The Water Pricing Problem in a Complex Water Resources System: A Cooperative Game Theory Approach

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Abstract The research presents a methodology to allocate water services costs in a water resources system among water users using a Cooperative Game Theory approach based on the integral river basin modelling. The proposed approach starts from the characterization of the system to be modelled. The Decision Support System WARGI [7, 4, 6] is then used to achieve the best water system performances and to calculate the least cost of each one of the users' coalitions that may arise using the resources. The cost allocation is evaluated by the Cooperative Game Theory methods. The aim of the work is to suggest a tool for decision makers to define water price policies in accordance with the sustainability and fairness principles settled by European Water Framework Directive [2].

1 Introduction

A central problem in planning the provision of public services when dealing with limited resources is how to determine a "fair" and "just" allocation of management costs. This problem is particularly relevant for water systems in Europe to comply with the Water Framework Directive 2000/60/CE [2], which addresses the recovery costs of water services considering adequate contributions and priorities when dealing with different water uses. The actual water pricing methods are mainly based on countable or historical cost (sunk costs) allocation corresponding to previous investments, and they are used as a simple cost recovery instrument.

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The cost allocation criterion is generally determined by legal imposition, and the users do not make any decisions, or they are simply consulted about the possible alternatives. Otherwise, a Cooperative Game Theory (CGT) approach is able to determine a sustainable, acceptable, rational and fair cost-sharing rule [12] and it is particularly appropriate for contexts like water services, in which it is important to define the agreements and to encourage cooperation among decision makers.

2 Cooperative Game Theory

CGT belongs to the mathematical science called Games Theory, developed in the last century by [10], and it searches cooperative solutions studying the individual decisions in situations in which there are some interactions among different decisional subjects. There are several applications of CGT also concerning water resources [9, 13, 3, 1]. To define a cooperative game we have to explain the following definitions, as in [12]. Let $N = \{1,2,3,\ldots,n\}$ be a set of players, for example different water users in a water resources system. Every subset S of N is called "coalition" and N is the "grand coalition". Let $c(i)$ be the cost of providing player i by itself and $c(S)$ the cost of providing players in S jointly. Consequently, $c(N)$ is the cost of grand coalition, i.e. in our contest the cost related to whole water service. By convention $c(\Phi) = 0$. The cost associated with a coalition represents the least cost to supplying the player in that coalition and the discrete function constituted by the least costs of every coalition is the so-called "characteristic function". An allocation is a vector $(x_1, x_2, \ldots, x_n)$ such that

$$\sum_N x_i = c(N) , \tag{1}$$

where x_i is the amount charged to player i. Allocations should respect two fundamental principles: the rationality and the marginality principle. By the first one, if cooperation among players is voluntary, then self-interest dictates that no participant Ð or group of participant Ð be charged more than their stand alone (opportunity) cost:

$$\sum_S x_i \leq c(S) , \tag{2}$$

otherwise they would have no incentive to agree to the proposed allocation. The second principle states that no player should be charged less than his marginal cost of including him in a coalition:

$$\sum_S x_i \geq c(N) - c(N-S) , \tag{3}$$

otherwise it could be said that the coalition $N-S$ is subsidizing S. The condition (2) provides incentives for voluntary cooperation and the condition (3) arises from considerations of equity. Given the formula (1) the conditions (2) e (3) are equivalent. There are two different types of solution of a cooperative game. The first one is the set of admissible solutions, the so-called "core". The core of a game is the set of all allocations $x \in \mathbf{R^N}$ such that (1) and (2), or equivalently (3), hold for all S of N [11]. The second type is represented by a single allocation. One of the most utilized single allocation method is the Shapley Value that responds to symmetry, additivity and monotonicity principles [8]. The Shapley Value is defined by this formula:

$$x_i = \sum_{S \subseteq N-i} \frac{|S|!\,(|N-S|-1)}{|N|!}\,[c(S+i)-c(S)]\ , \tag{4}$$

where $|S|$ is the cardinality of coalition S, i.e. the number of players involved in the coalition and, consequently, $|N| = n$.

3 Cost Allocation Methodology

The cost allocation methodology applied to water systems consists of the following main steps:

1. Water resource system analysis: analysis of hydrological, hydraulic, economic and environmental aspects of the system;
2. Cooperative Game definition: identification of independent agents and definition of coalitions;
3. Characteristic function calculation: evaluation of the least cost for every coalition using the optimization model WARGI;
4. Game solution: application of the CGT allocation methods.

The evaluation of the characteristic function is the base of cooperative games and requires a cost analysis associated with each possible coalition system, which implies an optimisation process whose magnitude grows exponentially with the number of system agents. The proposed approach starts characterizing the hydrologic, hydraulic, economic and environmental aspects of the system to be modelled. The characterization of the water system includes extended time-horizon data of surface hydrology, given by monthly runoff time series, the application of continuity equations and balance equations in the reservoirs and aquifers nodes. The determination of water costs is based on the construction, management and operation cost functions for the hydraulic infrastructures. The Decision Support System (DSS) WARGI [7, 4, 6] to optimize water resources systems is then used to achieve the best water system performances and to calculate the least cost of each one of the users' coalitions that may arise using the resources. The outputs of the optimization process give the characteristic function of the game and, so, it is possible to apply a CGT method to evaluate the cost-allocation.

3.1 CGT Application

The methodology is been applied in Flumendosa - Campidano water system in Sardinia, Italy. The system (Fig. 1) confers the resource to the three main users: municipal, irrigation and industrial, that can be supply using different infrastructures: dams, diversion site, channel and pipelines. Moreover, an interconnection to another water system with a pumping station exists. The water demands are reported in Fig. 2. In Flumendosa - Campidano application we considered the OMR (operating, maintenance and replacement) costs of the infrastructures given by the regional document Piano Stralcio di Bacino [5], given in Table 1. By the DSS WARGI the characteristic function is been estimated, as we report in Table 2. Through the characteristic function is possible to estimate the core of the game, calculating the minimum and maximum costs of each user (Table 3).

Consequently the core of the game is defined by the following mathematical system:

$$\begin{cases} Mun.+Irr.+Ind. = 533.48 \\ 0.00 \leq Mun. \leq 469.08 \\ 41.66 \leq Irr. \leq 533.48 \\ 1.42 \leq Ind. \leq 185.11 \end{cases} \tag{5}$$

Moreover, we are able to give a single allocation reference using the Shapley Value (Table 3).

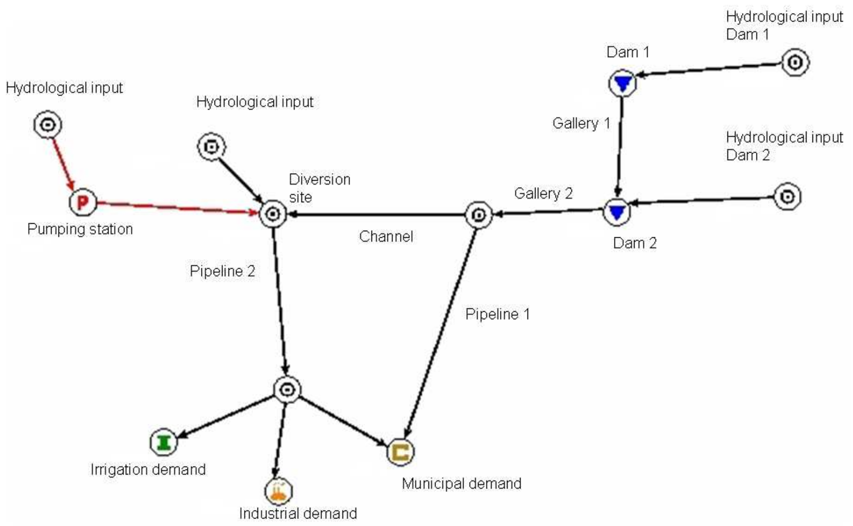

Fig. 1 Graph representing the water resource system.

Table 1 OMR costs of the infrastructures

Infrastructures	**OMR costs [€/ year]**
Dam 1	523748
Dam 2	498878
Gallery 1	64331
Gallery 2	116604
Channel	353951
Pipeline 1	874116
Diversion site	14000
Pipeline 2	1926526
Interconnection + Pumping station	873979

Table 2 Characteristic function

Coalitions	Mun.	Irr.	Ind.	Mun. + Irr.	Mun. + Ind.	Irr. + Ind.	Grand coalition
Cost [M€]	469.08	533.48	185.11	532.06	491.82	533.48	533.48

Table 3 Maximum and minimum values for each user and Shapley value (in M€)

Users	Minimum value	Maximum value	Shapley Value
Municipal	0.00	469.08	207.24
Irrigation	41.66	533.48	260.27
Industrial	1.42	185.11	65.97

4 Conclusions

The methodology, based on CGT, could be a valuable tool able to define water price policies and economical analyses that the water districts have to realize according

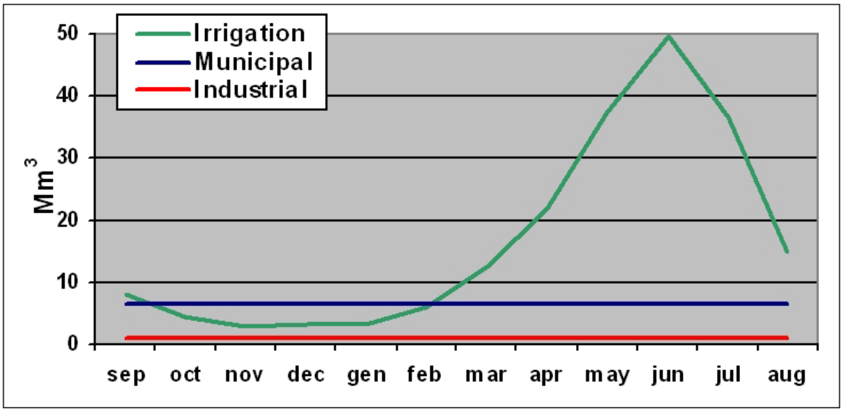

Fig. 2 Water demands.

to the European Water Framework Directive. The evaluation of the characteristic function is the base of cooperative games and it requires an optimization process through the use of DSS WARGI.

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