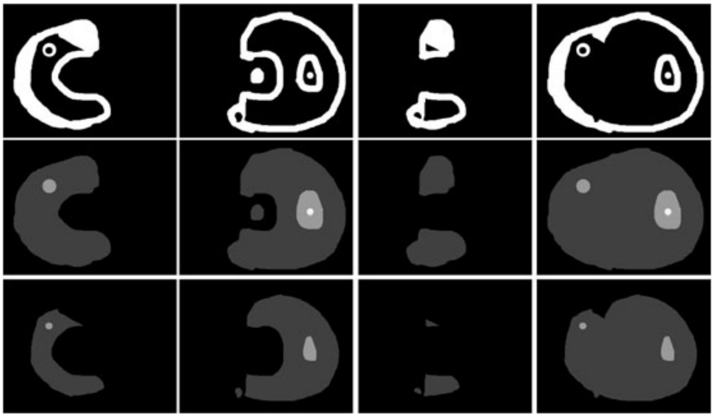

Fig. 1.
Columns 1 and 2 : Binary sets Z and Y and their decomposition
Column 3 : $Z \curlywedge Y$ and its decomposition
Column 4 : $Z \curlyvee Y$ and its decomposition

4.1 "Supremum" and "Infimum" of Two Functions

Let $(f_X^+,\ f_X^-)$ and (g_X^+, g_X^-) be the decomposition of two function into peaks and wells, given the view point X. For any composition law $\triangle$ on F_X, we may define $f \triangle g = f_X^+ \triangle g_X^+ - f_X^- \triangle g_X^-$.

Remark: We do not necessarily have $(f \triangle g)_X^+ = f_X^+ \triangle g_X^+$ and $(f \triangle g)_X^- = f_X^- \triangle g_X^-$. This means that the decomposition $(f_X^+ \triangle g_X^+, f_X^- \triangle g_X^-)$ is not necessarily a minimal decomposition of $f \triangle g$.

Based on the operator $\wedge$ and $\sqcup$ in F_X, we obtain $f \curlywedge g = f_X^+ \wedge g_X^+ - f_X^- \wedge g_X^-$ and $f \curlyvee g = f_X^+ \sqcup g_X^+ - f_X^- \sqcup g_X^-$.

Both operators are illustrated by the images I_{rc} disposed in a matrix in fig.1. Row 1 of the matrix shows the binary images, row 2 and 3 respectively their peak and wells components, taking as point of view set X the boundary of the image. Column1 and 2 show two sets Z and Y, column 3 their intersection $Z \curlywedge Y$ and column 4 their union $Z \curlyvee Y$. The images I_{23} and I_{24} represent respectively $Z_X^+ \wedge Y_X^+$ and $Z_X^+ \sqcup Y_X^+$. The images I_{33} and I_{34} represent respectively $Z_X^- \wedge Y_X^-$ and $Z_X^- \sqcup Y_X^-$. The images I_{13} and I_{14} represent respectively $Z \curlywedge Y = Z_X^+ \wedge Y_X^+ - Z_X^- \wedge Y_X^-$ and $Z \curlyvee Y = Z_X^+ \sqcup Y_X^+ - Z_X^- \sqcup Y_X^-$.

4.2 Operators Operating on the Peaks and Wells

Binary images. Given the decomposition $(f_X^+,\ f_X^-)$ of a function f into peaks and wells, and given an increasing operator ψ from F_X into F_X, we define $\widehat{\psi} f = \psi\left(f_X^+\right) - \psi\left(f_X^-\right)$. The decomposition is not necessarily minimal, but since $f_X^- < f_X^+$ and ψ increasing, $\widehat{\psi} f$ will be positive if f is positive.

Figure 2 presents in a matrix a number of operators on a binary image. The view point set X is the boundary of the image. The first row shows successively

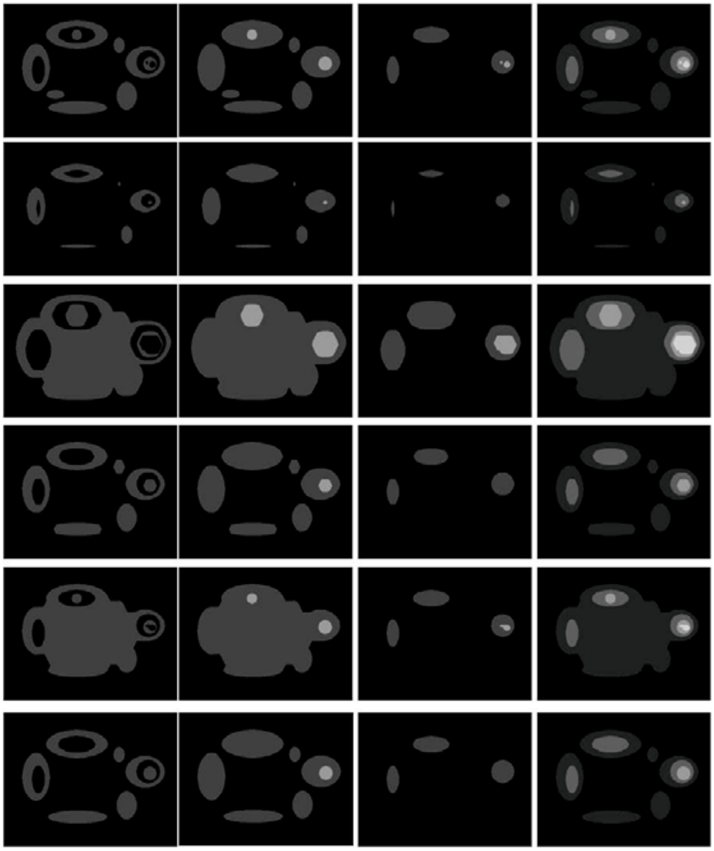

Fig. 2. Decomposition and transformations on binary images

the initial image f, its peak component f_X^+, its wells component f_X^-, and the total variation $f_X^+ + f_X^-$. Rows 2-6 present each a different tranform $\widehat{\psi} f = \psi\left(f_X^+\right) - \psi\left(f_X^-\right)$. First $\psi\left(f_X^+\right)$ and $\psi\left(f_X^-\right)$, represented in columns 2 and 3 are constructed, then $\psi\left(f_X^+\right) - \psi\left(f_X^-\right)$ represented in column 1 and $\psi\left(f_X^+\right) + \psi\left(f_X^-\right)$ represented in column 4. The transforms illustrated in each row are: 1) initial image; 2) erosion of size 11; 3) dilation of size 15; 4) opening of size 11; 5) closing of size 15; 6) reconstruction opening of size 11.

The matrix of images I_{rc} in figure 3 also presents an erosion of size 11 of the same binary image, but the decomposition is made with respect to a different view point set X, represented as a white dot superimposition to the binary set f in image I_{11}. X is contained in a white particle of Z, itself contained in a hole contained in a particle. Images I_{12} and I_{13} present the peaks and wells components of Z with respect to the point of view X, and image I_{13} the total variation. An erosion ε_{11} of size 11 is applied to the peak and wells components represented in images I_{22} and I_{23}. The transformed image $\widehat{\varepsilon_{11}}\,(f) = \varepsilon_{11}\left(f_X^+\right) - \varepsilon_{11}\left(f_X^-\right)$ is presented in image I_{21} and the total variation $\varepsilon_{11}\left(f_X^+\right) + \varepsilon_{11}\left(f_X^-\right)$ in figure I_{24}.

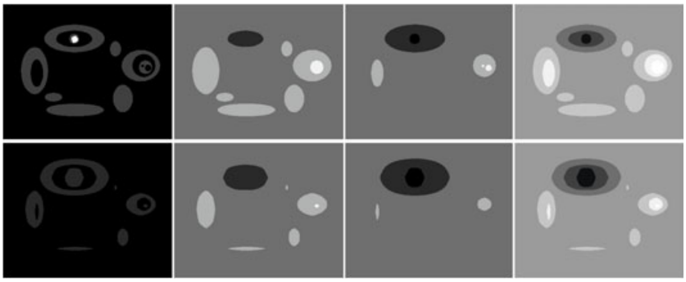

Fig. 3. Decomposition and transformations on binary images. The view point set is the white particle in the first image.

The second row of both figures 2 and 3 present an erosion of size 11, but with respect to a different point of view set X. The first example takes for X the boundary of the image, the second a dot contained within in a particle. In this last case, particles containing X are treated differently from particles which do not contain it. As the particles and holes containing X appear as holes both in the peak as in the wells image, their erosion enlarges them. On the contrary, particles and holes not containing X appear as peaks and are indeed eroded.

Grey tone images. Fig.4 presents the decomposition and transformation of grey tone images, if one takes as point of view the boundary of the image ; there are four matrices of images with an identical disposition. In each matrix the first row shows the positive variation σ^+ and negative variation σ^-, whereas the second row shows the initial image and the total variation $\sigma^\#$. The first matrix presents the initial image and its decomposition. The second matrix an erosion of size 3, the third a dilation of size 3 and the last an opening of size 3. Since the first letter "P" of "Paulus" intersects the boundary of the image, which plays the role of view point set X, it is not transformed as the other letters.

The last example in fig. 5 is taken from an image of the retina. The aim is to detect the small dots and the bloodings without detecting the vessel, although they have the same width. The images are organized as two matrices with the same structure. Both matrices differ by the choice of the view point set. For the left matrix the view point set is the left boundary of the image, and for the right matrix the white segment in superimposition with the vessel in image I_{11}. Both matrices are organized as follows. I_{11} the initial image, I_{12} its filtered version which is decomposed. The peaks are in I_{21}, the wells in I_{31}.

Consider the vessel which crosses the image. In the left matrix, where the left boundary of the image is the point of view set, the right boundary of the vessel produces an upwards transition in the peaks component in I_{21}. In the wells component in I_{31} it is the left boundary of the vessel which produces an upwards transition. Like that, with an appropriate choice of a point of view set, a thin dark structure like the vessel appears in each component as a step for climbing on a plateau. Small dark and round structures, here microaneurisms appear indeed as bright peaks in the wells component and not at all in the peaks component.

Fig. 4. Decomposition and transformation of grey tone images.

A reconstruction opening after a small erosion of both components produces duly suppresses the aneuvrism but leaves the step of the plateau unchanged (see images I_{22} and I_{32}). Subtracting the initial image in I_{12} from the reconstructed image after reconstruction opening of the components in I_{41} yields the final result shown in I_{42}.

Consider now the right matrix. As the point of view is inside the vessel, the vessel does not appear as peak in the wells components in I_{31}. On the contrary in the peaks component I_{21}, it appears as a well. Here again the behaviour of the aneurisms are different, they are not visible in the peaks component and appear as peaks in the wells component. An erosion applied to the components blows up the vessel in the peaks component in I_{22} and suppresses the aneurisms in the wells component. The recomposed image after erosion in I_{41} shows a blown up vessel and no aneurisms. Subtracting $I_{12} \vee I_{41}$ from the initial image I_{12} yields

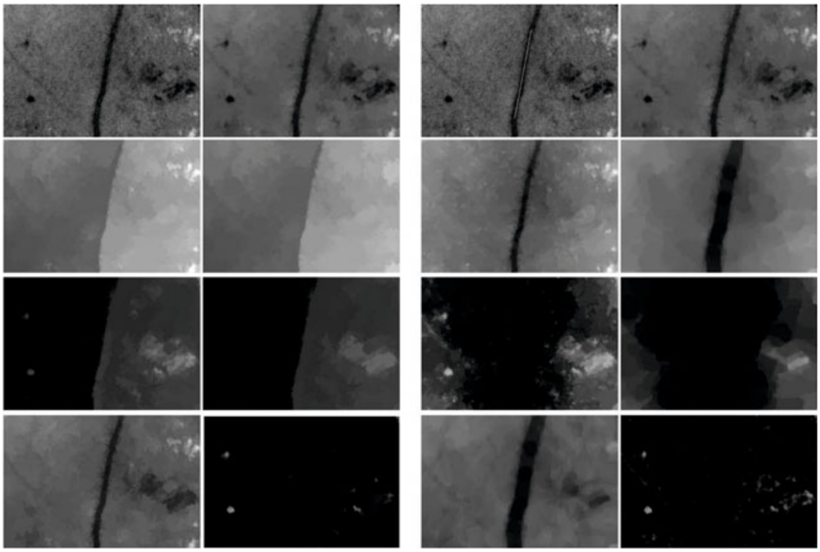

Fig. 5. Analysis of the retina in order to detect the micro-anevrysms and discard the vessel

a residual image in I_{42} where the vessel has disappeared and only the aneurisms are visible.

5 Conclusion

Decomposing an image into peaks and wells leads to interesting operators. We have several degrees of freedom. First chose the right point of view. Then apply the adequate operator on the peaks and wells components. The analysis of the lattice structure of the wells and peaks like images has shown that a large collection of operators are available. The construction of the decomposition is fast, as it only involves shortest paths algorithms. As the resulting component images turn the peaks and wells of the original image into peaks, they may advantageously be encoded as max-trees [7] in order to enlarge the scope of applicable operators to the peaks and wells components and increase the speed of processing.

It is also interesting to decompose the same image with respect to several distinct points of view. Changing the point of view produces a different decomposition of the same scene, with a different distribution of contrast, between the same objects, which may facilitate the segmentation.

More exotic variants can be envisaged. Changing the edge weights, for instance by keeping only the variations along each edge which are higher than a threshold will put the focus with contrasted contours and ignore the objects with a more fuzzy contour. The distribution of weights on the edges may also be directional, favoring some directions and discarding others. This gives the possibility to analyze anisotropies.

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