Chapter 7
Waveform Optimization for Integrated Radar and Communication Systems Using Meta-Heuristic Algorithms

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Abstract. Integration of multiple functions such as navigation and radar tasks with communication applications has attracted substantial interest in recent years. In this chapter, we therefore focus on the waveform optimization for such integrated systems based on Oppermann sequences. These sequences are defined by a number of parameters that can be chosen to design sequence sets for a wide range of performance characteristics. It will be shown that meta-heuristic algorithms are well-suited to find the optimal parameters for these sequences. The motivation behind the use of biologically inspired heuristic and/or meta-heuristic algorithms is due to their ability to solve large, complex, and dynamic problems.

7.1 Introduction

In recent years, integration of multiple functions such as navigation and radar tasks with communication applications has sparked a number of research initiatives. This includes research on future signals for hybrid receivers for Global Navigation Satellite Systems (GNSS)/communication and others tasks. The many benefits of multi-functionality range from reducing costs and probability of intercept to offering tolerable co-site interference. While navigation and radar applications require waveform designs that offer excellent autocorrelation characteristics, the target for communication applications is on sets of waveforms with minimum crosscorrelation among the sequences in the set. In the former case, typically only a single sequence

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is needed while in the latter case many sequences are required to support access of multiple users to the common transmission medium. As excellent autocorrelation properties come at the expense of crosscorrelation characteristics and vice versa, a related waveform optimization problem has to be posed and solved taking into account these conflicting requirements.

As far as the integration of radar and communication functionalities are concerned, the Office of Naval Research in 1996 launched the Advanced Multifunction Radio Frequency Concept (AMRFC) program [18,53]. This major program was motivated by the lack of integration of radar, communications, and electronic warfare functions which resulted in a significant increase of the number of topside antennas. Furthermore, it was realized that the lack of integration may also cause severe problems with antenna blockage and difficulties with own-ship electromagnetic interference. Also, a large number of antennas puts stress on maintenance resources. The concepts developed within the AMRFC program are centered around suitable broadband RF apertures that can cope with simultaneous operation of multiple functions and as such focuses on the rather expensive radio frequency (RF) front-end. A different approach on the basis of linear frequency modulated (LFM) waveforms, also known as chirps, has been proposed in [49]. In order to enhance the orthogonality among the signals and to support distinct separation of the different functions, it uses up-chirps for the communications component and down-chirps for the radar functionality of the integrated system. In this way, the suggested chirp signals allow for the radar and communication data to be simultaneously transmitted and received using some standard antenna array. Noting the inherent connection of the chirp-based integration concept to spread spectrum techniques, the work of [59, 60] investigated integrated radar and communication systems with the help of bipolar pseudo noise (PN) sequences, namely *m*-sequences [14, 63]. However, one of the severe drawbacks of *m*-sequences with respect to radar applications is their poor Doppler tolerance [32] and related problems of detecting multiple targets. These and related designs such as polyphase Barker sequences are optimized only with respect to the zero Doppler cut of the ambiguity function but produce much higher interference levels in the presence of Doppler shifted waveforms. As for the application to communications, large sets of *m*-sequences that would be needed to support multiple-access of many users have typically rather poor crosscorrelation properties [63]. As a consequence, they are generally only used as components of more complex designs such as Gold sequences. On the other hand, the large advances in modern integrated circuit technologies would facilitate an efficient implementation of more advanced sequence designs such as complex-valued sequences. Clearly, efficient optimization methods are needed to find suitable waveform and sequence designs for different applications.

Over the last few decades, researchers around the world have developed a vast number of algorithms to solve different optimization problems. Many of these algorithms are based on numerical linear and non-linear programming methods. As a result, the related algorithms require substantial gradient information and try to improve the solution in the proximity of an initial starting point. As a consequence, these methods provide useful strategies to find the global optimum for rather ideal and simple models. However, if the objective function and constraints have multiple or sharp peaks, these methods tend to become unstable. Most of the real world problems turn out to be too complex and difficult to solve using numerical based optimization methods as these tend to fail or are even unable to solve them. There exist also several direct search approaches which use no gradient information such as the Hooke and Jeeves method [17], Nelder-Mead simplex method [44], the Rosenbrock method [50], and the Powell method [47]. Common to these methods is that they take some basic approach of heading downhill from an arbitrary starting point but differ in deciding in which direction to and how far to move. Accordingly, the final outcome depends somewhat on the initial guess of the starting point. This would not be a major shortcoming if the parameter space is well behaved, i.e. if it contains a single, well-defined minimum. However, if the parameter space contains many local minima, as may be the case in waveform optimization, it can be more difficult for such traditional approaches to find the global minimum. In contrast to population based algorithms, these direct searches cannot explore the search space effectively in different directions simultaneously. Successive improvements can be made to speed up the downhill movement of the algorithms but this does not improve the algorithms ability to find the global minimum instead of converging to a local minimum.

The drawbacks of numerical methods motivated researchers to adopt ideas from nature and to translate them to solve problems in engineering sciences. This has led to the inception of many biologically inspired heuristic or meta-heuristic algorithms to solve challenging optimization problems. The word "meta" means beyond or higher and "heuristic" means to find or to discover by trial and error. These methods have proven to be efficient in handling computationally complex problems. They aim at defining effective general purpose methods to explore the solution space and avoid tailoring them to a specific problem. Due to their general purpose nature, they can be applied to a wide range of problems. Meta-heuristic algorithms are also referred to as black-box algorithms as they exploit limited knowledge about the problem to be solved. As no gradient or Hessian matrix information is required for their operation, they are also referred to as derivative-free or zero-order algorithms [5]. The term zero-order implies that only the function values are used to establish the search vector. Moreover, the function to be optimized does not necessarily have to be continuous or differentiable and may also be accompanied by a set of constraints. The choice of method for solving a particular problem depends primarily on the type and characteristics of the problem at hand. It must be stressed that the goal of a particular method used is to find the "best" solution of some sort to a problem compared to finding the optimal solution. In this context, the term "best" refers to an acceptable or satisfactory solution to the problem. This could be the absolute best solution from a set of candidate solutions or may be any of the candidate solutions. The requirements and characteristics of the problem determine if the overall best solution can be found [10, 54].

Nature has an evolution span of millions or even billions of years. In all these years, it has mastered the art of finding a perfect solution to almost all the problems it has been confronted with. As mentioned above, the development of nature inspired optimization algorithms has been an area of active research during recent years and resulted in many approaches such as genetic algorithms (GA), ant colony optimization (ACO), bee algorithms (BA), artificial bee algorithms (ABC), particle swarm optimization (PSO), simulated annealing (SA), harmony search (HS), firefly algorithms (FA), and artificial immune systems (AIS). The interested reader may be referred to [4, 10, 15, 31, 51, 56, 62] and the reference therein for more details and discussions on these topics.

Given the vast amount of available optimization methods, their application in waveform design also stretches from simple searches over more sophisticated and computational demanding realizations to the use of meta-heuristic algorithms. A simple computer search has been used in [45] to obtained sets of sequences with various combinations of sequence parameters. In [58], the optimization of orthogonal polyphase spreading sequences for wireless data applications is reported. It uses a built-in standard 'fmin' function provided in the numerical computing environment MATLAB. In particular, the related functions support multidimensional unconstrained nonlinear minimization including the Nelder-Mead direct search method. As the utilized cost functions in terms of average mean-square autocorrelation and crosscorrelation are very irregular and may have several local minima, the authors report the dependency of the optimization outcome on the starting point and corresponding convergence to different local minima. A similar optimization problem for complex-valued spreading sequences has been investigated in [9] using a global optimization method based on a modified bridging method. In order to solve the related complex optimization problem having a non-linear cost function and a non-linear constraint, a bridged function is used in the search for the global minimum such that the algorithm does not get stuck in a local minimum. Given that cost functions in waveform optimization are often highly irregular with many local minima or are even discontinuous, evolutionary algorithms have gained increased attention in the design of waveforms with respect to communication and radar applications. An evolutionary approach for designing complex spreading codes for direct sequence code-division multiple-access (DS-CDMA) systems has been proposed in [42, 43]. In particular, a multi-objective evolutionary approach is used to search for solutions that satisfy simultaneous objectives posed on autocorrelation and crosscorrelation properties. This approach turned out to be beneficial in the communications field for designing large number of spreading sequence sets with a wide range of correlation properties. In [7], genetic algorithms have been used to design PN sequence families with bounded correlation properties. It is claimed that this approach can produce sequences of any length and superior performance compared to the well-known Gold sequences. A number of recent works has also been reported for the use of evolutional algorithms in the field of radar applications. In [2], an evolutionary algorithm is applied to determine a suite of optimal waveforms to simultaneously perform different surveillance missions such as ground moving target indication, airborne moving target indication, and synthetic aperture radar. The authors have shown that evolutionary algorithms are well suited to design optimal waveforms for multi-mission objectives such as peak sidelobe levels, integrated sidelobe levels, pulse integration, and revisit time. The work reported in [38] used meta-heuristic algorithms to optimize waveforms with sparse spectrum for radar applications in the high frequency band. In particular, a genetic algorithm and particle swarm optimization are used to produce optimal waveforms with acceptable autocorrelation sidelobes. It is concluded that the particle swarm optimization is simpler and faster than the genetic algorithm. They are of the opinion that computational efficiency of particle swarm optimization is comparable or would be even better than the adaptive method of [40].

In view of the above, this chapter considers integrated radar and communication systems based on waveforms known as polyphase sequences. In order to account for the waveform design challenges associated with such integrated systems, we have compared performance and potential application scenarios of different classes of polyphase pulse compression sequences in our earlier studies reported in [25, 26]. Specifically, Oppermann sequences have been revealed in these studies to potentially better support the considered integration as these allow for the design not only of families with a wide range of correlations but also support a variety of characteristics with respect to the ambiguity function, i.e. delay-Doppler tolerance. These sequences provide a number of parameters that can be chosen to design sequences for a wide range of performance characteristics. It will be shown that meta-heuristic algorithms are well-suited to find the optimal parameters for these sequences. Numerical results will be provided for optimal Oppermann sequences obtained with meta-heuristic algorithms.

The rest of this chapter is organized as follows. In Section 7.2, an overview of meta-heuristic algorithms is presented. A brief discussion of polyphase sequences and the definition of Oppermann sequences is provided in Section 7.3. In Section 7.4, performance measures are introduced. Numerical examples are given in Section 7.5. In Section 7.6, conclusions are drawn.

7.2 Meta-Heuristic Algorithms

Meta-heuristic algorithms, also referred to as meta-heuristics for brevity, belong to a branch of stochastic optimization. They are utilized by both engineers and scientists wishing to optimize solutions to problems that are intractable by conventional methods. Meta-heuristic methods consist of two major components known as randomization and selection of the best solutions. The first component avoids that an algorithm gets trapped in a local optimum but also increases the diversity of the potential solutions while the latter component ensures convergence towards the optimal value [10, 61, 62]. A good combination of these two components usually ensures that the global optimum is achievable. The popularity of these algorithms stems from their ability to solve large, complex and dynamic problems. The efficiency of these algorithms or solutions they provide is a measure of their ability to reach an acceptable solution within a reasonable time frame.

The applications of meta-heuristics are broad, versatile and diverse. Application areas include controller design, applied mathematics, power systems, physics, data mining, fuzzy systems and many others. In this chapter, we will apply some of these algorithms to pseudo random signal processing with focus on waveform design for integrated radar and communication systems. For this purpose, meta-heuristic algorithms may be classified as being either population-based or flight/trajectory-based. Genetic algorithms, for example, can be classified as a population-based method while particle swarm optimization utilizes multiple particles to reach the optimal solution. On the other hand, simulated annealing uses a single solution that moves through the search space or design space in a piecewise manner. The essence of the algorithm is always to accept a better solution, whereas a not-so-good solution is accepted with certain probability. In the sequel, selected state-of-the-art zero order and meta-heuristic algorithms are presented.

7.2.1 Particle Swarm Optimization

The PSO is a population-based stochastic optimization technique which has been inspired by social behavior of a flock of birds, school of fishes and swarm of bees as proposed by Eberhart and Kennedy [30]. Since its inception, there have now as many as about 20 different variants of PSO been proposed while remaining still an active area of research. It shares many similarities with genetic and virtual ant algorithms including concepts such as population initialization with random solutions and search for a global optimum solution in successive generations. However, the evolution operators like mutation and crossover as well as encoding or decoding of the parameters into binary strings are not used with PSO algorithms. Instead, it uses a real-number randomness and global communication among the swarm population. Accordingly, each member in the swarm adapts its search patterns by learning from its own experiences of the other members. A member in the swarm is referred to as a particle and represents a potential solution which is a point in the search space. The global optimum is regarded as the location of food [37]. Each particle has a fitness value and a velocity to adjust its flying direction by learning from the best experiences of the swarm to search for the global optimum in the D-dimensional solution space. In our case, the dimension D of the problem is given by the number of parameters that are available for optimization for a given class of sequences. In order to avoid haphazard movements of the particles in the search space, upper and lower bounds are usually specified on the velocity. If the velocity v falls below the specified lower bound, it is set to v_{min} as a measure to prevent in-sufficient exploration of the search space. On the other hand, if the velocity exceeds the specified upper bound, it is set to v_{max} in order to avoid particles moving away from or past a good solution. Similarly, the actual search range for a D-dimensional problem is usually also constrained to a given interval $[c_{min}, c_{max}]^D$, in order to restrain the particles moving on the search boundary.

The standard PSO uses both the personal best, *pbest*, with respect to the location achieved by an individual particle and the global best, *gbest*, referring to the best solution/location among all particles in the swarm [10, 30]. The concept of personal best is primarily used to increase the diversity in finding a solution and to avoid pulling all the particles to the global best. This may cause the algorithm to converge prematurely without finding the overall best solution. However, such diversity can also be simulated by using some kind of randomness [61, 62]. Based on this observation, [62] argues that there is no need to use the personal best, unless the optimization problem is highly nonlinear and multi-modal. This version of the PSO is known as accelerated PSO (APSO) [61, 62].

7.2.2 Harmony Search

A new type of heuristic optimization algorithms known as harmony search (HS) was developed by Lee and Geem [31]. It formalizes the musician improvisation process, i.e. inventing music while performing, into a quantitative optimization process. It comprises of the following parts: (1) Usage of harmony; (2) pitch adjustment; and (3) randomization. In an HS algorithm, each musician (decision variable) plays (generates) a note (value) for finding a best harmony (global optimum). In other words, a harmony translates to an optimization solution vector and the musician's improvisation corresponds to local and global search schemes in terms of optimization. Solutions of the optimization process correspond to a musician while the harmony of the notes generated by a musician corresponds to the fitness of the solution. The pitch adjustment rate $r_{pa} \in [0.1, 0.5]$ and so-called harmony memory $r_{accept} \in [0.7, 0.95]$ ensure that the best harmonies established at some point will be carried over to a new harmony memory. For a detailed discussion on harmony search, the interested reader is referred to [31, 61, 62] and the references therein.

7.2.3 Adaptive Simulated Annealing

The classical SA algorithm [10, 54, 61, 62] relies on the Boltzmann sampling distribution. It comprises of components such as the probability density function of the state space $g(\gamma)$ with γ being the current solution, an acceptance probability function $h(\Delta E)$ with respect to the difference in system energy ΔE between two design vectors, and an annealing schedule for temperature $T(k)$ with annealing time k using Boltzmann annealing. An enhanced version of the classical SA known as adaptive SA (ASA) has been proposed in [20, 21, 22, 23] including comparisons, test case studies and applications. In contrast to SA, the annealing schedule for temperature $T(k)$ decreases exponentially in annealing time k. In addition, re-annealing and quenching is introduced with ASA that allows for adaptation to changing sensitivities in multidimensional parameter spaces.

7.2.4 Artificial Bee Colony Algorithm

The ABC algorithm was proposed by Karaboga [27] in 2005. It simulates the foraging behavior associated with bee colonies. A colony of honey bees can extend itself over long distances, sometimes more than 10 kilometers and in multiple directions simultaneously to exploit a large number of food sources. In a bee colony, tasks are divided among the specialized individuals or bees, namely employed, onlooker and scout bees. The population in a bee colony is divided into two halves. The first half of the population is comprised of employed bees while the second half includes the onlooker bees. The foraging process begins in a colony by scout bees being sent to search for promising food sources. Scout bees move from one food source to another in a random fashion. Employed bees perform duties of exploiting the possible food sources and passing on the information about the quality of the food source to the onlookers bee. The decision taken by onlooker bees to exploit a potential food source depends on the information provided by the employed bees. ABC algorithms have been used to solve both unconstrained and constrained optimization problems [3,27,28,29]. It requires only a few control parameters such as the colony size and maximum number of cycles [29].

7.2.5 *Preliminaries for Waveform Design*

From this point onwards, we will consider two-dimensional optimization problems unless otherwise specified. In the context of waveform design using Oppermann sequences, the term swarm in APSO, harmonies in HS, bees in ABC and candidate points in ASA relate to the parameters m and n which define a specific sequence family. In all these algorithms, the control parameters are defined in the initialization phase. Initially, all the algorithms start with a population randomly distributed except for ASA, which starts with the initial guess in the search space. In each step of the algorithms, there is always a solution or a set of solutions, representing the current state of the algorithm. These solutions are used to generate phases of the Oppermann sequences (see Section 7.3). In order to distinguish good waveform designs from inferior designs, waveform characteristics such as aperiod correlations, figure of merit, and integrated sidelobe measures are computed. The interested reader can find pseudo code of HS in [62], ASA in [52], and ABC in [27] while details of the APSO can be found in [61,62].

7.3 Polyphase Sequences and Their Applications

The history of complex-valued sequences ranges back as far as the 1950s when polyphase sequences where considered in many research laboratories. As the related research outcomes were reported mainly in classified documents with limited access, a broader audience was first reached with the work in [16] on phase shift pulse sequences. In the following decades, many complex-valued sequences have been proposed and analyzed with their applications ranging from radar systems to spread-spectrum communication systems. In particular, polyphase sequences have gained increased attention due to their ability to match regular phase shift keying modulation schemes. In addition, the advances in integrated circuit technologies have paved the way for moving from simple binary sequences to implementations of complex-valued sequences and related more involved pseudo random signal processing. In the sequel, we consider polyphase sequences and will shed some light on their potential to serve in integrated radar and communication systems. In particular, the family of Oppermann sequences [45] are considered in more detail as they offer the system designer large sets of sequences with a wide range of correlation properties compared to other classes of polyphase sequences.

7.3.1 Polyphase Sequences for Radar Systems

Pseudo random sequences and the related signal processing have emerged from space and military applications. In this context, the concept of pulse compression, i.e. expanded pulses with large time-bandwidth products, has been utilized in radar systems. This type of signals offer high range resolution as they can obtain high pulse energy and large pulse width. As an alternative to frequency-modulated signals, pulse compression sequences have been subject of many studies [14, 32]. Polyphase sequences are known to have better Doppler tolerance for a broader range-Doppler coverage than binary sequences [8, 32, 41, 46]. These sequences can be derived from the phase history of chirp or step chirp analog signals and can be processed digitally [36]. In radar applications, the performance of different polyphase sequences can be compared in terms of delay or range tolerance using measures such as the autocorrelation function, mainlobe-to-total-sidelobe ratio and peak-to-sidelobe ratio. The sensitivity of a particular waveform design towards Doppler shifts in case of moving targets can be characterized by using the ambiguity function. As there exist no analytical method that would allow for synthesizing the desired waveform given its desired ambiguity function, more practical optimization approaches are needed to facilitate such designs. For example, the design of a particular radar waveform may be first aiming for optimization of autocorrelation properties with respect to range characteristics followed by evaluating the ambiguity function to identify the Doppler tolerance of the deduced sequence.

As far as radar applications are concerned, Frank sequences [12] were the first polyphase sequences used in pulse compression radar [46]. They can only be designed for perfect square lengths, therefore, they have limited family size. Later in [34] modified versions of Frank sequences were obtained by permuting their phase history. The modified versions are referred to as P1 and P2 sequences. Rapajic and Kennedy in [48] proposed a new class of sequences, known as Px sequences. These sequences have superior performance in terms of integrated sidelobe levels compared to Frank, P1, and P2 sequences. However, for even square root sequence lengths, their performance is the same as for P2 sequences. In [35], the families of P3 and P4 sequences were proposed that can be constructed for any length. The authors of [6, 13] generalized the ideas behind Frank sequences resulting in Frank-Zadoff-Chu (FZC) sequences which can also be designed for any length. Several performance aspects of the aforementioned classes of polyphase sequences with respect to radar applications have been discussed in literature [34, 36, 48].

7.3.2 Polyphase Sequences for Communication Systems

A major boost for the application of pseudo random sequences in the field of communication systems was given by the development of cellular mobile communication systems and spread-spectrum based radios for indoor communication. In particular, the CDMA system for digital cellular phone applications by Qualcomm Incorporated and the family of IEEE802.11 standards for wireless local area networks (WLANs) has taken the theoretical concepts into practical systems. The main classes of sequences used with these systems are Walsh-Hadamard sequences [11, 55], m-sequences [11, 63], Barker codes [11, 63], and complementary code keying based modulation [19]. Subsequently, with the advent of the third generation of mobile communication systems, more advanced spread-spectrum techniques such as orthogonal variable spreading factor sequences [1] and complex-valued short scrambling sequences have been utilized. In contrast to radar applications where it is usual sufficient to have a single sequence with good autocorrelation characteristics, communication systems require a set of sequences to facilitate simultaneous channel access to a number of users. Clearly, minimum crosscorrelation among the sequences is a major design consideration in this case. Given the large advances in modern integrated circuit technologies, it has become feasible to implement complex-valued sequence designs including polyphase sequences such as Frank sequences, FZC sequences, and Oppermann sequences.

7.3.3 Application of Oppermann Sequences for Integrated Radar and Communication Systems

Given the insights from the brief overview on polyphase sequences from the viewpoint of radar and communication applications, it can be concluded that more flexible waveform designs are needed to address the conflicting objectives of these two applications. Our earlier research [25, 26] on this topic has revealed that Oppermann sequences may serve favorable in such integrated radar and communication systems compared to conventional waveform designs. This is mainly due to the fact that families of Oppermann sequences can be designed for a wide range of correlation properties. For any given sequence length, Oppermann sequences are defined by three parameters. These parameters can be used in an optimization process to control the progression of the autocorrelation function, crosscorrelation function, the power spectral density and characteristics of the ambiguity function. Due to space limitations, however, we will concentrate here on range (autocorrelation) and multiple access (crosscorrelation) characteristics. On the other hand, inclusion of moving targets and the related Doppler shifts into the framework of meta-heuristic algorithms may be addressed in our future research considering ambiguity and cross-ambiguity functions.

In this chapter, we consider weighted pulse trains that can be described by a complex envelope as

$$U_x(t) = \frac{1}{\sqrt{T}} \sum_{i=0}^{N-1} u_x(i)\,\mathrm{rect}\left(\frac{t-iT_c}{T_w}\right) \tag{7.1}$$

where $T = NT_c$ is the duration of the xth pulse train while T_c and $T_w \leq T_c$, respectively, denote the repetition period and the width of each rectangular pulse

$$\mathrm{rect}\left(\frac{t}{T_w}\right) = \begin{cases} 1 \text{ for } -\frac{T_w}{2} \leq t \leq \frac{T_w}{2} \\ 0 \text{ otherwise} \end{cases} \tag{7.2}$$

The elements $u_x(i)$, $i = 0, 1, \ldots, N-1$, of the xth complex-valued sequence $\mathbf{u}_x$ of length N represent the weights of the pulse train in (7.1). In general, these elements are given for a polyphase sequence as

$$u_x(i) = \exp\left[j\varphi_x(i)\right], \quad j = \sqrt{-1} \tag{7.3}$$

where the set of N phases $\{\varphi_x(0), \varphi_x(1), \ldots, \varphi_x(N-1)\}$ are referred to as phase sequence. In particular, the phase $\varphi_x(i)$ of the ith element $u_x(i)$ of the xth Oppermann sequence $\mathbf{u}_x = [u_x(0), u_x(1), \ldots, u_x(N-1)]$ of length N taken from a family or set $\mathscr{U}$ of sequences is given as

$$\varphi_x(i) = \frac{\pi}{N}\left[x^m(i+1)^p + (i+1)^n + x(i+1)N\right] \tag{7.4}$$

where $1 \leq x \leq N-1$, $0 \leq i \leq N-1$ and integers i are relatively prime to the length N. The maximum size of a family $\mathscr{U}$ of Oppermann sequences is obtained as $N-1$ when the length N of the sequences is a prime number. A particular family of Oppermann sequences is defined by the real-valued parameters m, n, and p. All the sequences in a family have the same magnitude of the autocorrelation function for a fixed combination of these three parameters. In [45], it has been shown that the magnitude of the autocorrelation function depends only on the parameter n if the parameter $p = 1$. For this case, the autocorrelation magnitude follows the expression

$$|C_x(l)| = \left|\frac{1}{N} \sum_{i=0}^{N-1-l} \exp\left\{\frac{j\pi}{N}\left[(i+1)^n - (i+l+1)^n\right]\right\}\right| \tag{7.5}$$

In the sequel, we therefore focus on the case of $p = 1$ which leaves us with m and n as free parameters for use in an optimized waveform design.

Due to the general definition of Oppermann sequences, they include some more specific sequences. For example, for the parameters $m = 2$, $n = -\infty$, $p = 1$, FZC sequences can be generated. As such, application of the considered meta-heuristic algorithms to these more specific sequences is straightforward.

7.4 Performance Measures

In the following sections, the definitions of the measures used in the performance comparison of the considered Oppermann sequences will be given. Specifically, let an Oppermann sequence of length N be denoted as $\mathbf{u}_x = [u_x(0), u_x(1), \ldots, u_x(N-1)]$ where subscript $1 \leq x \leq U$ relates to the xth sequence $\mathbf{u}_x$ taken from a given set $\mathscr{U}$ of size U.

7.4.1 Aperiodic Correlation Measures

In order to quantify the degree of similarity between different sequences from a given set or between a given sequence and a shifted version of it, respectively, autocorrelation and crosscorrelation measures are usual considered. In many fields, aperiodic signals need to be processed which occur only once within a considerable time span and appear to the application as more or less singular events. Accordingly, the aperiodic crosscorrelation (ACC) between two complex-valued sequences $\mathbf{u}_x = [u_x(0), u_x(1), \ldots, u_x(N-1)]$ and $\mathbf{u}_y = [u_y(0), u_y(1), \ldots, u_y(N-1)]$ of length N at discrete shift l is given as [11, 63]

$$C_{xy}(l) = \begin{cases} \frac{1}{N} \sum_{i=0}^{N-1-l} u_x(i) u_y^*(i+l), & 0 \leq l \leq N-1 \\ \frac{1}{N} \sum_{i=0}^{N-1+l} u_x(i-l) u_y^*(i), & 1-N \leq l < 0 \\ 0, & |l| \geq N \end{cases} \tag{7.6}$$

where $(\cdot)^*$ denotes the complex conjugate of the argument $(\cdot)$. In case of $\mathbf{u}_x = \mathbf{u}_y$, (7.6) is referred to as aperiodic autocorrelation (AAC) and is denoted as $C_x(l) = C_{xx}(l)$.

In addition to ACC and AAC, it is often more realistic to incorporate the whole range of possible correlation values into the performance evaluation of a given set of sequences rather than considering only peak values of aperiodic correlations. In this context, mean-square values from AAC and ACC may be used in favor of worst case scenarios. For this purpose, let us introduce the mean-square out-of-phase autocorrelation (MSAC), R_{ac}, and mean-square crosscorrelation (MSCC), R_{cc}, respectively, of a given set $\mathscr{U}$ of size U as

$$R_{ac} = \frac{1}{U} \sum_{x=1}^{U} \sum_{\substack{l=1-N \\ l \neq 0}}^{N-1} |C_x(l)|^2 \tag{7.7}$$

$$R_{cc} = \frac{1}{U(U-1)} \sum_{x=1}^{U} \sum_{\substack{y=1 \\ y \neq x}}^{U} \sum_{l=1-N}^{N-1} \left|C_{xy}(l)\right|^2 \tag{7.8}$$

7.4.2 Sidelobe Measures

The figure of merit (FOM) of a sequence $\mathbf{u}_x \in \mathscr{U}$, $1 \leq x \leq U$ of length N with aperiodic autocorrelation function $C_x(l)$ measures the ratio of energy in the mainlobe to the energy in the sidelobe of the autocorrelation function. It is defined as

$$FOM_x = \frac{C_x(0)}{2\sum_{l=1}^{N-1} |C_x(l)|^2}, \quad \forall x \tag{7.9}$$

Alternatively, the integrated sidelobe level (ISL) is often used for radar applications in the context of distributed target environments. The ISL of a sequence $\mathbf{u}_x \in \mathscr{U}$, $1 \leq x \leq U$ of length N is defined as

$$ISL_x = \frac{1}{FOM_x}, \quad \forall x \tag{7.10}$$

Another important measure in relation to radar applications is the peak-to-sidelobe ratio (PSLR) which relates to the ability of detecting targets without masking interfering targets. For example, if an AAC has large sidelobes, it will mask nearby targets and leave them undetected. Specifically, the PSLR of a sequence $\mathbf{u}_x$ measures the ratio of the in-phase value $C_x(0)$ to the maximum sidelobe magnitude $|C_x(l)|$ of the periodic autocorrelation function $C_x(l)$. It is defined as

$$PSLR_x = \frac{C_x(0)}{\max\limits_{1\leq l<N} |C_x(l)|}, \quad \forall x \tag{7.11}$$

7.5 Numerical Examples

In the sequel, some numerical examples are provided to illustrate the application of meta-heuristic algorithms for waveform optimization for integrated radar and communication systems. For this purpose, we consider the class of Oppermann sequences as defined in (7.4) of length $N = 31$. It is noted that the maximum number of $N - 1 = 30$ sequences in the designed set is obtained as N is chosen as a prime number. Furthermore, the considered sequence family offers parameters m and n for optimization given the case of parameter $p = 1$. Accordingly, the following optimization problems may be posed:

$$P1: \quad \min_{n\in[n_1,n_2]} ISL(\mathscr{U}) \tag{7.12}$$

$$P2: \quad \max_{n\in[n_1,n_2]} PSLR(\mathscr{U}) \tag{7.13}$$

$$P3: \quad \min_{m\in[m_1,m_2],n\in[n_1,n_2]} [R_{ac}(\mathscr{U}) + \alpha R_{cc}(\mathscr{U})] \tag{7.14}$$

where $m \in [m_1, m_2]$ and $n \in [n_1, n_2]$ are the search regions for m and n, respectively, and α is a weighting factor. While problems $P1$ and $P2$ given in (7.12) and (7.13), respectively, relate strongly to radar applications, problem $P3$ formulated in (7.14) can be used to find a trade-off between conflicting objectives of radar and communication applications. Especially, the weighting factor α may be chosen with respect to desirable system specifications. In contrast to [25], where we have used a two-step approach to first optimize autocorrelation properties by a simple brute-force search over parameter n followed by tuning m towards favorable delay-Doppler properties, we consider here two-dimensional optimization to simultaneously find the optimal values of n and m for problem $P3$. On the other hand, in view of the independence of the autocorrelation of Oppermann sequences on parameter m as shown in (7.5), problems $P1$ and $P2$ remain one-dimensional as PSLR and ISL only involve the aperiodic autocorrelation.

In order to solve the problems formulated in (7.12)-(7.14), we use APSO, HS ASA and ABC. The two-dimensional search space was constrained to the interval $m \in [0,4]$ and $n \in [0,4]$. The algorithms were executed on a laptop computer with Intel Pentium M 740 Processor running at 1.73 GHz and 2048 Megabytes of RAM. With the exception of ASA, where we used a C-routine called from MATLAB, all the other algorithms have been implemented in MATLAB. As for the translation of the notions from meta-heuristics to the optimization problem at hand, the following interpretation can be given.

- **APSO:** Initially, particles in a swarm are randomly distributed in a D-dimensional search space. In APSO, the parameter D refers to the dimension of the problem, swarm refers to a population, and particle is similar to an individual. Alternatively, each solution (or particle) flies through the search space and looks for an optimal position to land. In terms of Oppermann sequences, particles are represented by the values of m and n in a two-dimensional search space and are used to generate the phases of Oppermann sequences as defined in (7.4). The search for the optimal landing position, i.e. finding optimal values of m and n will continue until the criteria selected from (7.7) to (7.11) are met.
- **HS:** Initially, harmonies are randomly generated in a D-dimensional space and are stored in a harmony memory (HM). The use of HM ensures that the best harmonies will be carried over to the HM. As for the optimization of Oppermann sequences, the parameters m and n are represented by the obtained harmonies to generate phases as defined in (7.4). Then, pitch adjustment is used to control the convergence of the algorithm. Randomization introduced in the algorithm drives the algorithm to search previously unexplored areas in the search space until the criteria selected from (7.7) to (7.11) are met.
- **ASA:** This algorithm starts with the initial guess of the parameters in the D-dimensional search space. In terms of Oppermann sequences, the initial guess represents values of the parameters m and n. Each step of the ASA algorithm replaces the current solution by a random nearby solution. The obtained solutions are used to generate Oppermann sequences. The process of finding optimal values of m and n continues by generating feasible points in the search space and

acceptance probability including annealing and re-annealing temperatures until criteria selected from (7.7) to (7.11) are met.

- **ABC:** It is recalled that food sources are randomly distributed in the *D*-dimensional search space at the start of the search. Here, bees refer to a population of bees (employed, onlookers and scout) which are in the search of the best food position. Employed bees search for new food sources within their neighborhood that have more nectar compared to the food sources they have previously visited. These food sources represent the values of the parameters m and n of Oppermann sequences to generate the phases defined in (7.4). If during the optimization process the criteria set for (7.7) to (7.11) are not met, it will represent abandoned food source or bad sequence designs.. The search for the final food position represent optimal values of m and n that satisfy the criteria set for (7.7) to (7.11).

Figure 7.1 compares the performance of Oppermann sequences obtained through meta-heuristics in terms of PSLR with the brute-force search method with fixed step size reported in [25]. Clearly, the random search strategy employed in meta-heuristics widens the search area allowing the particles to explore the search space more effectively compared to an optimization using fixed step size. As can be seen from the figure, PSLR values can be improved for those prime length that would have inferior performance using brute-force search with fixed increment on n. In this case, meta-heuristic algorithms improve the performance of the designed set of Oppermann sequences to be comparable to other families such as the FZC sequences (see also [25]).

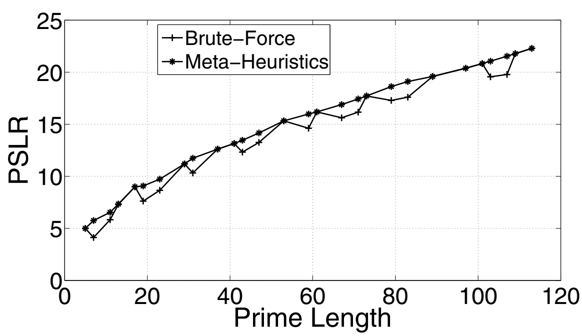

Fig. 7.1 Performance comparisons between brute-force search with fixed increment and meta-heuristic algorithms in terms of PSLR

The convergence behavior of the considered algorithms for the example of optimizing PSLR is illustrated in Fig. 7.2. It can be seen from the progressions in terms of iterations shown in the figure that ASA achieves the fastest convergence to the optimal values followed by APSO, ABC and HS. The fast convergence of ASA may be attributed to the fact that exponential annealing permits the algorithm to adaptively re-anneal and pacing the convergence in the search space in all dimensions. It

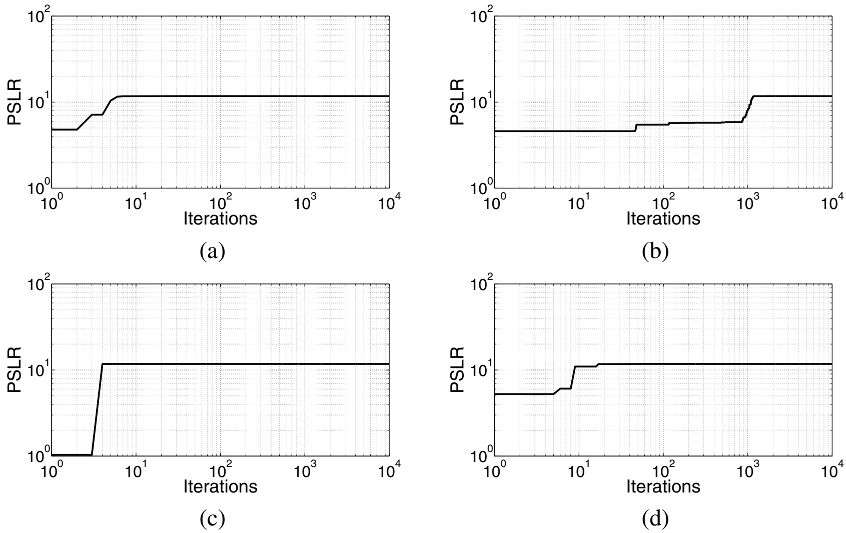

Fig. 7.2 Convergence of different meta-heuristic algorithms towards optimal PSLR: (a) APSO, (b) HS, (c) ASA, (d) ABC

should be mentioned that the similar convergence behavior and ranking among the algorithms can be observed when applied to optimize FOM, ISL, and mean-square aperiodic correlation measures.

Tables 7.1(a)-(e) show numerical results of optimal designs for Oppermann sequences of length $N = 31$ with respect to the optimization problems posed in (7.12), (7.13), and (7.14) using APSO, HS, ASA, ABC. As for the optimal designs presented in Table 7.1(a) and Table 7.1(b) for PSLR and ISL, respectively, it is sufficient to consider only the parameter n as these metrics involve only the AAC (see also (7.10) and (7.11)). It is recalled that according to (7.5), the AAC is independent of the parameter m for the considered case of parameter $p = 1$. Also, all $N-1 = 30$ Oppermann sequences in an optimized set achieve the same PSLR and ISL. Clearly, all considered meta-heuristic algorithms converge towards very similar results for these two classical design objectives of radar systems.

In order to illustrate the trade-off in waveform optimization for integrated radar and communication systems, let us focus now on the results presented in Tables 7.1(c)-(e) with respect to the optimization problem posed in (7.14). In particular, we have chosen $\alpha = 0$ relating to radar systems, $\alpha = 60$ emphasizing on communication systems, and $\alpha = 1$ as an example of an integrated radar and communication scenario. Clearly, the autocorrelation properties indicated by the small R_{ac} values in Table 7.1(c) are beneficial for radar systems and are independent of parameter m. On the other hand, good crosscorrelation characteristics are shown Table 7.1(d) for use with communication systems but these come at the expense of poor autocorrelation properties quantified by high values of R_{ac}. The results of

Table 7.1 Optimal designs for Oppermann sequences of length $N = 31$

(a) Peak-to-sidelobe ratio

Algorithm	n	PSLR
APSO	2.000	11.735
HS	2.000	11.734
ASA	2.000	11.735
ABC	2.000	11.735

(b) Integrated sidelobe level

Algorithm	n	ISL
APSO	2.007	0.110
HS	2.007	0.110
ASA	2.000	0.116
ABC	2.007	0.110

(c) MSAC; $\alpha = 0$

Algorithm	m	n	R_{ac}	R_{cc}
APSO	2.597	2.007	0.110	1.000
HS	2.744	2.007	0.110	1.001
ASA	2.000	2.000	0.116	1.000
ABC	0.614	2.007	0.110	1.005

(d) MSCC; $\alpha = 60$

Algorithm	m	n	R_{ac}	R_{cc}
APSO	1.003	1.002	19.676	0.341
HS	1.003	1.000	19.677	0.341
ASA	1.000	1.000	19.677	0.344
ABC	1.003	1.000	19.677	0.341

(e) MSAC+MSCC; $\alpha = 1$

Algorithm	m	n	R_{ac}	R_{cc}
APSO	0.930	2.007	0.110	0.997
HS	1.000	2.007	0.110	0.996
ASA	1.000	2.000	0.116	0.996
ABC	0.999	2.007	0.110	0.996

the trade-off example shown in Table 7.1(e) may perform favorable with integrated radar and communication systems keeping autocorrelation values low and driving crosscorrelation values smaller. An additional increase of α would result in an increase of autocorrlelation values and further reduce crosscorrelation values. Also, all four considered meta-heuristic algorithms provide very similar outcomes to the different optimization problems.

7.6 Conclusions

In this chapter, we have focused on the waveform optimization for integrated radar and communication systems. Given the conflicting requirements on autocorrelation and crosscorrelation characteristics, meta-heuristic algorithms are considered to basically perform a multidimensional optimization. Specifically, the selected class of

Oppermann sequences allows for designing families with a wide range of correlations with respect to a two-dimensional search space. The numerical results illustrate the potential of meta-heuristic algorithms for designing sequences for radar, communications, as well as integrated systems. By way of example with respect to PSLR, it is shown that meta-heuristics can improve performance compared to search methods with fixed increment.

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