Chapter 1
NS–F Equations and Modelling: A French Touch

This Overview is a brief outline of the events related to my rather long "RAM Adventure" during the years 1968–2009. In 1968–1969 my discovery of asymptotics and rational modelling of fluid dynamics problems was, for me, a revelation, and the Rational Asymptotics Modelling (RAM) Approach to these problems, governed by the Navier–Stokes–Fourier (NS–F) equations,[1] has been my main scientific activity during the last 40 years – the systematic, logical and well argued consistent approach via asymptotics, in perfect harmony with my idea about mathematically applied, but not ad hoc, theoretical researches in fluid dynamics, without any modern abstract, sophisticated, functional analysis!

This Overview first presents a short account of my first contribution to RAM in fluid dynamics, related to a justification of Boussinesq equations used in Chap. 1 of the original, version of my doctoral thesis, written in Moscow during 1965–1966. I then relate various events concerning my collaboration with Jean-Pierre Guiraud, working on asymptotic modelling of fluid flows at the Aerodynamics Department of ONERA[2] during the 16 years up to 1986, which resulted in the publication of 26 joint papers in various scientific journals. Finally, a few remarks are presented concerning my preceding seven books (three in French and four in English), published during the years 1986–2009, on modelling in Newtonian fluid flows.

Below we use "Navier" equations in place of "Navier–Stokes incompressible" equations. In fact, as main fluid dynamics equations we have Euler, Navier, and NS–F equations. Concerning the so-called "Navier–Stokes (isentropic)" equations – often used by mathematicians in their rigorous investigations – in reality these NS equations are unable to describe any real fluid flows! Note also that in a RAM

[1] Concerning the term "Navier–Stokes–Fourier" equations used in this book – NS–F equations, governing classical, Newtonian, viscous, compressible and heat-conducting fluid flows – it seems to me that it is better adapted than the term commonly used (mainly by mathematicians), "Navier–Stokes compressible" equations.

[2] Office National d'Études et de Recherches Aérospatiales, Châtillon-92320 (France).

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Approach, the Euler (vanishing viscosity case) and Navier (low compressibility case) equations are, in fact, derived consistently by limiting processes from NS–F full equations – but this is not the case for the NS (isentropic) equations!

Concerning my "Soviet Adventure" of 1947–1966... In 1954 I graduated from Yerevan State University with a Master of Sciences degree in pure mathematics (in the class of Sergey Mergelyan[3]); after which, during 1955–1956, I worked in the Institute of Water and Energy at the Armenian Academy of Sciences in Yerevan. I then had the opportunity for serious study in theoretical fluid dynamics, and in 1957 I chose dynamic meteorology as my main scientific research activity as a Ph.D. student in the Kibel Department of the Hydro-Meteorological Centre in Moscow.

Now, more than 50 years later, I am still proud to have been a student of Il'ya Afanas'evich Kibel[4] – an outstanding hydrodynamicist of the twentieth century who was active and creative throughout his entire career. Unfortunately, his life was too short. He died suddenly, at the age of 66, on 5 September 1970.

Mainly on the basis of my various publications in mesometeorology (linked with the lee waves downstream of a mountain in a baroclinic atmosphere and also with the free atmospheric local circulations above the various Earth sites) during the years 1957–1966, in the Kibel Department of the Hydro-Meteorological Centre in Moscow, in 1968 I had the opportunity to publish my first course in mesometeorology [1] for the engineering students at the École de la Météorologie in Paris.

In September 1966 I returned to Paris to write my thesis [2] on the basis of the results of research (1961–1965) into the lee waves 2D (non-linear) and 3D (linear) steady problems in non-viscous and adiabatic atmospheres, with the help of the Boussinesq approximation. In 1969 I was awarded the degree of Docteur d'État es Sciences Physiques by the University of Paris, which added to my Russian Ph.D. of 1960, from the University of Moscow and my SSSR Academy of Sciences Chief Scientific Research Worker degree in hydrodynamics and dynamic meteorology, obtained in 1964.

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[3] Sergey Nikitovich Mergelyan (1928–2008) was an Armenian scientist – an outstanding mathematician, and the author of major contributions in Approximation Theory (including his well-known theorem in 1951). The modern Complex Approximation Theory was based mainly on his work (see, for instance, the book *Real and Complex Analysis* by W. Rudin; French edition, Masson, Paris, 1978). He graduated from Yerevan State University in 1947, and in 1956 played a leading role in establishing the Yerevan Scientific Research Institute of Mathematical Machines (YerSRIMM). He became the first Director of this Institute, which today many refer to as the "Mergelyan Institute".

[4] Il'ya Afanas'evich Kibel (1904–1970), Member of the SSSR Academy of Sciences, was one of the leading Soviet scientist in the field of theoretical hydromechanics. He is famous as the founder of the hydrodynamic method of weather forecasting, and for implementation of mathematical methods in meteorology. See his pioneer monograph, *An Introduction to the Hydrodynamical Methods of Short Period Weather Forecasting*, published in Russian in Moscow (1957), and translated into English in 1963 (Macmillan, London). Some of his well-known works on the meteo-fluid are published in *Selected Works of I. A. Kibel on Dynamic Meteorology* (in Russian, GydrometeoIzdat, Leningrad, 1984).

In the original version of my thesis (hand-written in Moscow during 1965–1966), in Chap. 1 the Boussinesq approximate equations were derived in an ad hoc manner (*à la* Landau – as in [3, §56]). However, Paul Germain (future juryman during my thesis defence in March 1969) was unfavourable towards this method of deriving Boussinesq approximate equations for convection in fluids, and I was obliged to completely rewrite that chapter! Germain considered that it is possible to derive these Boussinesq equations by an asymptotic rational/consistent process (but by what method?), and in a letter[5] written in Paris and dated 8 March 1968, he wrote that "... I should understand the justification of our starting equations?"

1.1 My First Contribution to the RAM Approach in Fluid Dynamics

This "justification problem" was for me a difficult challenge – 1 year before my 1969 thesis defence – and I was in an awkward situation! For some time I did not fully understand the question in Germain's letter! Finally, however, I chose a bastardized method via the so-called isochoric model equations, when the density ρ is a conservative unknown function along the fluid flow trajectories in time–space (t, **x**), such that

$$D\rho/Dt = 0, \text{ with } D/Dt = \partial/\partial t + \mathbf{u}\cdot\nabla, \tag{1.1}$$

where ∇ denotes the gradient vector and **u** the velocity vector – this constraint being often used in fluid dynamics when gravity plays an active role. This above conservative condition on ρ is, in fact, an incompressible condition. In particular, it is systematically considered in Yih's monograph [4]; and see also the book by Batchelor, [5], p. 75.

For **u**, ρ, temperature T and thermodynamic pressure $p = R\rho T$, where R is the thermally perfect gas constant, when we consider a non-viscous, compressible and adiabatic atmospheric motion, we have the following Euler non-dissipative system of three equations:

[5] Paul Germain wrote to me (in French!): "J'ai pu regarder les feuilles que vous m'avez adressées sur la mise en équation de votre problème. Je prends note du fait que vous ne passez plus par la forme intermédiaire des équations de la convection qui figurait dans les documents que vous m'aviez antérieurement donnés. Je ne suis néanmoins pas satisfait, car je ne vois toujours pas comment est justifiée la cohérence de vos approximations et pourquoi, alors que vous supposer les perturbations de vitesses petites, en particulier la quantité: $u^2 + w^2 - U_\infty{}^2$, afin d'obtenir des équations linéaires, vous ne linéarisez pas les conditions aux limites. Vous devez me trouver un peu 'tâtillon'. Mais si je dois faire partie du jury de votre thèse, c'est à titre de mécanicien des fluides et comme tel, je souhaiterais comprendre le bien fondé des équations de départ. Or depuis votre exposé au séminaire, j'éprouve toujours la même difficulté et les variantes que vous m'avez proposées ne m'éclairent pas."

$$D\rho/Dt + \rho\nabla.\mathbf{u} = 0, \tag{1.2a}$$

$$DS/Dt = 0, \tag{1.2b}$$

$$\rho\, D\mathbf{u}/Dt + \nabla p + \mathbf{g}\rho = 0. \tag{1.2c}$$

For the specific entropy we have the relation:

$$S = Cv \log[p/\rho^{\gamma}], \text{ with } \gamma = Cp/Cv, \tag{1.3}$$

the ratio of specific heats, and in (1.2c) the gravity force (with a measure g) is taken into account

It is well known that an incompressible fluid motion is obtained (from a compressible fluid motion) as a result of the following formal limiting "incompressible process":

$$\mathrm{Lim}_{is} = \gamma \to \infty \text{ with Cp fixed.} \tag{1.4}$$

With (1.4), in place of Eq. 1.2b, according to (1.3), we recover the above mentioned isochoricity condition ($D\rho/Dt = 0$) which leads, from the equation of continuity (1.2a), to the usual incompressibility constraint:

$$\rho = \text{ constant } \Rightarrow \nabla.\mathbf{u} = 0.$$

Finally, in place of the Euler system of Eq. 1.2a–1.2c, with (1.3) – and as a consequence of (1.4) – we derive for the limit isochoric functions, $\mathbf{u}_{is}$, p_{is}, and ρ_{is}, the following simplified isochoric system of inviscid equations:

$$D\rho_{is}/Dt = 0, \quad \nabla.\mathbf{u}_{is} = 0, \tag{1.5a, b}$$

$$\rho_{is} D\mathbf{u}_{is}/Dt + \nabla p_{is} + \mathbf{g}\rho_{is} = \mathbf{0}. \tag{1.5c}$$

From (1.5a, b), $\nabla.\mathbf{u}_{is} = 0$, we have the possibility of introducing two stream functions, ψ and χ (as in [6]), such that in a 3D, steady case, $\partial\mathbf{u}_{is}/\partial t = 0$, $\partial S_{is}/\partial t = 0$, and $\partial\rho_{is}/\partial t = 0$, we obtain the following three relations:

$$\mathbf{u}_{is} = \nabla\psi \wedge \nabla\chi, \tag{1.6a}$$

$$\rho_{is} = \rho^*(\psi,\chi), \tag{1.6b}$$

$$(1/2)\mathbf{u}_{is}{}^2 + (p_{is}/\rho_{is}) + g\, z = I^*(\psi,\chi), \tag{1.6c}$$

where z ($\equiv$ **x. k** is directed above along the unit upward vector **k**) is the altitude, and the two functions, $\rho^*(\psi,\chi)$ and $I^*(\psi,\chi)$, are subject to a determination.

In particular, for the lee-waves problem, over and downstream of a mountain, this determination is performed via the boundary conditions at upstream infinity where, in a simple case, an uniform horizontal flow is assumed given.

From Eqs. 1.6a–1.6c we derive (again, according to [6]) two scalar equations for ψ and χ :

$$(\nabla \wedge \mathbf{u}_{is}).\nabla\psi = \partial I^*/\partial\chi + (p_{is}/\rho*^2)\partial\rho^*/\partial\chi; \quad (1.7a)$$

$$(\nabla \wedge \mathbf{u}_{is}).\nabla\chi = -\partial I^*/\partial\psi - (p_{is}/\rho*^2)\partial\rho^*/\partial\psi. \quad (1.7b)$$

If, now, $\mathbf{u}_{is}{}^{\infty} = U^{\infty}(z_{\infty})\mathbf{i}$ is the speed (along the axis of $x \equiv \mathbf{x}.\,\mathbf{i}$) far upstream of the mountain, which is simulated by the equation $z = \mu h(x)$, then at $x \to -\infty$, with $h\ (-\infty) \equiv 0$, the conditions are:

$$u_{is}{}^{\infty} = U^{\infty}(z_{\infty}),\ v_{is}{}^{\infty} = w_{is}{}^{\infty} = 0\ \rho_{is}{}^{\infty} = \rho^{\infty}(z_{\infty}); \quad (1.8a)$$

$$\psi = -\int_0^{Z\infty} U^{\infty}(z)dz\ = \psi_{\infty}(z_{\infty}), \quad (1.8b)$$

where $\mathbf{u}_{is}{}^{\infty} = (u_{is}{}^{\infty}, v_{is}{}^{\infty}, w_{is}{}^{\infty})$, and z_{∞} being, therefore, the altitude of a stream line in the basic non-disturbed two-dimensional far flow. In this particular, simple case, (1.8a,1.8b), the second stream function at infinity upstream is simply the plane (x, z), and $\chi_{\infty} \equiv y = \text{const}$.

We will suppose also, implicitly, that the solution of the considered lee-waves problem ought to be uniformly bounded at all points of the infinite plane (x, z). We assume also that $\psi = 0$ determines the wall of the mountain, and in a such case:

$$I^* = B(\psi) \text{ and } \rho^* = R(\psi) \quad (1.9a, b)$$

and in place of two Eqs. 1.7a, 1.7b, with the conditions (1.8a,1.8b), we derive a three-dimensional generalization of the 2D equation of Long, considered in his well-known paper [7]:

$$\nabla \wedge [\nabla\psi \wedge \nabla\chi].\nabla\psi = 0, \quad (1.10a)$$

$$\nabla \wedge [\nabla\psi \wedge \nabla\chi].\nabla\chi = -\ U^{\infty}(dU^{\infty}/d\psi) + (1/2)(d\log R/d\psi)(\nabla\psi \wedge \nabla\chi)^2 - (d\log R/d\psi)\{(1/2)U^{\infty^2} + g(z\ -\ z_{\infty})\}. \quad (1.10b)$$

In the case when (far upstream of the mountain):

$$U^{\infty} = (U^{\infty})^0 = \text{ const } \Rightarrow \psi_{\infty}(z_{\infty}) = -(U^{\infty})^0 z_{\infty}, \quad (1.11a)$$

$$\rho^{\infty}(z_{\infty}) = \rho^{\infty}(0)\exp[-\beta z_{\infty}], \tag{1.11b}$$

and if we introduce (see (1.12)) the non-dimensional quantities (H is a characteristic meso-length-scale)

$$\xi = x/H, \zeta = z/H, \Psi = \psi/H(U^{\infty})^{0}, X = \chi/H, \tag{1.12}$$

we obtain, as in our thesis [2], in place of Eq. 1.10b, the following dimensionless equation:

$$\nabla \wedge [\nabla\Psi \wedge \nabla X].\nabla X + \mathcal{D}(\Psi + \zeta) = \lambda[(\nabla\Psi \wedge \nabla X)^{2} - 1], \tag{1.13}$$

where

$$\mathcal{D} = \beta H^{2}[g/(U^{\infty})^{02}] \text{ and } \lambda = \beta(H/2), \tag{1.14a}$$

and we observe that the following relation

$$\lambda/\mathcal{D} = {Fr_{H}}^{2} \tag{1.14b}$$

is true, where ${Fr_H}^2(= (U^{\infty})^{02}/[gH])$ is the square of a Froude number.
But, ${Fr_H}^2 \ll 1$ when $H \gg (U^{\infty})^{02}/g$, and this is indeed the case for the usual meteo data.

The relation (1.14b) shows that the term proportional to λ, in the main Eq. 1.13 must be small ($\lambda \ll 1$), because it is necessary (in (1.13)) that

$$\mathcal{D} = \lambda/{Fr_{H}}^{2} = O(1), \tag{1.15}$$

as a ratio of two small parameters.

The parameter $\mathcal{D}$, being the main lee-waves parameter is the so-called Dorodnitsyn–Scorer parameter.

The relation (1.15) is, in fact, a similarity rule between two small parameters: λ and ${Fr_H}^2$ – the use of (1.15) being a key step in the derivation of our leading-order consistent Eq. 1.17 below.

Rigorously, the term proportional to λ can be neglected, in a first approximation, relative to the term with $\mathcal{D}$, which is assumed O(1), only when

$$\beta \ll 2/H, \tag{1.16}$$

and, in such a case, in leading-order approximate model Eq. 1.17, with subscript ‘$_B$’:

$$\nabla \wedge [\nabla\Psi_{B} \wedge \nabla X_{B}].\nabla X_{B} + \mathcal{D}(\Psi_{B} + \zeta) = 0, \tag{1.17}$$

where for X_B we have as the equation (according to (1.10a)):

$$\nabla \wedge [\nabla \Psi_B \wedge \nabla X_B].\nabla \Psi_B = 0, \tag{1.18}$$

The effect of the compressibility is present only in the last term of (1.17) proportional to $\mathcal{D}$.

This above approximation is just the well-known Boussinesq approximation of 1903 [8]: "The derivatives of $\rho^{\infty}(z_{\infty})$ can be neglected except when they intervene in the calculation of the force of Archimedes."

In particular, if we assume (2D case) that:

$$X_B \equiv \eta(= y/H) \text{ and } \Psi_B \equiv \psi_p(\xi, \zeta), \tag{1.19}$$

we derive, from (1.17), a linear Helmholtz (*à la* Long [7]) equation:

$$\partial^2\psi_p/\partial\xi^2 + \partial^2\psi_p/\partial\zeta^2 + \mathcal{D}(\psi_p + \zeta) = 0. \tag{1.20}$$

But, if (1.20) is a linear equation (derived without any linearization!) the slip boundary condition, along the wall of our mountain, remains non-linear – the slip condition being down a curvilinear surface of the mountain,

$$\psi_p(\xi, \zeta = \kappa\, h^*(\xi)) = 0, \tag{1.21a}$$

with

$$\kappa = \mu/H \tag{1.21b}$$

and

$$h^*(\xi) \equiv h(H\xi). \tag{1.21c}$$

The above results are, in fact, the main part of my first theoretical contribution to the RAM Approach in fluid dynamics, obtained during the rewriting of my Doctoral thesis in Paris during 1968–1969.

I do not see, in reality, whether Paul Germain was completely satisfied with my new derivation. But however that may be, my efforts in writing a new Chap. 2 for my thesis were successful, and on 10 March 1969, after the defence of this thesis in the Faculty of Sciences of the University of Paris, I obtained the degree of Docteur d'État es Sciences Physiques – Paul Germain and Jean-Paul Guiraud being members of my thesis jury, with, as President of the Jury, Paul Queney,[6] Professor at the Sorbonne.

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[6] The first theoretical investigations concerning 3D lee-waves problems in linear approximation was, in fact, carried out by Paul Queney. On the other hand, an excellent synthesis of theoretical developments on relief (lee) waves will be found in WMO Technical Note: "The Air flow over mountains", N° **34**, Geneva, 1960, by P. Queney et al.

In the last chapter of this book (in Sect. 9.1) the reader can find a more elaborate RAM Approach to fluid dynamics, for the 2D steady lee-waves problem, in the framework of low-Mach-number fluid flow (hyposonic) theory, which leads to a family of consistent, limiting leading-order model equations.

Concerning the full justification of the Boussinesq assertion and a satisfactory answer to Paul Germain's question – this justification of the Boussinesq approximate equations was for 5 years a major challenge for me, and I devoted considerable effort to the resolution of this problem.

Only in 1973, in the framework of low-Mach-number asymptotics, taking into account the existence of a hydrostatic reference state (function only of the altitude), did I well understand the way for a consistent non-contradictory RAM Approach.

In 1974 [9] these Boussinesq approximate equations for a viscous and non-adiabatic dissipative atmospheric motions were derived from the full unsteady NS–F dissipative equations.

To describe the atmospheric motions, which represent the departure from the hydrostatic reference state, I have considered the perturbations of pressure, density and temperature (these atmospheric perturbations being usually very small, relative to the hydrostatic reference state) and have rewritten (without any simplifications) the NS–F atmospheric equations relative to these thermodynamic perturbations and velocity vector.

This derived, very awkward, dimensionless system of equations is, in fact, a new (exact) form of the NS–F classical atmospheric equations well adapted for the application of our RAMA theory. In Chap. 4 we discuss a detailed RAMA of these Boussinesq approximate equations, inspired by my "Boussinesq's Centenary Anniversary paper" [10] of 2003, but for the sake of simplicity, only in the framework of a Euler non-viscous, compressible and adiabatic system of Eq. 1.2a–1.2c – this derivation being an instructive test problem for the formulation of our key steps in Chap. 6, devoted to the mathematics of the RAMA.

1.2 My Collaboration with Jean-Pierre Guiraud in the Aerodynamics Department of ONERA

In September 1967, thanks to the recommendation of Jean-Pierre Guiraud, I began a new career as a research engineer in the Aerodynamics Department of the Office National d'Études et de Recherches Aérospatiales, in Chatillon, near Paris. After working at ONERA for 5 years, in October 1972 I was – thanks to my Doctoral thesis (March 1969) – appointed Titular Professor of Fluid Mechanics at the University of Lille 1 – a position which I held until 1996.

I continued part of my theoretical researches in fluid-flow modelling as a 'Collaborateur Extérieur' at ONERA, and during 16 years there, from 1970 to 1986, for a full day once a week I worked with Jean-Pierre Guiraud in exchanging ideas and envisioning asymptotic modelling for various aerodynamics, stability/

turbulence and meteo problems. As a result of this collaboration, throughout this period we jointly published 26 papers in various scientific journals. (See, for instance, in the References, our 1986 paper [11], and references to other papers published either jointly or separately.) These works (published during 1971–1986) are devoted, with more or less success, to the application of the ideas that we discussed concerning various fields in fluid dynamics – all being motivated by the need for solving or understanding the basis of the solution of technological and geophysical problems involving fluid flows. These problems are related to:

Vortex flows in rotating machines (taking into account that the blades in a row are usually very close).

Rolled vortex sheets (in a region where the contiguous branches of the rolled sheet are so close to each other that they are very difficult to capture by numerical simulation).

Hydrodynamic stability (in a weakly non-linear domain, through perturbation techniques – the underlying mathematical theory being the so-called bifurcation theory).

Atmospheric flows (see Chap. 9 in this book, and our monograph, *Asymptotic Modeling of Atmospheric Flows* [12], published in 1990).

Flow at low Mach numbers (see our *Topics in Hyposonic Flow Theory*, Lecture Notes in Physics, vol. 672, 2006 [13]).

It was an extremely stimulating period of scientific research, for me. As far back as at the end of 1970 years it is evident that asymptotic techniques provide very powerful tools in the process of constructing working models for fluid dynamics problems, which are stiff, from the point of view of numerical analysis, coupled with a simulation via a powerful super-computer.

My approach differs from Van Dyke's exposition in [14], in the sense that: "Computational fluid dynamics is now a quite mature discipline, and for some time the growth in capabilities of numerical simulation will be dependent on, or related to, the development of rational asymptotic modelling approach – RAMA." If such is the case, then a simple definition of our RAMA is: "The art of a strongly argumentative, consistent, non-ad hoc and non-contradictory modelling assisted by the spirit of asymptotics."

It is my opinion that RAMA will remain for many years, or even decades – a quite powerful tool in deriving mathematically consistent models for numerical/simulation fluid research. By "mathematical", I mean that the models derived by RAMA, the approximate consistent models, under consideration, should be formulated as reasonably well-posed initial and/or boundary value problems, in place of the starting full NS–F extremely complex and stiff problems (as, for example, turbomachinery flows, which are arguably among the most complicated known to man and are of great technological importance – see Sect. 7.1).

I again observe that our RAMA is an extremely worthwhile objective, because most of the relevant engineering computations are based on relatively ad hoc models that are rife with internal inconsistencies.

1.3 A Few Remarks Concerning My Preceding Books on Modelling in Newtonian Fluid Flows

Concerning the joint efforts of Guiraud and myself in our tentative writing of a book on the RAMA in fluid dynamics, I must say first that in 1977, after several attempts to persuade him, we both worked intensively, up to 1982, on a hand-written (in French) manuscript entitled "The laminar flows at high Reynolds numbers: an essay on the asymptotic modelling of Newtonian dynamics of fluids." The possibility of publication, after a rewrite in English, became a reality – at least for me!

Unfortunately, at that moment our opinions diverged concerning the opportunity of such publication of our finished manuscript in its 1982 form. Guiraud wished to pursue a deeper investigation of some delicate and difficult questions requiring time and additional research. Contrary to Guiraud, I was of the opinion that further investigations would be of no benefit and, in particular, would not provide anything else to support our initial objective: to show the effectiveness of our RAMA!

Finally, in 1986 and 1987 I published alone (but by common consent) a course in two volumes, in French, in the Springer series Lecture Notes in Physics (LNP): *Les Modèles Asymptotiques de la Mécanique des Fluides*, I [15] and II [16] – more or less inspired by the manuscript produced by myself and Guiraud in 1982.

As Titular Professor at the Université de Lille 1, beginning in 1972, I systematically used, throughout almost 10 years, various parts of our manuscript in my teaching of theoretical fluid mechanics as a first Course and second Course, respectively, for final-year (M.Sc.) undergraduate students and post-graduate research workers, and for students preparing a doctoral thesis.

In the beginning, in 1977, my goal was, in fact, a monograph devoted to RAMA for Newtonian fluid flows, and I had in mind the derivation of various models corresponding to parameters (not only to Reynolds number) characterizing various (high or low) physical effects – viscosity, compressibility, heat conduction, gravity, Coriolis force, unsteadiness, geometrical constraints, and so on.

The above-mentioned two-volume Course was my first experience in opening a new way into the difficult field of theoretical (analytical) fluid mechanics via the NS–F equations, offering fresh ideas together with a first systematic presentation of asymptotic approach in fluid dynamics for both students and young researchers. In a short critical review (*J. Fluid Mech.*, 1991, vol. 231, p. 691), the following opinion was expressed concerning this two-volume Course:

> The text is in French. Equations are hand-written but very clearly done. In many of the areas covered in these two volumes there is a conspicuous lack of suitable expository material available elsewhere in the literature, and Professer Zeytounian's notes are to be welcomed for filling these gaps until fuller and more specialized accounts appear in book form.

In addition, the following appeared in *Mathematical Reviews*, 1988:

> A reader having acquired a practical knowledge of the asymptotic methods which are presented and used here may certainly benefit by the advanced material about Navier–Stokes equations provided in the main body of these two volumes.

Later, in 1994, a third volume was published, also in French: *Modelisation asymptotique en mécanique des fluides newtoniens* [17] – and here it seems judicious to quote several sentences extracted from a review (in *Appl. Mech. Rev*., vol. **49**(7), July 1996, p. 879) by J.-P. Guiraud:

> The purpose of this book is to present, through extensive use of dimensional analysis and asymptotic calculus, a unified view of a wide spectrum of mathematical models for fluid mechanics ... Usually, adequacy of a mathematical model is evaluated a priori through physical insight, experience, and inquiries about the topic. Here, the reader is proposed to become, on his own, an expert in adequacy, by systematic use of asymptotic approximation. Although a rather large spectrum of books on fluid mechanics and asymptotic methods may be found, it seems to this reviewer that the present one is rather exceptional by the extent and logical organization of the material ... A fascinating aspect is that the reader is led by the hand through a jungle of very different mathematical models, including Euler and Navier for incompressible fluid; Prandtl for boundary layer; Stokes, Oseen, and Rayleigh for various viscous effects; the usual regimes of aerodynamics, Boussinesq for the atmosphere and the ocean, primitive equations and quasi-geostrophic approximation for meteorology, gravity waves, amplitude equations of KdV or Schrodinger type, and low Mach number flows, including acoustics ... *Modelisation asymptotique en mécanique des fluides newtoniens* is a valuable book which is recommended both to individuals and libraries for the precise purpose indicated in the second sentence of this review. In principle, it is self-contained and might be a reference for students, engineers, and researchers who master computational aspects but want to be able to assess what kind of approximations are involved in the equations, as well as initial and boundary conditions with which they struggle.

Concerning, more precisely, the application of our RAMA to atmospheric motion, after my 1975 survey lecture,[7] published in 1976 [18], I decided to write a monograph devoted entirely to asymptotic modelling of atmospheric flows (see [12]), and in 1985 a manuscript (written in French) was ready. This manuscript was accepted by Prof. Dr. W. Beiglböck, of Springer-Verlag, Heidelberg, for publication in English. Unfortunately, the translation into English, by Lesly Bry, is infelicitous!

I think that my *Meteorological Fluid Dynamics* [19] is good preparation for the reading of [12], which was in fact published a little earlier than [19]. Here again, I quote part of a review of [12] (SIAM Review, vol. **33**(4), 1991, pp. 672–3) by Huijun Yang (University of Chicago):

> The present work is not exactly a 'course', but rather is presented as a monograph in which the author has set forth what are, for the most part, his own results; this is particularly true of Chaps. 7–13. (quoted from the Preface of this book):'In the book, the author viewed meteorology as a fluid mechanics discipline. Therefore, he used singular perturbation methods as his main tools in the entirety of the book ... The book consists of the author's more than 25 years work.' In the 32 references to his own work, fewer than one third were published in English, with the rest in Russian or French. Throughout the book, the reader can strongly feel the influence of Soviet works on the author. However, the author does

[7] Entitled *La Météorologie du point de vue du Mécanicien des Fluides*, written for the XIIth Symposium on Advanced Problems and Methods in Fluid Mechanics, Bialowieza (Poland), 8–13 September 1975.

> have his own character. The issues raised in the book, such as the initialization (initial layer) boundary layer treatment, and well-posedness and ill-posedness of the system, are very important problems facing researchers today in atmospheric sciences and other related sciences. The reader will find some valuable information on these issues ... The mathematically consistent treatment of the subject does give this book a unique place on shelves of libraries and offices of researchers ... This book is very different from recent books on the market [for example, Holton [20], Gill [21], Haltiner and Williams [22], Pedlosky [23], and Yang [24]]. I recommend that researchers in atmospheric dynamics and numerical weather prediction read this book to have an alternative view of deriving atmospheric flow models. Researchers in theoretical fluid mechanics might also be interested to see how singular perturbation methods can be used in atmospheric sciences. The book may be used as supplemental material for courses like numerical weather prediction or atmospheric dynamics. However, I do not think it is a suitable textbook for a regular class: as the author said in his Preface. 'I am well aware that this book is very personal – one might even say *impassioned*.'

This review seems rather favourable, but this does not seem to be the case with P. G. Drazin's review (*Journal of Fluid Mechanics*, 1992, 242):

> The author acknowledges that dynamic meteorology is too large a subject for him to attempt to cover completely. He 'has set forth what are, for the most part, his own results' in accord with his 'conception of meteorology as a fluid mechanics discipline which is in a privileged area for the application of singular perturbation techniques.' He applies the method of multiple scales or the method of matched asymptotic expansions to any equations he can find, systematically reviewing his own and his associates' extensive research. So it is a very personal view of meteorology, covering some areas of geophysical fluid dynamics with a formidable battery of notation. The nature of the subject demands a large and complex notation in any rational treatment. But the notation will allow only a few to benefit from this book. It is a barrier which I found hard to cross, not having the time and will to work through the book line by line, as a result with which I am familiar was difficult for me to follow. The approximations of dynamic meteorology are mostly singular. Nonetheless, their essentials are well understood by meteorologists now, albeit in a rough and ready way, and meteorologists are unlikely to be [influenced] by Zeytounian's approach. Yet the approximations of meteorology are subtle and deserve a more rational development than is commonly understood. This is the achievement of the book. The author derives many equations systematically, albeit not rigorously, from the primitive equations, rather than solving equations governing particular problems. For all these reasons, I feel that the book will be studied intensively by a few specialists but neglected by others.

Obviously, the last two sentences of this critical review are very controversial – especially the assertion of 'albeit not rigorously'. It seems clear that for Drazin, my "French touch" is not to everyone's taste! Indeed, the publication of this monograph in 1990 was possibly premature, despite the publication, in 1985, of my survey paper [25], *Recent Advances in Asymptotic Modelling of Tangent Atmospheric Motions*, devoted to an asymptotic rational theory of the modelling of atmospheric motions in a 'flat earth' closely related with the so-called β–plane approximation.

In [12], [19], and [25] my clear purpose was to initiate a process which does not seem to have sufficiently attracted the attention of scientists. This process involves the use of the RAMA for carrying out models; that is, for building approximate simplified (but consistent) well-posed (at least from a fluid dynamician's point of view) problems based on various physical situations and concerned with one or several high or low parameters.

I do not, of course, affirm that this is the only method, nor even the most efficient one, for deriving such problems in view of a numerical/computational simulation. I do, however, feel that when such a procedure is feasible it should be undertaken. As a matter of fact, the application of this approach makes it possible, in principle, to improve, at least, a second-order model problem obtained from the NS–F full problem, used by advancement in the associated asymptotic expansion.

After 1996 – having retired from the University of Lille 1, and with more time to write – I decided to return to my first (1977) idea concerning a monograph devoted to RAMA for Newtonian fluid flows and the derivation of models corresponding to various parameters (not only to Reynolds number) characterizing various (high or low) physical effects. I (partly) realized this objective in 2002 with my monograph [26] *Asymptotic Modelling of Fluid Flow Phenomena* – the first book in English devoted entirely to asymptotic modelling of fluid flow phenomena, dealing with the art of asymptotic modelling of Newtonian laminar fluid flows. In Chaps. 2–12 of this work I consider several important topics involved in the accomplishment of my objective in determining how simplified rational consistent asymptotic simplified models can be obtained for the most technologically important fluid mechanical problems.

According to Marvin E. Goldstein (of NASA Glenn Research Center) in his detailed, scrupulous, and rigorous review (SIAM Review, vol. **45**(1), pp. 142–146, 2003): "There are enough of the selected topics that accomplish the author's objective to make this book an important contribution to the literature." Goldstein also writes:

> Applied mathematicians have always found fluid mechanics to be a rich and interesting field, because the basic equations (i.e., the Navier–Stokes equations) have an almost unlimited capacity for producing complex solutions that exhibit unbelievably interesting properties, and because the dimensionless form of these equations contains a parameter (called the Reynolds number) which is usually quite large in technologically and geophysically interesting flows. This means that asymptotic methods can be used to obtain approximate solutions to some very interesting and important flow problems. These solutions usually turn out to be of non-uniform validity (i.e., they break down in certain regions of the flow), and matched asymptotic expansions have to be used to construct physically meaningful results. However, advances in computer technology have led to the development of increasingly accurate numerical solutions, and have thereby diminished the interest in approximate analytical results. But real flows (especially those that are of geophysical or engineering interest) are extremely complex and exhibit an enormous range of length and time scales whose resolution will probably remain well beyond the capabilities of any computer that is likely to become available in the foreseeable future. So simplification and modelling are still necessary, not only to meet the engineer's requirement for generating numbers but also for developing conceptual models that are simple enough to be analyzed and understood. The asymptotic scaling techniques and the reduced forms of the general equations that emerge from the matched asymptotic expansion process (as well as from other singular perturbation techniques) provide a rational and systematic method for obtaining the necessary simplified flows model, which, in most cases, still have to be solved numerically. The author states that the goal of this book is to promote the use of asymptotic methods for developing simplified but rational model for the Navier–Stokes equations which can then be solved numerically to obtain appropriate descriptions of the flow.

> This is an extremely worthwhile objective, because most of the relevant engineering computations are based on relatively ad hoc models that are rife with internal inconsistencies. To my knowledge, this is the first book devoted to accomplishing the author's stated objective, and it is, therefore, unfortunate that it is not as well executed as it could be.

To mitigate this last critical sentence, however, he continues:

> However, there are enough of the selected topics that accomplish this objective to make this book an important contribution to the literature. It contains an excellent chapter (Chap. 11) in which the classical model equations for large-scale atmospheric motion are derived in a fairly rational fashion. The author also devotes a full section (Sect. 6.6) to turbomachinery flows, which are arguably among the most complicated known to man and are of great technological importance. It is remarkable that he was able to make some progress toward developing an asymptotic basis for some of the more prominent engineering models of these flows.

Goldstein concludes: "It is this reviewer's hope that the deficiencies in this work will encourage others to write new and improved books with similar themes" – but unfortunately, it seems that for the present this is not the case! On the other hand, in 2006 and 2009 Springer published my two monographs, *Topics in Hyposonic Flow Theory* [13], devoted to hyposonic (low Mach numbers) flows, and *Convection in Fluids: A Rational Analysis and Asymptotic Modelling* [27], mainly related to the well-known Bénard convection problem in a layer of weakly expansible liquid heated from below.

Concerning the first of these, a considerable amount of stimulation and encouragement was derived from my collaboration with Jean-Pierre Guiraud, who, over a period of 20 years, has played an important role in asymptotic modelling of the various low-Mach-number flow problems presented in that book. The reader should take into account that is the first book devoted to hyposonic flow theory, and it is the author's hope that the various unavoidable 'deficiencies' (noted by Goldstein in his review of [13]) will persuade others to work on similar themes.

On the other hand, in [27] the main motivation was a rational analysis of various aspects (in particular, the influence of the viscous dissipation, free surface and surface tension) of the Bénard convection (heated from below) problem. It presents a careful investigation of three significant approximate models (see, for instance, Chap. 8 in the present book) related to the Bénard (1900) experiments, by which he discovered his well-known Bénard cells! It is evident that Professor Manuel G. Velarde was influential when I wrote my book on convection in fluids [27], as I benefited greatly from our collaboration and many discussions relating to Marangoni thermocapillary convection during my sojourn at the Instituto Pluridisciplinar UCM de Madrid in 2000–2004.

1.4 Conclusion

More than 50 years ago, with the works of Kaplun, Kaplun and Lagerstrom, and Proudman and Pearson, asymptotic techniques provided a new impetus for research in theoretical fluid dynamics. Twenty years later, a much more powerful revival was possible due to the dramatic influence of high-speed computers and the numerical analysis and simulation of fluid flow problems. The survey paper by Birkhoff [28] includes, through a series of case studies, a detailed assessment of the status, development, and future prospects of numerical/computational fluid dynamics.

During the early times, asymptotic techniques were mostly used in order to derive approximate solutions in closed form. Perhaps of more significance for the progress of understanding and also of research, however, was the use of asymptotic techniques in order to settle, on a rational basis, a number of approximate models which much earlier were often derived by ad hoc non-rational procedures. One of the most well-known examples is Kaplun's celebrated paper on boundary-layer theory, which provided a firm theoretical basis for some 50 years of boundary-layer research.

From this early example it is clear that my 1977 idea, relative to asymptotic techniques as a well-suited and invaluable tool for the derivation of mathematically consistent models (from full fluid dynamics equations – NS–F equations) which are amenable to numerical treatment rather than for obtaining closed form solutions, was a perspective of scientific activity in interaction with numerical simulation – even though, in the 1970s, numerical fluid dynamics was almost non-existent due to the lack of high-speed computers.

It is now evident that asymptotic techniques serve as very powerful tools in the process of constructing rational consistent mathematical simplified models for problems which are stiff, from the point of view of numerical analysis. Here, Chap. 6 is devoted to the mathematics of the RAM Approach, which seems a good basis for a practical use of this RAMA in simulation/computation via high-speed computers.

As a matter of fact J. P. Guinaud, who read a large part of the Chaps. 1 to 6 of the present book suggest me to quote what follows: "While having been absent from the Community of fluid mecanicians fifteen years from now, it was a pleasure for me to read the report by Zeytounian, of a long coworking with him. It is mere justice to mention that, during this active collaboration, a number of ideas were initiated by Zeytounian. My main contribution was the result of ten years of struggle with asymptotics before I had the good fortune to meet Zeytounian." A significant contradiction is obvious in the scientific activity of J. P. Guinaud. He never published any book whilst having written a number of documents, corresponding to many courses he teached manuscripts which were much more carefully written than simple notes to be distributed for the students. In particular, the Guirand Notes ("Topics in hypersonic flow theory", Department of Aeronautics and Astronantics, Stanford University; SUDAER n° **154**, may 1963, Stanford, California, USA) are Published in Russian by MiR, Moscow Editions, as a book in 1965.