Chapter 16
The World According to de Finetti: On de Finetti's Theory of Probability and Its Application to Quantum Mechanics

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Abstract Bruno de Finetti is one of the founding fathers of the subjectivist school of probability, where probabilities are interpreted as rational degrees of belief. His work on the relation between the theorems of probability and rationality is among the corner stones of modern subjective probability theory. De Finetti maintained that rationality requires that degrees of belief be coherent, and he argued that the whole of probability theory could be derived from these coherence conditions. De Finetti's interpretation of probability has been highly influential in science. This paper focuses on the application of this interpretation to quantum mechanics. We argue that de Finetti held that the coherence conditions of degrees of belief in events depend on their verifiability. Accordingly, the standard coherence conditions of degrees of belief that are familiar from the literature on subjective probability only apply to degrees of belief in events which could (in principle) be jointly verified; and the coherence conditions of degrees of belief in events that cannot be jointly verified are weaker. While the most obvious explanation of de Finetti's verificationism is the influence of positivism, we argue that it could be motivated by the radical subjectivist and instrumental nature of probability in his interpretation; for as it turns out, in this interpretation it is difficult to make sense of the idea of coherent degrees of belief in, and accordingly probabilities of unverifiable events. We then consider the application of this interpretation to quantum mechanics, concentrating on the Einstein-Podolsky-Rosen experiment and Bell's theorem.

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16.1 The Background and Motivation

The foundations of this paper were laid in 1988/1989, when I worked on a seminar paper for Itamar Pitowsky's course in the philosophy of probability.[1] The question that motivated the paper was whether subjective probability, and more specifically de Finetti's subjectivist interpretation, could successfully be applied in quantum mechanics (QM). This question, which was raised by Itamar, may seem a bit anachronistic now that the subjective interpretation of quantum probabilities is gaining popularity. But back then this interpretation was undeveloped.[2]

In de Finetti's interpretation, probabilities have no objective reality. They are the expressions of the uncertainties of individuals. Itamar's question was not whether such a radical subjective interpretation could constitute an adequate interpretation of probabilities in quantum mechanics. Rather, it was the question whether de Finetti's interpretation could be reconciled with the apparent non-classical character of these probabilities. We explain this concern in Sect. 16.1.3, and discuss it in more detail in Sect. 16.2. To prepare the ground for this discussion, we now turn to present Bell's theorem and two different interpretations of it. In Sects. 16.3 and 16.4, we introduce the main ideas of de Finetti's theory of probability, and in Sects. 16.5–16.7 we discuss the application of this theory to the quantum realm.

16.1.1 Bell's Theorem and Its Common Interpretation

Recall the Einstein-Podolsky-Rosen (EPR) experiment. Pairs of particles are emitted from the source in opposite directions. When the particles are spacelike separated, they each encounter a measurement apparatus that can measure their position or momentum. The distant measurement outcomes are curiously correlated. Einstein et al. [6] thought that this kind of correlation reflects the incompleteness of QM rather than non-local influences. They argued that the QM state-description is incomplete, and they believed that a more complete description would render the distant measurement outcomes probabilistically independent. The idea is that the correlations between such distant outcomes could be explained away by a local common cause: the complete pair-state at the emission. Given this state, the joint probability of the outcomes would factorize into their single probabilities (see Factorizability below), and so the correlations between them would not entail the existence of non-locality.

[1] See Berkovitz [1, 2].

[2] For applications of the subjective interpretation to QM, see for example, Caves, Fuchs and Schack [3–5] and Pitowsky [59]. While these applications appeal to de Finetti's subjective theory of probability, both the interpretation of de Finetti and the focus of its application are substantially different from the ones offered below.

In his celebrated theorem, Bell [7–11] considers models of the kind EPR may have had in mind, but he focuses on Bohm's [12] version of the experiment (henceforth, the EPR/B experiment), where the measured quantities are spins in various directions. These models postulate the existence of "hidden variables" that are supposed to constitute a (more) complete pair's state, and this state is supposed to determine the measurement outcomes or their probabilities in a perfectly local way. Bell's theorem demonstrates that such models are committed to certain inequalities concerning the probabilities of measurement outcomes, the so-called "Bell's inequalities," which are violated by the predictions of QM and (granted very plausible assumptions) actual experimental results. In Clauser and Horne's [13] version, the inequalities are concerned with the probabilities of measurement outcomes of spins in two different directions in each wing of the EPR/B experiment, henceforth the "Bell/CH inequalities" (for details, see Sect. 16.2).

The common view is that Bell's theorem demonstrates that local hidden-variables models cannot reproduce the predictions of QM [7–11, 13–16]. On this view, the derivation of Bell/CH inequalities involves the following premises.

(i) The distribution of the complete pair-state is determined by the QM pair-state, and is independent of the settings of the measurement apparatuses. That is, for any QM pair-state ψ, complete pair-states λ, and setting of the L- and R-apparatus to measure spins in the directions x and y, respectively, we have:

$$(\lambda - \text{independence}) \quad \rho_{\psi xy}(\lambda) = \rho_{\psi}(\lambda);$$

where $\rho_{\psi}(\lambda)$ and $\rho_{\psi xy}(\lambda)$ are the probability distributions of λ given ψ and given $\psi \& x \& y$, respectively. Note that in our notation for conditional probabilities, we place the conditioning events in the subscript rather than after the conditionalization stroke. Unlike Kolmogorov's [17] axiomatization, in this approach conditional probability is not defined as a ratio of unconditional probabilities. Rather, conditional probability may be thought of as a conditional, which does not necessarily entail the corresponding conditional probability *a la* Kolmogorov (for more details, see Sect. 16.3.7). In this concept of conditional probability, the conditioning events ψ and $\psi \& x \& y$ are not part of the probability spaces referred by $\rho_{\psi}(\ )$ and $\rho_{\psi xy}(\ )$, respectively. To highlight this fact, we place them in the subscripts. As we shall see later, this alternative concept of conditional probability is in line with de Finetti's theory of probability. Arguably, it is also a more appropriate representation of the basic idea of conditional probability in other interpretations of probability [18, 19]. Yet, while this representation is important for pedagogical reasons, it is not essential for our analysis of Bell's theorem and the feasibility of interpreting probabilities in the quantum realm along de Finetti's theory.

(ii) For each complete pair-state λ and apparatus settings x and y, the model prescribes probabilities of single and joint measurement outcomes: $P_{\lambda x}(O_x)$, $P_{\lambda y}(O_y)$ and $P_{\lambda xy}(O_x \& O_y)$, where O_x is the outcome "up" in x-spin measurement on the L-particle; and similarly, *mutatis mutandis*, for the outcome O_y in the R-wing.

(iii) The joint probability of distant outcomes given the complete pair-state and apparatus settings factorizes into the single probabilities of the outcomes. The idea here is that the correlation between the distant outcomes are explained by a common cause, i.e. the complete pair-state, so that conditionalization on the common cause renders the outcomes probabilistically independent. More precisely, for any $\lambda,\ x,\ y,\ O_x$ and O_y:

$$\text{(Factorizability)} \quad P_{\lambda xy}(O_x \& O_y) = P_{\lambda x}(O_x) \cdot P_{\lambda y}(O_y).$$

(iv) The QM probabilities of outcomes are reproduced as statistical averages over the model probabilities of outcomes – namely, as sum-averages over the model probabilities according to the distribution of the complete pair-state. That is, granted *λ-independence*, for any $\psi,\ x$ and y, we have:

$$P_{\psi x}(O_x) = \int_\lambda P_{\lambda x}(O_x)\, d\rho(\lambda),\ P_{\psi y}(O_y) = \int_\lambda P_{\lambda y}(O_y)\, d\rho(\lambda),\ P_{\psi xy}(O_x \& O_y) = \int_\lambda P_{\lambda xy}(O_x \& O_y)\, d\rho(\lambda).$$

Bell's theorem demonstrates that in any model that satisfies (i)-(iv), the probabilities of measurement outcomes in the EPR/B experiment are constrained by the Bell/CH inequalities (see Sect. 16.2). Thus, granted the plausibility of *λ-independence* and the overwhelming evidence for the empirical adequacy of QM (in its intended domain of application), the consensus has it that *Factorizability* fails in this experiment. The failure of this condition is commonly thought of as indicating some type of non-locality (for a recent review of quantum non-locality, see [20] and references therein).

16.1.2 Fine's Interpretation of Bell's Theorem

Following Bell [10], the above analysis of Bell's theorem relies on a principle of causal inference which is similar to Reichenbach's [21] principle of the common cause. That is, it is assumed that non-accidental correlations have causal explanation, and the kind of explanation is as spelled out in (iii) and (iv) above. While this kind of inference is common, there are dissenting views. Fine [22–24] denies that non-accidental correlations must have causal explanation, and he argues that the correlations between the distant measurement outcomes in the EPR/B experiment do not call for causal explanation; and Cartwright [25, Chaps. 3 and 6] and Chang and Cartwright [26] challenge the assumption that common causes must render their joint effects probabilistically independent.

More important to our consideration, Fine [27, p. 294] argues that

(F) What hidden variables and the Bell/CH inequalities are all about are the requirements that make "well defined precisely those probability distributions for non-commuting observables whose rejection is the very essence of quantum mechanics."

The idea is that Bell's theorem focuses on models that presuppose the existence of joint probability over non-commuting spin observables in the EPR/B experiment – a distribution that does not exist according to standard QM. In more detail, Fine [27] argues that:

I. (Corresponding to "Proposition 1") "The existence of a deterministic hidden-variables model is strictly equivalent to the existence of a joint distribution probability function $P(AA'BB')$ for the four observables of the experiment, *one that returns the probabilities of the experiment as marginals*." [27, p. 291]
II. ("Proposition 2") "Necessary and also sufficient for the existence of a deterministic hidden-variables model is that Bell/CH inequalities hold for the probabilities of the experiment." [27, p. 293]
III. ("Proposition 3") "There exists a factorizable stochastic hidden-variables model for a correlation experiment if and only if there exists a deterministic hidden-variables model for the experiment." [27, p. 293]

Fine believes that (I)–(III) entails (F), and this suggests that the common interpretation of Bell's theorem – namely, that (granted the very plausible assumption of *λ-independence*) the violation of Bell/CH inequalities entails quantum non-locality – is misguided.

16.1.3 Subjective Probability, Joint Distributions and Verifiability

De Finetti held that for degrees of belief to be coherent they have to be probabilities, i.e. they have to satisfy the probability axioms. It is commonly presupposed, albeit implicitly, that a person's coherent degrees of belief concerning all the propositions she considers are to be represented by a joint probability distribution, which returns these degrees of belief as marginals; for notable examples, see Lewis's [28] "A Subjectivist's Guide to Objective Chance" and Carnap's [29] "On Inductive Logic." If the subjectivist interpretation were committed to such an assumption, and the view that the Bell/CH inequalities follow from the assumption of a joint distribution over non-commuting observables in the EPR/B experiment were correct, followers of this interpretation would be bound to have probabilities that are constrained by Bell/CH inequalities, and accordingly incompatible with the predictions of QM.

Indeed, followers of the subjectivist interpretation may agree with Fine's analysis of Bell's theorem, yet reject the view that a person's degrees of belief are to be represented by a single probability distribution. The question is whether they have non-*ad hoc* reasons to reject this view. Based on a reconstruction of de Finetti's probability theory in Sects. 16.3 and 16.4, we shall argue in Sect. 16.5 that followers of de Finetti have such reasons in the context of the EPR/B experiment and Bell's theorem. That is, we shall argue in Sect. 16.4 that de Finetti's notion of coherent degrees of belief embodies a certain verifiability condition. Consequently: (a) Degrees of belief in events that are not verifiable have no definite coherence conditions, and accordingly have no probability. (b) There are no joint probability distributions over events that are not jointly verifiable. (c) The coherence conditions of degrees of belief in events that are not jointly verifiable are weaker than they would have been had the events been jointly verifiable. Thus, the coherence conditions of degrees of belief in events that are not jointly verifiable are weaker than the familiar coherence conditions discussed in the literature on subjective probability. Accordingly, the inequalities that constrain the probabilities of such events are weaker than those that constrain the probabilities of events that are jointly verifiable.

In Sects. 16.5 and 16.7, we shall consider the implications of these consequences for the structure of probabilities in models of the EPR/B experiment in which probabilities are interpreted along de Finetti's theory of probability. These sections reflect the implications of de Finetti's theory, as reconstructed in Sects. 16.3 and 16.4. De Finetti himself struggled to understand the nature of the QM probabilities and their relation to "classical" probabilities. In Sect. 16.6, we shall briefly look at de Finetti's own analysis of the QM probabilities. But first we turn to present the Bell/CH inequalities and to consider Fine's claim that these inequalities follow from, and are equivalent to the assumption of a joint distribution over non-commuting observables in the EPR/B experiment.

16.2 Joint Distributions, Probabilistic Inequalities and Bell's Theorem

The term "Bell/CH inequalities" is ambiguous. It refers to different kinds of inequalities. The first kind is a theorem of probability theory:

$$(\text{Bell/CH} - \text{prob})$$
$$-1 \leq P_\lambda(X\&Y) + P_\lambda(X'\&Y) + P_\lambda(X\&Y') - P_\lambda(X'\&Y') - P_\lambda(X) - P_\lambda(Y) \leq 0.$$

Indeed, this inequality obtains for any joint probability distribution over any four events X, X', Y, Y' (or propositions about them). In the context of the hidden-variables models of the EPR/B experiment, it is natural to think about λ as the complete pair-state, and X (Y) and X' (Y') as referring to spin properties of the particles, or properties that determine their dispositions to spin in measurements. For example, X (Y) may be the event of the L- (R-) particle spinning "up" in the direction x (y), or some other property that determines the disposition of the L- (R-) particle to spin "up" along the direction x (y) in a spin measurement along this direction.

The second and third kinds of Bell/CH inequalities are not theorems of probability theory:

$$\text{(Bell/CH - phys - }\lambda\text{)} \quad -1 \leq P_{\lambda xy}(O_x \& O_y) + P_{\lambda x'y}(O_{x'} \& O_y) + P_{\lambda xy'}(O_x \& O_{y'}) - P_{\lambda x'y'}(O_{x'} \& O_{y'}) - P_{\lambda x}(O_x) - P_{\lambda y}(O_y) \leq 0$$

$$\text{(Bell/CH - phys - }\psi\text{)} \quad -1 \leq P_{\psi xy}(O_x \& O_y) + P_{\psi x'y}(O_{x'} \& O_y) + P_{\psi xy'}(O_x \& O_{y'}) - P_{\psi x'y'}(O_{x'} \& O_{y'}) - P_{\psi x}(O_x) - P_{\psi y}(O_y) \leq 0$$

where, as before, ψ is the QM pair-state, x (y) is the setting of the L- (R-) apparatus to measure spin in the direction x (y), and O_x (O_y) is the outcome "up" in x- $(y$-) spin measurement on the L- (R-) particle; and similarly, *mutatis mutandis*, for x' (y') and $O_{x'}$ $(O_{y'})$. (Bell/CH – physics – λ) is an inequality of probabilities of the hidden-variables model, whereas (Bell/CH – physics – ψ) is an inequality of QM probabilities. The latter inequality is derived from the former by integrating over all the complete pair-states λ while assuming λ-independence.

In (Bell/CH – prob) all the probabilities belong to the same probability space, whereas in (Bell/CH – phys – λ) and (Bell/CH – phys – ψ) each of the probabilities belongs to a different probability space. This should be clear from the fact that each of the probabilities in these latter inequalities has a different subscript. Thus, unlike the former inequality, these inequalities cannot be derived purely on the basis of considerations of coherence or consistency.

Indeed, (Bell/CH – phys – λ) and (Bell/CH – phys – ψ) are sometimes represented in terms of conditional probabilities *a la* Kolmogorov with the conditioning events placed after the conditionalization stroke rather than in the subscript, where in each inequality all the probabilities are embedded in one "big" probability space:

(Bell/CH –phys – λ – big)

$$-1 \leq P(O_x \& O_y / \lambda \& x \& y) + P(O_{x'} \& O_y / \lambda \& x' \& y) + P(O_x \& O_{y'} / \lambda \& x \& y') - P(O_{x'} \& O_{y'} / \lambda \& x' \& y') - P(O_x / \lambda \& x) - P(O_y / \lambda \& y) \leq 0$$

(Bell/CH – phys – ψ – big)

$$-1 \leq P(O_x \& O_y / \psi \& x \& y) + P(O_{x'} \& O_y / \psi \& x' \& y) + P(O_x \& O_{y'} / \psi \& x \& y') - P(O_{x'} \& O_{y'} / \psi \& x' \& y') - P(O_x / \psi \& x) - P(O_y / \psi \& y) \leq 0.$$

Yet, these inequalities are not theorems of probability theory. Unlike (Bell/CH – prob), they cannot be derived from the assumption of a joint distribution over the measurement outcomes, the (QM or complete) pair-state and apparatus settings.

We shall discuss the relationships between (Bell/CH – prob) and (Bell/CH – phys – λ) below and in Sect. 16.3.7.

In hidden-variables models of the EPR/B experiment that postulate the existence of definite values for all the four spin quantities that are involved in the Bell/CH inequalities, it is natural (though not necessary) to suppose a joint probability over these probabilities.[3] Thus, in such models, it is plausible to expect (Bell/CH – prob). But (Bell/CH – prob) is neither necessary nor sufficient for (Bell/CH – phys – ψ) or (Bell/CH – phys – ψ – big). Indeed, unless we make some assumptions about the relationships between the probabilities of the spin quantities in (Bell/CH – prob) and the probabilities of their measurement outcomes, the assumption of joint probability over these quantities will do little to constrain the probabilities of their measurement outcomes. Two natural assumptions are *λ-independence* and the assumption that the probabilities of spin-measurement outcomes "mirror" the probabilities that the particles' spins have before the measurements: for any spin properties X and Y, apparatus settings x and y to measure these properties, and the corresponding measurement outcomes O_x and O_y,

$$\text{(Mirror)}\quad P_{\lambda x}(O_x) = P_\lambda(X), \quad P_{\lambda y}(O_y) = P_\lambda(Y), \quad P_{\lambda xy}(O_x \& O_y) = P_\lambda(X \& Y).$$

Although these assumptions may seem natural, models of the EPR/B experiment that postulate joint probability over the values of the particles' spin in various directions violate at least one of these assumptions; and their violation bears directly on the question whether the quantum realm involves some kind of non-locality. *λ-independence* fails in models of the experiment that postulate retro-causal influences from the measurement events to the source at the emission, so that the distribution of the complete pair-state depends on the measured quantities (for recent discussions of such models, see [31–35], and references therein). In such models, the QM statistics may be accounted for by such retro-causal influences rather than non-locality.

Mirror may be violated in various "hidden-variables" theories. For example, it is violated in Bohmian mechanics, if X and X' (Y and Y') are respectively the positions of the L- (R-) particle relative to planes aligned along the directions x and x' (y and y') at the emission. In Bell's [36] "minimal" Bohmian mechanics spins are not intrinsic properties of the particles, and the positions of the particles at the emission influence their spin dispositions, i.e. their behavior in spin measurements: X (Y) determines the spin disposition of the L- (R-) particle in the direction x (y) in a measurement of spin x (y), if the L- (R-) measurement occurs first; and similarly for X' (Y') and $x'(y')$. Yet, due to non-local influences, the distribution of these dispositions is different from the distribution of the outcomes of the corresponding spin measurements. If, for example, at the emission both particles are disposed to spin "up" in a z-spin measurement, and the L-measurement occurs first, this

[3] Svetlichny et al. [30] argue that if probabilities are interpreted as infinitely long-run frequencies in random sequences, such a joint probability distribution need not exist.

measurement will change the z-spin disposition of the R-particle: after the L-measurement, it will be disposed to spin "down" on z-spin measurement ([36, 37], Chap. 7, [20], Sect. 5.3.1).

While the joint distribution over the spin quantities of the particle-pair in the EPR/B experiment (the "hidden variables") is neither necessary nor sufficient condition for (Bell/CH – physics – ψ) or (Bell/CH – physics – ψ – big), the question arises whether some other joint distributions are. The most comprehensive, relevant joint probability distribution in the context of these inequalities is a distribution over the QM and complete pair-state, the various relevant apparatus settings and the corresponding measurement outcomes,[4] and such distribution is neither necessary nor sufficient for these inequalities. (Bell/CH – phys – ψ) follows from *Factorizability* and *λ-independence* [13],[5] and as it is not difficult to see these conditions do not presuppose a joint distribution over the pair-state, apparatus settings and measurement outcomes. Similarly, (Bell/CH – phys – ψ - big) follows from factorizability and λ-independence expressed in terms of conditional probabilities *a la* Kolmogorov – for any QM and complete pair-states, λ and ψ, apparatus settings x and y to measure the particles' spins along the directions x and y, and the corresponding measurement outcomes O_x and O_y,

$$\text{(Factorizability*)} \quad P(O_x\&O_y/\lambda\&x\&y) = P(O_x/\lambda\&x) \cdot P(O_y/\lambda\&y)$$

$$(\lambda \text{ - independence*}) \quad \rho(\lambda/\psi\&x\&y) = \rho(\psi)$$

– and these conditions do not presuppose such a joint distribution. Indeed, each particular case of *Factorizability** presupposes a joint distribution over the complete pair-state, two measurement outcomes (O_x and O_y) and two apparatus settings (x and y), and each particular case of *λ-independence** presupposes a distribution over the QM and complete pair-state and two apparatus settings. But these conditions do not presuppose a joint distribution over the QM and the complete pair-state and all the four apparatus settings and four corresponding measurement outcomes that are involved in (Bell/CH – phys – ψ – big). Thus, a joint probability over the QM and complete pair-state, apparatus settings and measurement outcomes is not a necessary condition for (Bell/CH – phys – ψ – big). It is also

[4] In fact, one may also add to this list the complete states (the "hidden variables") of the apparatus settings. While such a distribution will be even more comprehensive, it will not change the conclusion of the analysis below.

[5] The derivation of (Bell/CH – phys – ψ) from *Factorizability* and *λ-independence* is straightforward. $-1 \leq a \cdot b + a' \cdot b + a \cdot b' - a' \cdot b' - a - b \leq 0$ obtains for any real numbers $0 \leq a, a', b, b' \leq 1$. Substituting $a = P_{\lambda x}(O_x)$, $a' = P_{\lambda x'}(O_{x'})$, $b = P_{\lambda y}(O_y)$, $b' = P_{\lambda y'}(O_{y'})$ and applying *Factorizability* we have (Bell/CH – phys – λ), and integrating over λ while assuming *λ-independence* we obtain (Bell/CH – physics – ψ).

not sufficient for (Bell/CH – phys – ψ – big), as it is easy to construct such a distribution that violates the inequality.[6]

Fine [27] discusses a fourth kind of Bell/CH inequality, where the probabilities are supposed to be "the observed distributions for each of the four observables involved in the EPR/B experiment plus the joint observed distributions for each of the four compatible pairs" of these observables. (Fine [27], p. 291)

(Bell/CH – Fine)

$$-1 \leq P(O_x \& O_y) + P(O_{x'} \& O_y) + P(O_x \& O_{y'}) \\ -P(O_{x'} \& O_{y'}) - P(O_x) - P(O_y) \leq 0;$$

where, presumably, P is a probability function that depends on the QM pair-state ψ. (Bell/CH - Fine) follows from the assumption of a joint probability over the measurement outcomes. The question is what could motivate such an assumption. Surely, the probabilities in this inequality need to depend on the apparatus settings, so that they either belong to different spaces (each characterized by different apparatus settings), as in (Bell/CH – phys - ψ), or are in the same probability space but are conditional on the QM pair-state and apparatus settings, as in (Bell/CH – phys - ψ - big). In the first case, the motivation for (Bell/CH – Fine) should probably include assumptions like *Mirror* and *λ-independence*, and as we have seen the violation of such assumptions is relevant to the question whether the quantum realm involves non-locality. In the second case, one may assume a joint distribution for the QM pair-state, apparatus settings and measurement outcomes, but as we argued above such a distribution would not entail (Bell/CH – phys - ψ - big). So in either case, (Bell/CH – Fine) has to be motivated by assumptions about the physical nature of the systems involved in the EPR/B experiment – in particular, assumptions about the state of the particles at the source, the causal relations between this state and the state of the measurement apparatuses during the measurements, and the causal relations between measurements in the two distant wings of the experiment. And granted such assumptions, the violation of (Bell/CH – Fine) will have bearings on the causal relations in the EPR/B experiment in general, and the question of quantum non-locality in particular.

It is also noteworthy that in the derivation of the Bell/CH inequalities, or more precisely (Bell/CH – Fine), Fine [27] in fact presupposes *λ-independence* and some factorizability conditions. That he presupposes λ-independence is clear from the

[6] For example, (Bell/CH – phys – ψ – big) fails for any joint distribution that returns the following probabilities as marginals for apparatus settings that satisfy $|x-y| = |x'-y| = |x-y'| = 60°$ and $|x'-y'| = 180°$: $P(\psi \& x \& y) = P(\psi \& x' \& y) = P(\psi \& x \& y') = P(\psi \& x' \& y') = 1/4$, $P(\psi \& x) = P(\psi \& y) = 1/2$, $P(O_x \& O_y \& \psi \& x \& y) = P(O_{x'} \& O_y \& \psi \& x' \& y) = P(O_x \& O_{y'} \& \psi \& x \& y') = 1/32$, $P(O_{x'} \& O_{y'} \& \psi \& x' \& y') = 1/8$, $P(O_x \& \psi \& x) = P(O_y \& \psi \& y) = 1/4$.

fact that he takes the distribution of λ to be the same for all spin measurements; and as it is not difficult to see from equations (2) and (11) in his paper, his characterization of hidden-variables models embody factorizability conditions. Recalling footnote 5, it is not difficult to show that λ-independence and these factorizability conditions are sufficient for the derivation of Bell/CH inequalities. So the question arises as to the role that the assumption of joint distribution plays in Fine's derivation of these inequalities. It may be tempting to argue that such an assumption is necessary for the physical plausibility of the hidden-variables models. But, first, this argument is not open to Fine, who holds that the rejection of such an assumption is the very essence of QM. Second, even if we suppose that the assumption of joint distribution were important for the ontological status of the hidden-variables models (an assumption that Bell, Clauser and Horne and many others reject), the violation of this assumption *per se* is not sufficient to vindicate Fine's claim that "what the hidden-variables models and the Bell/CH inequalities are all about are the requirements that make well defined precisely those probability distributions for non-commuting observables." [27, p. 291] Since factorizability fails in the EPR/B experiment, Fine has to appeal to the view that the violation of this condition has no implications for the question of non-locality [22–24]. For if we suppose that the failure of factorizability involves some kind of non-locality, as a broad consensus has it, then the fact that factorizability fails in standard QM as well as in any alternative interpretation or hidden-variables model in which *λ-independence* obtains, will entail that the common interpretation of Bell's theorem is on the right track.

In any case, as we shall see in Sect. 16.5, if probabilities are interpreted along de Finetti's probability theory, (Bell/CH – Fine) cannot be derived from the assumption of joint probability distribution over the measurement outcomes since such distribution does not exist. Similarly, *λ-independence* and *Mirror* do not entail (Bell/CH – phys – ψ) since (Bell/CH – prob) does not hold; for the joint probability distribution over the spin quantities in this latter inequality does not exist. But before turning to discuss the application of de Finetti's theory to the quantum probabilities, we introduce the highlights of this theory in Sects. 16.3 and 16.4.

16.3 De Finetti's Theory of Probability

Our aim here is to offer a new reading of de Finetti's theory of probability and, assuming that quantum probabilities are interpreted along this theory, to study their logical structure – i.e. the inequalities that constrain them. Thus, for lack of space, the presentation of de Finetti's theory will be uncritical.

16.3.1 The Probability Axioms Are Not Merely Formal Conventions

De Finetti held that "probability theory is not merely a formal, merely arbitrary construction, and its axioms cannot be chosen freely as conventions justified only by mathematical elegance or convenience. They should express all that is necessarily inherent in the notion of probability and nothing more." [38, pp. xiii–xiv] He thought of probability as a guide of life under uncertainty. Having been influenced by positivism, he held that probability, like other notions of great practical importance, should have an operational definition, namely "a definition based on criterion which allows us to measure it." [39, p. 76] Also, being a guide of life under uncertainty, de Finetti maintained that probability should be closely related to rational decisions under uncertainty. ([38], pp. xiii–xiv, Chaps. 1 and 2; [39], 76–89) The decision framework that he had in mind is Bayesian, where a person's degrees of belief reflect her uncertainty concerning the things she cares about, her utilities reflect her subjective preferences, and the outcomes of rational decisions are actions that maximize her expected utility.[7] De Finetti thought of probability as reflecting rational degrees of belief, and of coherence as a necessary condition for degrees of belief being rational, and he argued that all the theorems of probability theory could be derived from the coherence conditions of degrees of belief. ([39], pp. x, 72–75, 87–89; [38], Chaps. 1 and 2)

16.3.2 The Domain of Probability Is the Domain of Uncertainty

De Finetti made a distinction between the domain of certainty, i.e. that which one takes as certain or impossible, and the domain of uncertainty, i.e. the range over which one's uncertainty extends. The distinction between these domains is very important and fundamental to de Finetti's philosophy of probability, as his long and detailed discussion of this topic demonstrates [39, Chap. 2]. The domain of uncertainty depends on one's (actual and/or hypothetical) background knowledge [39, pp. 27, 47] and one's reasoning, and thus it may include events that are logically impossible or certain, e.g. complicated contradictions or tautologies that one fails to recognize. The domain of probability is the domain of uncertainty. This domain is supposed to include all the atomic uncertain events (or the propositions that such events occur) and their logical combinations, which may be certain (for example, if *A* is an uncertain event, the domain of uncertainty will also include the certain event *A* or not-*A*). Whether an event is atomic is a pragmatic matter, which does not depend on metaphysical questions. It is noteworthy that for de

[7] It is noteworthy that unlike Frank Ramsey [40], another founding father of the modern school of subjective probability, de Finetti held that probability is not strictly related to rational preferences.

Finetti, there is a sharp distinction between being certain about an atomic event, and having a degree of belief one in it. The former belongs to the domain of certainty, whereas the latter belongs to the domain of uncertainty.

16.3.3 Probabilities Are Subjective and Instrumental

Many friends of the subjective interpretation of probability think that coherence is a necessary but not sufficient condition for the rationality of degrees of belief. They hold that for degrees of belief to be rational, they also have to be constrained by knowledge of objective facts about the world. In particular, it is frequently maintained that when objective probabilities are available, they should constrain the corresponding subjective probabilities. Thus, many hold that rationality requires that a person's subjective probability of E given that the objective probability of E is p, and she assumes, believes or knows nothing else about the prospects of E, should be p. An influential expression of this idea is Lewis's [28] "principal principle."

De Finetti rejected the idea that subjective probabilities are supposed to be guesses, predictions or hypotheses about the corresponding objective probabilities, or based on such probabilities or any other objective facts. Indeed, he argued that probabilities are inherently subjective, and that none of the objective interpretations of probability makes sense. He held that objective probability does not exist, and that recognition of its inexistence would constitute a progress in scientific thinking. "The abandonment of superstitious beliefs about the existence of Phlogiston, the Cosmic Ether, Absolute Space and Time, … , or Fairies and Witches, was an essential step along the road to scientific thinking. Probability, too, if regarded as something endowed with some kind of objective existence, is no less misleading misconception, an illusory attempt to exteriorize or materialize our true [i.e. actual] probabilistic beliefs." [39, p. x][8]

De Finetti [39, pp. x–xi] argued that probability and probabilistic reasoning should always be understood as subjective. Probability only reflects uncertainty, and accordingly no fact could prove or disprove a degree of belief. He did not deny, however, the psychological influence that facts may have on degrees of belief. "I find no difficultly in admitting that any form of comparison between probability evaluations (of myself, or of other people) and actual events may be an element influencing my further judgment, of the same status as any other kind of information … But, as with any other experience, these modifications would not be governed by a mechanical rule; it is, in each case, my personal judgment that is responsible for giving a weight to the facts (for instance, according to my feelings

[8] The addition of the word "actual" in the square brackets is mine, as the translation from Italian seems incorrect. The word "vero" could be translated as "actual" or "true", and it is clear that in this context it should be translated as "actual."

about the success of the other person being due to his skill and competence or merely due to a meaningless chance).” [38, p. 21]

The source of uncertainty is immaterial. “It makes no difference whether the uncertainty relates to unforeseeable future, or to an unnoticed past, or to a past doubtfully reported or forgotten; it may even relate to something more or less knowable (by means of a computation, a logical deduction, etc.) but for which we are not willing or able to make the effort; and so on. . . . The only relevant thing is uncertainty – the extent of our knowledge and ignorance. The actual fact of whether or not the events considered are in some sense *determined*, or known by other people, and so on, is of no consequence.” [39, pp. x–xi] The important thing for de Finetti is that in all these different states of uncertainty, subjective probability could be useful as a guide. The role of probability is *purely instrumental*, and its value should be determined solely on the basis of its potential to serve as a guide in everyday and science. De Finetti went to great pains in his attempt to show that his subjective theory of probability could serve as such a guide.

As de Finetti’s *Philosophical Lectures on Probability* suggest, he was instrumentalist about probabilistic theories [41, pp. 53–54], interpreting their probabilities as subjective, representing nothing but degrees of expectations. [41, p. 52] And he held that distributions brought to us by probabilistic theories, such as Statistical Mechanics and Quantum Mechanics, “provide more solid grounds for subjective opinions.” [41, p. 52]

Like other instrumental views, de Finetti thought that subjective probability could play its role as a guide, independently of our metaphysical assumptions about the world. “[P]robabilistic reasoning is completely unrelated to general philosophical controversies, such as Determinism versus Indeterminism, Realism versus Solipsism – including the question of whether the world ‘exists’, or is simply the scenery of ‘my’ solipsistic dream.” [39, p. xi]

16.3.4 Intuition, Prudence and Learning from Experience

It is common to portray probability in de Finetti’s radical subjective interpretation as unconstrained, too permissive and possibly whimsical (see, for example, [42, Sect. 3.5.4]. On the other hand, de Finetti held that assigning or “evaluating” probabilities is an inductive reasoning, and as such it is based on learning from experience; and “to speak about inductive ‘reasoning’ means, however, to attribute a certain validity to that mode of learning, to consider it not as a result of a capricious psychological reaction, but as a mental process susceptible of an analysis, interpretation and justification.” [38, p. 147] Indeed, he warned against superficiality in assigning probabilities, which is frequently associated with subjective probability. In particular, he warns against two common patterns of superficiality. “On the one hand You may think that the choice, being subjective, and therefore arbitrary, does not require too much of an effort in pinpointing one particular value rather than a different one; on the other hand, it might be thought that no mental effort is required, since it can be avoided by the mechanical application of some standardized procedure." [39, p. 179] He recommended various features that must underlie each probability evaluation, like for example to "think about every aspect of the problem. . .try to imagine how things might go. . .encompass all conceivable possibilities; and also take into account that some might have escaped attention. . .identify those elements which, compared to others, might clarify or obscure certain issues. . .enlarge one's view by comparing a given situation with others. . .attempt to discover the possible reasons lying behind those evaluations of other people. . ." [39, pp. 183–4]. This is not surprising given that de Finetti held that "the (subjectivistic) theory of probability is a normative theory, not a descriptive one," and the value of probability theory is "precisely as an aid to the avoidance of plausible and frequently serious shortcomings and errors." [38, p. 151]

De Finetti's philosophy of probability presupposes that people have the intuitive faculty to form reasonable opinions about uncertain events and, with the aid of probability theory, the capacity to form reasonable probabilistic opinions. De Finetti held that people need to develop and refine this faculty, and apply reason to learn to guard it against the tendency to form superficial probabilistic opinions. Yet, he cautioned against the misunderstanding of the role of reason. In particular, he warned that "the tendency to overestimate reason – often in an exclusive spirit – is particularly harmful. Reason, to my mind, is invaluable as a supplement to the other psycho-intuitive faculties, but never a substitute for them. Figuratively, reason is a pole that may keep the plant of intuitive thought from growing crooked, but it is not itself either a plant or a valid substitute for a plant." [38, pp. 147–8]

Learning from experience is important for assigning both "prior" and posterior' probabilities. De Finetti held that every probability is conditional "not only on the mentality or the psychology of the individual involved, at the time in question, but also, and essentially, on the state of information in which he finds himself at the moment," though in many cases there is no need to mention explicitly the background information, and accordingly it is suppressed [39, p. 134]. So both prior and posterior probabilities are conditional probabilities. The prior probabilities are conditional on some prior background information, and they are updated according to the increase or change in background knowledge/beliefs/assumptions. De Finetti makes a distinction between updating and changing opinions. When one conditionalizes on new information, one keeps the same opinion yet updates it to a new situation [41, p. 35]. And when one revises one's probability function, one changes one's opinion. Change of opinion could result from reconsideration of neglected, inaccurate or ambiguous information, or change of mind about the relevance of information, or superficial or careless evaluations, etc. Thus, de Finetti held that realistically the evolution of one's subjective probabilities involves both updating and changing opinions [41, pp. 39–40].

Due to the disparity in subjective evaluations, prior probabilities are expected to vary significantly. Yet, de Finetti held that the effects of "the disparity between the initial judgments of people or of vagueness in the initial judgments of one person are often largely eliminated," if the additional information gathered between the prior and the posterior evaluations is sufficiently revealing and the prior probabilities are "sufficiently gentle or diffuse," i.e. not too opinionated [38, p. 145].

16.3.5 Probabilities Are Coherent Degrees of Belief

Probabilities are not just any degrees of belief. They are coherent degrees of belief in (propositions about) events that belong to the (agent's) domain of uncertainty. The notion of coherent degrees of belief is commonly understood in terms of Dutch books, i.e. bets that results in loss come what may. The idea is that incoherent degrees of beliefs are subjected to Dutch books. ([39, 40, 60], Chaps. 3 and 4) Coherence is thus characterized in a betting framework, where a person is subjected by a clever bookie to series of bets. The person assigns certain odds to these bets according to her degrees of belief, and the bookie prescribes the possible gains and losses according to these odds.

In his later work, de Finetti preferred a different decision-theoretic framework (for the motivation, see Sect. 16.3.6). ([38], Chaps. 1 and 2; [39], Chaps. 3 and 4) In this alternative framework, there is no bookie. Individuals express their degrees of belief, and they are subjected to fixed gains and variable monetary losses, the so-called "loss functions," which are functions of their degrees of belief about events and the occurrence of these events. That is, letting E being any verifiable event, d a degree of belief in E, and $\mathbf{E}$ an indicator function denoting whether E occurs ($\mathbf{E} = 1$ if E occurs, and $\mathbf{E} = 0$ otherwise), the loss L that the individual is subjected to is:

$$\text{(L1)} \quad L = \frac{(\boldsymbol{E} - d)^2}{k};$$

where k is an arbitrary constant which is fixed in advance and which may differ from one case to another. In the case of multiple degrees of belief, the total loss is the sum of the losses incurred by each degree of belief. For example, the loss function for the degrees of belief $d_1,\ d_2,\ d_3$ in the events $E_1,\ E_2,\ E_3$, respectively, is:

$$\text{(L2)} \quad L = \frac{(\boldsymbol{E}_1 - d_1)^2}{k_1} + \frac{(\boldsymbol{E}_2 - d_2)^2}{k_2} + \frac{(\boldsymbol{E}_3 - d_3)^2}{k_3}.$$

In this alternative decision-theoretic scheme, coherent degrees of belief are explicated in terms of admissible decisions. The "decisions" are the individual's degrees of belief in various events, and they are admissible if they are not dominated by any other decisions, i.e. by any other degrees of belief in the same events; where a set of degrees of belief in events is dominated by another set of degrees of belief in the same events, if it leads to higher losses come what may. A set of degrees of beliefs is coherent just in case it is not dominated by any other set of degrees of belief in the same events.

16.3.6 Measurements of Degrees of Belief

De Finetti assigned a great importance to the measurement of degrees of belief. He thought that since probability is supposed to be a guide of life, it should have a meaning that renders it effective as such. Being influenced by positivism, he held that "in order to give an effective meaning to a notion – and not only an appearance of such in a metaphysical-verbalistic sense – an operational definition is required." By operational definition, he meant "a definition based on a criterion which allows us to measure it." [39, p. 76] His inspiration came from early twentieth century physics. "The notion of probability, like other notions of practical significance, ought to be operationally defined (in the way that has been particularly stressed in physics following Mach, Einstein, and Bridgman), that is, with reference to observations, in experiments that are at least conceptually feasible. In our case, the experiments concern the behavior of an individual (real or hypothetical) facing uncertainty." [38, p. xiv]

The main reason why de Finetti preferred the loss-functions decision-theoretic scheme is that the Dutch-book framework involves a bookie, an "opponent," the presence of whom may intrude with the measurement of degrees of belief. In particular, de Finetti mentioned the possibility that the bookie or the individual take advantage of differences of information, competence or shrewdness [39, p. 93]. The presuppositions of this scheme are that individuals strive to maximize their expected utility, and that utility is linear with money, where k is supposed to warrant this linearity. Granted these assumptions, it is not difficult to show that it is in the best interest of individuals to express their actual degrees of belief; for any other degrees of belief will lower their (subjective) expected utility.

Since de Finetti defines probability in terms of betting or measurement contexts, it may be tempting to interpret him as behaviorist about degrees of belief, holding that degrees of belief, and accordingly probabilities, do not exist outside these contexts [43, 185–9]. This interpretation is particularly suggestive given the inspiration that de Finetti took from Bridgman's [44] operationalism, where theoretical terms are defined in terms of the operational procedures of their measurements. Yet, de Finetti did not intend the betting and the loss-function decision-theoretic frameworks as Bridgman-like operational definitions of degrees of belief. Indeed, he held that degrees of belief exist independently of the contexts of their measurement. "The criterion, the operative part of the definition which enables us to measure it, consists in this case of testing, through the *decisions* of individual (which are observable), his opinions (previsions, probabilities), which are not directly observable." [39, p. 76] Moreover, as Eriksson and Hájek [43, p. 190] point out, de Finetti's worries about the relation between utility and money and about agents who care too much or too little about their bets, do not make sense if degrees of beliefs are interpreted along Bridgman's operationalism. The operational procedure is supposed to provide a reliable measurement of degrees of belief, not a definition of them. Yet, as we shall see in Sect. 16.4, the operational procedure plays an important role in explicating the coherence conditions of degrees of belief and to that extent it plays an important role in defining subjective probabilities.

16.3.7 Conditional Probability

Following Kolmogorov's [17] influential axiomatization of probability, it is common to define conditional probability in terms of unconditional probabilities: $P(B/A) \equiv P(B\&A)/P(A)$. De Finetti rejected this axiomatic approach. He thought that probability theory should be derived from the analysis of the meaning of probability. He held that every probability is conditional "not only on the mentality or the psychology of the individual involved, at the time in question, but also, and essentially, on the state of information in which he finds himself at the moment," though in many cases there is no need to mention explicitly the background information, and accordingly it is suppressed [39, p. 134]. Thus, he maintained that conditional probability is the fundamental object of probability theory, and unconditional probability does not make sense (except when it is a conditional probability in disguise).[9]

In introducing the concept of conditional probability, de Finetti says that "we shall write $P(E|H)$ for the *probability* '*of the event E conditional on the event H*' (or even the *probability* '*of the conditional event* $E|H$'), which is the probability that You attribute to E if You think that in addition to your present information, i.e. the H_0 which we understand implicitly, *it will become known to You that H is true* (*and nothing else*)." [39, p. 134] This characterization is ambiguous. On the one hand, conditional probability is characterized as a conditional with a probabilistic consequent, whereas on the other it is likened to unconditional probability of a "conditional event."

The association of conditional probability with a "called-off" bet in the betting decision-theoretic framework, and the loss function for conditional probability in the loss-function decision-theoretic framework both suggest the first interpretation. The loss function for the probability of E given H and the background knowledge H_0 is:

$$\text{(L3)} \quad L = \frac{\boldsymbol{H}_0\mathbf{H}(\boldsymbol{E} - d)^2}{k}$$

where d is a degree of belief in E, $\mathbf{E}$ and $\mathbf{H}$ are indicator functions, denoting the truth value of E and H, and $\mathbf{H_0}$ is an indicator function denoting the truth value of H_0. Based on (L3), the proposition that the conditional probability of E given H and H_0 equals p may be characterized by the following conditional:

(CP1) If you have the background knowledge H_0 and you come to know H (and nothing else), then your degree of belief in E will be p.

[9] In fact, the idea that conditional probability is the fundamental object of probability theory could also be defended in other interpretations of probability. [18, 19, 45]

The idea is that a person with such a conditional probability is subjected to a loss of $(\mathbf{E} - p)^2/k$ on the condition that she has the background knowledge/beliefs H_0 and she comes to know H and nothing else; and the loss is zero, if she does not have the knowledge/beliefs H_0 or does not come to know H. This is very similar to the idea of a called-off bet, where the probability of E given $H\&H_0$ being p is explicated by a bet in which a person pays pS dollars on the condition that she knows $H\&H_0$ for the opportunity to earn S dollars if E occurs and zero otherwise, and the bet is called off if she does not know $H\&H_0$.

The notion of conditional probability applies not only to cases where one knows H_0 and H, but also to cases where one assumes or believes H_0 and H. We may thus extend the meaning of conditional probability as follows:

(CP2) If you know, believe or assume H_0 and you come to know, believe or assume H (and nothing else), then your degree of belief in E will be p.

Further, the conditioning event and the background knowledge may be counterfactual rather than actual. In such cases, conditional probability may be characterized by the following counterfactual conditional:

(CP3) If you had the background knowledge or beliefs H_0 and you had come to know, believe or assume H (and nothing else), then your degree of belief in E would have been p.

Beware! (CP2) is neither the material nor the strict conditional. It is true if one knows, believes or assumes H and nothing else beside one's background knowledge H_0, and one's degree of belief in E is p; it is false when one has the background H_0 and comes to know, believe or assume H but one's degree of belief in E is not p; and it is indeterminate when one does not have the background H_0 or does not come to know, believe or assume H. (CP3) is not the Stalnaker–Lewis counterfactual conditional, though it may be interpreted as being true in case one's degree of belief in E is p in the most similar relevant worlds or scenarios in which one holds H_0 and H. For a more detailed discussion of these conditionals, see Berkovitz [45].

In order to distinguish the above notion of conditional probability from that of Kolmogorov, we shall place the conditional event in the subscript: $P_{H_0H}(E)$ will denote the conditional probability of E given H and the background knowledge H_0. De Finetti ([38], Chap. 2, [39], Chap. 4) demonstrates that coherence entails that:

$$\text{(C1)} \quad P_{H_0H}(E) \cdot P_{H_0}(H) = P_{H_0}(E\&H);$$

where $P_{H_0}(H)$ and $P_{H_0}(E\&H)$ are respectively the probability of E given H_0 and the probability of $E\&H$ given H_0. When $P_{H_0}(H)$ is definite and non-zero, we obtain Kolmogorov's definition of conditional probability as a coherence condition on degrees of belief.

Recall (Sect. 16.2) the two different ways of representing the Bell/CH inequalities: in terms of conditional probabilities with the conditions (namely, the pair-state and the apparatus settings) in the subscript, as in (Bell/CH – phys – ψ); and in terms of conditional probabilities *a la* Kolmogorov, where the conditions are placed after the conditionalization stroke, as in (Bell/CH – phys – ψ – big). (C1) suggests a way to relate these different representations.

De Finetti's proposal that the probability of E given H may be seen as the probability of the "conditional event" $E|H$ suggests another interpretation of conditional probability. Conditional events (or "tri-events") are in effect three-valued propositions about events, the truth-value of which depends on the condition ([39], p. 139, [46], Appendix, pp. 307–11). In particular, $E|H$ is the proposition that E occurs, but its truth-value depends on whether H occurs. If H occurs, then $E|H$ is true if E occurs and false if E does not occur; and if H does not occur, then $E|H$ has indeterminate truth-value. The idea is then to assign probabilities only to conditional events that are true or false, so that indeterminate conditional events have no probabilities.

We shall return to consider the implications of the two different interpretations of de Finetti's concept of conditional probability in our discussion of his verificationism in Sect. 16.4, and in the application of his theory of probability to QM in Sect. 16.5.

16.3.8 Symmetry and Exchangeability

Judgments of equally probable events, and accordingly of symmetries, are central to all interpretations of probability. In objective interpretations of probability, the symmetries concern the way things are. For de Finetti, the relevant symmetries concern one's opinions and judgments. De Finetti held that any evaluation of equally probable events is based on subjective judgments, and that the notion of exchangeability is central to such judgments. A collection of events is said to be exchangeable if the probability p_h that h of them occur depends only on h and is independent of their order of appearance [38, p. 229]. Followers of de Finetti's interpretation and friends of the Bayesian interpretation of quantum probabilities attribute a great importance to exchangeability. Indeed, the notion of exchangeability, and the related notion of partial exchangeability are bound to play a central role in the interpretation of the quantum probabilities along de Finetti's probability theory. For example, Caves et al. [4] apply de Finetti's work on exchangeability to the interpretation of the notion "unknown quantum states" and the related notion of "unknown quantum probabilities" from a subjectivist Bayesian perspective. Yet, as the notion of exchangeability is not central to our main focus – the study of the coherence conditions of degrees of belief in the context of QM – we postpone its discussion to another opportunity.

16.4 Verifiability, Coherence and Contextuality

De Finetti [39, p. 34] held that the events in probability assignments have to be verifiable. "In general terms, it will always be a question of examining, if, and in which sense, a statement really constitutes an 'event,' permitting in a more or less realistic acceptable form, and in unique way, the 'verification' of whether it is 'true' or 'false'... *A* and *B* are events (observables), but it is not possible to observe both of them, and, therefore, it is not possible to call the product *AB* an event (observable)."

An important implication of this view is that the constraints on probabilities of events that are not jointly verifiable are weaker. For example, if *A* and *B* are jointly verifiable, their probabilities are subjected to the inequality $P(A)+P(B)-P(A\&B)\leq 1$. But if *A* and *B* are not jointly verifiable, they have no joint probability, and accordingly their probabilities are not subjected to this inequality.

De Finetti [46, p. 260] acknowledged that verifiability is "a notion that is often vague and illusive" and thought that it is necessary "to recognize that there are various degrees and shades of meaning attached to [it]." He took a pragmatic attitude toward the kind and degree of verifiability that is actually required for events to have a definite probability [46, Appendix]. To simplify things, we shall focus on verifiability in principle, and by "verifiable events" we shall mean events that are verifiable in theory according to one's beliefs.

Unlike probabilities, de Finetti was not antirealist about events. Yet, he held that notions of great practical importance should have "operational definitions," namely definitions based on criteria that render them measurable. If events are not verifiable, they cannot have such an operational definition. Further, the prospects of adequate measurements of degrees of belief in such events are dim, thus undermining the idea that probability should also have an operational definition. The most obvious explanation for de Finetti's verificationism is the influence of positivism. De Finetti [39, p. 76] was worried that events that are not verifiable may appear to be sensical but in fact be meaningless, and accordingly degrees of belief in such events will be useless.

In the context of de Finetti's philosophy of probability, there is a different reason to motivate his verificationism. It is difficult to make sense of the idea of coherent degrees of belief in, and accordingly probabilities of unverifiable events. This is clear in the betting decision-theoretic framework. Bets on events that are in principle unverifiable could never be concluded. Accordingly, no Dutch book could be based on such bets, and the idea that Dutch book could be used to explicate the notion of "coherent degrees of belief" collapses. Things are not so obvious in the loss-function decision-theoretic framework, as this framework appears to provide a way to explicate this notion even in the case of unverifiable events; for the notion of "admissible decision," which is used to explicate coherence in this framework, seems applicable even in the case of unverifiable events. But a little reflection on the nature of probabilities in de Finetti's theory suggests that this appearance is deceptive. In this theory, there are no objectively correct probability assignments. Probabilities are subjective opinions that only reflect uncertainty about things. The value of probabilities reside solely in their instrumental role as a guide for decisions under uncertainty, and this role could only be measured in terms of verifiable "gains" and "losses," or more generally verifiable consequences. In the case of unverifiable events, the instrumental value of probabilities vanishes because the consequences of probability assignments are in principle unverifiable. This lack of instrumental value reflects on the prospects of explicating the notion of coherent degrees of belief. Incoherent degrees of belief in unverifiable events have no verifiable harmful consequences, and so radical subjectivists about degrees of belief, like de Finetti, who deny the existence of objective probabilities, have no incentive to have coherent degrees of belief in such events. Accordingly, like in the betting decision-theoretic framework, the idea that the loss-function decision-theoretic framework could be used to explicate the notion of coherent degrees of belief collapses in the case of unverifiable events.

De Finetti proposes to make the verifiable nature of events explicit by assigning probabilities to "conditional events" $E|H$ (see Sect. 16.3.6) rather than to the events themselves; where H is an observation that enables to verify the event E [46, pp. 266–7, 307–313]. The idea is to assign probabilities only to conditional events $E|H$ with determinate truth-values, so that unverifiable events E have no probabilities. This idea is easily generalized to complex "conditional events," i.e. logical combinations of conditional events. Consider, for instance, $E_{12}|H_{12}$, the conjunction of the conditional events $E_1|H_1$ and $E_2|H_2$; where H_i is an observation that enables to verify whether E_i is true, and E_{12} is the event that denotes the conjunction of the events E_1 and E_2. $E_{12}|H_{12}$ is true if H_{12} and E_{12} are both true, false if H_{12} is true and E_{12} false, and has indeterminate truth-value if H_{12} is false. By restricting probabilities to conditional events, "complex" conditional events (like E_{12}) may fail to have definite probabilities, even when the "atomic" events that constitute them (E_1 and E_2) do. In this approach, a person's probabilities are represented by a "big" probability space with "gaps" in the place of some complex events (henceforth, DF-big-space). The logic and probability of conditional events seem to require some kind of three-valued logic, and indeed de Finetti discussed various three-valued logics that could serve as a basis for such probability theory [46, Appendix, pp. 302–313]. For de Finetti's early thoughts about conditional events and their logic, see De Finetti [47] and Mura [48].

De Finetti also entertained the idea of representing probabilities of verifiable events in terms of classical, two-valued logic. In fact, as we shall see in Sect. 16.6, he preferred such an approach. This alternative approach is in line with our proposal in Sect. 16.3.7 that conditional probability *a la* de Finetti may be characterized as a conditional with a probabilistic consequent. Indeed, this interpretation of de Finetti suggests a natural way of representing probabilities of verifiable events in terms of two-valued events. The main idea is to suppose that the "unconditional" probability of an event E being p has in effect a logical structure of a conditional with a probabilistic consequent: if an observation H that enables to verify E occurs (occurred), the probability of E is (would be) p. Recall (Sect. 16.3.7) that in our suggested notation, this conditional is represented as $P_H(E) = p$, i.e. as a conditional probability with the conditioning event in the subscript; and probabilities with different subscripts, i.e. conditionals with different antecedents, correspond to different probability spaces. That is, we could represent de Finetti's verificationism by supposing that a person's subjective probabilities are represented by multiple probability spaces, in each of which probabilities of events are conditional (implicitly) on an observation that enables to jointly verify all the events in the space. On this view, a person's coherent degrees of belief are represented by many "smaller" probability spaces (henceforth, DF-many-spaces), each contains events that could be jointly verified.

Although the two approaches are different, in de Finetti's philosophy of probability they are closely related. In both approaches, probabilities of events are conditional on observations that enable to verify them. This is not obvious in the DF-big-space, where probabilities appear to be unconditional. But recall (Sect. 16.3.7) that for de Finetti probabilities of "conditional events" are closely connected, if not equivalent, to the corresponding conditional probabilities. The similarity between conditional probability, represented as a conditional with probabilistic consequent, and the corresponding probability of conditional event is hindered by de Finetti's formal notation, which is similar to the common notation for conditional probability *a la* Kolmogorov. Yet, in both cases only verifiable events E have probabilities, and the observations H that enable their verification have no probability, as they are not events in the probability space. To highlight this feature, in our representation of conditional probability as a conditional with a probabilistic consequent, we have placed the conditioning events H in the subscript rather than after the conditionalization stroke, $P_H(E)$; and, as de Finetti [38, p. 104] remarks, the conditional event $E|H$ "must be considered as a whole," and accordingly H is not part of the probability space. Indeed, the inclusion of H in the probability space while maintaining de Finetti's verificationism would lead to an infinite regress, where H would have to be a conditional event, the condition of which would have to be represented by a conditional event, and so forth.

Finally, as represented above de Finetti's verificationism is very stringent. Conditionalizing probabilities of events on observations that enable to verify them would severely restrict the range of events that have probabilities. First, this brand of verificationism restricts probabilities to observational contexts. Second, in various cases the required observations are actually impossible to carry out. Third, it threatens to render de Finetti's philosophy of probability extremely operationalist, as the probability of an event may vary according to the kind of observation that enables to verify it. Yet, it is possible to sustain the main thrust of de Finetti's verificationism while avoiding the above undesired consequences by conditionalizing probabilities of events on the proposition that the events are verifiable in principle, rather than on the proposition that observations that enable to verify them have been performed. In fact, this weaker version of verificationism is what de Finetti seemed to have in mind. We shall discuss the implications of the weaker and the stronger versions of verificationism in the next section.

16.5 Coherent Degrees of Belief for the EPR/Bohm Experiment

The most important implication of de Finetti's verificationism is that the coherence conditions on degrees of belief in events that are *not* jointly verifiable are weaker than they would have been had the events been jointly verifiable. Let's consider again (Bell/CH – prob) (see Sect. 16.2). In de Finetti's theory, (Bell/CH – prob) is a necessary condition for coherent degrees of belief in, and accordingly for probabilities of X, Y, $X\&Y$, $X'\&Y$, $X\&Y'$ and $X'\&Y'$ *only when* these events (propositions) are jointly verifiable. But in various hidden-variable models of the EPR/B experiment, X and X' (Y and Y') are values of non-commuting spin observables, which are not jointly verifiable. Similarly, the measurement outcomes in (Bell/CH – Fine) are not jointly verifiable, and so they are not necessary conditions for coherent degrees of belief, and accordingly for probabilities of the measurement outcomes involved in this inequality. Thus, if probabilities are interpreted along de Finetti's theory, (Bell/CH – prob) and (Bell/CH – Fine) do not apply to the EPR/B experiment.

Recalling (Sect. 16.4) that de Finetti formalizes his verificationism in terms of conditional probabilities, the failure of these inequalities can be manifested in two different ways, corresponding to the two different interpretations of de Finetti's concept of conditional probability. Consider, for example, (Bell/CH – prob). In the DF-big-space approach, probabilities are assigned only to conditional events. In our case, the relevant conditional events are X/H_X, Y/H_Y, $X'/H_{X'}$, $Y'/H_{Y'}$, $X\&Y/H_{XY}$, $X'\&Y/H_{X'Y}$, $X\&Y'/H_{XY'}$, $X'\&Y'/H_{X'Y'}$, $X\&X'/H_{XX'}$ and $Y\&Y'/H_{YY'}$; where, as before, H_i is either a measurement that enables to verify the event i, or the proposition that the event i is verifiable (we shall consider below the differences between these two interpretations of H_i). Since it is impossible in principle to jointly observe X and X' (Y and Y'), individuals who are familiar with this feature of the quantum realm will not assign a determinate truth-value to $X\&X'/H_{XX'}$ ($Y\&Y'/H_{YY'}$), and so $X\&X'/H_{XX'}(Y\&Y'/H_{YY'})$ and any conjunction that includes it has no probability. Consequently, a (Bell/CH - prob)-like inequality is not a necessary condition for the probabilities of the conditional events X/H_X, Y/H_Y, $X\&Y/H_{XY}$, $X'\&Y/H_{X'Y}$, $X\&Y'/H_{XY'}$ and $X'\&Y'/H_{X'Y'}$. In the DF-many-spaces approach, the assumption that X and X' (Y and Y') are not jointly verifiable entails that the events X, X', Y and Y' are not in the same probability space. There are four smaller probability spaces, each contains two of these events: $\{X, Y\}$, $\{X', Y\}$, $\{X, Y'\}$ and $\{X', Y'\}$. So (Bell/CH - prob) is not a necessary condition for coherent degrees of belief in, and accordingly for the probabilities of the events X, Y, $X\&Y$, $X'\&Y$, $X\&Y'$ and $X'\&Y'$. The upshot is that followers of de Finetti, who assume that the spins of a particle in different directions are not jointly verifiable, are not committed to (Bell/CH – prob). Thus, they may assume *Mirror* (e.g. that the probability distribution of spin-measurement outcomes reflects the probability distribution of the particles' spins before the measurements) and *λ-independence* (e.g. that the distribution of the particles' spins is independent of the measurements), yet assign probabilities that are not constrained by (Bell/CH – phys – ψ). Similarly, followers of de Finetti will not see (Bell/CH – Fine) as a necessary constraint on the probabilities of the four spin-measurement outcomes involved in each of the Bell/CH inequalities.

Two challenges may be posed for de Finetti's verificationism. The first is for the DF-many-spaces approach. In this approach, the same event may have different probabilities in different spaces: e.g. event X may have the probability p_1 in the probability space S_1 that is constituted by the "atomic" events X and Y, and p_2, $p_2 < p_1$, in the space S_2 that is constituted by the "atomic" events X and Y'. For recall that the probabilities in S_1 are conditionalized on a measurement H_{XY} that enables to verify whether X and Y occur, and the probabilities in S_2 are conditionalized on a measurement $H_{XY'}$ that enables to verify whether X and Y' occur. If H_{XY} and $H_{XY'}$ are incompatible measurements, there is no Dutch-book argument to dictate that the probability of X should be the same in both probability spaces.

Things are different, however, in our suggested interpretation of de Finetti's verificationism, where events are conditionalized on their verifiability rather than on measurements that enable their verification (see Sect. 16.4). In this version, the probability of X has to be the same in S_1 and in S_2 on pain of a Dutch book, where a bookie offers to sell a bet on X for $\$p_1$ and buy it back for $\$p_2$, thus "pumping" money out of any individual who holds that the probability of X in S_1 is different from the probability of X in S_2. The reasoning is as follows. An individual who holds the above probabilities should consider as fair a bookie's offer to (i) sell a conditional bet on X given that X and Y are jointly verifiable at the price of $\$p_1$, and (ii) buy a conditional bet on X given that X and Y' are jointly verifiable at the price of $\$p_2$. Since in each of these cases the bet is conditional on the relevant events being *verifiable*, rather than on actually being verified by measurements, the two bets could be jointly realized. Thus, if the individual accepts both bets as fair, she is destined to lose money come what may.

The second challenge is for both approaches, and it is related to the Kochen and Specker's (1967)'s no-go theorem. Due to its verificationism, de Finetti's theory of probability prescribes weaker constraints on probabilities in the EPR/B experiment. This provides followers of de Finetti's theory with some flexibility that is lacking in other interpretations of probability. Thus, for example, hidden-variables models of this experiment in which probabilities are interpreted along de Finetti's theory may postulate the existence of definite values for non-commuting spin observables, i.e. values of spins in various directions, even if they assume *Mirror* and *λ-independence*. Yet, Kochen and Specker's theorem and other similar theorems impose heavy constraints on assignments of definite values to such non-commuting observables (for a review of these theorems, see [49]), which substantially limit the scope of such flexibility. The reasoning is as follows.

In their theorem, Kochen and Specker consider a spin-1 particle and triples of the square values of spins in three orthogonal directions, S_x^2, S_y^2, S_z^2. The observables S_x^2, S_y^2, S_z^2 commute and accordingly their values are jointly verifiable (though the observables S_x, S_y, S_z do not commute and so their values are not jointly verifiable).

Kochen and Specker demonstrate that granted the following assumptions, there is no coherent way of distributing the values of spins in 117 directions.

Values: All physical quantities of a quantum system, i.e. all the observables that pertain to it, have definite values at all times.

Non-contextuality: Properties that a system possesses, i.e. the values of the observables that pertain to it, are non-relational to other properties or the measurement context.

More recently proofs involving less observables have been given (for references, see [49]). The upshot is that any "hidden-variables" model that satisfies these assumptions cannot provide a coherent assignment to a particle's spins in more than a limited number of directions. Indeed, the challenge that Kochen and Specker's theorem raises is not particular to the interpretation of probabilities along de Finetti's theory; it is posed for any interpretation of the probabilities of "hidden-variables" models. Yet, these theorems substantially restrict the advantages that de Finetti's interpretation provides.

De Finetti was also verificationist about events (see Sect. 16.4), and his verificationism may provide a way around Kochen and Specker's theorem. The proof of the theorem requires a truth-value assignment to propositions about events that are not jointly verifiable, and given de Finetti's verificationism about events the assignment of truth values to propositions about events that are not jointly verifiable may be more flexible, so as to avoid a Kochen and Specker-like contradiction; for such an assignment may violate *Non-contextuality*. Recall (Sect. 16.4) that de Finetti argued for verificationism on the grounds that the instrumental value of notions depends on their verifiability, and that this reasoning relies heavily on a positivist philosophy. Recall also that in the case of probabilities of events, de Finetti's verificationism can be motivated on different grounds – namely, by the radical subjectivist and instrumental nature of probability in his theory; for due to this nature, it is difficult to make sense of the notion of *coherent* degrees of belief, and accordingly of probabilities of unverifiable events. Such a motivation does not seem to exist in the case of events *per se*, as Finetti was not antirealist about events.

Followers of de Finetti's interpretation of probability who do not wish to adhere to de Finetti's positivism may circumvent Kochen and Specker's theorem by rejecting *Values*. They may for example follow the orthodox interpretation and accordingly reject *Values*; for recall that in this interpretation, the particles in the EPR/B experiment have no definite spins before the measurements. While the rejection of *Values* does not entail the failure of *Mirror*, it is more difficult to motivate the later premise when the former fails. Alternatively, followers of de Finetti may hold *Values* but reject *Non-contextuality*. For instance, they may hold that the values of spin quantities are *relational* to the values of other spin quantities,[10] so that the value of the particle's spin along the direction x relative

[10] For an example of interpretation of QM that postulates such relationalism, see Berkovitz and Hemmo's [50] relational modal interpretation.

to the values of its spins in the (mutually) orthogonal directions y and z is different from its value relative to the values of its spins in different (mutually) orthogonal directions y' and z'. Given such contextuality, there exist coherent assignments for the values of all the spin quantities that are involved in the Kochen and Specker theorem. The question whether such contextuality is compatible with *Mirror* is rather delicate and go beyond the scope of our current discussion. But, in any case, the above reasoning seems to suggest that the challenges that the Kochen and Specker theorem poses limit the advantage that de Finetti's interpretation of probability may have over other interpretations of probabilities.

16.6 De Finetti on the Nature of Quantum Probabilities

De Finetti found QM both fascinating and challenging. He dedicated a substantial part of the long appendix of his *Theory of Probability* to the analysis of QM probabilities [46, pp. 302–333]. Unlike his analysis of the foundations of probability, the discussion of the nature of QM probabilities lacks incisiveness and clarity. De Finetti refers frequently to von Nenumann's [51] *Mathematical Foundations of Quantum Mechanics*, Bodieu's [52] *Theorie dialectique des probabilities* and Reichenbach's [53] *Philosophic Foundations of Quantum Mechanics*. He models his analysis as a simplified version of Bodieu's and Reichenbach's. Like Bodieu, de Finetti believes that quantum probabilities are a special case of a general calculus of probability. Yet, he thinks that Reichenbach presents "the questions most lucidly from the logical and philosophical point of view," and he thus uses Reichenbach's comments as guidelines for developing his own analysis of the QM probabilities. The aim of de Finetti's analysis is "finding the logical constructions which will prove suitable for resolving the difficulties we find ourselves" in trying to interpret QM. He believes that "the correct path is straightforward and simple" and "it is obscured precisely by preconceived ideas about what it is that constitutes a necessary prerequisite for any logic," and the key for resolving the difficulties is to recognize that the logic of events should be three-valued [46, p. 303, 305–9].

Reichenbach presented the three truth-values in reference to observations: E is true if the observation H has given the result E; E is false if the observation H has given the result not-E; and E is indeterminate or meaningless if the observation H has not been made. De Finetti [46, p. 307] thinks that Reichenbach's presentation corresponds to his conditional three-valued events, the only difference being that in his framework the third value is called "void." Following Reichenbach, he seems to favor the view that the third truth value lies between true and false; for "[t]his is, in fact, the requirement that must be satisfied if something is to be called a mathematical structure or, in particular, a logical structure." Yet, later, in his philosophical lectures on probability, he [41, p. 169] explicitly rejects this view when he says that denoting the third truth-value by "$1/2$" instead of "$\emptyset$" "is not appropriate because it somewhat suggests that it is an intermediate value between true and false." This later view of the indeterminate truth-value is more in line with our interpretation of de Finetti, where indeterminate conditional events have no determinate truth-value and accordingly have no probability.

In any case, de Finetti [46, p. 308] thinks that all the logical construction of Reichenbach's three-valued logic "could be expressed in terms of two-valued logic", so as to avoid "creating a number of symbols and names of operations and consequent rules (which are difficult to remember and sort out, and difficult to use without confusion arising). Above all, one avoids creating the tiresome and misleading impression that one deals with mysterious concepts which transcend ordinary logic." De Finetti thinks that the conceptual scheme of the three-valued event, expressed in ordinary binary logic, could account for the quantum puzzles. In particular, he argues that this framework could serve as the basis for understanding the problem of complementarity in QM. He characterizes complementarity in terms of indeterminate three-valued events – two events are complementary if at least one of them "remains certainly indeterminate (but it is not known which…)" [46, p. 311] – and then proceeds to argue that complementary events also arise in classical phenomena though "the most celebrated example is undoubtedly that of complementarity in quantum mechanics." [46, p. 312]

We argued above that de Finetti's theory of probability could serve as a basis for interpretation of the quantum probabilities. Yet, we believe that de Finetti's discussion of QM probabilities and their relationships to classical probabilities does not do justice to the difficulties that are involved in such an endeavor. In particular, de Finetti seems to be unaware of Bell's and Kochen and Specker's theorems and the heavy constraints they impose on assignments of probabilities in the quantum realm.

16.7 Conclusions

De Finetti held that a theory of probability has to express what is inherent in the notion of probability and nothing more. Probability is a rational guide of life under uncertainty. Probabilities are coherent degrees of belief in verifiable events, and the theorems of probability are supposed to follow from the coherence conditions of degrees of belief. Unlike other subjective interpretations, probability is not supposed to be ignorance about objective probabilities. Probability reflects only subjective uncertainty, and its value is purely instrumental. We argued that in de Finetti's instrumental philosophy of probability, coherence embodies a certain kind of verificationism, and accordingly the coherence conditions of degrees of belief in events depend on their verifiability. Indeed, in the context of this philosophy it is difficult to make sense of coherent degrees of beliefs in events that are unverifiable.

We argued that de Finetti's verificationist conception of coherence has important implications. A common view has it that in the subjective interpretation, probabilities are coherent degrees of belief and in principle every event (or proposition about it) may have a probability. In de Finetti's theory, there are many degrees of belief that have no corresponding probability; for degrees of belief in unverifiable events have no coherence conditions, and accordingly no probability. The restriction of probabilities to verifiable events also entails that the coherence conditions of degrees of belief in events that are not jointly verifiable are weaker than the (familiar) coherence conditions that such events would have had, had they been jointly verifiable.

The idea that verifiability is relevant for probability was also highlighted in Pitwosky's [54, 55] discussion of George Boole's [56] "conditions of possible experience." Boole thought of probabilities as relative frequencies in a finite sample, and of the conditions of possible experience as inequalities concerning such probabilities. Pitowsky [55, p. 105] notes that "*none of Boole's conditions of possible experience can ever be violated when all the relative frequencies involved have been measured in a single sample*. The reason is that such a violation entails a logical contradiction ... But sometimes, for various reasons, we may choose or be forced to measure the relative frequencies of (logically connected) events, in several distinct samples. In this case a violation of Boole's conditions may occur."

We proposed that the restriction of probabilities to verifiable events in de Finetti's theory entails that the probability space of these events is "non-classical" (see de Finetti's big-space approach in Sect. 16.4), or that probabilities are represented by multiple, smaller probability spaces, each of which contains events that are jointly verifiable (see de Finetti's many-spaces approach in Sect. 16.4). In either case, the implication is that the inequalities that constrain the probabilities of the values of spin observables in the EPR/B experiment are different from the inequalities that would have obtained had these events been jointly verifiable; and similarly, *mutatis mutandis,* for spin-measurement outcomes. This different probability structure provides followers of de Finetti's theory with some extra flexibility. Thus, for example, their probability assignments for the values of spin observables in "hidden-variables" models for the EPR/B experiment will not be constrained by (Bell/CH – prob) (see Sect. 16.2). Accordingly, they may suppose that the probabilities of spin-measurement outcomes in the EPR/B experiment "mirror" the probabilities of the corresponding spin observables before any measurement occur (*Mirror*) and that the distribution of the values of these spin observables is independent of the measurement settings (*λ-independence*) (see Sect. 16.2), yet their probabilities of spin-measurement outcomes will not be subjected to the (Bell/CH – phys - ψ) or (Bell/CH – phys - ψ - big) (see Sects. 16.2 and 16.5). However, the heavy constrains that Kochen and Specker's and similar theorems impose substantially limit the scope of such advantages (see Sect. 16.5).

Finally, it is noteworthy that in the context of de Finetti's theory of probability it is more difficult to reconstruct Bell's argument for non-locality. First, in this context it is more difficult to relate probabilities to causality, and accordingly it is hard to motivate the violation of *Factorizability* (see Sect. 16.1.1) as a locality condition. Second, it may be impossible to formulate *λ-independence*, another main premise of Bell's theorem; for if probability is interpreted along de Finetti's theory, in some hidden-variables theories the probability of the complete pair-state in the EPR/B experiment will not exist because this state is unverifiable. Whether this is the case will depend on both the nature of the complete pair-state, which varies from one hidden-variables theory to another, and the concept of verifiability one has in mind. Yet, that it is more difficult to reconstruct Bell's argument in the context of such radical subjective theory of probability should not be surprising, as probabilities in this theory are purely subjective and instrumental and accordingly are not supposed to reflect objective facts about the world. In de Finetti's interpretation, quantum probabilities are not supposed to reflect the ontological nature of the quantum realm; they only serve as a guide for policing uncertainty and forming anticipations about events in this realm.

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