Avoiding Probability Saturation during Adjustment of Markov Models of Ageing Equipment

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Abstract. Markov models are well established technique used widely for modeling equipment deterioration. This work presents an approach where Markov models represent equipment ageing and also incorporate various maintenance activities. Having available some basic model it is possible to adjust its parameters so that it represents some hypothetical new maintenance policy and then to examine impact that this new policy has on various reliability characteristics of the system. The paper deals with a method of model adjustment and specifically investigates its one particular problem: avoiding probability saturation in a model which is tuned towards increased repair frequencies. The text describes the adjustment method in a general case, identifies specific risk of probability saturation that may take place during the iterative procedure and proposes a new extension to the method that overcomes this problem with minimal intervention in the internal structure of the model, in a specific class of cases.

1 Introduction

Selection of an efficient maintenance strategy plays a very important role in the management of today's complex systems. When searching for an optimal strategy, numerous issues must be taken into account and, among them, reliability and economic factors are often equally important. Finding a reasonable balance between them is the key point in efficient maintenance management and to facilitate finding such a balance some measures should be available that allow quantitative evaluation of the deterioration process of a system in a case when it is subjected to various maintenance actions (inspections, repairs, replacements, etc.).

This work deals with development that aims at providing a computer tool for a person deciding about maintenance activities, which would help in evaluation of both the risks and the costs associated with selection of various possible maintenance strategies. Rather than searching for a solution to a problem: "what maintenance strategy would lead to the best reliability and dependability parameters of the system operation", in this approach different maintenance scenarios can be examined in the "what-if" type of studies and then, using the tool, their reliability

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and economic effects can be automatically estimated so that the person responsible for the maintenance is assisted in making an informed decision ([5], [7], [11], [20]).

The proposed application of Markov models in representation of deterioration and maintenance processes has been presented initially in [4], while in [14] and [15] the procedure of model adjustment to modified repair frequencies was discussed. Efficiency of this method with its possibly weak points was further investigated in [17] and [18]. In this work, we extend the method so that it can properly deal with a class of cases when so called probability saturation takes place during adjustment towards increased frequencies of repairs.

2 Adjusting the Deterioration Model

There are three major factors that decide about equipment deterioration: its physical characteristics, operating practices, and the maintenance policy. Of these three aspects, especially the last one relates to the events and actions that should be properly modeled.

2.1 Construction of the Model

The method discussed in this work is based on the model in which the equipment will deteriorate in time and, if not maintained, will eventually fail. If the deterioration process is discovered, preventive maintenance is performed which can restore the condition of the equipment ([1], [10]). Such a maintenance activity will return the system to a specific state of deterioration, whereas repair after failure will restore to "as new" condition. The maintenance components that must be recognized in the model are: monitoring or inspection (how the equipment state is determined), the decision process (which determines the outcome of the decision), and finally, the maintenance actions (or possible decision outcomes). These elements can be properly incorporated in an suitable state-space (Markov) model ([6], [8], [9], [12], [13], [19]) which consists of the states the equipment can assume in the process, and the possible transitions between them. In a Markov model, the rates associated with the transitions are assumed to be constant in time.

The method described in this work uses specific model developed for the Asset Maintenance Planner (AMP) ([2], [3]). The AMP model is designed for equipment exposed to deterioration but undergoing maintenance at prescribed times. It computes the probabilities, frequencies and mean durations of the states of such equipment. The basic ideas in the AMP model are the probabilistic representation of the deterioration process through discrete stages, and the provision of a link between deterioration and maintenance. For structure of a typical AMP model see Fig. 1.

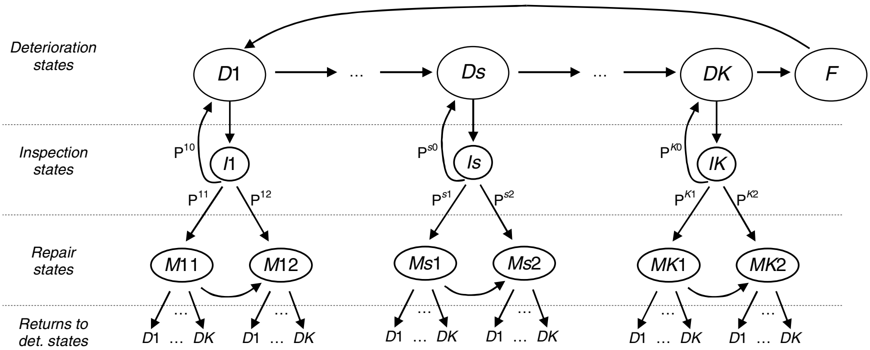

Fig. 1 The state-transition model representing the deterioration process with inspection and repair states (an example with two repair types

In this model, the deterioration progress is represented by a chain of *deterioration states* $D1 \dots DK$ which leads to the *failure state* F. In most situations, it is sufficient to represent deterioration by three stages: an initial ($D1$), a minor ($D2$), and a major ($D3$) stage ($K = 3$). This last is followed, in due time, by equipment failure (F) which requires extensive repair or replacement.

In order to slow deterioration and thereby extend equipment lifetime, the operator will carry out maintenance according to some pre-defined policy. In the model of Fig. 1, regular inspections (Is) are performed which result in decisions to continue with minor ($Ms1$) or major ($Ms2$) maintenance or do nothing (more than two types of repairs can also be included). The expected result of all maintenance activities is a single-step improvement in the deterioration chain; however, allowances are made for cases where no improvement is achieved or even where some damage is done through human error in carrying out the maintenance, which results in returning to the stage of more advanced deterioration.

The choice probabilities (at transitions from inspection states) and the probabilities associated with the various possible outcomes are based on user input and can be estimated, e.g., from historical records or operator expertise.

Mathematically, the model expressed in Fig. 1 can be represented by a semi-Markov process, and solved by the well-known procedures. The solution will yield all the state probabilities, frequencies and mean durations. Another technique, employed for computing the so-called first passage times (FPT) between states, will provide the average times for first reaching any state from any other state. If the end-state is F, the FPTs are the mean remaining lifetimes from any of the initiating states.

2.2 Adjusting the Model to Requested Repair Frequencies

Preparing the Markov model for some specific equipment is not an easy task and requires expert intervention. The goal is to create the model representing closely the real-life deterioration process known from the records that usually describe equipment operation under a regular maintenance policy with some specific frequencies of inspections and repairs. The model itself permits calculation of the repair frequencies and compliance of the computed and recorded frequencies is a very desirable feature that verifies trustworthiness of the model.

In this section, we will summarize the method of model adjustment proposed in [14] and [15] that aims at reaching such a compliance. It can be used also for a different task: fully automatic generation of a model for some new maintenance policy with modified frequencies of repairs. Such a task needs to be done fairly often during evaluation of various maintenance scenarios.

Let K represents the number of deterioration states and R the number of repairs in the model under consideration. Also, let P^{sr} = probability of selecting maintenance r in state s (assigned to the decision after state Is) and P^{s0} = probability of returning to state Ds from inspection Is (situation when no maintenance is scheduled as a result of the inspection). In Fig. 1 the probabilities are located nearby respective transitions. Then, for all states $s = 1 \ldots K$:

$$\mathrm{P}^{s0} + \sum_r \mathrm{P}^{sr} = 1 \tag{1}$$

Let F^r represent the frequency of repair r acquired through solving the model. The problem of model tuning can be formulated as follows:

> Given an initial Markov model M_0, constructed as above and producing the initial frequencies of repairs $\mathbf{F}_0 = \left[\mathrm{F}_0^1, \mathrm{F}_0^2, \ldots \mathrm{F}_0^R\right]$, adjust the probabilities P^{sr} so that some goal frequencies $\mathbf{F}_\mathrm{G}$ are achieved.

The vector $\mathbf{F}_\mathrm{G}$ usually represents the observed historical values of the frequencies of various repairs.

In the proposed solution, a sequence of tuned models M_0, M_1, … M_N is evaluated with each consecutive model approximating desired goal with a better accuracy. The procedure consists of the following steps repeated in an iterative loop with i denoting the iteration counter:

1° For the current model M_i, compute the vector of repair frequencies $\mathbf{F}_i$.

2° Evaluate an error of M_i as a distance between vectors $\mathbf{F}_\mathrm{G}$ and $\mathbf{F}_i$.

3° If the error is within the user-defined limit, consider M_i as the final model and stop the procedure ($N = i$); otherwise continue with the next step.

4° Construct a new model M_{i+1} through adjusting values of P_i^{sr} and compute P_{i+1}^{s0} from equation (1).

5° Proceed to step 1° with the next iteration.

The error computed in step 2° can be expressed in many ways. As the frequencies of repairs may vary in a broad range within one vector $\mathbf{F}_i$, yet the values of all are significant in model interpretation, the relative measures work best in practice. The most restrictive formula evaluates maximum relative error over all frequencies and this was used in this work:

$$\left\|\mathbf{F}_{\mathrm{G}} - \mathbf{F}_i\right\| = \max_r \left|\mathrm{F}_i^r / \mathrm{F}_{\mathrm{G}}^r - 1\right| \tag{2}$$

2.3 Tuning Repair Probabilities

Of all the steps outlined in the previous point, it is clear that adjusting probabilities P_i^{sr} in step 4° is the heart of the whole procedure.

In general, the probabilities represent $K \cdot R$ free parameters and their uncontrolled modification could lead to serious deformation of the model. To avoid this, a restrictive assumption is made: if the probability of some particular maintenance must be modified, it is modified proportionally in all deterioration states, so that at all times

$$\mathrm{P}_0^{1r} : \mathrm{P}_0^{2r} : \ldots : \mathrm{P}_0^{Kr} \;\sim\; \mathrm{P}_i^{1r} : \mathrm{P}_i^{2r} : \ldots : \mathrm{P}_i^{Kr} \tag{3}$$

for all repairs ($r = 1 \ldots R$).

This assumption also significantly reduces dimensionality of the problem, as now only R scaling factors $\mathbf{X}_{i+1}=[\mathrm{X}_{i+1}^1, \mathrm{X}_{i+1}^2, \ldots \mathrm{X}_{i+1}^R]$ must be found to compute new probabilities for the model M_{i+1}:

$$\mathrm{P}_{i+1}^{sr} = \mathrm{X}_{i+1}^r \cdot \mathrm{P}_0^{sr}, \quad r = 1 \ldots R,\; s = 1 \ldots K \tag{4}$$

Moreover, although the frequency of a repair r depends on the probabilities of all repairs (modifying probability of one repair changes, among others, state durations in the whole model; thus, it changes the frequency of all states), it can be assumed that, in a situation of a single-step small adjustment, its dependence on repairs other than r can be considered negligible and F_i^r can be considered to be a function of just one variable:

$$\mathrm{F}_i^r = \mathrm{F}_i^r\left(\mathrm{X}_i^1, \mathrm{X}_i^2 \ldots \mathrm{X}_i^R\right) \approx \mathrm{F}_i^r\left(\mathrm{X}_i^r\right) \tag{5}$$

With these assumptions, generation of a new model is reduced to the problem of solving R non-linear equations in the form of $\mathrm{F}_i^r\left(\mathrm{X}_i^r\right) = \mathrm{F}_G^r$ and this task can be accomplished with one of the standard root-finding algorithms.

One point of the procedure requires additional attention, though: applying equation (4) with $X_{i+1} > 1$ may violate condition

$$\sum_{r=1}^{R} \mathrm{P}_{i+1}^{sr} \le 1 \tag{6}$$

in some deterioration state s. This situation needs special tests that would detect such illegal probability values and reduce them proportionally so that their sum does not exceed 1: a so called *scale-down transformation* needs to be applied. As practical studies show such conditions do occur during model tuning towards repair frequencies that are remarkably higher than F_0^r from the initial model M_0. In its simplest form, the scale-down operation consists in dividing each probability P^{sr} in the offending state s by the sum of all repair probabilities in this state:

$$\mathrm{P}^{sr} = \mathrm{P}^{sr} \big/ S_{Ds}\,, \quad S_{Ds} = \sum_{r=1}^{R} \mathrm{P}^{sr} \tag{7}$$

This will also lead to $\mathrm{P}^{s0} = 0$ which means that every inspection ends with some repair and there are no direct returns from *Is* state to *Ds*. Moreover, this obligatory correction mechanism can result in violation of the proportionality rule (3) as an inevitable side effect.

The following three approximation algorithms were implemented in the task of solving equation (5): Newton method working on a linear approximation of $\mathrm{F}_i^r()$ functions (the NOLA method), the secant method and the false position (*falsi*) method. For their detailed presentation please refer to [14] and [15].

Generally, the practical tests have shown that although simplifications of the NOLA solution may seem critical, it is reasonably efficient and stable in real-world practical cases because it has one advantage over its more sophisticated rivals: since it does not depend on previous approximations, selection of the starting point is not so important and the accuracy during the first iterations is often better than in the secant or *falsi* methods. Superiority of the latter methods, especially of the *falsi* algorithm, manifests itself in the later stages of the approximation when the potential problems with an initial selection of the starting points have been diminished.

3 Automatic Correction of the Model in the Case of Probability Saturation

As practical applications of the adjustment method described in the previous section have shown, the procedure must be carefully applied to the models that represent real-life deteriorating processes because it is relatively easy to arrive at the solution that correctly realizes the optimization goal, i.e. produces the requested repair frequencies, but the internal structure of the model is modified to the degree which harms the relation between the new unit and the original equipment. In this section we will propose an approach that aims at one specific problem related to this issue which may arise in practical cases when increasing the repair frequencies is requested.

Adjusting the model to the repair frequencies that are substantially higher than the original ones may lead to *model saturation* – a condition in which repair probabilities reach the limit (6) and there is no room for further increase if the adjustment procedure is limited only to the simple probability scaling expressed by equation (4). In this situation bringing together the two requirements: tuning the model towards high repair frequencies and, at the same time, keeping the modifications of the internal structure within a safe range that does not break proper relation with the original, is a challenge that needs a new, careful development.

3.1 Application Context

The method of model adjustment that is being considered in this work has been practically implemented in the Asset Risk Manager (ARM) software system which uses the concept of a life curve and discounted cost to study the effect of equipment ageing under different hypothetical maintenance strategies ([4], [18]). As noted in section 2, the method uses semi-Markov models of the Asset Maintenance Planner (AMP) ([2], [3]). For the ARM program to automatically generate the life curves for different requested maintenance policies (with, among other parameters, different repairs frequencies), default Markov model for the equipment has to be built and stored in the computer database. This is done through the prior running of the AMP program by an *expert user*. Therefore, both AMP and ARM programs are closely related, and usually, should be run consecutively.

Implementation details of Markov models, tuning their parameters and all other internal particulars should not be visible to the non-expert *end user* who actually operates the ARM software in order to investigate various potential modifications of the present (default) maintenance policies associated with the model and evaluates their economic and reliability costs. All final results are visualized either through an easy to comprehend idea of a life curve or through other well-known concepts of financial analysis. Still, prior to running the analysis some expert involvement is needed, largely in preparation, importing and adjusting AMP models. After that the adjustment method should run automatically in the background and the end user should be presented with results that come from the tuned models. In this context it is vital that the method can generate correctly adjusted models reliably and without human intervention.

Discussion included in [17] and [18] investigated main challenges that are brought by this task. It has shown that, while tuning the model towards decreased repair frequencies usually succeeds without additional specific requirements, some special rules in model construction should be respected if the model is to be tuned towards *increased* frequencies. The two main factors that were recognized were as follows: (1) although it may seem that in the initial (minor) deterioration state no repairs are performed after inspections, still some non-zero probabilities are required in $D1$ if purely hypothetical questions like “What if I start some repair twice as often as previously?” shall be allowed; (2) including an option of not doing any repair after inspection in the later deterioration states, albeit with small probability, is also desirable because it increases ability of the model to represent diverse maintenance configurations found in the studies.

3.2 The Problem of Model Saturation

For practical illustration two real-world Markov models were selected that are especially prone to the problems of probability saturation. Specifically, they do not follow rule (1) from the previous point: they assume that no repair is performed after inspections in the first deterioration state, i.e. $P^{1r} = 0$ and $P^{10} = 1$. Such assumption is common for AMP models created according to actual historical records describing equipment operation.

Both models have the same general structure with $K = R = 3$, i.e. they include three deterioration states ($D1 \div D3$) and three repairs: minor (index = 1), medium (2) and major (3). The main difference between them lies in distributions of repair probabilities P^{sr} in the deterioration states (or, strictly speaking, in inspection states $I1 \div I3$ associated with the deterioration states; see Fig. 1). These probabilities are given in Table 1.

Table 1 Repair probabilities in Markov models used as examples A and B

Deterioration state:	*D*1			*D*2			*D*3		
Probability of repair:	P^{11}	P^{12}	P^{13}	P^{21}	P^{22}	P^{23}	P^{31}	P^{32}	P^{33}
Model A:	0.000	0.000	0.000	0.80	0.15	0.05	0.20	0.50	0.30
Model B:	0.000	0.000	0.000	0.64	0.12	0.04	0.18	0.45	0.27

The model A has been created with assumption that although there are no repairs in the first state *D*1, when the equipment is in subsequent states *D*2 and *D*3 every inspection leads to some sort of repair and in these states the totals $S_{D2} = S_{D3} = 1$ ($P^{20} = P^{30} = 0$). Looking at the probability distribution in each state it can be seen that in the medium deterioration *D*2 the minor repair is evidently the most often chosen one ($P^{21} = 0.80$) while in the major deterioration *D*3 the distribution is to some extent more balanced with medium repair taking half of the chances ($P^{32} = 0.50$).

The model B is a sibling of A with just one but important difference: repair probabilities in *D*2 and *D*3 are lower by, respectively, 20% and 10% than the values of model A, which also means that after inspections in these states it is possible to return to *Ds* without undertaking any repair ($P^{20} = 0.2$ and $P^{30} = 0.1$). In other words, model B, as opposite to model A, has been created according to the requirement (2) introduced in the previous point.

In the coming analyses series of models will be generated from the initial model M_0 in both cases A and B for a sequence of goal frequencies

$$\mathbf{F}_G = \left[\alpha \cdot F_0^1, F_0^2, F_0^3\right] \tag{8}$$

with factor α increasing from 0.5 (frequency of minor repair reduced by half) to 2.0 (minor repair performed twice as often) in steps of 0.1. Values of α in the figures and in the following discussion will be expressed as %. Frequency of the minor repair (no. 1) was selected as the varying parameter of $\mathbf{F}_G$ just as an example with frequencies of other repairs remaining constant, but equivalent results could be demonstrated with changing frequencies of medium or major repairs. The figures will include graphs presenting variations of repair probabilities P^{sr} and their sums S_{Ds} in deterioration states of the final adjusted models as functions of the α factor.

The problem of probability saturation is illustrated in Fig. 2 which includes minor repair probabilities in all states (probability of other repairs are not included to preserve space) for models A and B tuned with the standard procedure described in the previous section. Both models can be successfully adjusted only up to the point of saturation which is reached for $\alpha = 100\%$ for model A (i.e. the initial model is already saturated) and 130% for model B (as it turns out, in this particular case $P^{20} = 0.2$ and $P^{30} = 0.1$ leave space enough for 30% increase in frequency of the minor repair). In both cases for these goals probabilities in states $D2$ and $D3$ sum up to unity and cannot be further increased, while in $D1$ the P^{11} is zero and applying the scaling factor as in equation (4) cannot produce any increase. On the other hand, the procedure has no problems with adjustment towards lower frequencies and in such cases the probabilities are scaled accordingly ([14], [15], [17]).

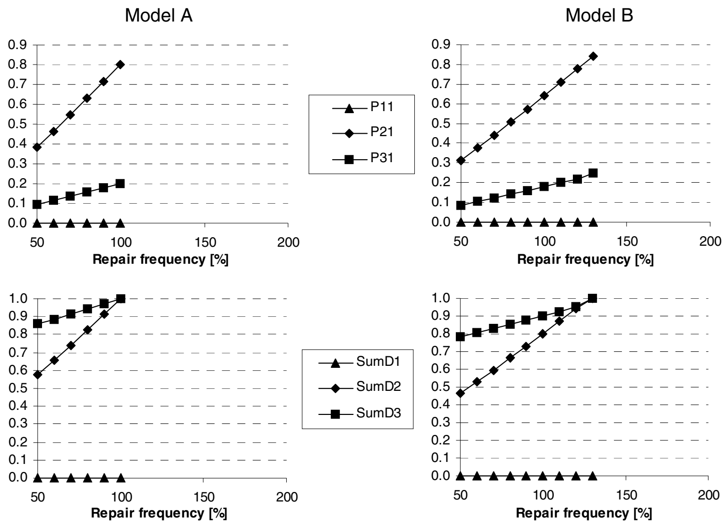

Fig. 2 Unsuccessful adjustment of the models with the standard (unmodified) procedure: probability of the minor repair in all three states (above) and sum of probabilities per state (below)

3.3 Challenges of Model Alteration

The above example of unsuccessful tuning can be used also for illustration of the main idea of the proposed extension to the algorithm: if the model gets saturated during the adjustment iteration but there is still some state with null repair probability, the process can be continued in the same iterative way after some non-zero probability is added into this state. Such modification, though, goes far beyond the restrictive assumption expressed by equation (3) and, being a more serious invasion into the model structure, must be applied in a cautious and thoughtful manner.

In particular, the following two issues must be taken into account: (1) forcing non-zero probability in some state before it is not absolutely necessary, i.e. prior to model saturation, instantly changes reaction to the adjustment iterations, hence may change the final result of the tuning also in cases when the standard procedure would be able to produce the correct result; (2) replacing the null value of P^{sr}, even if delayed up to the moment of saturation, but with probability which is too high for the needs of the adjustment also may affect the final result in a way that is against the general idea of the conservative tuning which tries to preserve the structure of the original model with minimal possible modifications.

Figure 3 illustrates these two problems using model B as an example. The two upper graphs show adjustment results when null P^{sr} is replaced with a non-zero value right from the first iteration only if respective frequency needs to be increased in the goal vector, i.e. without waiting until the model gets saturated. In this case it means that the model is modified for cases where $\alpha > 100\%$ instead of $\alpha > 130\%$. As the graphs show, forcing $P^{11} > 0$ prematurely causes evident instabilities in growths of P^{21} and P^{31} and even more significant instabilities in distributions of sums S_{D1}, S_{D2} and S_{D3}. In fact the model does not reach saturation in state $D3$ even for $\alpha = 200\%$, while the Fig. 2 indicates that this model should saturate for $\alpha = 130\%$.

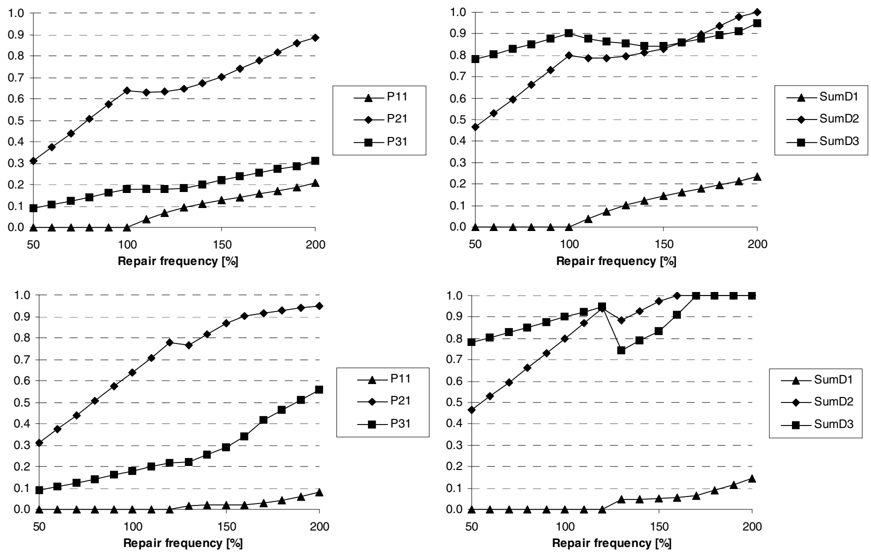

Fig. 3 Incorrect modifications generated by the adjustment procedure: non-zero repair probabilities introduced before model saturation (above) and too high probabilities forced after model saturation (below)

The two lower graphs in Fig. 3 present the results when the moment of probability increase is properly delayed until model saturation ($\alpha = 130\%$) but P^{11} is assigned with a value which exceeds the needs of tuning. Again, as a result the growths of P^{21} and P^{31} are noticeably disturbed and even more evident instabilities can be seen in graphs of the sums S_{Ds}: the exaggerated intervention applied for $\alpha =$ 130% drives the model out of the saturation state until $\alpha = 170\%$, and only after this point the procedure continues with the expected linear growth of P^{11}.

3.4 Extension of the Adjustment Procedure

After analyses of case studies like the above two examples, the following modification of the adjustment procedure has been found to be the most flexible and efficient solution that gives optimal results in broad range of practical cases. It not only delays the increase of null probability until the moment of model saturation, but also scales its value adequately.

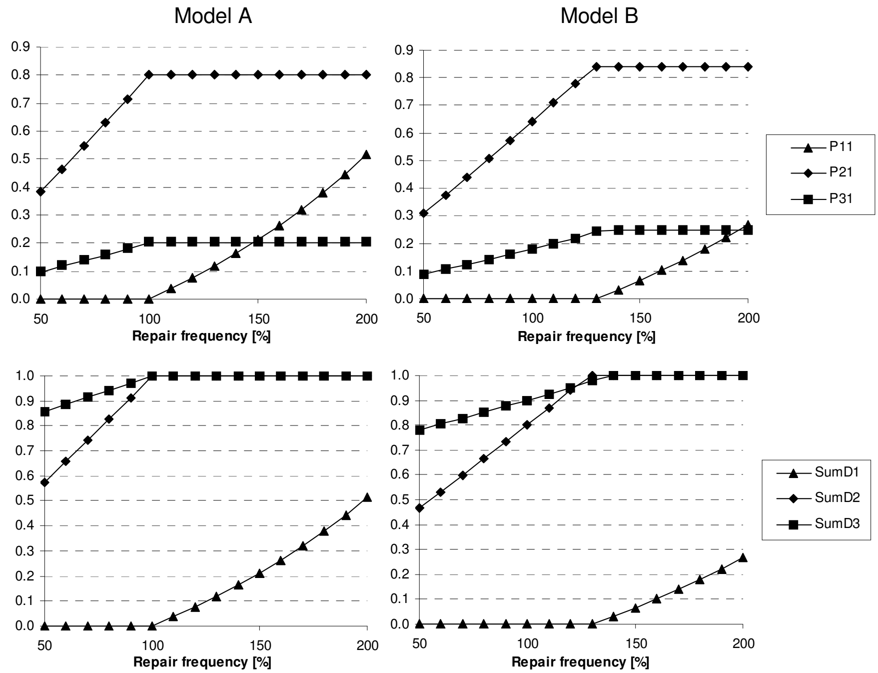

Fig. 4 Tuning the models A (left) and B (right) by the proposed extension of the adjustment procedure

The modification does not amend the general iterative scheme defined in point 2.2 (steps 1° ÷ 5°); the changes are limited only to internal details of step 4° which computes new probability values for the next model M_{i+1}. The modified implementation of this operation detects and deals in a different way with the following two cases:

(a) If the model is not saturated, i.e. there is a state with $0 < S_{Ds} < 1$, the standard approach is applied: in all states the values of P^{sr} are multiplied by the scaling factors X^r (equation (4)) and then, if required, they are scaled down as in equation (7).

(b) If the model is saturated but there is a state with $P^{sr} = 0$ (a chance for probability increase), this particular null probability is replaced with a predicted average increase of P^{sr} in other states computed by the normal method as above; after this the model is no longer saturated and the iterative scaling of this probability can be continued with the standard algorithm.

It should be noted that in case (b) the new value that replaces the null probability is computed as an average of predicted *actual* increases of probabilities for given repair in other states: these increases will be scaled down with equation (7) because these states, by virtue of the method, will be saturated. As a result, the applied value of the increase will be proportional to the needs of particular situation but, at the same time, it will be additionally constrained.

Figure 4 presents the results obtained after application of such extended procedure to models A and B. In both cases the models can be successfully adjusted in the full examined range, i.e. up to the doubled frequencies of the minor repair. Also, as it can be seen, the final results are virtually identical to the outcomes of the standard (unmodified) procedure in cases when model saturation does not take place (as compared to Fig. 2), while the growth of the probabilities after the saturation point follows the expected course without any instabilities.

4 Conclusions

The purpose of the method presented in this paper is to extend the adjustment algorithm which was proposed in [14] and [16]. The main idea is to modify the model during the iteration by forcing a value greater than zero for a repair probability in situation when this probability reach the limit in other states, i.e. the model saturates. This extension allows to evaluate a class of cases that was not properly handled by the original method.

The proposed approach strives to be as conservative as possible with regard to the amount of alterations introduced to the existing model. While the original method constrains the adjustment operations so that the distribution of the repair probabilities over all deterioration states is not altered, the modification introduced by this extension is more significant and must be applied in a very cautious manner in order to avoid deformation of the model and corruption of the produced results.

In this situation there is a growing need for methods that would evaluate trustworthiness of the generated results. The future work should include development of new metrics that would be able to quantitatively assess modification of the model and to estimate the range of its valid use.

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