Geodesic Attributes Thinnings and Thickenings

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Abstract. An attribute opening is an idempotent, anti-extensive and increasing operator that removes, in the case of binary images, all the connected components (CC) which do not fulfil a given criterion. When the increasingness property is dropped, more general algebraic thinnings are obtained. We propose in this paper, to use criteria based on the geodesic diameter to build algebraic thinnings for greyscale images. An application to the extraction of cracks is then given to illustrate the performance of the proposed filters. Finally, we will discuss the advantages of these new operators compared to other methods.

Keywords: Geodesic attributes, diameter, elongation, tortuosity, thinnings, thickenings.

1 Introduction

A pre-processing step consists of filtering out the noise and the unwanted features while preserving, as much as possible, the desired information. Mathematical morphology [6], [9] is based on a set approach and classically uses structuring elements (SE) to obtain information on the morphology of the objects. In [10] and in [8], an overview of morphological filtering is presented. We notice that simple openings and closings with square, disk or hexagon SEs, are often good enough for the filtering task. However, if the structuring elements are able to adapt their shapes and sizes to the image's content, the noise reduction and feature enhancement properties are even better (see for example [2] and [3]). The openings and closings by reconstruction can also be considered as a part of adaptive morphology. This leads Vincent [13] to propose area openings, and more generally, Breen and Jones to introduce attributes openings [4].

Here, we start from Lantuéjoul and Maisonneuve's work, [5], to introduce new attributes based on the geodesic diameter. These attributes are particularly useful to measure the length of thin structures. Many papers provide methods to extract thin structures: morphological top hats, supremum of openings by segments, path openings [11] but none of them has the flexibility of the method proposed here.

This work is a part of an industrial project where our goal is to highlight all the defects from metallic surfaces. These cracks are usually long, narrow and

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not necessarily straight. Standard filters often fail to extract them and these new operators have been developed for this task. More generally, the framework of this study is the detection of cracks.

This paper provides the background to construct algebraic thinnings based on geodesic attributes. Sections 2 and 3 are a review of attribute thinnings and geodesic binary attributes. Section 4 explains how to construct geodesic attribute thinnings, whereas section 5 highlights some practical considerations. Lastly, section 6 illustrates their interest through an application.

2 Background: Attribute Thinnings

2.1 Connected Components and Attributes

Let $I : D \rightarrow V$ be a binary image, with $D \subseteq \mathcal{Z}^2$ typically being a rectangular domain and V the set of values: $V = \{0; 1\}$. The object X included in I is $X = \{x \in D | I(x) = 1\}$ and we denote X^c, the complementary set. We associate to I, a local neighbourhood describing the connexion between adjacent pixels. In this study, each pixel will be connected to its eight nearest neighbours. With this 8-connectivity, we define $\{X_i\}$ the set of the connected components of X.

An attribute operator is defined for all connected components X_i in the following way:

$$Att_\lambda(X_i) = \begin{cases} X_i & \text{if } X_i \text{ satisfies } C_\lambda, \\ \emptyset & \text{otherwise.} \end{cases} \tag{1}$$

with C_λ, a criterion parameterised by λ.

2.2 Attribute Thinnings

On the basis of the definition of the attribute operator, a filter ρ^{Att_λ}, called an attribute thinning, can be introduced:

$$\rho^{Att_\lambda}(X) = \bigcup \{Att_\lambda(X_i), i \in I\} \tag{2}$$

Attribute thinnings are anti-extensive and idempotent (see [4] for the proof). Moreover, if these operators are also increasing, they become attribute openings, denoted γ^{Att_λ}.

The dual transform of a thinning is called a thickening and is defined by duality. In what follows, we restrict our study to thinnings as the computation of thickenings is straightforward.

The non increasingness of these filters could cause some problems, especially when we compute granulometries, ultimate thinnings or greyscale thinnings. In the literature, some solutions have been proposed to solve these issues and we will discuss this point when we will extend these operators to grey level images.

2.3 Grey Level Operator

The extension of attribute thinnings to grey level images is not straightforward, since these operators are not always increasing. First, we will talk about the classical method for openings. Then, we will describe the procedure for thinnings.

The openings extend to the grey level domain in the usual way, by thresholding the initial image to obtain N binary images (with N the number of grey levels in the image). Thus, an opening in grey level may be constructed explicitly by stacking the result of each binary opening, computed from each threshold of the original image. With f an image, $f : D \rightarrow V$ with $V = \{0, \ldots, N\}$, the grey level attribute opening is given by:

$$(\gamma^{Att_\lambda}(f))(x) = \sup \left\{ h \in \{0, \ldots, N\} \,|\, x \in \gamma^{Att_\lambda}(T_h(f)) \right\} \tag{3}$$

where $T_h(f)$ stands for the threshold of f at value h. Throughout this paper, this method will be referred as the “opening binary to grey extension” (OBGE).

For thinnings, at least two methods are available in the literature to construct this extension. They are both presented by Breen and Jones, in [4].

The first one is a method which preserves the information for lower threshold values, once the criterion is fulfilled. Then, it locates the threshold set that satisfy the criterion. Hereafter, this extension is referred to as the “thinning binary to grey extension” (TBGE), and this method has the favour of Breen and Jones.

However, another solution consist in applying exactly the same method as for openings (See OBGE equation 3). Figure 1 (curve b), presents the result of a greyscale thinning with the non increasing criterion: *have a length equal to* λ. In this example, this criterion is fulfilled for a high threshold value whereas it is not true for lower thresholds. The one-dimensional signal is truncated and some edges are emphasised. Therefore, we have filtered out all the unwanted information. Regarding curve c, this is a thinning using the TBGE method and it behaves as a morphological reconstruction by dilation of the curve b.

A real example is presented in section 4.2 where we will discuss the consequences of each method on the result.

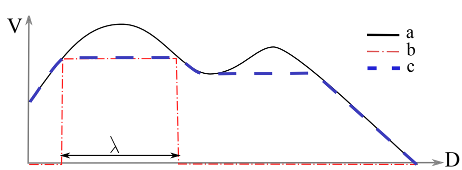


Fig. 1. Example of a grey level thinning with the criterion: *have a length equal to* λ: (a) initial signal, (b) result of the thinning using the OBGE method. (c) result of the thinning using the TBGE method. This curve can be seen as the morphological reconstruction by dilation of the thinned signal (curve b).

3 Geodesic Attributes

From now on, an "object" will be a connected component. The following definitions are valid in continuous or discrete contexts. In practice, as previously said, computations are done in $\mathcal{Z}^2$, with an 8-connectivity. Hereafter, we will define the geodesic attributes on an arbitrary object.

3.1 Geodesic Diameter

Lantuéjoul and Maisonneuve, in [5], asked a question: "What is the length of an object?" The first idea is to measure the length of the segment between its end points (Figure 2(a) and 2(b)); though, it is not a satisfactory definition, as this segment is not always a path included within the object. Moreover, defining the end points is not a trivial question. Another measurement can be considered: the length of the set of points corresponding to a homothetic skeleton of the object (Figure 2(c) and 2(d)). Since this is a part of the object, its length is a more representative measurement. However, skeleton computation methods are difficult to use. Fluctuations can be inserted when small modifications are involved, especially when the objects have rough boundaries.

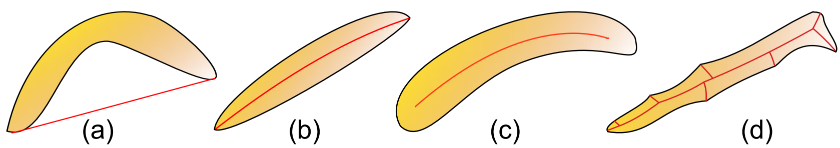


Fig. 2. (a) and (b): the length of the segment between its ends points. (c) and (d), the measurement of its skeleton. These definitions are not always suitable.

Lantuéjoul and Maisonneuve use the notion of geodesic arcs, which are the shortest paths between two points of an object. Let X be an object and x, y two points from X. Figures 3(a) and 3(b) illustrate two paths between these two points and their corresponding geodesic arc, whose length is written $dX(x, y)$. Thus, measuring the length of an object is measuring the length of its longest geodesic arc (Figure 3(c)):

$$L(X) = \sup_{x,y \in X} dX(x, y) \tag{4}$$

$L(X)$ is the geodesic diameter of X and has mainly three advantages: it is a general definition, as it is valid for every object. It is a robust definition, as a small change in the shape of the object will cause, at most, a small change of the measure of the geodesic diameter, if the topology of the object is not changed. Finally, the computation of $L(X)$ leads to other attributes such as the geodesic elongation and the geodesic tortuosity.

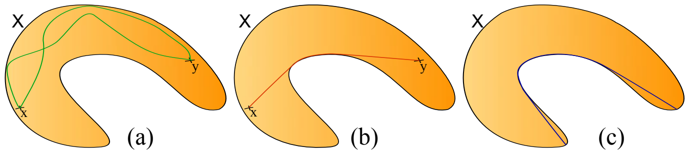


Fig. 3. (a), two paths between x and y; (b), geodesic arc between these two points; (c), geodesic diameter of X

3.2 Geodesic Elongation

The geodesic diameter is the first available attribute and it gives a satisfactory definition of the length of an object. However, we do not have many details on its shape. By combining the length factor with the area of the CC, we do have some information on its tendency to be elongated. The longer and narrower an object is, the higher the elongation factor will be. On the contrary, any disk will have a value of 1. The elongation factor, introduced in [5], is computed as follows:

$$E(X) = \frac{\pi L^2(X)}{4S(X)} \tag{5}$$

where $S(X)$ denotes the area of X. Note that this definition can naturally be generalised to higher dimensions.

3.3 A New Geodesic Attribute: The Geodesic Tortuosity

We propose a new descriptor derived from the geodesic diameter: the geodesic tortuosity. A pair of points $\{x, y\}$ is called a pair of geodesic extremities of X if and only if $dX(x, y) = L(X)$. Note that some objects may have more than one pair of geodesic extremities (ie. a disk). Let $E_x(X) = \{\{x_0, y_0\}, \{x_1, y_1\}, \ldots\}$ be the set of geodesic extremities of X. Then we define $L_{Eucl}(X)$ as the minimal Euclidian distance between geodesic extremities:

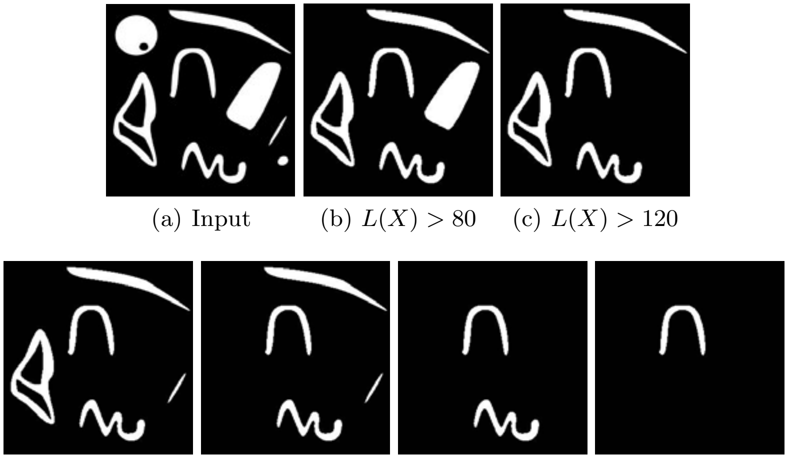
$$L_{Eucl}(X) = \min_{(x,y)\in E_x(X)} \|x, y\| \tag{6}$$

The tortuosity is the ratio between the geodesic diameter and $L_{Eucl}(X)$ (Equation 7). The more twisted the object is, the higher its tortuosity will be. On the contrary, any straight object will be valuated by 1.

$$T(X) = \frac{L(X)}{L_{Eucl}(X)} \tag{7}$$

3.4 Geodesic Attribute Properties and Comments

All these attributes are rotation invariant. Moreover, the geodesic elongation and tortuosity attributes are scale invariant. Other attributes could be derived from the computation of the geodesic diameter; we can name one, which is



(a) Input (b) $L(X) > 80$ (c) $L(X) > 120$

(d) $E(X) > 5$ (e) $E(X) > 10$ (f) $T(X) > 1.5$ (g) $T(X) > 2$

Fig. 4. Filtering result with geodesic attributes criteria C_λ: (a) initial image; (b) and (c): geodesic diameter; (d) and (e): geodesic elongation; (f) and (g): geodesic tortuosity

scale and rotation invariant: the circularity attribute. It is, in fact, the inverse of the geodesic elongation. In comparison, Urbach and Wilkinson, in [12] and [14], used the moment of inertia instead of the geodesic diameter to compute the elongation attribute. However, they are two very different attributes and the geodesic diameter is, in our opinion, a better representation of the length of an object.

4 Geodesic Attributes Thinnings

The main idea of this paper is to combine geodesic attributes with thinnings to obtain a new powerful family of filters.

4.1 Binary Images

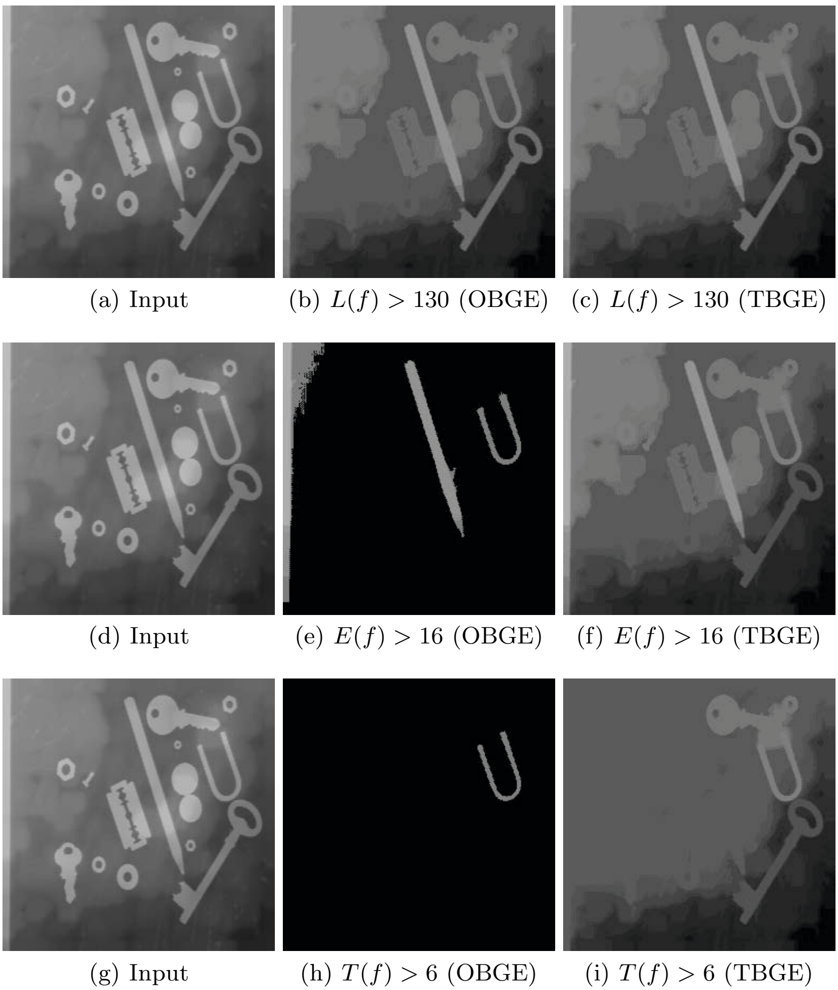
Figure 4(a) is a toy example where we can apply our different operators. It is a set of objects which look like fibres and we want to filter out these objects, with the following criteria:

- Suppress the particles whose geodesic diameter is smaller than 80 pixels in Figure 4(b) and smaller than 120 pixels in Figure 4(c);
- Suppress the particles which are not elongated, i.e. whose geodesic elongation is smaller than 5 in Figure 4(d) and smaller than 10 in Figure 4(e);
- Suppress all particles which are not tortuous, i.e. whose geodesic tortuosity is smaller than 1.5 in Figure 4(f) and smaller than 2 in Figure 4(g).

Hence, we can characterise the shape of these structures with a good accuracy.

4.2 Grey Level Images

Two methods have been presented in section 2.3, to extend this algorithm to grey level images. Figure 5 shows the differences between these approaches. For a segmentation task, using the OBGE method, yields the best results (Figures 5(b), 5(e) and 5(h)) since the tools are correctly isolated from the background and a simple threshold often leads to the wanted segmentation. However, to isolate all the filtered objects of the image, a top hat has to be computed on the thinned image using the TBGE method.



(a) Input (b) $L(f) > 130$ (OBGE) (c) $L(f) > 130$ (TBGE)

(d) Input (e) $E(f) > 16$ (OBGE) (f) $E(f) > 16$ (TBGE)

(g) Input (h) $T(f) > 6$ (OBGE) (i) $T(f) > 6$ (TBGE)

Fig. 5. Filtering by geodesic attributes: diameter, elongation and tortuosity. The first column is the initial image. For the second (resp. third) column, the extension for greyscale image is the OBGE method (resp. TBGE method)), discussed in 2.3.

We notice for the geodesic diameter, these two methods give exactly the same result. Thus, this thinning based on the geodesic diameter behaves as an opening for this image. This is due to the fact that most of the objects in figure 5(a) have a convex shape.

The choice of the method will depend on our applications. In the following, the OBGE method is used, as we want to isolate the cracks from the background.

5 Practical Considerations and Optimisation

5.1 Computation of the Geodesic Diameter

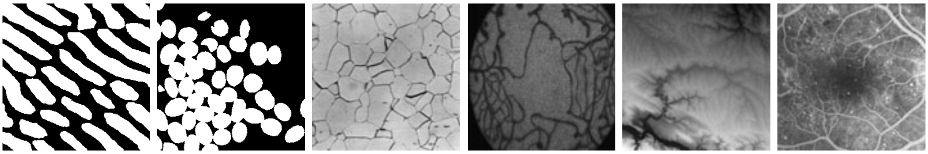
The geodesic diameter has to be computed for every CC of every threshold of the image. Hence, the complexity of this algorithm depends mainly on the number and the area of these CCs. Maisonneuve and Lantuéjoul in [5] designed an efficient parallel implementation for binary images to compute the geodesic diameter in a hexagonal grid. Let Y be a set of emitting sources and X be an object simply connected with Y a subset of X. The computation of $L(X)$ requires a propagation step from the emitting sources into X. If Y is correctly chosen (e.g. the boundaries of the objects), the geodesic diameter would be deduced from the last wave iteration. However, this algorithm does not support holes; the CC has to be simply connected, otherwise the propagation wave would never end, turning infinitely around the holes. This is a real limitation to the use of this algorithm.

Classical attribute filters are often based on a tree representation, as presented by Salembier et al. in [7]. However, we could not find a fast way to update the geodesic diameter value, when a new pixel is added to a CC. Hence, the connected component tree representation is not as efficient as for simple attributes (area, width, height).

Then, we prefer using a direct implementation where the greyscale image is converted into binary images. Each connected component is isolated using a stack of pixels as container. Each pixel belonging to the boundary of the object is a starting point to a region growing process in order to build a distance map. The highest value of all the distance maps is the geodesic diameter.

5.2 Optimisation

A possible acceleration is available for the geodesic diameter and the geodesic elongation thinnings. During, the region growing process, when the front wave becomes larger than the attribute, it is useless to compute the real value of this attribute. The criterion is passed, we keep the current connected component and we can stop the propagation step. The time saved is huge (see table 1 and figure 6) but it does depend on λ.



(a) Eutectic (b) Coffee (c) Grains3 (d) Macula (e) Relief (f) Retina2

Fig. 6. Images used to build table 1

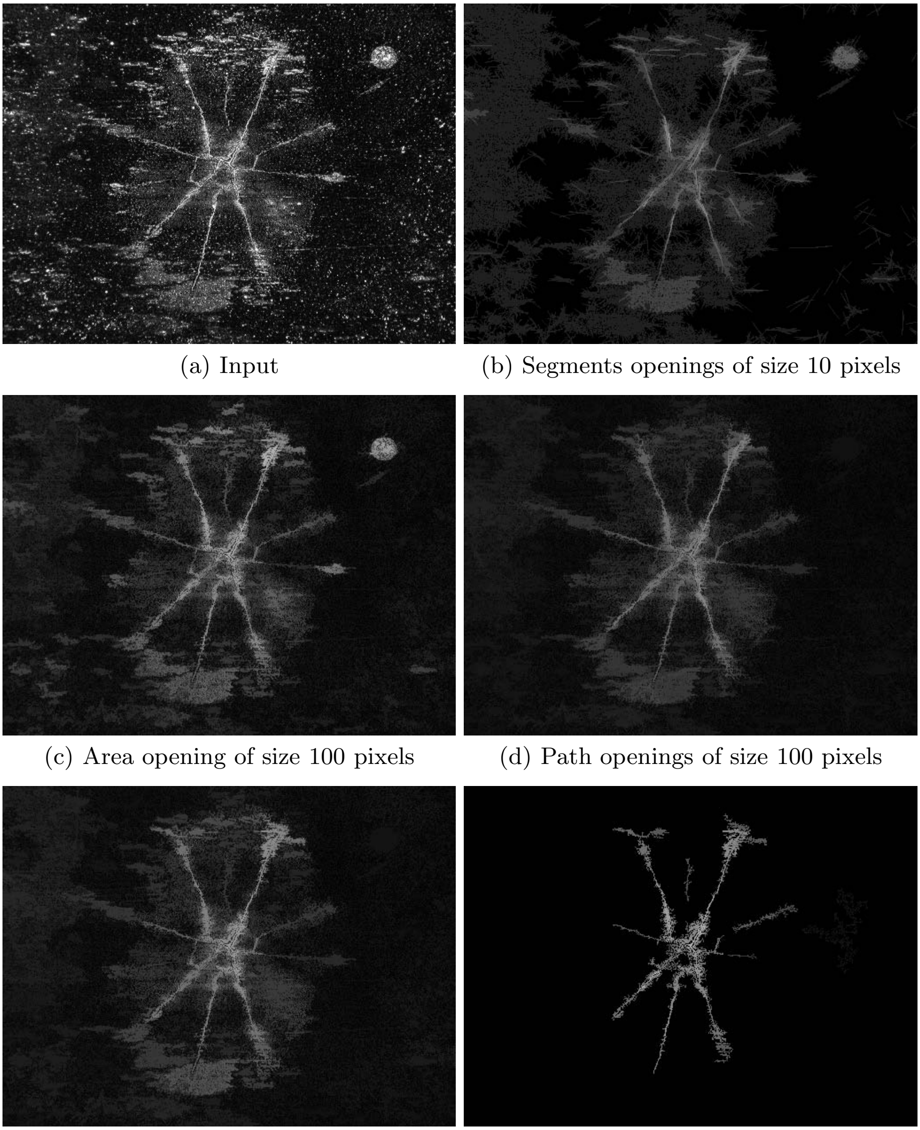
Table 1. Running times for different images for a geodesic diameter thinning or thickening with the OBGE method and for $\lambda = 20$. Timings are in seconds. Laptop computer: Intel Core2 Duo T7700 @ 2.40GHz

Images	Direct method	Accelerated method (see 5.2)
Coffee (256 x 256 x 1-bit)	36	0.079
Eutectic (256 x 256 x 1-bit)	37	0.015
Grains3 (256 x 256 x 8-bits)	3650	5.4
Macula (256 x 256 x 8-bits)	3850	2.7
Relief (256 x 256 x 8-bits)	1024	0.99
Retina2 (256 x 256 x 8-bits)	1820	1.56

6 Results

The new operators have been applied to real images in the framework of our project. Here, the geodesic attribute thinnings are used to detect long and narrow structures. The proposed image, is a crack and we want to extract it (Figure 7(a)). We use and compare five different methods in order to do it:

- The supremum of openings by segments of size 10 pixels oriented every 2 degrees. Figure 7(b) presents the result and we see that, only the linear part of the crack is preserved. This method is used to extract linear features, and when a crack is not straight, this method is not efficient.
- An area opening of size 100 pixels (Figure 7(c)). Here we observe that the noise is correctly filtered out. However, the circle structure is preserved, as well as the compact noise which is larger than 100 pixels.
- A maximum path opening of size 100 pixels (Figure 7(d)). For each pixel, the graph has 3 predecessors and 3 successors according to [1]. The result is much better than the previous method. However, not all the branches of the crack are extracted. When the path is too tortuous, this algorithm is not able to follow the entire crack and fails to estimate its length.
- A geodesic diameter thinning of size 100 pixels yields a better result since all the branches are correctly extracted (Figure 7(e)). This algorithm is a connected operator. The tortuosity of the CCs has no influence over its length.
- A geodesic elongation thinning of value 20 (Figure 7(f)). This method filters out all the noise and offers a very efficient detection.



(a) Input

(b) Segments openings of size 10 pixels

(c) Area opening of size 100 pixels

(d) Path openings of size 100 pixels

(e) Geodesic diameter thinning of size 100 pixels

(f) Geodesic elongation thinning of size 20

Fig. 7. Crack detection: to detect these thin structures, we use 5 different methods. The geodesic attribute thinning yields the best detection.

7 Conclusion and Future Work

We have presented new attributes thinnings based on geodesic criteria. The geodesic elongation, the geodesic diameter and the geodesic tortuosity are non increasing criteria which offer good filtering capabilities. Thus, the extraction of long and elongated structures is easy and is made in an efficient way. It offers more flexibility compared to other methods. Moreover, we can have a representation of the elongation and the tortuosity, which is not possible with path openings. An acceleration is proposed for thinnings based on the geodesic diameter and the geodesic elongation. Therefore, these operators are fast enough for many applications.

Speed up the computation of the geodesic diameter seems to be difficult. However, we are working on the elaboration of a new strategy to approximate the geodesic diameter with a high accuracy. In practice, we get very similar results but the final algorithm is several time faster. In average, the running times are divided by a factor of 20, compared to the accelerated method presented in 5.2. Hence, the extension to 3D images will be straightforward. Future work will also include granulometries and ultimate thinnings with geodesic attributes.

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