Pattern Spectra from Partition Pyramids and Hierarchies

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Abstract. Constrained connectivity relations partition the image definition domain into connected sets of maximal extent. The homogeneity and maximality of the resulting cells is subject to non-connective criteria that associate to logical predicates. The latter are based on attribute metrics, and are used to counter the leakage effect of the single-linkage clustering rule. Linkage is controlled by some dissimilarity measure and if unconstrained, may be used to generate regular connectivity classes. In this paper we introduce a hierarchical partition representation structure to map the evolution of components along the dissimilarity range in the absence of constraints. By contrast to earlier approaches, constraints may be put in place on the actual structure in the form of filters. This allows for custom and interactive segmentation of the image. Moreover, given an instance of the dissimilarity measure, one can retrieve all connected sets making up the corresponding connectivity class, directly from the hierarchy. The evolution of linkage relations with respect to the attributes on which the predicates are based on is used to compute a new type of pattern spectrum that is demonstrated on two real applications.

1 Introduction

Image segmentation in the context of connected morphology [18] is the partitioning of the image definition domain into homogeneous cells of maximal extent. Maximality is guaranteed through a connective criterion of homogeneity, i.e. one that generates a connection [17, Chap. 2] from the homogeneous image regions. Connective criteria however, introduce leakage effects on the connected components and examples are discussed in [12].

Constrained connectivity [19] provides a segmentation framework that relies on a sub-connection of the canonical path-based graph connectivity. This is referred to as *dissimilarity-based* or α-connectivity [19] and can be constrained through a series of logical predicates. Utilizing appropriate constraints minimizes the leakage effect in the process of generating homogeneous image regions by

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preventing the creation of a maximal partition [18]. That is because the binary criterion associated to each predicate is not connective. This works in the benefit of many applications since it allows for strict controls over the segmentation process.

The α-connected components (α-CCs) [19], also known as single-linkage components [7], are equivalence classes on the image definition domain, thus defining a unique partition. Constrained α-partition cells are non maximal in the sense of a connective criterion, and an example is discussed in Section 3. α-partitions however, irrespective of any constraints, are totally ordered with respect to the dissimilarity range [15] and can be organized in a hierarchical structure that we refer to as a *partition pyramid*. This can be further reduced to a compact equivalent, the *partition hierarchy*, which is characterized by strict inclusion, thus resolving redundancies caused by replicating components over multiple pyramid levels. Both structures are discussed in Section 4. Running queries based on cell attributes [2] allows for customizing the segmentation in ways not supported by the conventional connective segmentation scheme. The work in this paper deals with computing pattern spectra [5] from both types of partition representation structures. This is described in Section 5 and it is followed by two applications given in Section 6. A short discussion and conclusions are given in Section 7.

2 Preliminaries

Let I be a gray-tone image and E be its definition domain, i.e. a Euclidean subspace. A partition $\mathbf{P}$ of E is its division into a set of non overlapping and non-empty *cells*, the union of which is equal to E. The cells of $\mathbf{P}$ are both collectively exhaustive and mutually exclusive with respect to the set being partitioned. The formal definition as given in [17, Chap. 1], is the following:

Definition 1. *Let E be the definition domain of an image. A partition $\mathbf{P}$ of E is a mapping $x \rightarrow \mathbf{P}(x)$ from E into the power set of E, denoted by $\mathcal{P}(E)$, such that:*

1. $\forall x \in E \Rightarrow x \in \mathbf{P}(x)$;
2. $\forall x, y \in E \Rightarrow \mathbf{P}(x) = \mathbf{P}(y)$ *or* $\mathbf{P}(x) \cap \mathbf{P}(y) = \emptyset$.

The term $\mathbf{P}(x)$ above indicates a cell of $\mathbf{P}$ marked by/containing a point $x \in E$. It follows that:

$$\bigcup_{x \in E} \mathbf{P}(x) = E. \tag{1}$$

A partitioning scheme frequently encountered in image analysis is the separation of the image content into foreground and background components. This dichotomy can be realized by connected operators, examples of which are the connectivity openings and closings. Given a point $x \in E$ that marks a set $X \subseteq E$, the connectivity opening Γ_x extracts the set of maximal extent containing x or returns $\emptyset$ otherwise. A set $C \subseteq E$ is of maximal extent if there is no other set $C' \supset C$ such that $C' \subseteq E$ and $C' \in \mathcal{C}$. In this case C is referred to as a *connected component* according to the connectivity class $\mathcal{C}$.

A connectivity class is the family of all the sets of a space E that are connected according to some notion of connectivity. The axiomatic definition given in [17, Chap. 2] states:

Definition 2. *Let E be an arbitrary non-empty space. A* connectivity class *or* connection $\mathcal{C}$ *is any family in $\mathcal{P}(E)$ such that:*

1. $\emptyset \in \mathcal{C}$;
2. $\forall\, x \in E, \{x\} \in \mathcal{C}$;
3. *for each family $\{C_i, i \in L\} \subseteq \mathcal{C}$, $\bigcap_i C_i \neq \emptyset$ implies $\bigcup_i C_i \in \mathcal{C}$, where L is an index set.*

Connectivity openings are directly related to connectivity classes [17, Chap. 2], and can be customized to address more general notions of connectivity such as clustering, contraction [14, 1], mask-based connectivity [11] or partition-induced connectivity [18, 12]. They are anti-extensive, increasing and idempotent operators and form the basis of attribute filters [2]. The latter are edge preserving operators that extract connected components which satisfy some attribute criterion. Attribute filters and other connected operators can be organized in families known as *granulometries* [17], with respect to some scale or attribute parameter s from a totally ordered set, from which one can study the distribution of image detail with respect to the concerned attribute.

Let $S = \{0, s_1, ..., s_{\max}\}$, be a set of totally ordered thresholds for some increasing attribute. Formally, a granulometry of a binary image X can be defined as a decreasing family of attribute openings $\{\Gamma^s \mid s \in S\}$ for which:

$$\forall s, s' \in S \Rightarrow \Gamma^s(\Gamma^{s'}(X)) = \Gamma^{s'}(\Gamma^s(X)) = \Gamma^{\max(s,s')}(X). \tag{2}$$

Granulometries for non-increasing operators have been investigated in [22]. The order in that case is preserved by utilizing appropriate filtering rules. The distribution of image detail that is often given by the sum of pixels, with respect to one or more attributes is a histogram that is referred to as the granulometric curve or *pattern spectrum* [5]. An attribute class or histogram bin to which a point $x \in X$ contributes, is the smallest value of s for which $x \notin \Gamma^s(X)$. The pattern spectrum by $\mathtt{PS}_{\Gamma^s}(X)$ applying an attribute-specific granulometry $\{\Gamma^s\}$ to a binary image X is defined as [22]:

$$(\mathtt{PS}_{\Gamma^s}(X))(u) = -\left.\frac{\mathrm{d}\xi(\Gamma^s(X))}{\mathrm{d}s}\right|_{s=u}. \tag{3}$$

The term ξ is the Lebesgue measure, which for $n = 2$ is the set area.

3 Constrained Connectivity

Consider a gray-tone image I projected on a graph space in which, vertices (nodes) correspond to atomic elements, and edges to pairs of adjacent vertices. A path $\{x \rightsquigarrow y\}$ between any two elements x to y is a chain of pairwise adjacent elements commonly given in the form of $\{x \rightsquigarrow y\} \equiv \langle x_0 = x, x_1, ..., x_n = y\rangle$.

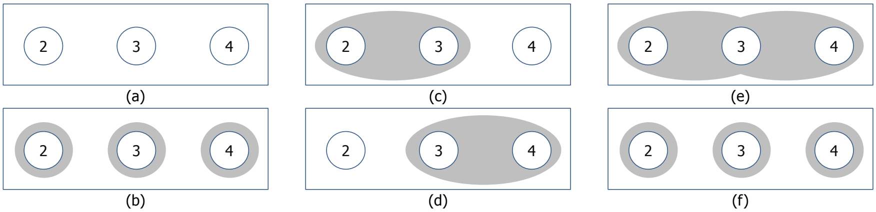


Fig. 1. Three points on the input space labeled by some attribute value, (a). Setting α=1 yields 6 connected sets; the 3 singletons in (b), the set $\{2, 3\}$ in (c), the set $\{3, 4\}$ in (d), and the set of maximal extent, i.e. α-connected component, in (e). For the last three, the difference between neighbors is no greater than 1. The total range constraint $\omega = \alpha$ yields three singleton sets since the ω range of the 1-CC in (e) is 2, i.e. ω prevents its creation, (f).

Links between adjacent nodes are called edges. Assuming that $\mathtt{d}$ is the slope, i.e. the intensity difference, between any two adjacent elements, then setting $\mathtt{d} = 0$ leads to the definition of *flat zones* [16], which are connected image regions of constant intensity. Note that $\mathtt{d}$ is not necessary intensity-specific. The term flat-zone refers to components with members of 0 dissimilarity between them, and in the more general case, single-linkage [7] or α-connected components [19], also known as *quasi-flat zones* [6], can be defined as follows:

$$C^{\alpha}(x) = \{x\} \cup \{y \mid \exists\{x \rightsquigarrow y\} : \forall x_i \in \{x \rightsquigarrow y\} \ \wedge \ x_i \neq y, \ \mathtt{d}(x_i, x_{i+1}) \leq \alpha\}. \quad (4)$$

That is, all atomic elements are connected sets themselves, and a connected set of maximal extent, i.e. a connected component, marked by a point $x \in E$ is the union of all points $y \in E$, such that for each one there exists a path to x in which all adjacent elements have a dissimilarity measure less than or equal to α. The α parameter in the above definition is a threshold; if the difference between some attribute of x and y is less than or equal to α, the two are directly connected, i.e. there exists an edge between x and y, thus are members of the same α-connected component $C^{\alpha}(x)$. The case in which $\mathtt{d}(x, y) > \alpha$ does not imply that x and y do not belong to the same $C^{\alpha}(x)$ but only that there is no direct linkage between them. Note that any α-CC given by (4), satisfies the two last conditions of Def. 2, i.e. α-connectivity is a sub-connection of the canonical path-wise connection on a graph space. Fig. 1 shows an example of a single 1-CC consisting of three points. In image (e) the union of two α-connected sets $\{2, 3\}$ and $\{3, 4\}$ that have a non-empty intersection, is α-connected too.

Connectivity relations based on dissimilarity measures are known to suffer from leakage effects through paths in which adjacent elements differ less than α. One solution, proposed explicitly for this problem, is the introduction of constraints through a sequence of logical predicates based on various attributes [21]. An example is the ω range [19], i.e. the total intensity variation allowed within a component, which leads to the following definition:

$$C^{(\alpha,\omega)}(x) = \max\Big\{C^{\alpha_j}(x) \mid \alpha_j \leq \alpha \text{ and } \mathrm{R}(C^{\alpha_j}(x)) \leq \omega\Big\}. \quad (5)$$

A $C^{(\alpha,\omega)}(x)$ connected component is essentially the maximal indexed $C^{\alpha_j}(x)$ containing the point $x \in E$ that does not exceed the local and global dissimilarity threshold α and ω respectively. The latter is given by the R() functional.

The two conditions of (5) can be expressed as Boolean valued functions, each returning **true** when the associated argument satisfies the corresponding predicate and **false** otherwise. Consider a binary criterion σ [18] enumerating the logical output of each predicate in the general case of constrained connectivity.

Definition 3. *Let E and T be two arbitrary sets and let $\mathcal{F}$ be a family of functions from E into T. A criterion σ on the class $\mathcal{F}$ is a binary function from $\mathcal{F} \times \mathcal{P}(E)$ into $\{0,1\}$ such that for each function $f \in \mathcal{F}$, and for each set $X \in \mathcal{P}(E)$:*

1. *$\sigma[f, X] = 1$, if the predicate returns **true**,*

2. *$\sigma[f, X] = 0$, if the predicate returns **false**.*

Moreover, it is assumed that for all functions the respective criteria are satisfied on $\emptyset$, i.e.:

$$\sigma[f, \emptyset] = 1, \forall\, f \in \mathcal{F}. \tag{6}$$

In the interest of segmentation as described in [18], searching for the largest partition in conjunction to the connectivity class axiomatics, Serra concludes to the more explicit *connective criterion* which is defined as follows:

Definition 4. *A criterion $\sigma : \mathcal{F} \times \mathcal{P}(E) \rightarrow \{0,1\}$ is connective if for each set $f \in \mathcal{F}$ the sets X for which the predicate(s) is (are) satisfied, generate a connection, i.e.:*

1. *for the class of singletons $\mathcal{S}$ and $\forall\, f \in \mathcal{F}, \{x\} \in \mathcal{S} \Rightarrow \sigma[f, \{x\}] = 1$;*
2. *$\forall\, f \in \mathcal{F}$ and $\forall \{X_i\} \in \mathcal{P}(E) : \sigma[f, \{X_i\}] = 1$ the following holds: $\bigcap X_i \neq \emptyset$ and $\bigwedge \sigma[f, \{X_i\}] = 1 \Rightarrow \sigma\left[f, \bigcup X_i\right] = 1$.*

Returning to the example of Fig. 1, it is seen that the three points of image (a) define 6 α-connected sets for $\alpha = 1$; that is the 3 singletons, the sets $\{2,3\}$ and $\{3,4\}$, and the 1-CC consisting of all 3 points, i.e. the connected set of maximal extent. The conditions for the criterion σ to be connective, are satisfied. In particular, the sets $\{2,3\}$ and $\{3,4\}$ have a non-empty intersection and the α predicate is satisfied in their union. This however, is not the case when a further constraint is added. Image (f) shows an example for the case that $\omega = \alpha$ (ω is the total range). The corresponding criterion σ is satisfied on each of the two overlapping connected sets but fails on their union because the total range is 2. Thus, σ does not generate a connection for $\alpha = \omega = 1$, i.e. it is not a connective criterion.

4 Partition Pyramids and Hierarchies

Creating a partition of the image definition domain based on constrained connectivity relations was so far dealt with by constraining the evolution of regular α-CCs. In this section we propose an alternative method, in which the evolution of components is computed under the absence of constraints, and represented in a hierarchical structure. Constraints are put in place in a separate stage, through binary attribute criteria on the respective α-CCs. Creating the desired partition becomes an equivalent process to regular attribute filtering, and allows for interactive segmentation.

Recall that the notion of connectivity defines an equivalence relation on the image definition domain. Like with regular adjacency-based connectivity, dissimilarity-based connectivity was shown to be reflexive, symmetric and transitive [21, 19], thus leading to a partition of any given I to a finite set of cells that correspond to α-CCs. The cells of $\mathbf{P}^\alpha$ are also known as *equivalence classes.* Note that all elements in a given cell are equivalent among themselves and no element is equivalent to any other element from a different cell.

Given an image I, let $\Pi^A(E)$ be the set of all α-partitions of its definition domain E. A is called the alpha dissimilarity range and is defined as a vector of thresholds $A = [0, 1, ..., \alpha_{\max}]$ whose upper bound depends on the dissimilarity measure $\mathtt{d}$. Given a point $x \in E$ marking a cell of a partition $\mathbf{P}^\alpha \in \Pi^A(E)$, with $\alpha \in A$ and assuming that $|A| > 1$, then for any other $\alpha' \in A$:

$$\forall x \in E, \text{if } \alpha' < \alpha \Rightarrow C^{\alpha'}(x) \subseteq C^\alpha(x) \Rightarrow \mathbf{P}^{\alpha'} \preccurlyeq \mathbf{P}^\alpha. \tag{7}$$

The relation $\preccurlyeq$ denotes a notion of order with respect to $\alpha \in A$ [19]. The family of ordered partitions of E for the entire α dissimilarity range is defined as follows:

Definition 5. *A partition pyramid of E assuming that $|A| > 1$, is a mapping $\triangle^A : E \rightarrow \Pi^A(E)$ given by:*

$$\triangle^A = \left\{\mathbf{P}^{\alpha=0}, \mathbf{P}^{\alpha=1}, ..., \mathbf{P}^{\alpha_{\max}}\right\} \mid \mathbf{P}^{\alpha'} \preccurlyeq \mathbf{P}^\alpha, \forall\, \alpha' < \alpha \text{ with } \alpha', \alpha \in A. \tag{8}$$

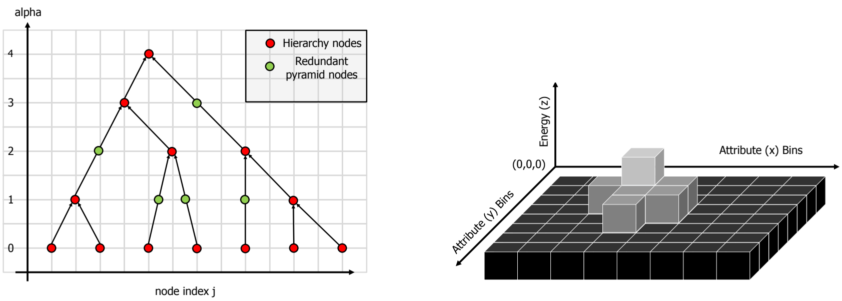
A pyramid level $\triangle^A_\alpha \in \triangle^A$ is a partition $\mathbf{P}^\alpha$ of E, with $\alpha \in A$. Note that the base of the pyramid corresponds to the finest, and the tip to the coarsest partition of E, with respect to the dissimilarity measure $\mathtt{d}$.

Consider a variable $j \in J^\alpha$, in which $J^\alpha \subseteq \mathbb{Z}$ is an index set, employed to address the α-CCs making up $\mathbf{P}^\alpha$. Given a point $x \in E$:

$$\exists! j \in J^\alpha : x \in C^\alpha_j. \tag{9}$$

This complies with the first condition of Def.1, i.e. each point belongs to a unique cell of the partition, while the second condition follows, i.e. $\bigcup_{j \in J^\alpha} C^\alpha_j = E$.

A partition pyramid is a multi-scale partition representation structure which is often characterized by large redundancies. This is due to the persistence of some α-CCs in more that one level of $\triangle^A$. To counter this, an index mapping of α-CCs is introduced, that leads to a hierarchical representation configured with strict inclusion.



(a) (b)

Fig. 2. Example of a 5-level α-partition hierarchy and pyramid (a). The union of all nodes defines the pyramid structure. The red nodes alone are the resulting hierarchy. The green nodes represent the redundancy of the pyramidal representation. Example of a 2D pattern spectrum (b).

Definition 6. *Let $\triangle^A$ be an α-partition pyramid of a gray-tone image I, defined for a dissimilarity measure* d *over a set of threshold values A. An α-partition hierarchy $\pitchfork^A$ is a family of ordered mappings $\pitchfork^A_\alpha$: $J^\alpha \rightarrow K^\alpha$ with $K^\alpha \subseteq J^\alpha$, given by:*

$$\pitchfork^A = \left\{ \pitchfork^A_{\alpha=0}, \pitchfork^A_{\alpha=1}, ..., \pitchfork^A_{\alpha_{\max}} \right\} \mid \pitchfork^A_{\alpha'} \prec \pitchfork^A_\alpha, \; \forall \; \alpha' < \alpha \textit{ with } \alpha', \alpha \in A, \tag{10}$$

and $\forall \alpha \in A \setminus 0$ *and* $\forall j \in J^\alpha$*:*

$$\pitchfork^A_\alpha = \left\{ C^\alpha_j \mid \left(C^\alpha_j \in \triangle^A_\alpha \right) \wedge \left(C^\alpha_j \notin \triangle^A_{\alpha-1} \right) \right\}. \tag{11}$$

In words, each level of the hierarchy $\pitchfork^A$ contains explicitly only those elements of the corresponding pyramid level, that appear for the first time.

The α-partition hierarchy $\pitchfork^A$ can be viewed as a lossless compression of an α-partition pyramid. Each level $\triangle^A_\alpha$ can be restored as follows:

$$\triangle^A_\alpha = \left\{ \bigvee_{\alpha' \in [0,...,\alpha],\; j \in J^{\alpha'}} C^{\alpha'}_j \mid C^{\alpha'}_j \in \pitchfork^A \right\}, \tag{12}$$

i.e. it is the set of all maximal $C^{\alpha'} : \alpha' \leq \alpha$, which further define a partition of E. Fig. 2(a) shows an example of a five-level color-coded partition and hierarchy, to differentiate between redundant and essential (unique) nodes.

5 Pattern Spectra

Pattern spectra are typically computed from a set of operators, ordered with respect to some global variable, and an example is the morphological granulometries. α-partition pyramids by contrast to regular connected granulometries, provide an order with respect to the local dissimilarity parameter α. To compute the α-connected pattern spectra of a gray-tone image I, one needs to compute the stack of all possible α-partitions.

Given any partition $\mathbf{P}$ of E with cells indexed by a variable $j \in J$, the u entry or bin of the pattern spectrum is equivalent to the sum of the areas of all partition cells with attribute measures contained within the bounds of u, i.e.:

$$(\mathtt{PS}_{\mathbf{P}}(I))(u) = \sum_{j:\mathrm{Bin}(\mathbf{P}_j)=u} \mathrm{area}(\mathbf{P}_j), \tag{13}$$

in which “Bin” is the binning function described later, and “area” is the set area. Extending (13) for an α-partition pyramid we arrive at:

$$(\mathtt{PS}_{\triangle^A}(I))(u) = \sum_{\alpha \in A} \sum_{j \in J^\alpha:\mathrm{Bin}(C_j^\alpha)=u} \mathrm{area}(C_j^\alpha). \tag{14}$$

It can be seen that any α-CC that remains invariant for a limited range of dissimilarity values, is accessed in each iteration of the inner sum. This type of redundancy is typical for the pyramidal partition representation and an example is shown in Fig. 2(a). The union of all nodes corresponds to $\triangle^A$, with $A = [0, 1, 2, 3, 4]$, and the green nodes represent the redundancy.

This, in the case of an α-partition hierarchy, also referred to as the α-tree [9], is countered by introducing the multiplier term $\Delta\alpha$, also known as component *lifetime* or *persistence*.

Assume two nested α-CCs: $C^{\alpha_p} \subset C^\alpha$ with α_p given by:

$$\alpha_p = \vee \alpha' \in A \ : \ (\alpha' < \alpha) \ \wedge \ (C_j^\alpha, C_{j'}^{\alpha'} \in \pitchfork^A). \tag{15}$$

The component lifetime is defined as $\Delta\alpha = \alpha - \alpha_p$, and assuming a new index set K derived from J, with $k \in K^\alpha$, $\forall\ \alpha \in A$, the pattern spectrum computed from the α-partition hierarchy is given by:

$$(\mathtt{PS}_{\pitchfork^A}(I))(u) = \sum_{\alpha \in A \setminus 0} \sum_{k \in K^\alpha:\mathrm{Bin}(C_k^\alpha)=u} \mathrm{area}(C_k^\alpha)\Delta\alpha. \tag{16}$$

An example of a 2-D pattern spectrum is shown in Fig. 2(b). Each dimension matches a specific attribute [22] with the axes normalized based on some mapping function. This, for the experiments that follow, is the simple area mapper from [22]. After initializing the n-dimensional pattern spectrum matrix, the binning function configured for each attribute separately, scales the component attribute based on the chosen mapping function and identifies the bin to which it contributes. The scaled contribution of each component, i.e. the term “area(C_k^α)$\Delta\alpha$” from (16), is accumulated to the energy counter of each bin. The latter is gray level coded.

6 Applications

Pattern spectra have been used as feature vectors to describe image objects or regions in both classification and regression problems. An example highlighting

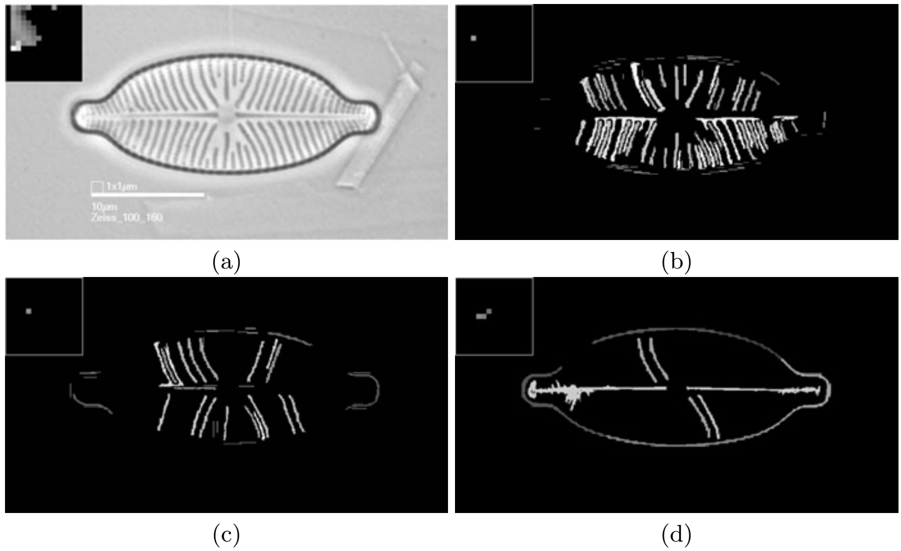


Fig. 3. Diatom ornament analysis: The original image in (a) with its pattern spectrum at the top left corner. The attributes along the x and y-axis are non-compactness and ω-range respectively. The image structures corresponding to bins (3,5), (4,5) and $(6,5) \cup (4,6) \cup (5,6)$ are highlighted in images (b), (c) and (d) respectively with the selected bins of the pattern spectra displayed at their top left corner.

the strength of this method in comparison to other descriptors, is in the classification of diatoms from microscopy imaging [3].

The work in [22] reports on the performance of 2-D pattern spectra computed based on the non-compactness and area attributes of connected components. The method outperforms all others from [3] but reaches an upper bound in classification success due to limitations in its descriptive power. The problem, that is addressed in [12], is essentially the inability of connected operators configured with regular adjacency-based connectivity, to deal effectively with component subregions that are more 'loosely' connected. Operators configured with partition-induced connectivity deal with this issue more effectively [12] and yield reacher spectra. This is at the cost of a higher computational demand due to the absence of nesting properties.

Following is a demonstration of a pattern spectrum computed based on the non-compactness and total range (ω) attributes of α-CCs. Image (a) in Fig.3 shows a sample from the ADIAC database [3] with its pattern spectrum at the top left corner. Images (b-d) show selected object features (diatom ornament and contour fragments) that correspond to some indicative bins of the α-connected pattern spectrum. We observe robust extraction of distinct features which suggests a better descriptive power for the proposed method.

The second application concerns rubble detection and follows the basic strategy from [10], only dealing with a selection of α-CCs instead of standard connected components. In brief, the method can be summarized in two steps; the

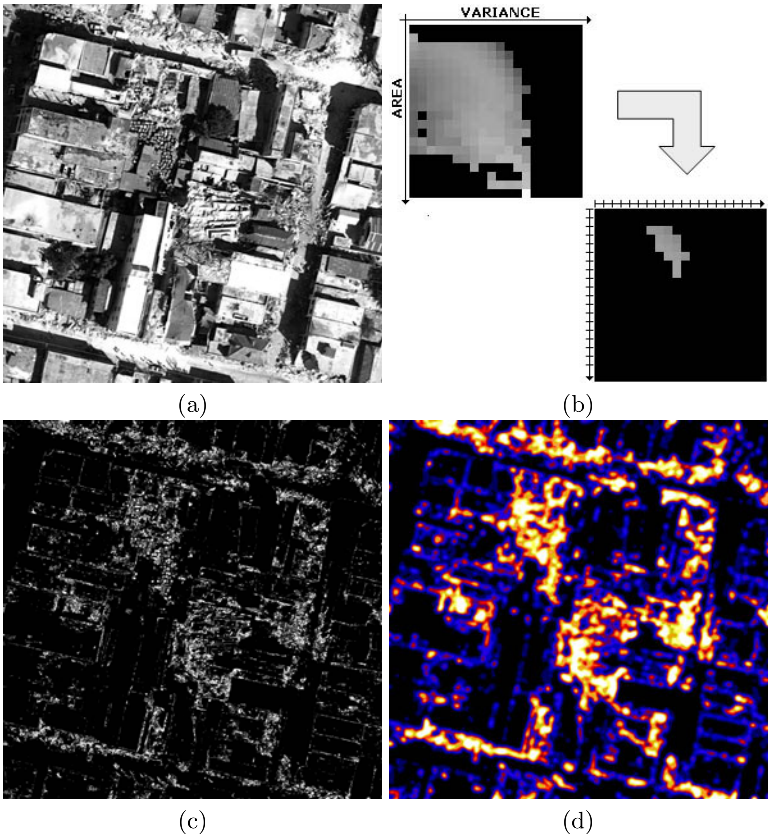


Fig. 4. Rubble detection based on selected image features from the α-connected pattern spectrum. The original image in (a), its pattern spectrum (left) and selected bins (right) in (b). The image features corresponding to the selected bins (c) and their color coded spatial aggregation in (d).

accumulation of the first plane of the opening and closing instance of a DMP vector field [13] configured with the area attribute [8] followed by spatial aggregation by a Gaussian low pass filter.

In this example, an α-connected pattern spectrum is computed based on the variance and area attributes. Rather than dealing with explicit attribute classes we chose a range of spectral bins. For the size attribute the bins range from the size corresponding to a quarter of the regular building size (estimated empirically) up to its full size. The range of the variance attribute is kept short since the rubble fragments are expected to be of poor texture. It is only the local background that is highly textured. Image (a) of Fig. 4 shows a sample of aerial, very high resolution imagery (approximately 15cm) of Port-au-Prince after the earthquake in Haiti in January 12^{th} 2010. The image is courtesy of Google 2010 and is available at the Google Crisis Response web-site at `http://www.google.com/relief/haitiearthquake/imagery.html`. Image

(b) shows the original α-connected pattern spectrum (top) and the selected bins (bottom). Image (c) shows the features corresponding to the selected bins from (b). Note that the color of each α-CC is the average of all 0-CCs it contains. The result of a low pass Gaussian filter with a square kernel size of 41×41 in thermal palette display is shown in image (d). The kernel size is set to match the estimated length of the average building size, i.e. approximately 6.5m.

7 Conclusions

Constraining dissimilarity-based connectivity relations is a way of controlling the leakage effect induced by connective criteria, discussed in Section 3. Constraints were used to control the evolution of single-linkage components along a dissimilarity range [19]. This approach however was shown to be limited to specific attribute thresholds and does not allow interactivity. In this paper we presented two hierarchically-ordered partition representation structures that map the sequence of progressive partitions along the dissimilarity range in the absence of constraints. In this approach, the type and number of constraints and the threshold value associated to each one separately, may be adjusted interactively. Moreover, through the hierarchical representation, we retain information on the evolution of the single-linkage independent on the threshold value. This was utilized in the the computation of α-connected pattern spectra and two application examples were given. We observed a high descriptive power of the proposed type of spectra, which remains to be further verified by large scale experimentation. Moreover, with both applications in the previous section using the slope as a dissimilarity measure, investigating the suitability and usage of others such as those presented in [4, 20] is an open challenge. The hierarchical partition representation that was introduced suggests that a new tree-based representation algorithm can be developed that would allow the rapid processing of large datasets. This is currently under development [9], while in future work we aim at investigating the morphological-profile [13, 8] equivalent feature vectors directly from α-hierarchies.

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